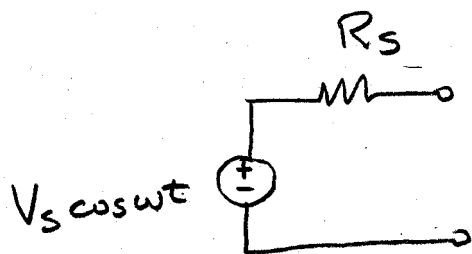


Sinusoidal Voltages and the Sinusoidal Steady State

Self-Learning Topic: Use these notes to review A.C. circuit analysis, phasors and impedance.



- In the second segment of the course, we will study transmission line circuits where the generator is sinusoidal

$$v(t) = V_s \cos \omega t$$

- The “amplitude” of the voltage is V_s .
- The “RMS value” of the voltage is $\frac{V_s}{\sqrt{2}}$.
- The frequency of operation is f Hertz, and the “radian frequency” is $\omega = 2\pi f$.
- The RF circuits we study are “linear time-invariant systems” or LTI systems which you studied in ELEC361 “Signals and Systems”.
- The generator “turns on” at $t = 0$, and then there is a “transient” period in which all the voltages and currents in the circuit change from zero (before $t = 0$) to become sinusoidal at frequency $\omega = 2\pi f$.
- In an LTI system, when the generator is sinusoidal, then at “steady state” all the voltages are sinusoidal *at the same frequency as the generator* and have the form

$$v(t) = A \cos(\omega t + \phi)$$

where

A is the *amplitude* of the voltage

ϕ is the *phase* of the voltage

- So all we need to find is the amplitude A and the phase ϕ of the voltage.
- “Phasor Analysis” from ELEC 273 “Basic Circuit Analysis” is an efficient way to solve the circuit.

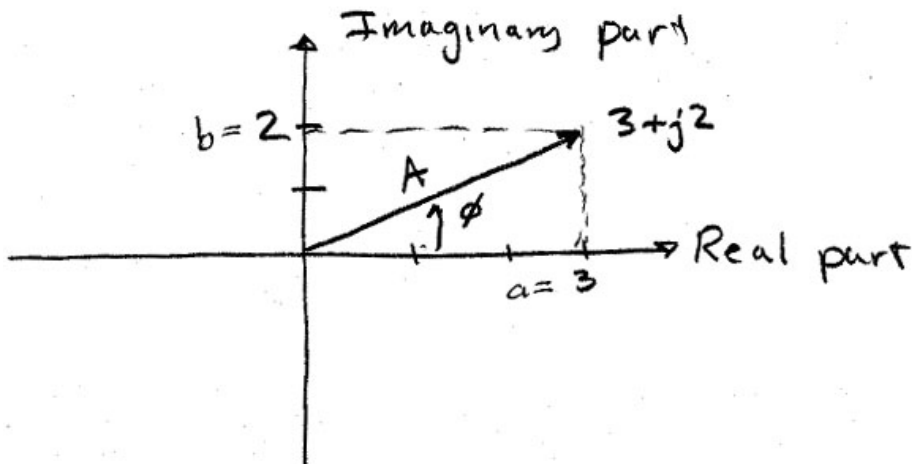
Complex Numbers

- A “complex number” has a “real part” and an “imaginary part”, and can be written down in “rectangular form” as

$$V = a + jb$$

where a and b are real numbers, and $j = \sqrt{-1}$.

- The “real part” is a .
- The “imaginary part” is b .
- We often draw complex numbers in the “complex plane”. For example, $V = 3 + j2$ can be represented in the complex plane as



- Complex numbers are often represented in “polar form” as $V = A\angle\phi$ where A is the magnitude, ϕ is the angle, and the symbol “ \angle ” is read as “angle”.
 - The magnitude is $A = \sqrt{a^2 + b^2}$
 - The angle is $\phi = \tan^{-1}\left(\frac{b}{a}\right)$
 - For the example above, $A = \sqrt{3^2 + 2^2} = 3.6056$, and angle $\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.7$ degrees, so $V = 3.6056\angle 33.7^\circ$.
- If we are given the magnitude A and the angle ϕ , then we can convert back to the “rectangular” form using:
 - Real part $a = A \cos \phi$
 - Imaginary part $b = A \sin \phi$
 - So the complex number is $V = a + jb$
 - Recall Euler’s Identity: $e^{j\phi} = \cos \phi + j \sin \phi$
 - It is often convenient to write the complex number in “exponential” format as follows:

$$V = A \cos \phi + jA \sin \phi = A(\cos \phi + j \sin \phi) = Ae^{j\phi}$$
 - So the notations $V = A\angle\phi$ and $V = Ae^{j\phi}$ are equivalent.
- **Addition** of complex numbers: add the real parts, add the imaginary parts.
 - If $V_1 = a_1 + jb_1$ and $V_2 = a_2 + jb_2$ then the sum is

$$V = V_1 + V_2 = (a_1 + jb_1) + (a_2 + jb_2) = (a_1 + a_2) + j(b_1 + b_2)$$
 - Find $V = V_1 + V_2$ if $V_1 = 3 + j2$ and $V_2 = -4 + j6$:

$$V = (3 + j2) + (-4 + j6) = -1 + j8$$
- **Multiplication** of complex numbers. If $V = a_1 + jb_1$ and $I = a_2 + jb_2$, then find $S = VI$:
 - Either multiply the rectangular forms algebraically:

$$S = VI = (a_1 + jb_1)(a_2 + jb_2) = a_1a_2 + j^2b_1b_2 + ja_1b_2 + ja_2b_1$$
 and since $j^2 = (\sqrt{-1})^2 = -1$, we can write this as

$$S = (a_1a_2 - b_1b_2) + j(a_1b_2 + a_2b_1)$$
 - Or multiply the polar forms by finding the product of the magnitudes and the sum of the angles. Thus $V = A_1e^{j\phi_1}$ and $I = A_2e^{j\phi_2}$ so

$$S = VI = (A_1e^{j\phi_1})(A_2e^{j\phi_2}) = A_1A_2e^{j\phi_1}e^{j\phi_2} = (A_1A_2)e^{j(\phi_1+\phi_2)}$$
 The magnitude is (A_1A_2) .

The angle is $\phi_1 + \phi_2$.

- Division of complex numbers. If $V = a + jb$ and $Z = r + jx$, then find $I = \frac{V}{Z}$:

- Either divide the rectangular forms algebraically:

$$I = \frac{V}{Z} = \frac{a + jb}{r + jx}$$

Multiply the top and bottom by the “complex conjugate” of the denominator $r - jx$:

$$I = \frac{(a + jb)(r - jx)}{(r + jx)(r - jx)} = \frac{(ar + bx) + j(br - ax)}{r^2 + x^2}$$

which can be written as real part + imaginary part as

$$I = \frac{ar + bx}{r^2 + x^2} + j \frac{br - ax}{r^2 + x^2}$$

- Or divide the polar forms by taking the quotient of the magnitudes and the difference of the angles. Thus if $V = A_1 e^{j\phi_1}$ and $Z = A_2 e^{j\phi_2}$ then

$$I = \frac{V}{Z} = \frac{A_1 e^{j\phi_1}}{A_2 e^{j\phi_2}} = \frac{A_1}{A_2} \frac{e^{j\phi_1}}{e^{j\phi_2}} = \frac{A_1}{A_2} e^{j(\phi_1 - \phi_2)}$$

so the magnitude is $\frac{A_1}{A_2}$ and the angle is $\phi_1 - \phi_2$.

Review of A.C. Circuit Analysis

- Circuits in which the generator is “ $\cos \omega t$ ” are called “alternating current” circuits or “A.C.” circuits.
- Circuits in which the generator is “ $\cos \omega t$ ” are called “alternating current” circuits or “A.C.” circuits.

$$V_s(t) = V_o \cos \omega t$$

- The repetition rate is call the frequency and is f Hz or “cycles per second”; the radian frequency is $\omega = 2\pi f$.
- The “period” is the repetition rate of the sinusoidal voltage and is $T = \frac{1}{f} = \frac{2\pi}{\omega}$.
- “Alternating” refers to the fact that first the voltage is positive for half a cycle and then negative for half a cycle. That is, the sign of the voltage alternates.
- This is in contrast to “D.C.” circuits in which the voltage is constant and so does not change sign as time advances.
- People describe a $\cos \omega t$ signal as a “time harmonic” signal.
- When we first switch on the generator there is a *transient* period as the leading edge of the cosine voltage travels along the transmission line to the load, reflects back to the source, re-reflects to the load, and so forth.
- During the transient period, the voltage at any point in the circuit gradually stabilizes to have a constant amplitude and phase, where “constant” means not changing with time, and the circuit reaches “steady state”.
- We are not interested in the *transient* voltages and currents that are present just after the generator is switched on.
- We are interested in the voltages and currents in the *sinusoidal steady state* after the switching transients have died away, a long time after the generator is switched on. (A “long time” means about five times the longest time constant in the circuit.)

- We will find the solution in the sinusoidal steady state using complex numbers called *phasors*.

Review of “Phasors” and “Impedance”

Phasors

- Linear Time-Invariant (LTI) System: In an LTI system, if the generator is a sinusoidal voltage at radian frequency ω , such as

$$V_s(t) = V_o \cos \omega t$$

then in the “*sinusoidal steady state*” all the voltages and currents in the circuit are sinusoidal at the same frequency:

$$v(t) = A \cos(\omega t + \phi)$$

- Since we know the frequency, all we have to find is the **amplitude** A and the **phase angle** ϕ for each voltage or current.
- It is convenient to “code” the amplitude and phase into a complex number called a “phasor”:

$$V = A \angle \phi$$

where the symbol “ \angle ” means “angle” and $A \angle \phi$ is read “A angle ϕ ”.

- So if $v(t) = 10 \cos(\omega t + \pi/6)$ then the phasor representing this voltage is $V = 10 \angle \pi/6$.
- I usually use degrees for calculation so I would write $V = 10 \angle 30^\circ$
- We can use the exponential form to write $V = A \angle \phi$ as

$$V = A e^{j\phi}$$

- The correspondence between phasors and time functions is

$$V = A e^{j\phi} \leftrightarrow v(t) = A \cos(\omega t + \phi)$$

- In words we can say that:
 - The **magnitude** of the phasor is the **amplitude** of the cosine.
 - The **angle** of the phasor is the **phase angle** of the cosine.
 - In mathematical terms we have defined a mapping from “sinusoidal time functions” $v(t) = A \cos(\omega t + \phi)$ into complex numbers call phasors of the form $V = A e^{j\phi}$. The mapping is “one to one”: for each complex number there is a unique sinusoid, and for each sinusoid there is a unique complex number.
- We can “recover” the cosine wave $v(t)$ from the phasor V with the formula

$$v(t) = \text{Re}(V e^{j\omega t})$$

where the function $\text{Re}(\dots)$ means “take the real part of”, so

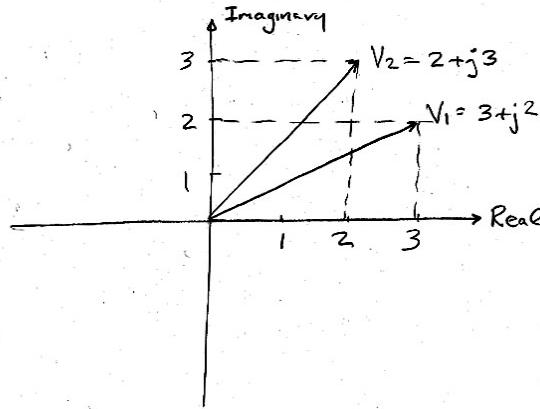
$$v(t) = \text{Re}(A e^{j\phi} e^{j\omega t}) = \text{Re}(A e^{j(\omega t + \phi)}) = \text{Re}[A \cos(\omega t + \phi) + jA \sin(\omega t + \phi)]$$

and taking the real part

$$v(t) = A \cos(\omega t + \phi)$$

Phasor Diagram

- A “phasor diagram” is a complex plane in which we draw each phasor as an arrow starting at the origin.
- Example: Draw a phasor diagram showing the phasor voltages $V_1 = 3 + j2$ volts and $V_2 = 2 + j3$ volts:



Phasor diagram showing voltages $V_1 = 3 + j2$ volts and $V_2 = 2 + j3$ volts.

- To draw the phasor $V_1 = 3 + j2$, find the point with real part 3 and imaginary part 2 and join it to the origin with an arrow.
- Later in the course, our phasors will be a function of *position* z on a transmission line.
- Then a phasor diagram will be useful to show how the phasor changes as the position or location on the transmission line changes.

Adding Voltages as Phasors

- Given phasor voltages $V_1 = 3 + j2$ volts and $V_2 = 2 + j3$ volts, find the sum,

$$V = V_1 + V_2$$
- Find the amplitude and phase of the sum voltage, $V = V_1 + V_2$.
- Find the time function $v(t)$
- Draw a phasor diagram showing the sum $V = V_1 + V_2$.

Solution

- Find the sum of $V_1 = 3 + j2$ and $V_2 = 2 + j3$:

$$V = V_1 + V_2 = (3 + j2) + (2 + j3) = 5 + j5$$
- To find the amplitude and phase of V , express V in polar form:
 - the magnitude is $|V| = \sqrt{5^2 + 5^2} = 7.071$ volts
 - the phase is $\phi = \tan^{-1}\left(\frac{\text{imaginary}}{\text{real}}\right) = \tan^{-1}\left(\frac{5}{5}\right) = 45^\circ$
 - So $V = 5 + j5 = 7.071 \angle 45^\circ$ volts.
 - Your calculator has a “rectangular to polar” conversion button. Learn to use it!
- To find the time function $v(t)$:

○ Either use $v(t) = \text{Re}(Ve^{j\omega t})$ so

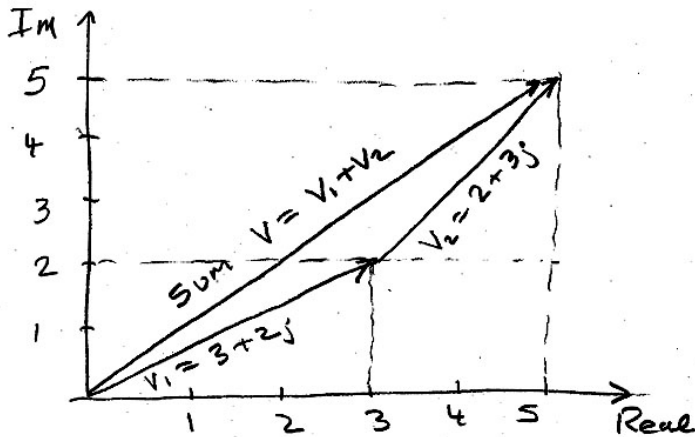
$$v(t) = \text{Re}(Ve^{j\omega t}) = \text{Re}(7.071e^{j45^\circ} e^{j\omega t}) = \text{Re}(7.071e^{j(\omega t + 45^\circ)}) = 7.071 \cos(\omega t + 45^\circ)$$

(This is naughty because the units in the argument of the cosine are mixed: ωt is radians but 45° is degrees.)

- Or use the “rules” that the magnitude of the phasor, 7.071, is the amplitude of the cosine, and the angle of the phasor, 45° , is the phase angle of the cosine, so we can write by inspection that

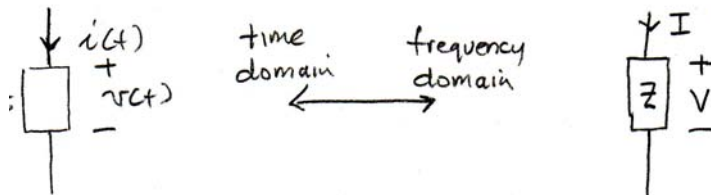
$$v(t) = 7.071 \cos(\omega t + 45^\circ)$$

- We can add voltages on a “phasor diagram”:



- To draw the sum:
 - first draw the arrow for $V_1 = 3 + j2$ volts,
 - then start at the tip of V_1 , and draw the arrow for $V_2 = 2 + j3$ volts.
 - the sum, $V = V_1 + V_2$, is the arrow from the origin to the end of the V_2 arrow.
- “Visualizing” the sum of phasors in this way sometimes provides insight into a problem that is hard to understand otherwise!

Impedances



- Let the voltage across a component be $v(t) = V_o \cos(\omega t + \theta)$ so that the phasor representing the voltage is $V = V_o e^{j\theta}$.
- Let the current flowing through the component be $i(t) = I_o \cos(\omega t + \phi)$ so that the phasor representing the current is $I = I_o e^{j\phi}$.
- Then the “impedance” of the component is defined as the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I}$$

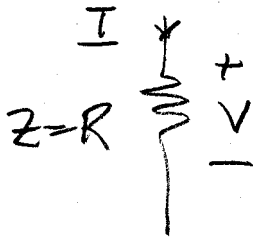
so

$$Z = \frac{V_o}{I_o} e^{j(\theta - \phi)}$$

- The magnitude of the impedance is $|Z| = \frac{V_o}{I_o}$

- The angle of the impedance is $(\theta - \phi)$

Resistor



- The resistor obeys $v(t) = Ri(t)$
- The voltage is $v(t) = V_o \cos(\omega t + \theta)$ with phasor representation $V = V_o e^{j\theta}$.
- The current is

$$i(t) = \frac{v(t)}{R} = \frac{V_o \cos(\omega t + \theta)}{R} = \frac{V_o}{R} \cos(\omega t + \theta)$$

which has amplitude $\frac{V_o}{R}$ and phase angle θ , so the phasor for the current is

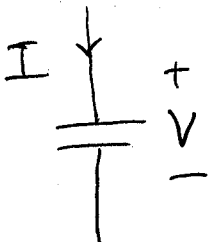
$$I = \frac{V_o}{R} e^{j\theta}$$

- Then the impedance is the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I} = \frac{V_o e^{j\theta}}{\frac{V_o}{R} e^{j\theta}} = R$$

- So the “impedance” of a resistor is $Z = R$ ohms. Simple!

Capacitor



- The capacitor obeys $i(t) = C \frac{d}{dt} v(t)$
- The voltage is $v(t) = V_o \cos(\omega t + \theta)$ with phasor representation $V = V_o e^{j\theta}$.
- The current is

$$i(t) = C \frac{d}{dt} V_o \cos(\omega t + \theta) = -\omega C V_o \sin(\omega t + \theta)$$

- Since we are writing our phasors relative to cosine, we need to change the sine into a cosine using the trig identity

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos\left(A + \frac{\pi}{2}\right) = \cos A \cos \frac{\pi}{2} - \sin A \sin \frac{\pi}{2}$$

$$\cos\left(A + \frac{\pi}{2}\right) = -\sin A$$

$$\cos\left((\omega t + \theta) + \frac{\pi}{2}\right) = -\sin(\omega t + \theta)$$

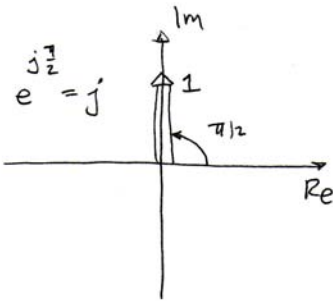
so

$$i(t) = -\omega CV_o \sin(\omega t + \theta) = \omega CV_o \cos(\omega t + \theta + \frac{\pi}{2})$$

- The amplitude is ωCV_o and the phase angle is $\left(\theta + \frac{\pi}{2}\right)$ so the phasor for the current is

$$I = \omega CV_o e^{j(\theta + \pi/2)} = \omega CV_o e^{j\pi/2} e^{j\theta}$$

- We can replace $e^{j\pi/2}$ by j :



- Since $e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} = j$, we can write

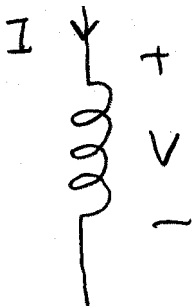
$$I = j\omega CV_o e^{j\theta}$$

- Then the impedance is the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I} = \frac{V_o e^{j\theta}}{j\omega CV_o e^{j\theta}} = \frac{1}{j\omega C}$$

- So the “impedance” of a capacitor is $Z = \frac{1}{j\omega C}$ ohms.

Inductor



- The inductor obeys $v(t) = L \frac{d}{dt} i(t)$
- The voltage is $v(t) = V_o \cos(\omega t + \theta)$ with phasor representation $V = V_o e^{j\theta}$.
- The current is

$$i(t) = \frac{1}{L} \int v(t) dt = \frac{1}{L} \int V_o \cos(\omega t + \theta) dt = \frac{1}{\omega L} V_o \sin(\omega t + \theta)$$

- Since we are writing our phasors relative to cosine, we need to change the sine into a cosine using the trig identity

$$\cos\left(A - \frac{\pi}{2}\right) = \cos A \cos\left(-\frac{\pi}{2}\right) - \sin A \sin\left(-\frac{\pi}{2}\right)$$

$$\cos\left(A - \frac{\pi}{2}\right) = \sin A$$

$$\cos\left(\omega t + \theta - \frac{\pi}{2}\right) = \sin(\omega t + \theta)$$

so

$$i(t) = \frac{1}{\omega L} V_o \sin(\omega t + \theta) = \frac{V_o}{\omega L} \cos\left(\omega t + \theta - \frac{\pi}{2}\right)$$

- The amplitude is $\frac{V_o}{\omega L}$ and the phase angle is $\left(\theta - \frac{\pi}{2}\right)$ so the phasor for the current is

$$I = \frac{V_o}{\omega L} e^{j(\theta - \pi/2)} = \frac{V_o}{\omega L} e^{-j\pi/2} e^{j\theta}$$

- Since $e^{-j\pi/2} = \cos\left(-\frac{\pi}{2}\right) + j \sin\left(-\frac{\pi}{2}\right) = -j = \frac{1}{j}$, we can write

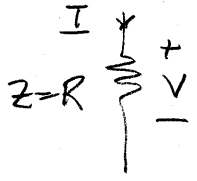
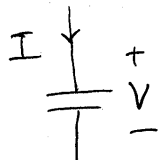
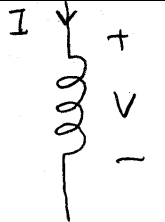
$$I = \frac{V_o}{j\omega L} e^{j\theta}$$

- Then the impedance is the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I} = \frac{V_o e^{j\theta}}{\frac{1}{j\omega L} V_o e^{j\theta}} = j\omega L$$

- So the “impedance” of an inductor is $Z = j\omega L$ ohms.

Impedance

Component		Time Domain	Frequency Domain “Impedance”
Resistor		$v(t) = Ri(t)$	$Z = R$
Capacitance		$i(t) = C \frac{d}{dt} v(t)$	$Z = \frac{1}{j\omega C}$
Inductance		$v(t) = L \frac{d}{dt} i(t)$	$Z = j\omega L$

Phasors and Time Differentiation

If the phasor representing $v(t) = V_o \cos(\omega t + \theta)$ is $V = V_o e^{j\theta}$, then

prove that the phasor representing $\frac{dv}{dt}$ is $j\omega V$.

Proof

- If $v(t) = V_o \cos(\omega t + \theta)$ then find the phasor V_1 that represents $\frac{dv}{dt}$.
- Calculate $\frac{dv}{dt} = \frac{d}{dt} v(t) = \frac{d}{dt} (V_o \cos(\omega t + \theta)) = -\omega V_o \sin(\omega t + \theta)$
- To represent $\frac{dv}{dt}$ with a phasor we need to change “cos” into “sin” with the trig identity

$$\cos\left(\omega t + \theta + \frac{\pi}{2}\right) = -\sin(\omega t + \theta)$$

hence

$$\frac{dv}{dt} = -\omega V_o \sin(\omega t + \theta) = \omega V_o \cos\left(\omega t + \theta + \frac{\pi}{2}\right)$$

- The phasor V_1 that represents $\frac{dv}{dt}$ is

$$V_1 = \omega V_0 e^{j(\theta + \pi/2)} = \omega V_0 e^{j\theta} e^{j\pi/2} = e^{j\pi/2} \omega V_0 e^{j\theta}$$

- We can replace $e^{j\pi/2}$ by j . So the phasor V_1 that represents $\frac{dv}{dt}$ is

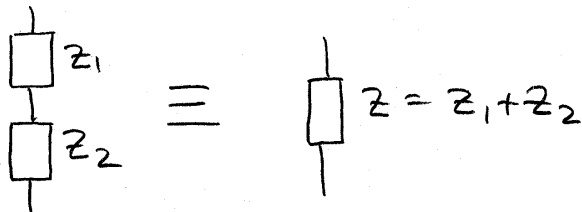
$$V_1 = e^{j\pi/2} \omega V_0 e^{j\theta} = j\omega V_0 e^{j\theta}$$

and since the phasor representing $v(t) = V_0 \cos(\omega t + \theta)$ is $V = V_0 e^{j\theta}$,

the phasor representing $\frac{dv}{dt}$ is

$$V_1 = j\omega V$$

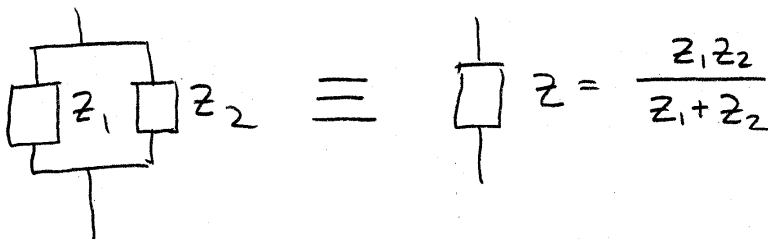
Series and Parallel Impedances



- Impedances in series add.
- If Z_1 is in series with Z_2 then the equivalent single impedance is

$$Z = Z_1 + Z_2$$

(Homework: prove this.)

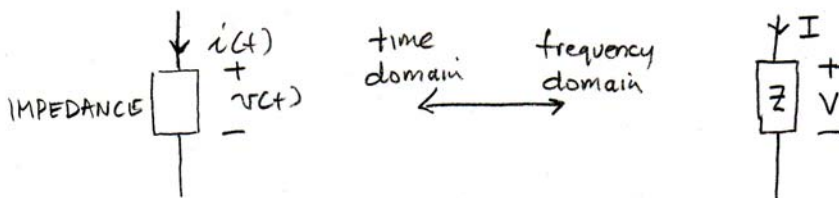


- Impedances in parallel combine like resistances in parallel.
- If Z_1 is in parallel with Z_2 then the equivalent single impedance is

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

(Homework: prove this.)

Power in A.C. Circuits



- If the voltage across an impedance is $v(t) = V_0 \cos(\omega t + \phi)$ and the current is $i(t) = I_0 \cos(\omega t + \theta)$, then what is the power delivered to the impedance?
- The "instantaneous power" is

$$p(t) = v(t)i(t) = V_0 \cos(\omega t + \phi) I_0 \cos(\omega t + \theta)$$

- The “average power” is the instantaneous power averaged over one A.C. cycle:

$$P_{av} = \frac{1}{T} \int_0^T p(t) dt$$

- You can evaluate the integral with trig identities to show that the average power is

$$P_{av} = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos(\phi - \theta)$$

where $\frac{V_0}{\sqrt{2}}$ is the root-mean-square value or “RMS value” of the A.C. voltage, $\frac{I_0}{\sqrt{2}}$ is the “RMS value” of the A.C. current, and $\cos(\phi - \theta)$ is called the “power factor”. (Homework: prove this.)

- The “RMS” value of a voltage $v(t)$ is given by

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

You can prove that if $v(t) = V_0 \cos(\omega t + \phi)$, then the RMS value is $V_{RMS} = \frac{V_0}{\sqrt{2}}$

- It is more convenient to calculate the power in an A.C. circuit directly from the phasors than it is by

using $P_{av} = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos(\phi - \theta)$.

- The phasor for $v(t) = V_0 \cos(\omega t + \phi)$ is $V = V_0 e^{j\phi}$
- The phasor for $i(t) = I_0 \cos(\omega t + \theta)$ is $I = I_0 e^{j\theta}$
- The “complex power” is defined as

$$S = \frac{1}{2} VI^*$$

where I^* is the complex conjugate of the current phasor, $I^* = I_0 e^{-j\theta}$.

(To get the complex conjugate, replace j by $-j$.)

- The complex power S is decomposed into a real part and an imaginary part:

$$S = P_{av} + jQ$$

- The real part P_{av} is the average power:

$$P_{av} = \text{Re}(S) = \frac{1}{2} \text{Re}(VI^*)$$

- The imaginary part Q is called the “reactive power”.

- So we can calculate the average power directly from the phasors using $P_{av} = \frac{1}{2} \text{Re}(VI^*)$ to get

$$P_{av} = \frac{1}{2} \text{Re}(V_0 e^{j\phi} I_0 e^{-j\theta}) = \frac{1}{2} \text{Re}(V_0 I_0 e^{j(\phi - \theta)})$$

$$P_{av} = \frac{1}{2} \text{Re}[V_0 I_0 (\cos(\phi - \theta) + j \sin(\phi - \theta))]$$

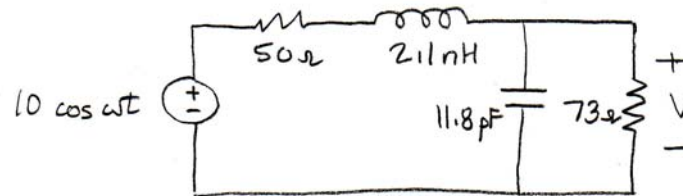
$$P_{av} = \frac{1}{2} V_0 I_0 \cos(\phi - \theta)$$

- This is the same as the formula given above.

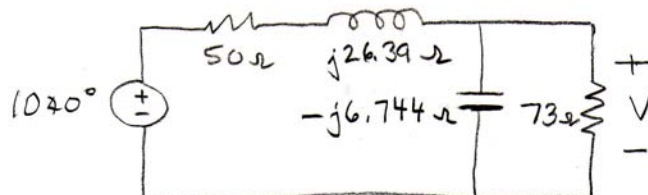
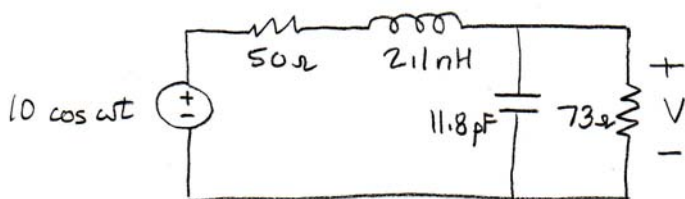
Solving A.C. Circuits Using Phasors

- Replace the $\cos \omega t$ generator by the corresponding phasor.
- Replace the R , L , and C circuit elements by impedances R , $j\omega L$ and $\frac{1}{j\omega C}$.
- Write KVL and KCL equations for the phasor voltages and currents.
- Solve the equations using complex arithmetic.

Example



- Solve this circuit to find the amplitude and phase of the load voltage V at $f = 2$ GHz.
- The generator in this circuit is $10 \cos \omega t$ volts at a frequency of $f = 2$ GHz, so $\omega = 2\pi f = 1.2566 \times 10^{10}$ rad/sec
- With phasors, we can solve an A.C. circuit the same way as we solve a D.C. circuit:
 - Represent the voltages and currents with complex numbers called “phasors”.
 - Change the R, L and C elements to impedances.
 - Use KVL and KCL to write equations for the circuit:
 - mesh equations
 - node equations
 - (Actually all the circuits we will solve are really simple and we won't need the powerful, formal methods called “node analysis” and “mesh analysis”.)
 - Solve the circuit equations using complex arithmetic.
 - The answer is a phasor:
 - the amplitude of the A.C. voltage is the magnitude of the phasor
 - the phase of the A.C. voltage is the angle of the phasor



Step 1: Convert to phasors and impedance

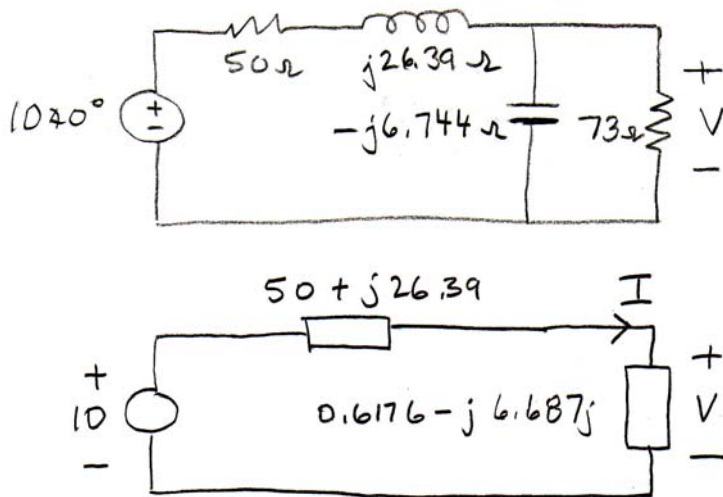
- The generator “ $10 \cos \omega t$ ” is represented by the phasor $10 \angle 0^\circ = 10$ volts.
- The impedance of the 50-Ω resistor is $R = 50$ ohms.
- The impedance of the inductance is $j\omega L = j26.39$ ohms.
- Combine the resistor and inductance in series into a single impedance of $50 + j26.39$ ohms.

- The impedance of the 73-Ω resistor is $R = 73$ ohms.
- The impedance of the capacitance is $\frac{1}{j\omega C} = -j6.744$ ohms.
- Combine the parallel capacitor and resistor into a single impedance of

$$= \frac{73(-j6.744)}{73 - j6.744} = 0.6176 - j6.687 \text{ ohms}$$

$$Z = 73 \parallel (-j6.744) = \frac{73(-j6.744)}{73 - j6.744} = \frac{-j492.02}{73 - j6.744} = \frac{492.02 \angle -90^\circ}{73.31 \angle -5.28^\circ}$$

$$= 6.715 \angle -84.7^\circ = 0.6178 - j6.687 \text{ ohms}$$
- About complex arithmetic:
 - Learn how to use the “rectangular to polar” conversion on your calculator, so that you can easily change the polar form $6.715 \angle -84.7^\circ$ into the rectangular form $0.6178 - j6.687$



Step 2: Find the current from a KVL equation:

$$10 - (50 + j26.39)I - (0.6176 - j6.687)I = 0$$

$$I = \frac{10}{(50 + j26.39) + (0.6176 - j6.687)}$$

Step 3: Solve the circuit using complex arithmetic:

$$I = \frac{10}{50.67 + j19.71} = \frac{10}{54.37 \angle 21.26^\circ} = 0.1839 \angle -21.26^\circ = 0.1714 - j0.0667$$

- Then find the voltage:

$$V = (0.6176 - j6.687)I = (0.6176 - j6.687)(0.1714 - j0.0667)$$

It is easier to do this in polar form:

$$V = (6.708 \angle -84.72^\circ)(0.1839 \angle -21.26^\circ) = 1.233 \angle -106^\circ$$

(In rectangular form, $V = -0.3398 - j1.185$, but we actually want the polar form because we want the amplitude and phase.)

- The amplitude of the voltage across the resistor is 1.233 volts.
- The phase angle of the voltage across the resistor is -106.0 degrees.