ELEC353 Lecture Notes Set 19

Tutorial WB Friday April 8, 2011
The room is changed to H633-1.

The homework assignments are posted on the course web site.

Homework #11: Do this assignment by March 25.
Homework #12: Do this assignment by April 1.

Course Evaluation: don't forget to do the course evaluation (nag, nag) 21-March-2011 to 9-April-2011

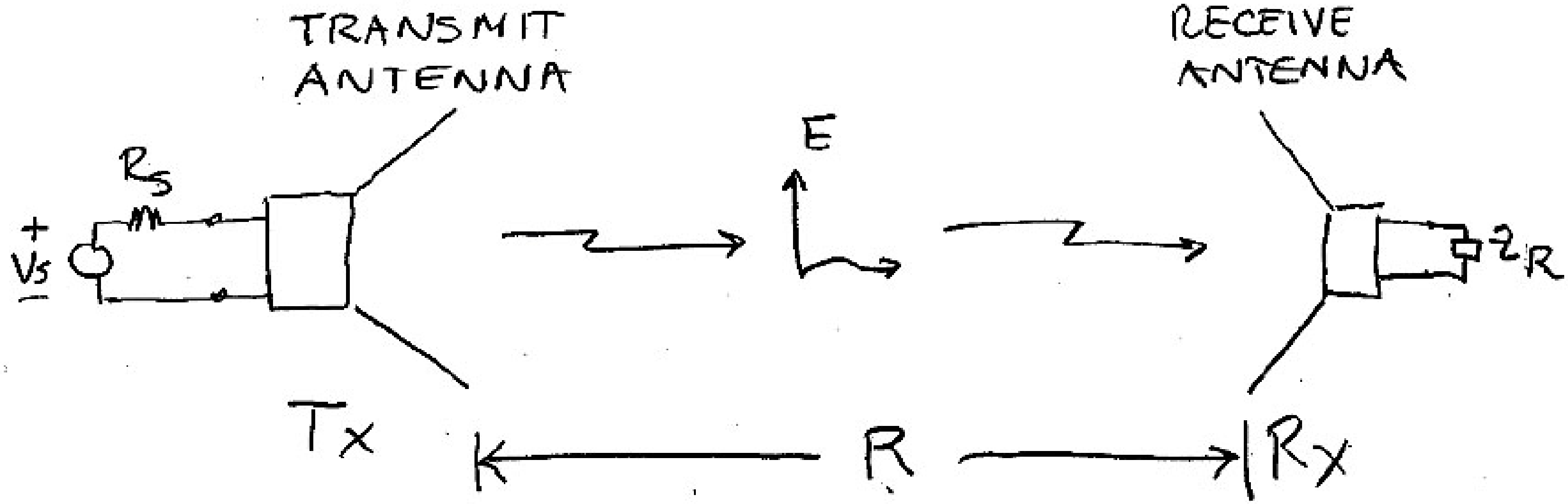
The final exam is

Wednesday April 27th, 2011 from 9 to 12.

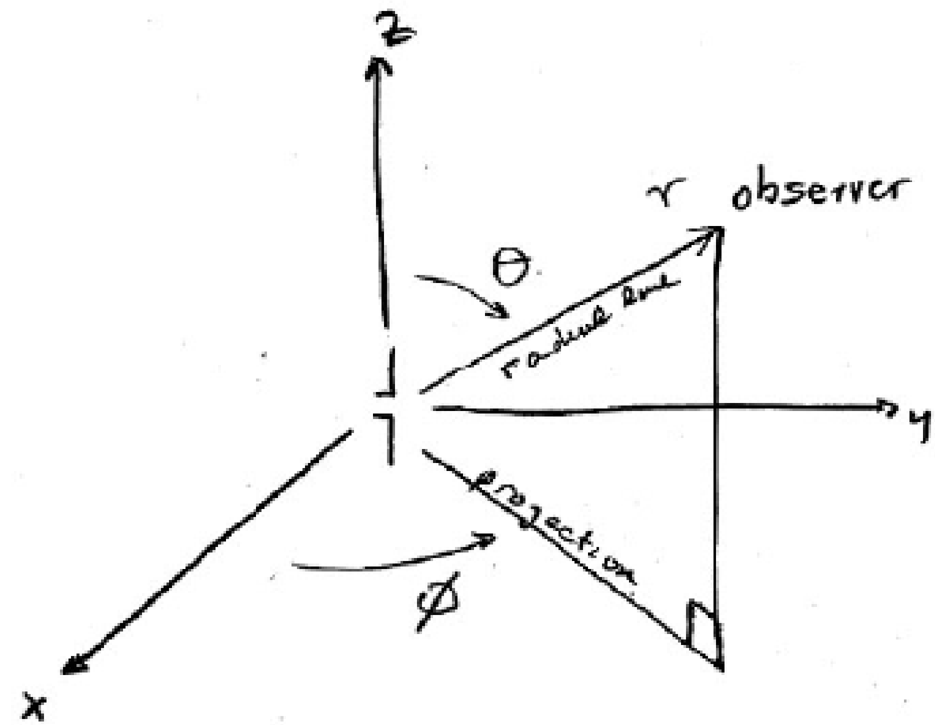
Old exam questions: posted on the web site.

Antennas and Wireless Links

Good reference: C.R. Paul, K.W. Whites, and S.A. Nasar, "Introduction to Electromagnetic Fields", 3rd edition, McGraw-Hill, 1998.



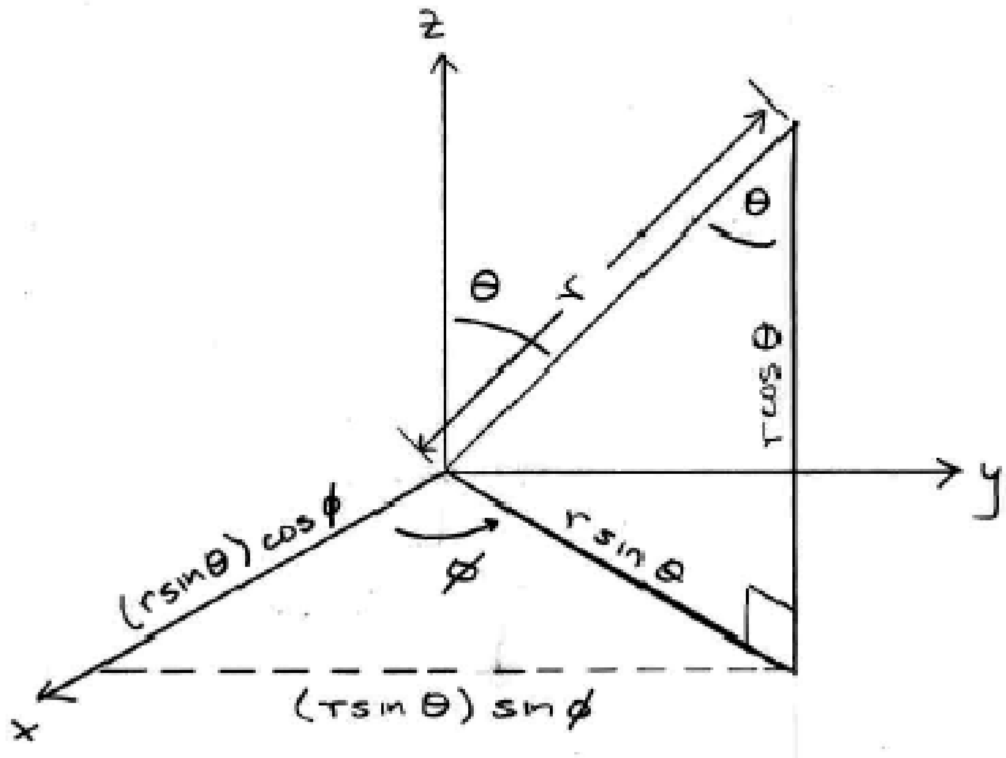
Spherical Coordinates



r = the distance from the origin
θ = the angle that the radial line from the origin makes to the z axis
ϕ = the angle that the projection of the radial line onto the xy plane makes to the x axis.

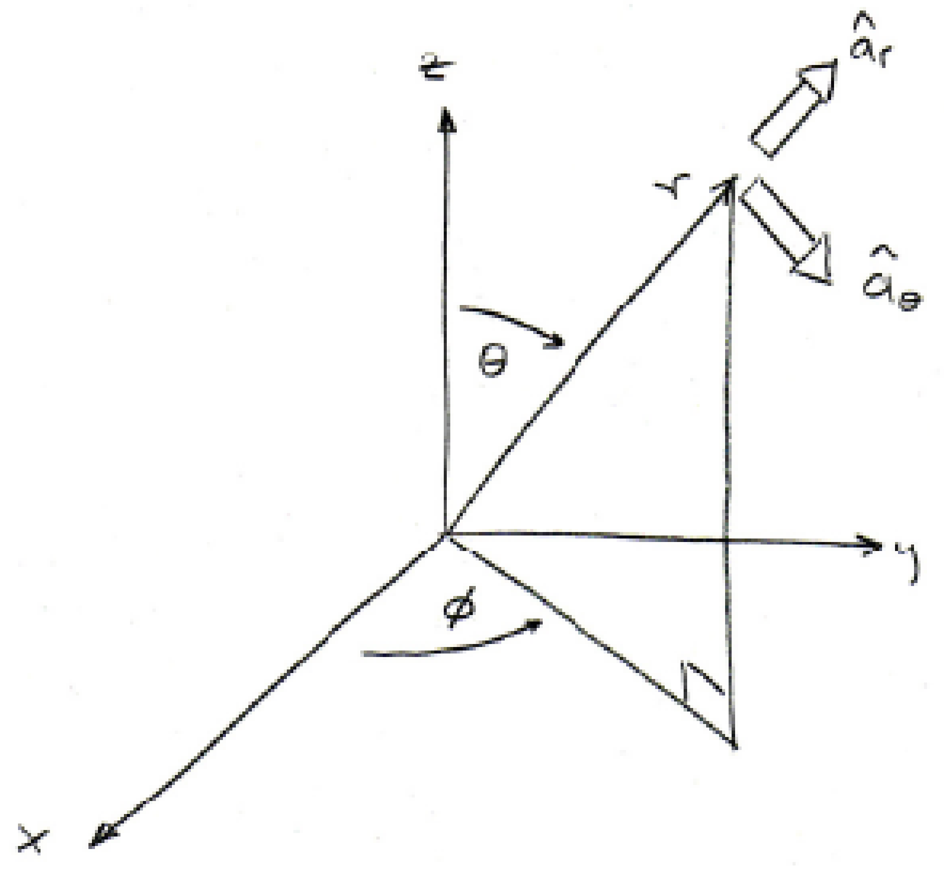
5

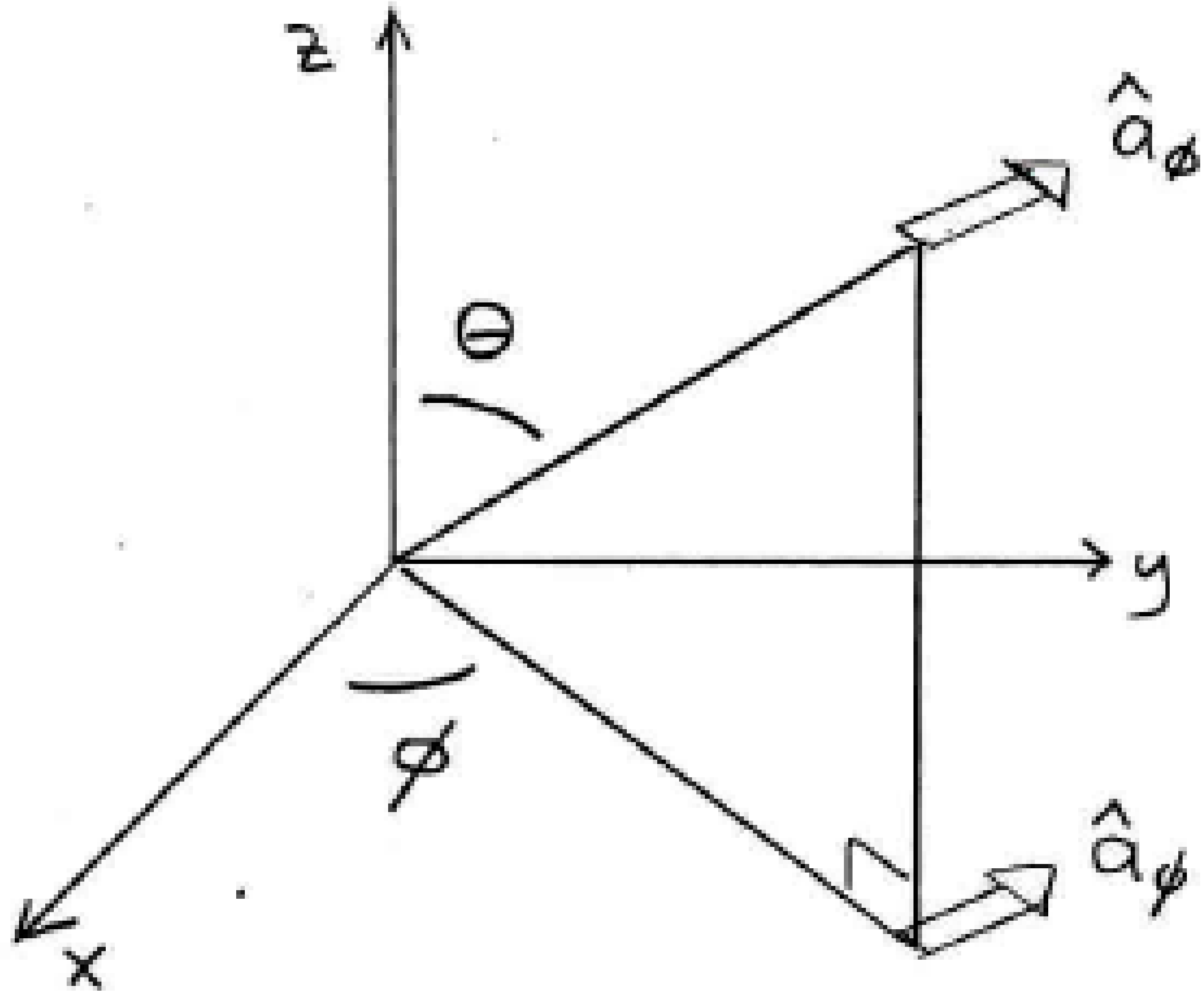
Relation to Rectangular Coordinates



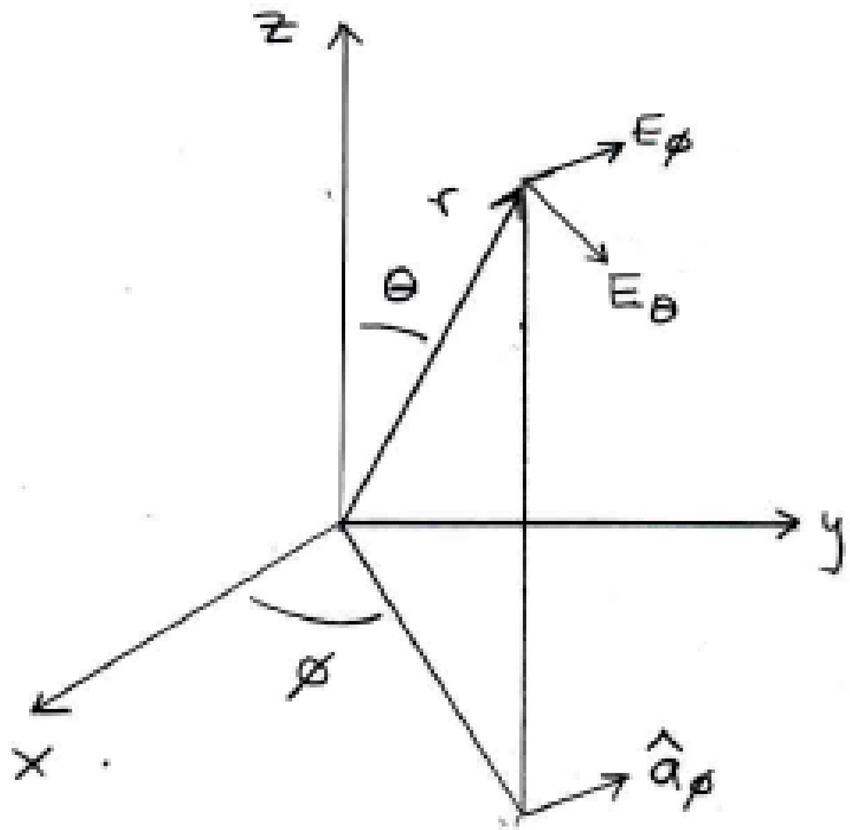
$$z = r\cos\theta$$
$$x = (r\sin\theta)\cos\phi$$
$$y = (r\sin\theta)\sin\phi$$

Spherical Unit Vectors



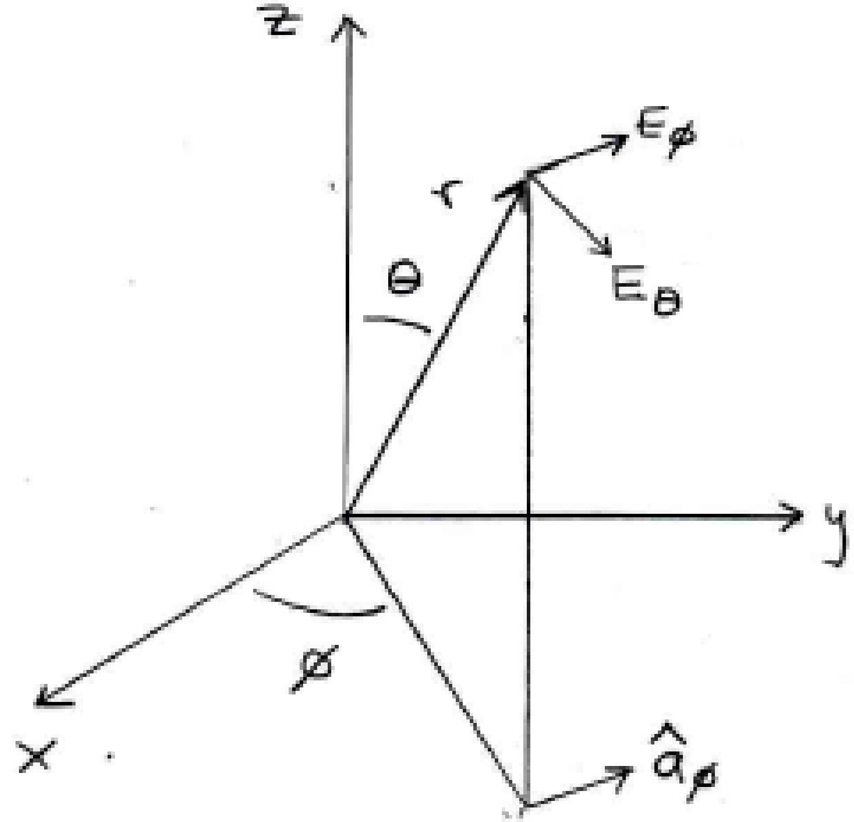

z
$\hat{a}_\phi$
Θ
y
ϕ
$\hat{a}_\phi$
x

Electric Field Vector in Spherical Coordinates



$$\overline{E}(r,\theta,\phi)=\hat{a}_r E_r(r,\theta,\phi)+\hat{a}_\theta E_\theta(r,\theta,\phi)+\hat{a}_\phi E_\phi(r,\theta,\phi)$$

The Far Fields of an Antenna



$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

The Magnetic Field

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Faraday's Law: $\nabla x \overline{E} = -j\omega\mu\overline{H}$

so

$$\overline{H} = \frac{-1}{j\omega\mu}\nabla \times \overline{E}$$

$\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$$\overline{H}(r,\theta,\phi) = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Homework: Prove this, but neglect terms in $\frac{1}{r^2}$ and keep only those terms proportional to $\frac{1}{r}$.

Power Flow Density

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi) = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\overline{E} \times \overline{H}^*\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right) \times \left(-\hat{a}_\theta \frac{e_\phi}{\eta}\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{e_\theta}{\eta}\frac{e^{-j\beta r}}{r}\right)^*\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right) \times \left(-\hat{a}_\theta \frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r} + \hat{a}_\phi \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r}\right)\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\left(\hat{a}_\theta e_\theta \frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi \frac{e^{-j\beta r}}{r}\right) \times \left(-\hat{a}_\theta \frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r} + \hat{a}_\phi \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r}\right)\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ 0 & e_\theta \frac{e^{-j\beta r}}{r} & e_\phi \frac{e^{-j\beta r}}{r} \\ 0 & -\frac{e_\phi^*}{\eta}\frac{e^{j\beta r}}{r} & \frac{e_\theta^*}{\eta}\frac{e^{j\beta r}}{r} \end{vmatrix}\right]$$

$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_r\left(\frac{1}{\eta} e_\theta e_\theta^* \frac{e^{-j\beta r}}{r}\frac{e^{j\beta r}}{r} + \frac{1}{\eta} e_\phi e_\phi^* \frac{e^{-j\beta r}}{r}\frac{e^{j\beta r}}{r}\right)\right]$$

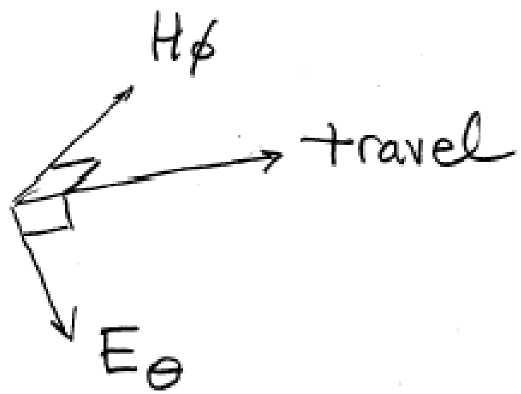
$$\overline{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\hat{a}_r\left(\frac{|e_\theta|^2}{\eta}\frac{1}{r^2} + \frac{|e_\phi|^2}{\eta}\frac{1}{r^2}\right)\right]$$

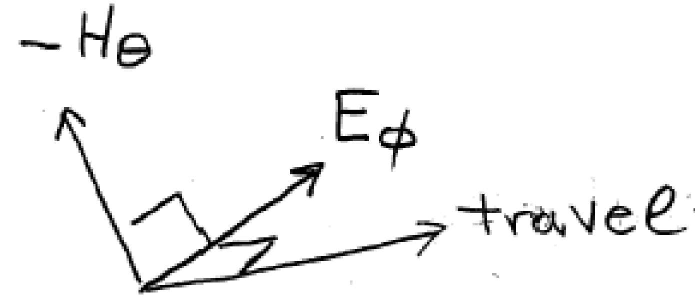
$$\overline{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right]$$ Watts per square meter

Quasi-Plane Waves

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi) = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$





E_θ and H_ϕ

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi) = \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta}\right]$$

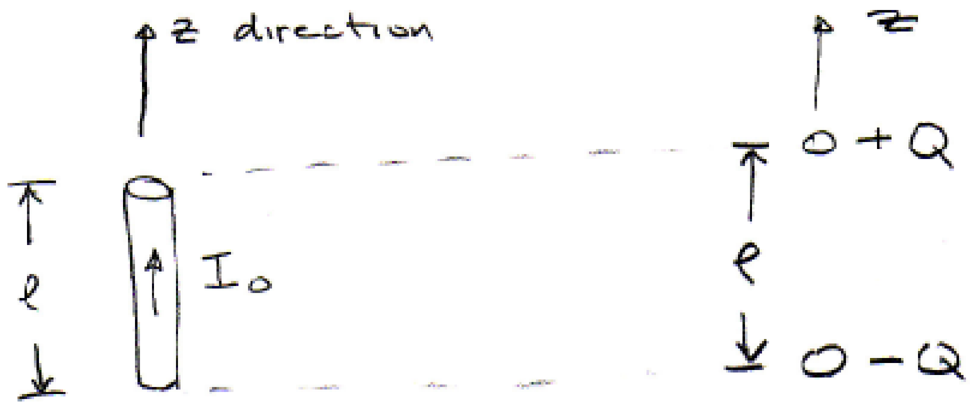
E_ϕ and $-H_\theta$

$$\overline{E}(r,\theta,\phi) = \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi) = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\phi|^2}{2\eta}\right]$$

The Hertzian Dipole



$$\ell << \lambda$$

$$i(t) = I_0 \cos(\omega t).$$

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi} \eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

Near Fields and Far Fields

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi} \eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r}\left[\frac{j}{\beta r} + \frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

These formulas contain three sets of terms:

- $\dfrac{1}{\beta r} = \dfrac{1}{2\pi} \dfrac{1}{r/\lambda}$
- $\dfrac{1}{(\beta r)^2} = \dfrac{1}{(2\pi)^2} \dfrac{1}{(r/\lambda)^2}$
- $\dfrac{1}{(\beta r)^3} = \dfrac{1}{(2\pi)^3} \dfrac{1}{(r/\lambda)^3}$

The Near Field

- Consider "close" distances, where $r << \lambda$
- then $\beta r = 2\pi \frac{r}{\lambda}$ is "small"
- since $\frac{1}{(\text{small})^2} >> \frac{1}{\text{small}}$, we have $\frac{1}{(\beta r)^2} >> \frac{1}{\beta r}$
- and $\frac{1}{(\text{small})^3} >> \frac{1}{\text{small}}$, we have $\frac{1}{(\beta r)^3} >> \frac{1}{\beta r}$
- The dominant terms are those in $\frac{1}{(\beta r)^2}$ and $\frac{1}{(\beta r)^3}$
- Then the fields are given by

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2}\right]$$

$$E_r = \frac{2\beta^2 I_0 \ell}{4\pi}\eta_0 \cos\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi}\eta_0 \sin\theta e^{-j\beta r}\left[\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right]$$

$$e^{-j\beta r} \approx e^{j0} = 1$$

The Far Field

- Consider "far" distances, where $r >> \lambda$
- then $\beta r = 2\pi \frac{r}{\lambda}$ is "large"
- since $\frac{1}{(\text{large})^2} << \frac{1}{\text{large}}$, we have $\frac{1}{(\beta r)^2} << \frac{1}{\beta r}$
- and $\frac{1}{(\text{large})^3} << \frac{1}{\text{large}}$, we have $\frac{1}{(\beta r)^3} << \frac{1}{\beta r}$
- The dominant terms are those in $\frac{1}{\beta r}$
- Then the "far fields" of the Hertzian dipole are given by

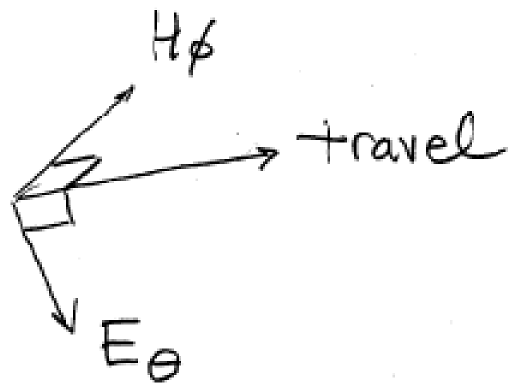
$$E_r = 0$$

$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

Quasi-Plane Wave

The far field is a quasi-plane wave.



$$E_\theta = \frac{\beta^2 I_0 \ell}{4\pi} \eta_0 \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

$$H_\phi = \frac{\beta^2 I_0 \ell}{4\pi} \sin\theta e^{-j\beta r} \frac{j}{\beta r}$$

so

$$H_\phi = \frac{1}{\eta_0} E_\theta$$

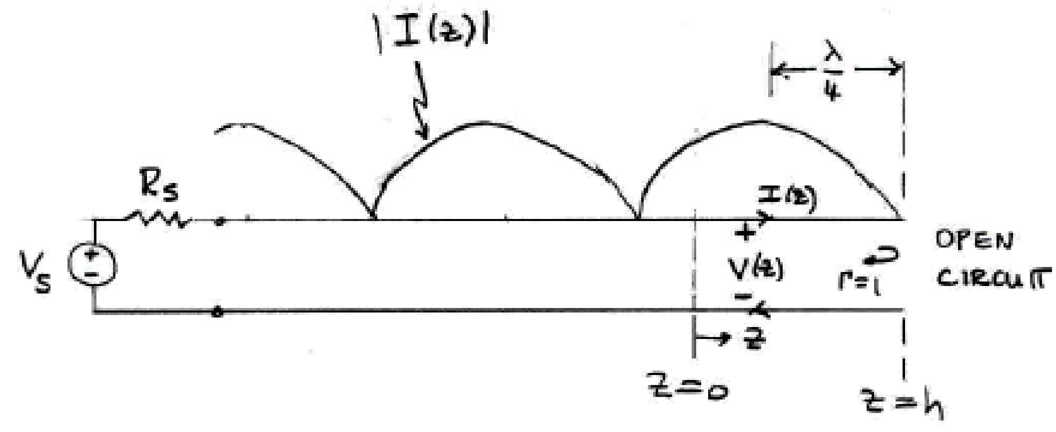
Angle Dependence

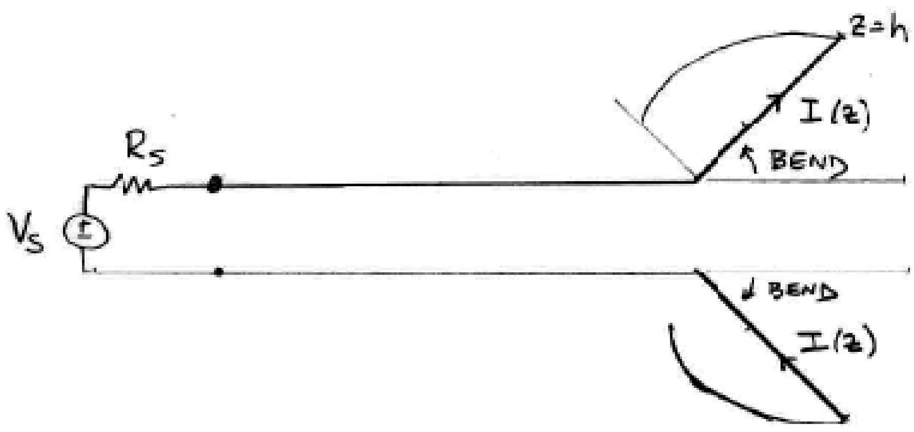
$$E_\theta = \frac{j\eta_0 \beta I_0 \ell}{4\pi} \sin\theta \frac{e^{-j\beta r}}{r}$$

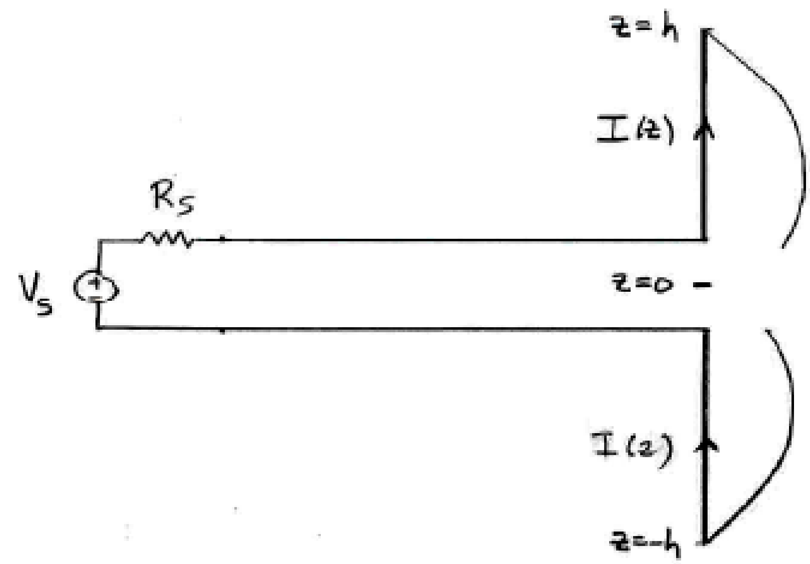
$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$e_\theta(\theta,\phi) = \frac{j\eta_0 \beta I_0 \ell}{4\pi} \sin\theta$$ and $$e_\phi = 0$$

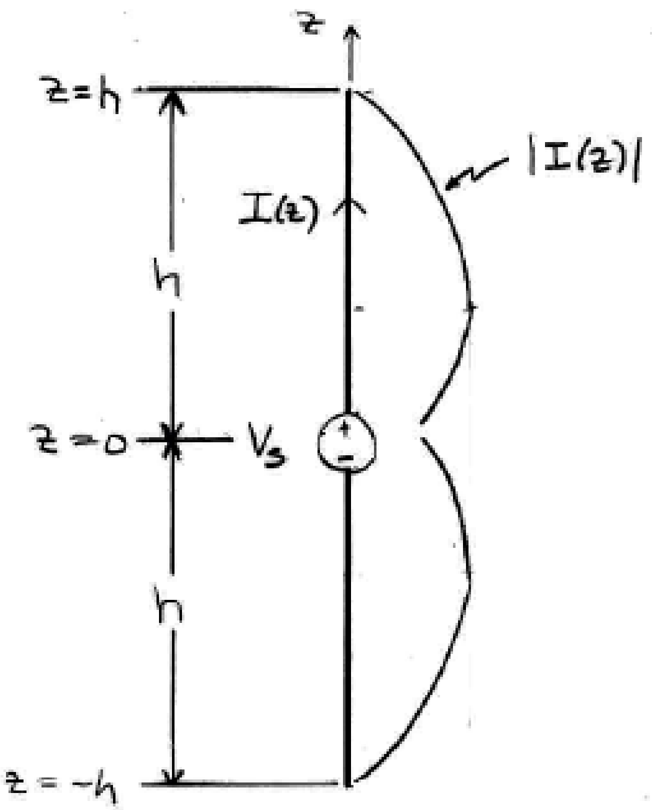
The Dipole Antenna







The Far Fields of a Dipole Antenna



$$I(z) = I_0 \sin(\beta(z-h))$$

It May Be Shown that:

$$E_\theta(\theta) = \frac{j\eta_0 I_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi) \frac{e^{-j\beta r}}{r}$$

$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi} F(\theta)$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

Example

A vertical, half-wave dipole antenna is used to radiate a signal at 2450 MHz. At a perpendicular distance of 1 m from the center of the dipole, the field strength is 10 V/m.

1) What is the amplitude I_0 of the current on the dipole?

2) What is the field strength 30 m from the antenna, in the horizontal plane?

3) What is the field strength 200 m from the antenna, at an elevation of 30 degrees above the horizontal plane?

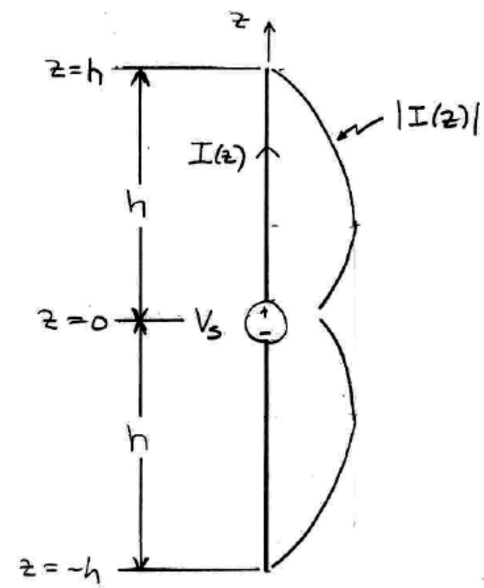
Solution

- The field of the dipole antenna is

$$E_\theta(r,\theta) = \frac{jI_0\eta_0}{2\pi}\frac{e^{-j\beta r}}{r}F(\theta)$$

where the angular dependence is

$$F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos(\beta h)}{\sin\theta}$$



•A “half wave” dipole is a dipole whose length is half a wavelength.

•Thus $2h = \frac{\lambda}{2}$ and $h = \frac{\lambda}{4}$

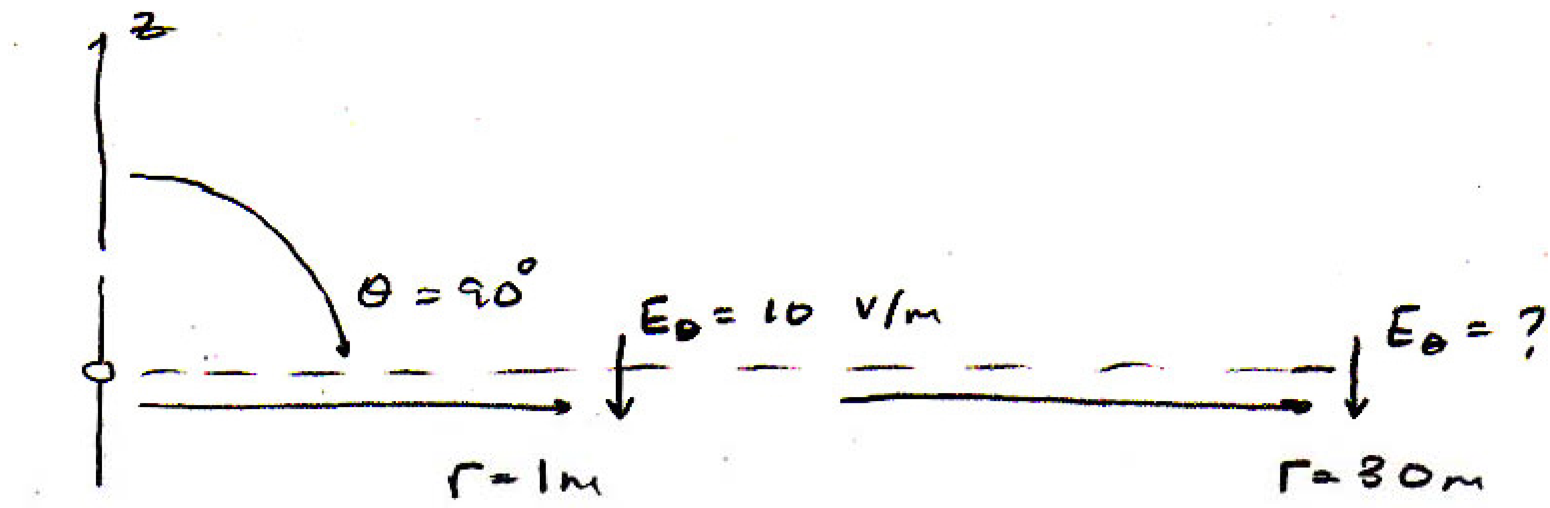
Half-wave dipole: $h = \dfrac{\lambda}{4}$

$$\beta = \frac{2\pi}{\lambda} \qquad \beta h = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\theta}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$



- We are told that the dipole is "vertical" so the "horizontal plane" is the θ=90 degrees plane.
- We can specialize the field to the θ=90 by evaluating $F(\theta=90)$ to get

$$F(\theta)=\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}=\frac{\cos\left(\frac{\pi}{2}\cos\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}=\frac{\cos 0}{1}=1$$

- So the field in the horizontal plane is given by

$$E_\theta(r,\theta=90)=\frac{jI_0\eta_0}{2\pi}\frac{e^{-j\beta r}}{r}$$

- What is the value of I_0?
    - We are told that the electric field strength one meter from the dipole is 10 V/m:
    $\left|E(r=1,\theta=90)\right|$=10 V/m
- At 1 meter distance,

$$\left|E(r=1,\theta=90)\right| = \left|\frac{jI_0\eta_0}{2\pi}\right|\left|\frac{e^{-j\beta r}}{r}\right|_{r=1} = \frac{I_0\eta_0}{2\pi} = 10$$

so

$$I_0 = \frac{20\pi}{\eta}$$

- Then in the $\theta = 90$ plane the field is given by

$$E_\theta(r,\theta=90) = j10\frac{e^{-j\beta r}}{r}$$

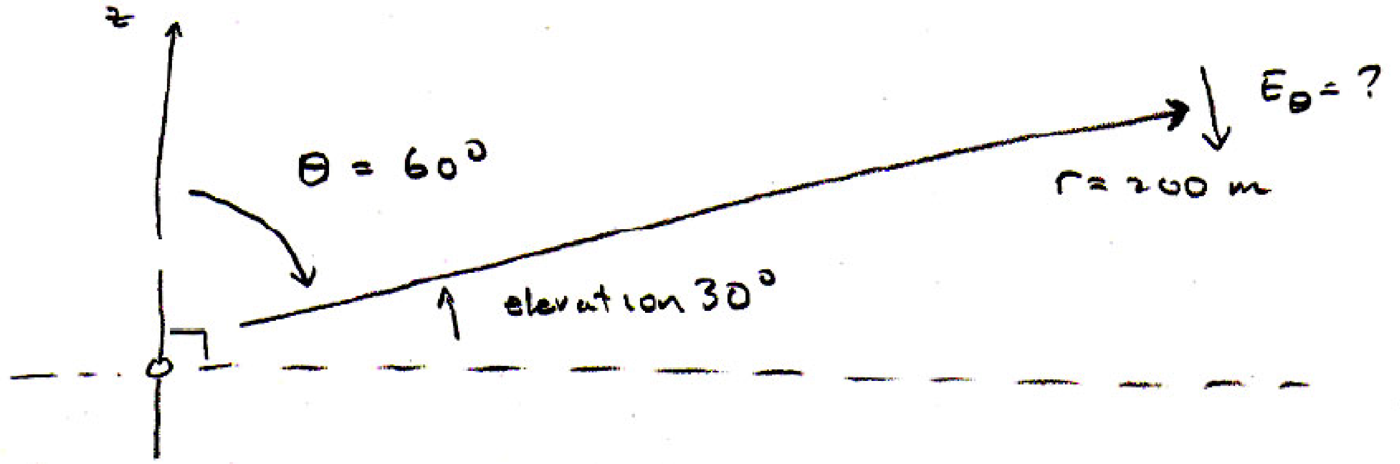
and the amplitude of the field is

$$\left|E(r,\theta=90)\right| = \frac{10}{r}$$

- Then the field strength at $r = 30$ m and θ=90 degrees is

$$\left|E\right| = \frac{10}{30} = 0.333 \text{ V/m}$$

What is the field strength 200 m from the antenna, at an elevation of 30 degrees above the horizontal plane?



- In general the field is given by

$$E_\theta(r,\theta) = \frac{jI_0\eta_0}{2\pi}\frac{e^{-j\beta r}}{r}F(\theta)$$

and with $\frac{I_0\eta_0}{2\pi} = 10$ we have

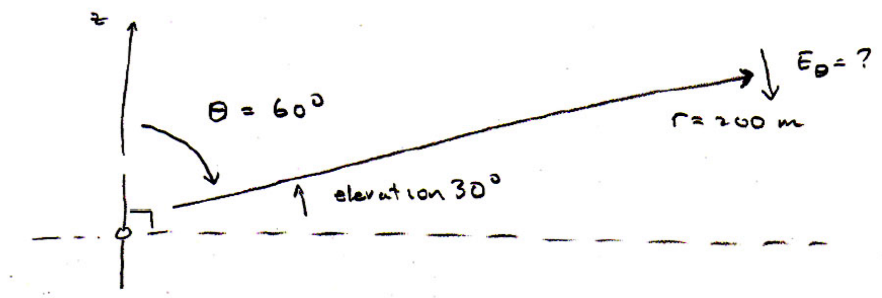
$$E_\theta(r,\theta) = j10\frac{e^{-j\beta r}}{r}F(\theta)$$

- The amplitude of the field is

$$\left|E_\theta(r,\theta)\right| = \frac{10}{r}F(\theta)$$

where

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$



- An elevation of 30 degrees above the horizontal plane corresponds to an angle θ of 60 degrees.
- To find the field strength at θ=60 degrees at r=200 m distance, evaluate

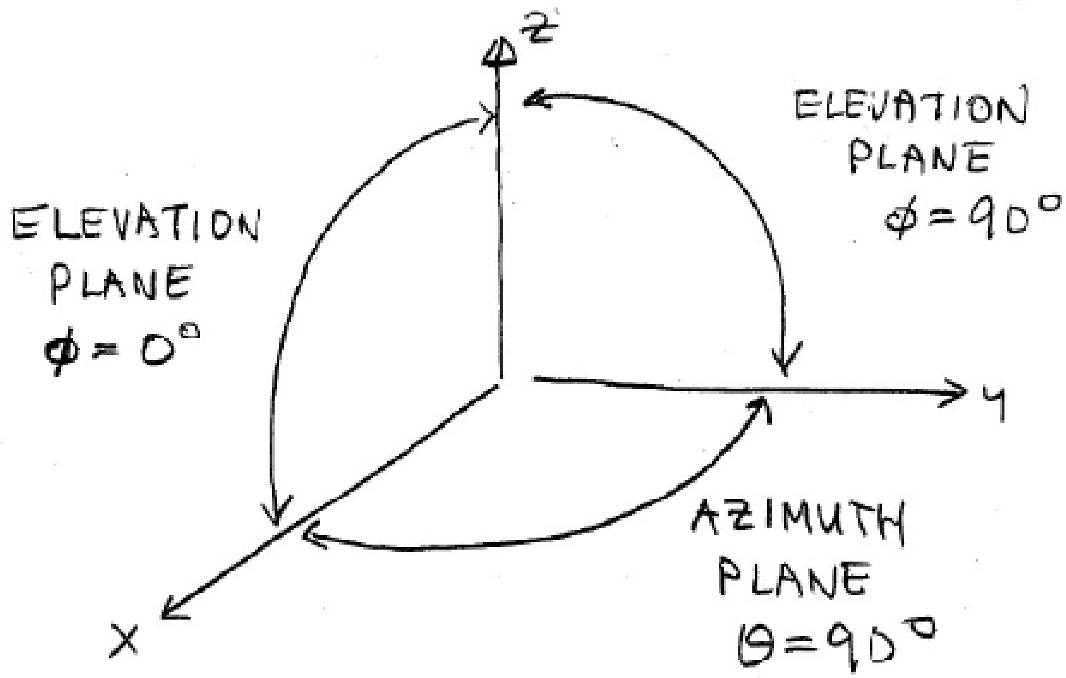
$$|E| = \frac{10}{r} F(\theta) = \frac{10}{200} F(\theta = 60)$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\frac{\pi}{3}\right)}{\sin\frac{\pi}{3}} = 0.8165$$

$$|E| = \frac{10}{200} \cdot 0.8165 = 0.04082 \text{ V/m} = 40.82 \text{ mV/m}$$

Radiation Patterns



$$\overline{E}(r,\theta,\phi)=\hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}+\hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

Azimuth Pattern for: $\theta = 90°$: $|e_\theta|$ and $|e_\phi|$ vs. ϕ (xy plane)

Elevation pattern for $\phi = 0°$: $|e_\theta|$ and $|e_\phi|$ vs. θ (xz plane)

Elevation pattern for $\phi = 90°$: $|e_\theta|$ and $|e_\phi|$ vs. θ (yz plane)

Example – Dipole Antenna

- Plot the azimuth and elevation patterns for a half-wave dipole antenna..
- The radiation patterns are given by

$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi} F(\theta)$$

where

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta}$$

- For a "half-wave dipole", the length is $2h = \frac{\lambda}{2}$ so $h = \frac{\lambda}{4}$ and

$$\beta h = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

so

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right) - \cos\left(\frac{\pi}{2}\right)}{\sin\theta} = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

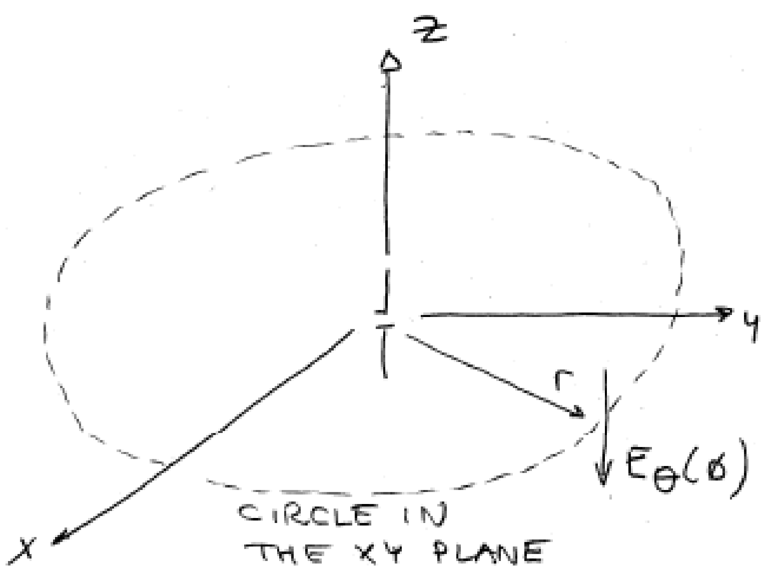
Azimuth Pattern:

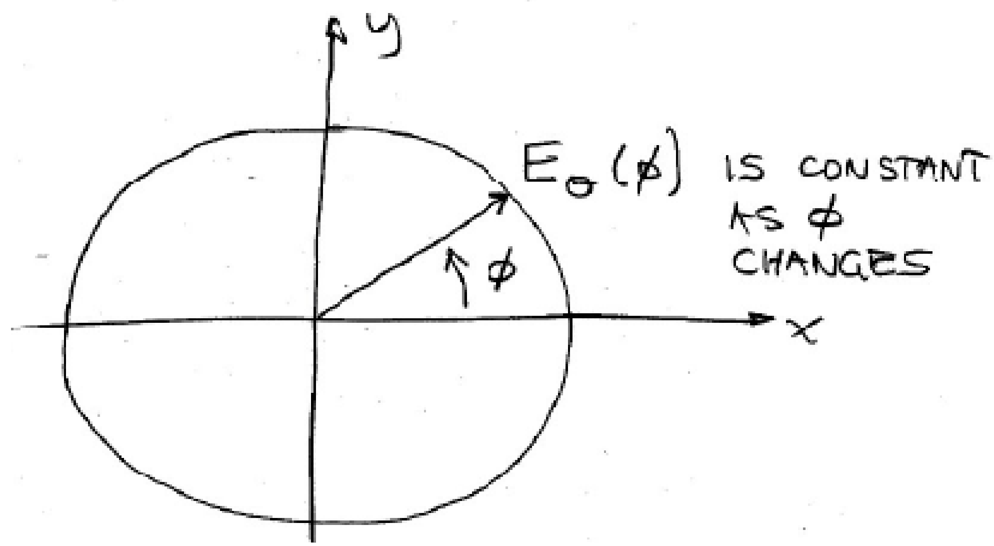
- The "azimuth pattern" is obtained by setting $\theta = \dfrac{\pi}{2}$ to get

$$F(\theta) = \frac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta} \qquad \text{so} \qquad F\left(\theta = \frac{\pi}{2}\right) = \frac{\cos\left(\dfrac{\pi}{2}\cos\dfrac{\pi}{2}\right)}{\sin\dfrac{\pi}{2}} = \frac{\cos 0}{1} = 1$$

$$e_\theta\left(\theta = \frac{\pi}{2}\right) = \frac{jI_0\eta_0}{2\pi} F\left(\frac{\pi}{2}\right) = \frac{jI_0\eta_0}{2\pi}$$

and so E_θ is constant and independent of angle ϕ.



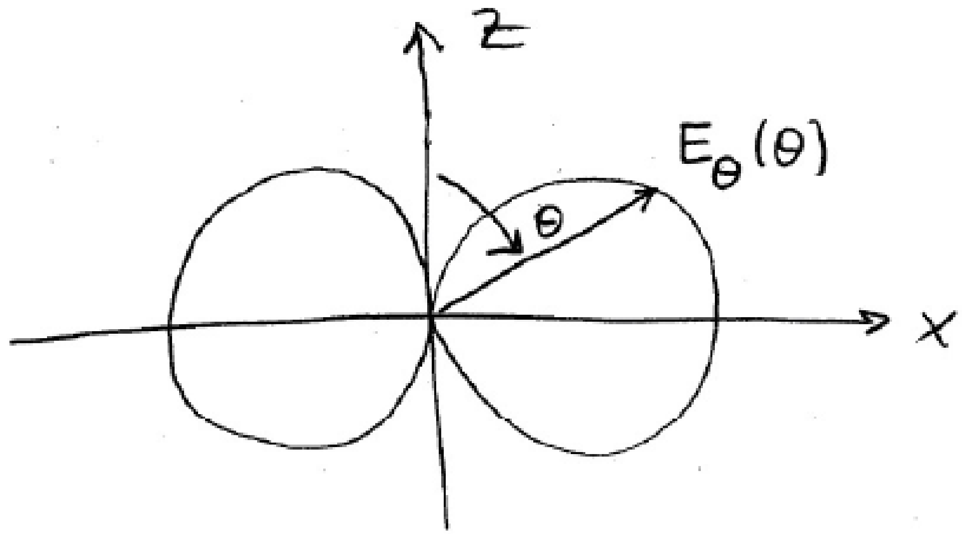


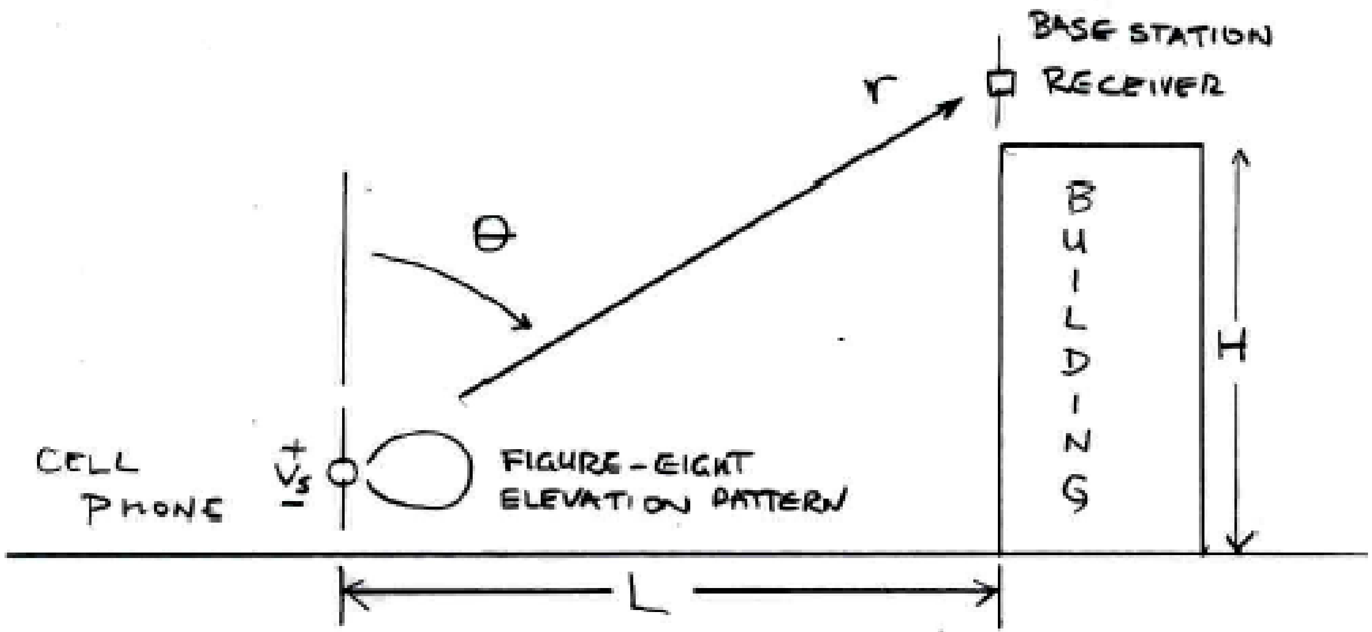
Elevation Patterns:

Plot the dipole's elevation patterns.

- The "elevation pattern" sets $\phi = 0$ or ϕ=90, and graphs $|E_\theta|$ as a function of angle θ.
- Since $F(\theta) = \dfrac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$, we have

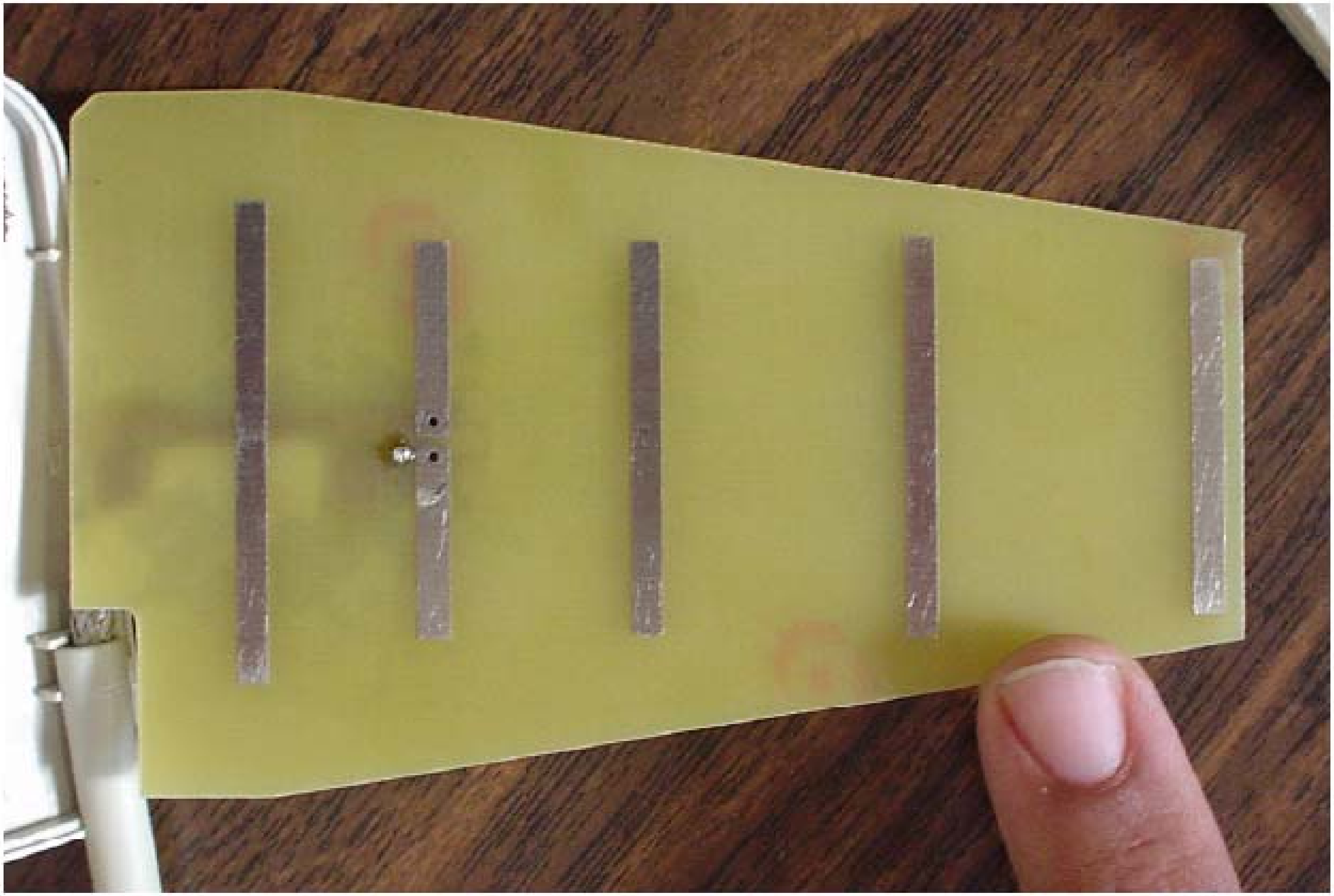
$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi}F(\theta) = \frac{jI_0\eta_0}{2\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$



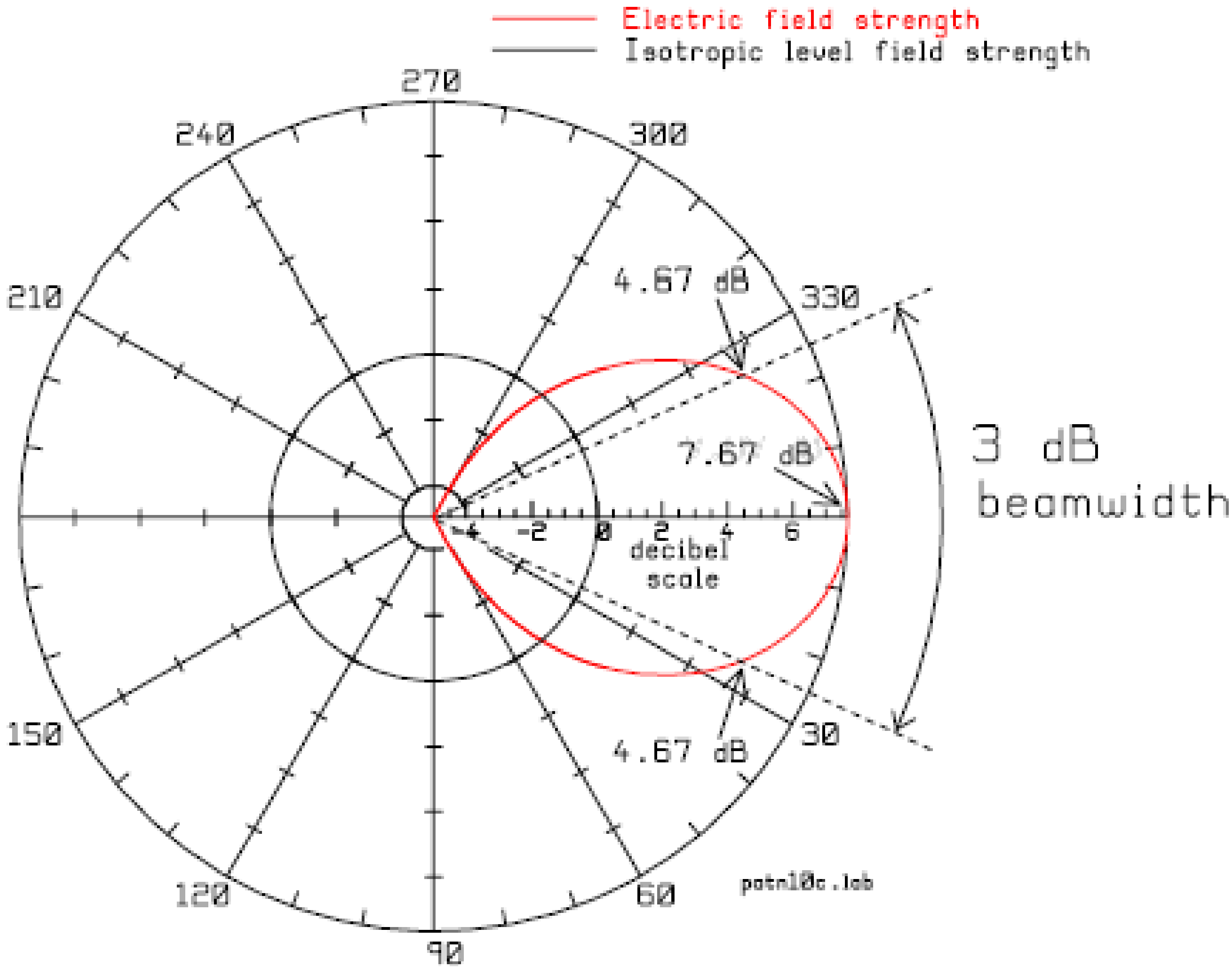


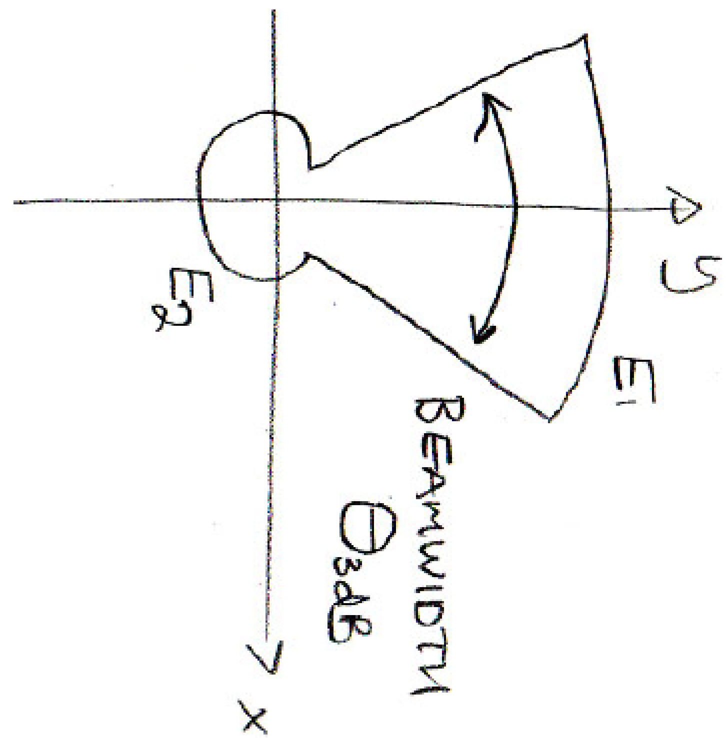
If we "model" a cell phone's antenna as a vertical dipole, then the "elevation plane" radiation pattern tells us that there may be problems communicating with a base station on top of a nearby building.

Directional AP Antenna



Radiation Pattern of the Directional AP Antenna

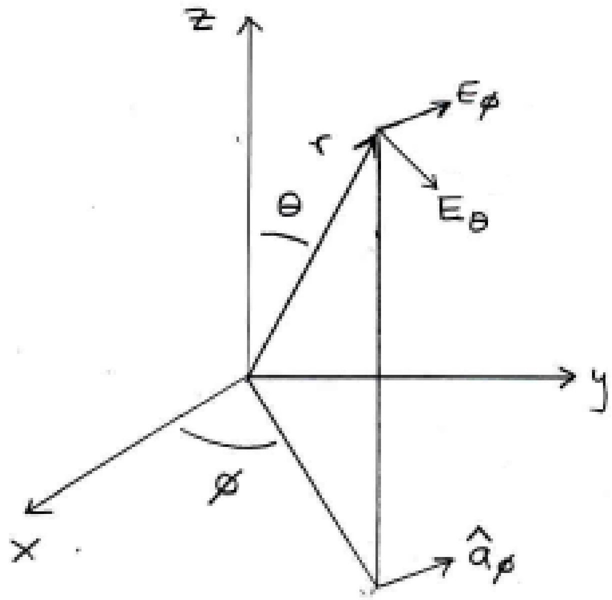




"Keyhole" Approximation of an Antenna Pattern

- This is a simple approximation of the radiation pattern of a Yagi antenna with 3 dB beamwidth θ_{3dB}
- The "keyhole" pattern has a "main lobe" of width θ_{3dB} and field strength E_1
- The "side lobes" and "back lobes" are approximated with field strength $E_2 << E_1$
- The total radiated power in the "keyhole" pattern is the same as the radiated power of the access point antenna.
- The "keyhole" is often further approximated by using $E_2 = 0$

Review: Far Field and Power Flow

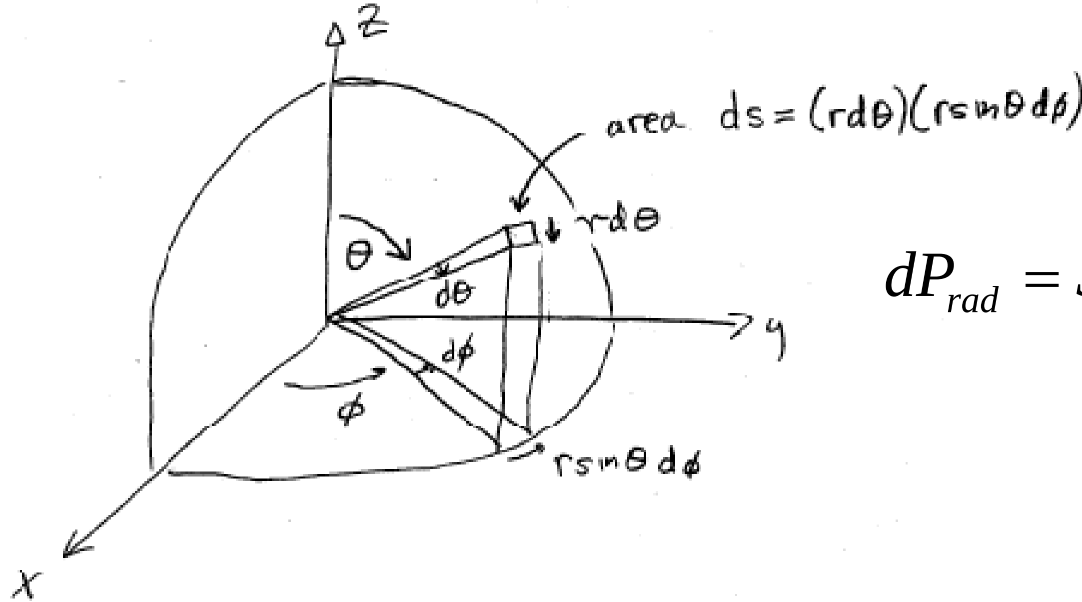


$$\overline{E}(r,\theta,\phi) = \hat{a}_\theta e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{H}(r,\theta,\phi) = -\hat{a}_\theta \frac{1}{\eta} e_\phi(\theta,\phi)\frac{e^{-j\beta r}}{r} + \hat{a}_\phi \frac{1}{\eta} e_\theta(\theta,\phi)\frac{e^{-j\beta r}}{r}$$

$$\overline{S}_{av}(r,\theta,\phi) = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right]$$

The Radiated Power



$$dP_{rad} = S_{av}(\theta,\phi)ds$$

To trace out a sphere we must vary θ from 0 to π, and for each θ value we must trace out a full circle by letting ϕ increase from 0 to 2π.

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi)ds$$

$$ds = r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$\bar{S}_{av} = \hat{a}_r \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right]$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{1}{r^2}\left(\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right) r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \left(\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right) \sin\theta d\theta d\phi$$

The Isotropic Power Density

Suppose that we have an antenna that radiates the same power density S_{iso} uniformly in all directions.

$$S_{av}(r,\theta,\phi) = S_{iso}(r)$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{iso} r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = S_{iso} r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi$$

$$P_{rad} = 4\pi r^2 S_{iso}$$

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$ "isotropic power density"

The Isotropic Level Field Strength

"isotropic power density" $$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

In general $$S_{av} = \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta} + \frac{|e_\phi|^2}{2\eta}\right]$$

Suppose $e_\theta(\theta,\phi) = e_{iso}$ and $e_\phi(\theta,\phi) = 0$

$$S_{av} = \frac{1}{r^2}\left[\frac{e_{iso}^2}{2\eta}\right] = \frac{P_{rad}}{4\pi r^2}$$

$$\frac{e_{iso}^2}{2\eta} = \frac{P_{rad}}{4\pi}$$

$$e_{iso} = \sqrt{\frac{\eta P_{rad}}{2\pi}}$$

"isotropic level" field strength

System-Level Antenna Definitions

Directive Gain

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{S_{iso}}$$

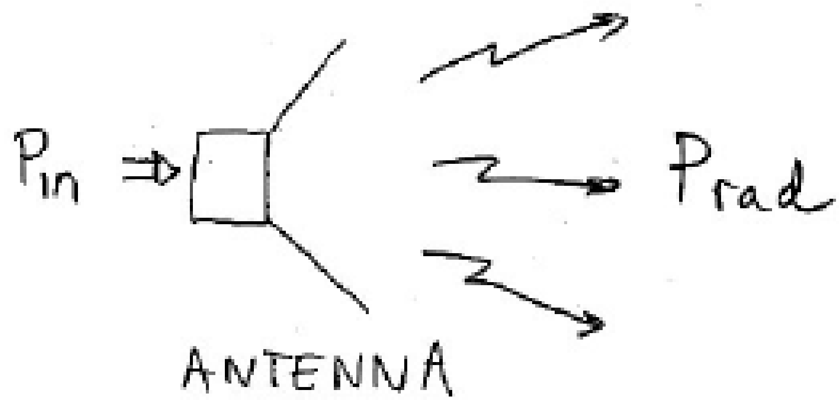
$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{rad}/4\pi r^2}$$

P_{rad} = the radiated power

Directivity = the maximum value of the directive gain:

$$D_{\max} = \max\left[D(\theta,\phi)\Big|_{\substack{0\le\theta\le\pi \\ 0\le\phi\le2\pi}} \right]$$

Lossy Antennas and Gain



Efficiency: $e = \frac{P_{rad}}{P_{in}}$

P_{rad} = the **radiated** power

P_{in} = the **input** power

Power Gain: $G(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{in}/4\pi r^2}$

Relation of the Power Gain to the Directive Gain:

$$G(\theta,\phi) = \frac{S_{av}}{P_{in}/4\pi r^2} = \frac{S_{av}}{\frac{1}{e}P_{rad}/4\pi r^2} = e\frac{S_{av}}{P_{rad}/4\pi r^2} = eD(\theta,\phi)$$

$$G(\theta,\phi) = eD(\theta,\phi).$$

The “Gain” of an Antenna

The “gain” of an antenna is the maximum value of the power gain:

$$G_{max} = \max_{\substack{0 \le \theta < \pi \\ 0 \le \phi < 2\pi}} G(\theta, \phi)$$

$$G_{max} = \max_{\substack{0 \le \theta < \pi \\ 0 \le \phi < 2\pi}} \left[eD(\theta, \phi) \right] = eD_{max}$$

Directivity of a Half-Wave Dipole

$$S_{av} = \frac{1}{r^2}\left[\frac{|e_\theta|^2}{2\eta_0} + \frac{|e_\phi|^2}{2\eta_0}\right]$$

$$S_{av} = \frac{1}{2\eta_0 r^2}\left(|e_\theta|^2 + |e_\phi|^2\right)$$

$$e_\theta(\theta,\phi) = \frac{jI_0\eta_0}{2\pi}F(\theta) \qquad F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$S_{av} = \frac{1}{2\eta_0 r^2}\left(\frac{I_0\eta_0}{2\pi}\right)^2 \left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

$$S_{av} = \frac{I_0^2\eta_0}{8\pi^2 r^2}\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

$$S_{av} = \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S_{av}(r,\theta,\phi) r^2 \sin\theta d\theta d\phi$$

$$P_{rad} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2 r^2 \sin\theta d\theta d\phi$$

This can be integrated numerically to obtain

$$P_{rad} = 36.5 I_0^2$$

$$P_{rad} = 36.5 I_0^2$$

directive gain

$$D(\theta,\phi) = \frac{S_{av}(\theta,\phi)}{P_{rad} / 4\pi r^2}$$

Half-wave Dipole:

$$S_{av} = \frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2$$

so

$$D(\theta,\phi) = \frac{\frac{I_0^2 \eta_0}{8\pi^2 r^2} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right|^2}{36.5 I_0^2 / 4\pi r^2}$$

From the previous slide:

$$D(\theta,\phi)=\frac{\frac{I_0^2\eta_0}{8\pi^2 r^2}\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2}{36.5 I_0^2 / 4\pi r^2}$$

$$D(\theta,\phi)=\frac{\eta_0}{36.5(2\pi)}\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|^2$$

The "directivity" is the maximum value of the directive gain:

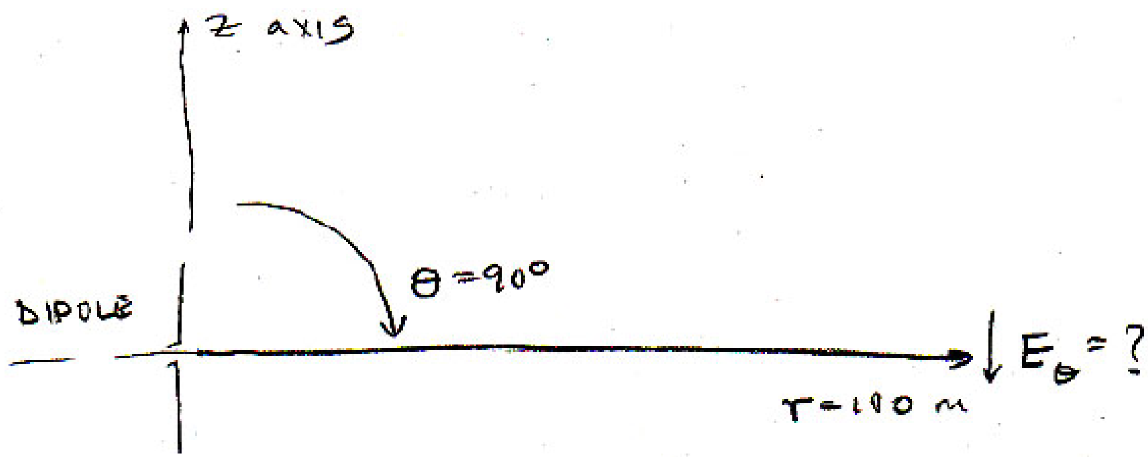
As angle θ varies the maximum value of $\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right|$ is unity.

$$D_{\max}=\frac{\eta_0}{36.5(2\pi)}=1.64$$

Example

What is the electric field strength 100 m from a half-wave dipole which radiates 600 mW at 850 MHz? Assume that the observer is in a plane perpendicular to the axis of the dipole.

Solution



- An isotropic antenna that radiates P_{rad} W has a power density of

$$S_{iso} = \frac{P_{rad}}{4\pi r^2}$$

- Since $D(\theta,\phi) = \dfrac{S_{av}(\theta,\phi)}{S_{iso}}$, the power density of the dipole is larger according to the directive gain of the dipole $S_{av}(\theta) = D(\theta)S_{iso}$

$$S_{iso}=\frac{P_{rad}}{4\pi r^2}$$

$$S_{av}(\theta)=D(\theta)S_{iso}$$

Since the observer is on a line perpendicular to the axis of the dipole, we have $\theta=90$ degrees, and we can use the directivity of the dipole, $D_{\max}=D(90)=1.64$

$$S_{av}(90)=D(90)S_{iso}=1.64\frac{P_{rad}}{4\pi r^2}$$

$$S_{av}=\frac{E_\theta^2}{2\eta}$$

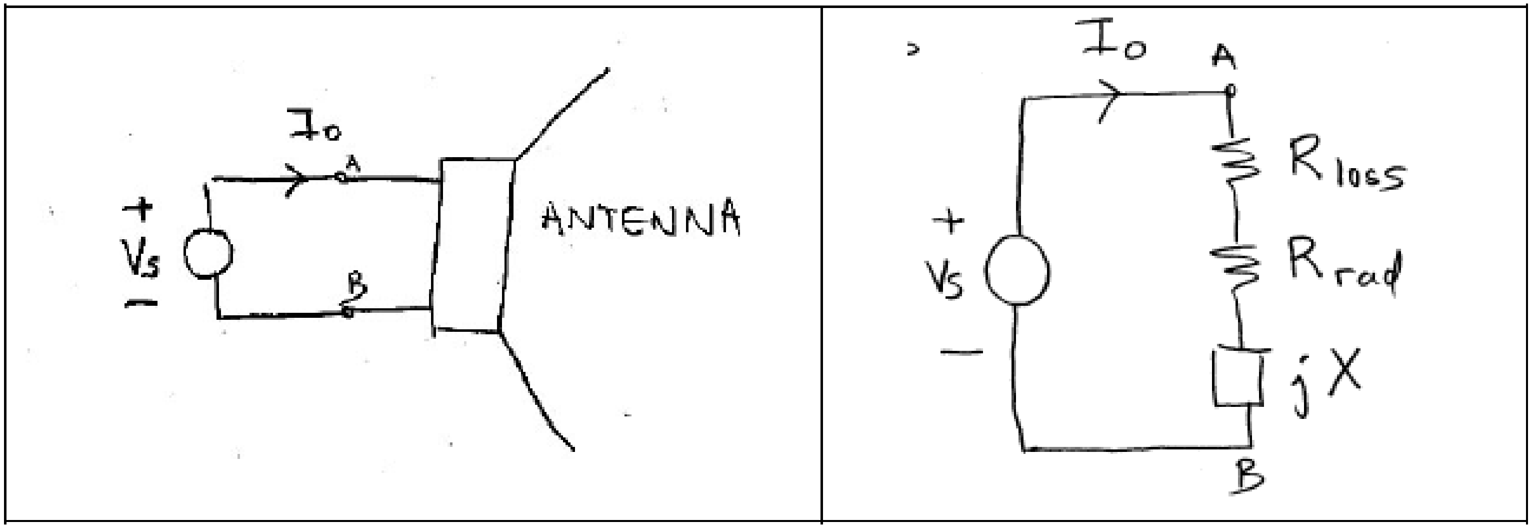
$$E_\theta(r)=\sqrt{2\eta S_{av}}=\sqrt{2\eta\left(1.64\frac{P_{rad}}{4\pi r^2}\right)}$$

$$\eta_0\approx 120\pi$$

$$E_\theta=\sqrt{2\cdot 120\pi\left(1.64\frac{P_{rad}}{4\pi r^2}\right)}=\frac{9.92}{r}\sqrt{P_{rad}}$$

$$E_\theta=\frac{9.92}{100}\sqrt{0.6}=76.8\ \text{mV/m}$$

Radiation Resistance



The "radiation resistance" is defined as the resistor value R_r which dissipates P_{rad} watts with a current of I_0 amps (amplitude):

$$\frac{1}{2}|I_0|^2 R_r = P_{rad}$$

so

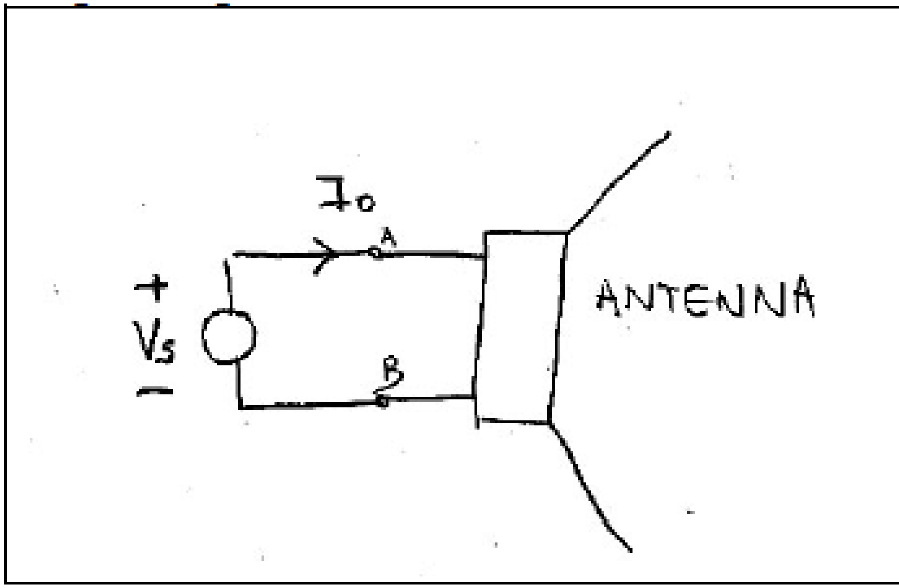
$$R_r = \frac{2P_{rad}}{|I_0|^2}$$

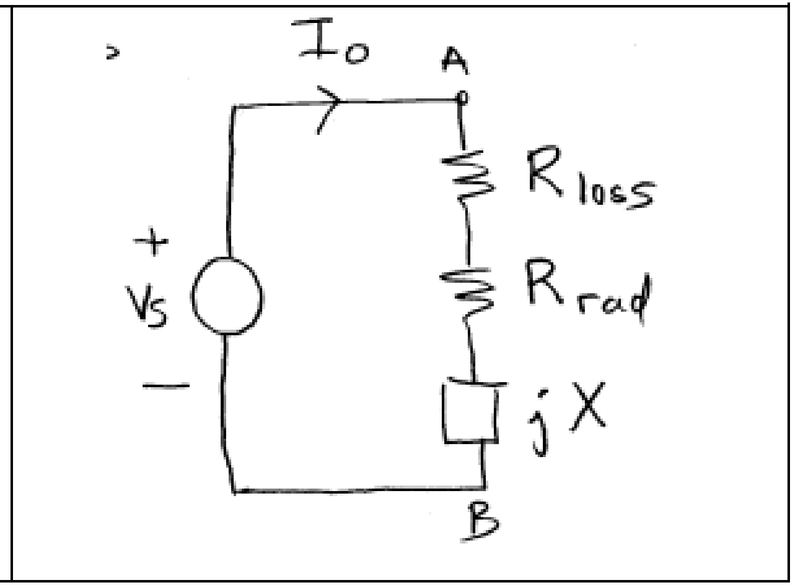
Radiation Resistance of a Half-Wave Dipole

$$P_{rad} = 36.5|I_0|^2$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{2x36.5|I_0|^2}{|I_0|^2} = 73 \text{ ohms}$$

Input Impedance





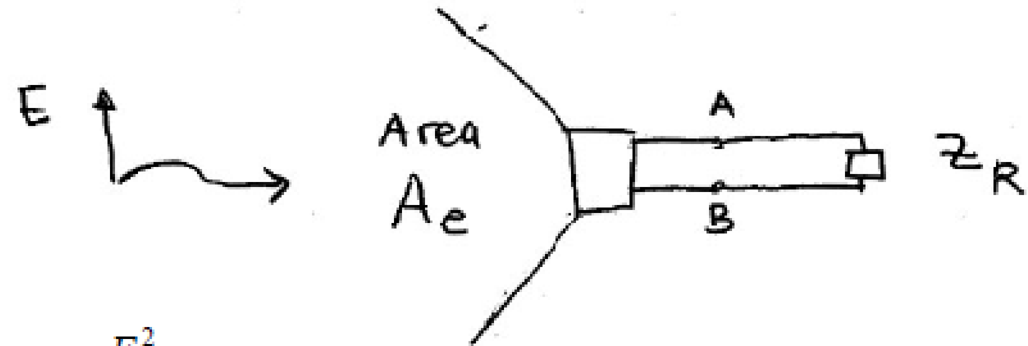
$$Z_{in} = \frac{V_s}{I_0} = (R_{loss} + R_{rad}) + jX$$

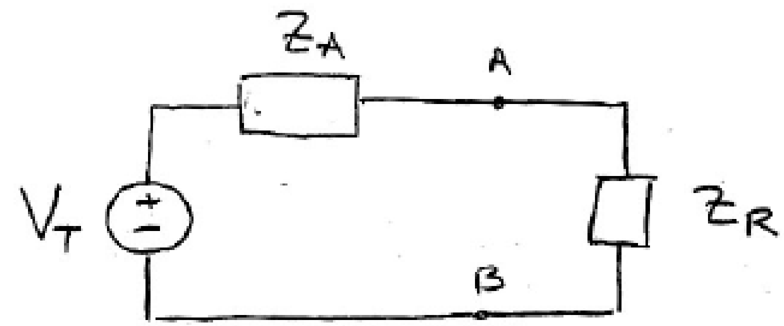
input power

$$P_{in} = \frac{1}{2}\mathrm{Re}\left(V_s I_0^*\right) \text{ watts}$$

The term X is the reactance of the antenna and represents energy stored in the "near field" of the antenna. This energy moves in and out of the antenna during the AC cycle and is not dissipated or radiated away.

Receiving Antennas





$S_{av} = \frac{E^2}{2\eta}$ watts/meter^2.

"Matched" load: $Z_R = Z_A^*$

$$P_{rec} = A_e S_{av}$$

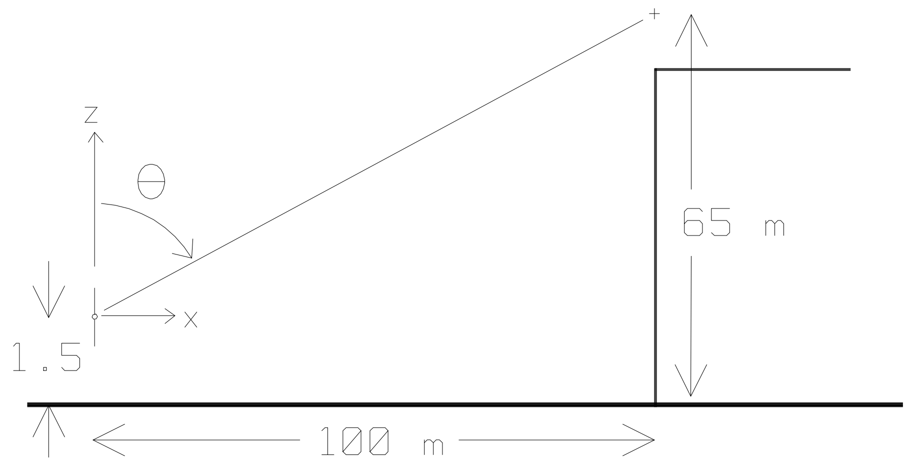
A_e = "effective area" or " aperture" of the antenna.

The effective area is related to the gain in a simple way:

$$A_e = \frac{\lambda^2}{4\pi} G$$

Lossless antennas: gain = directivity so $A_e = \frac{\lambda^2}{4\pi} D$

Directive Gain and Aperture



To account for the directional properties of the antenna, we can use the "directive gain":

$$A_e(\theta,\phi) = \frac{\lambda^2}{4\pi} D(\theta,\phi)$$

Example

What is the maximum effective area of a half-wave dipole antenna operating at 1900 MHz? If the field strength of the incident plane wave is 100 mV/m, what is the power that the dipole receives into a matched load? Assume that the dipole is a lossless antenna.

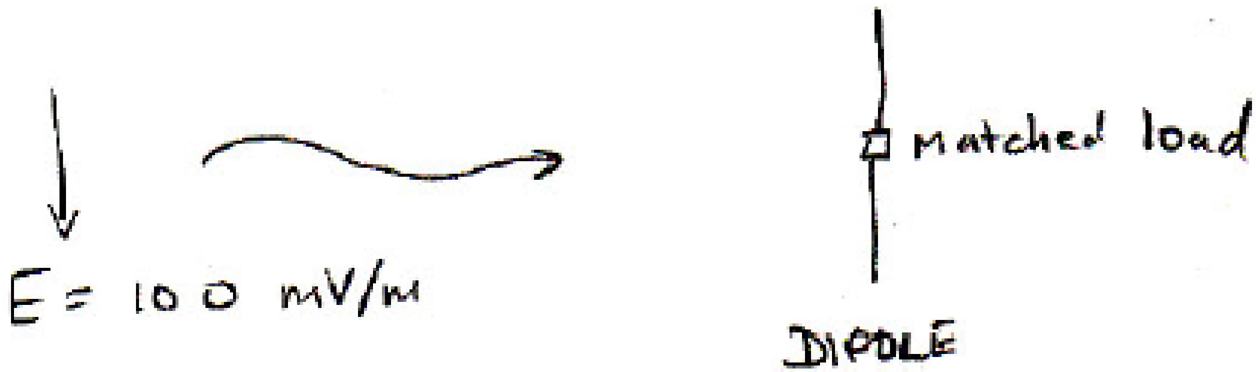
Solution

- The effective area of the dipole is $A_e = \frac{\lambda^2}{4\pi} G$
- The dipole is lossless so the power gain is equal to the directive gain, $G(\theta) = D(\theta)$. ("Lossless" means that the efficiency is unity, $e = 1$.)
- So the effective area is

$$A_e(\theta) = \frac{\lambda^2}{4\pi} D(\theta)$$

- The maximum directive gain of the half-wave dipole is the "directivity" and is $D_{max} = 1.64$
- The wavelength at 1900 MHz is $\lambda = \frac{c}{f} = \frac{300}{1900} = 0.1579$ m
- So the maximum effective area is

$$A_e = \frac{\lambda^2}{4\pi} D_{max} = \frac{(0.1579)^2}{4\pi} x1.64 = 3.25x10^{-3} \text{ square meters}$$



- The power density of a plane wave of field strength $E = 100$ mV/m amplitude is

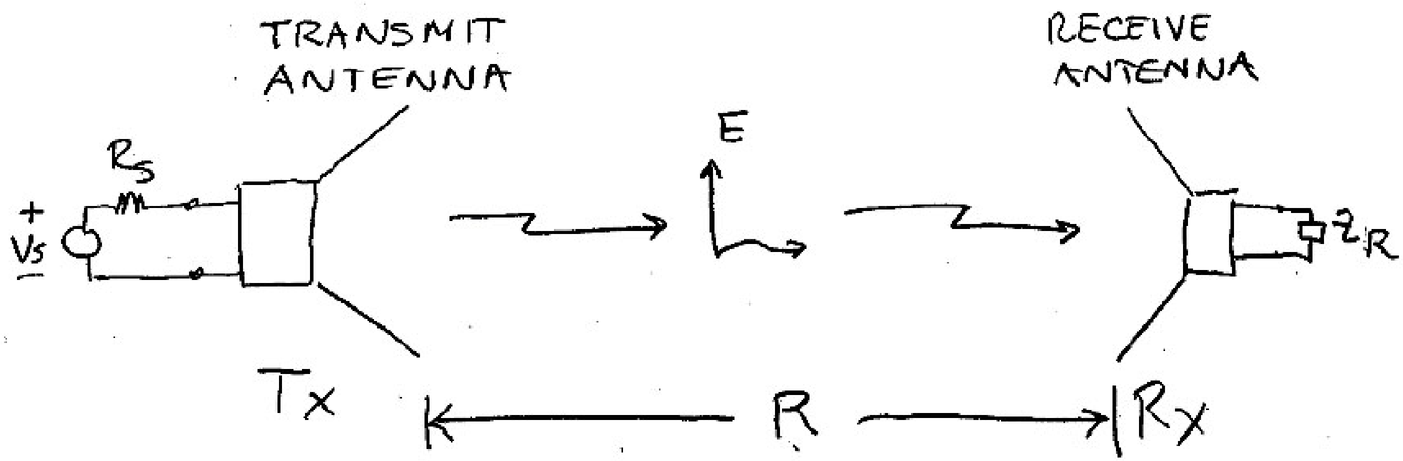
$$S_{av} = \frac{E^2}{2\eta} = \frac{(0.1)^2}{2 \cdot 377} = 13.26 \text{ microwatts per square meter}$$

where the approximation $\eta \approx 377$ ohms has been used.

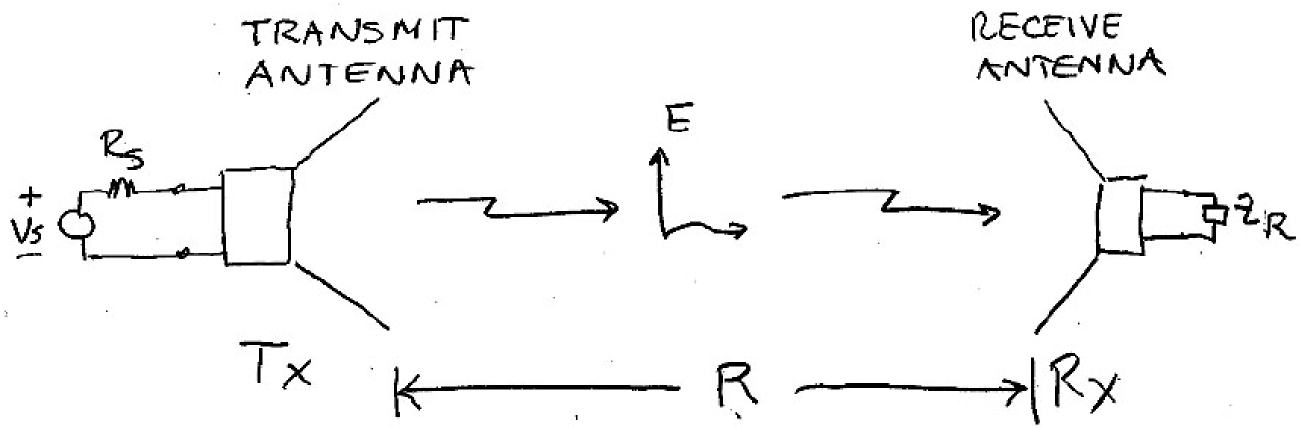
- The power received into a matched load is

$$P_{rec} = A_e S_{av} = \left(3.25x10^{-3}\right)\left(13.26x10^{-6}\right) = 43.1 \text{ nanoWatts}$$

The Friis Transmission Equation



- A transmit antenna (Tx) has input power P_{in} and gain G_T .
- It is located at a distance R from a receiving antenna.
- The receiving antenna (Rx) has gain G_R and is terminated in a matched load.
- Find the received power P_R into the matched load.



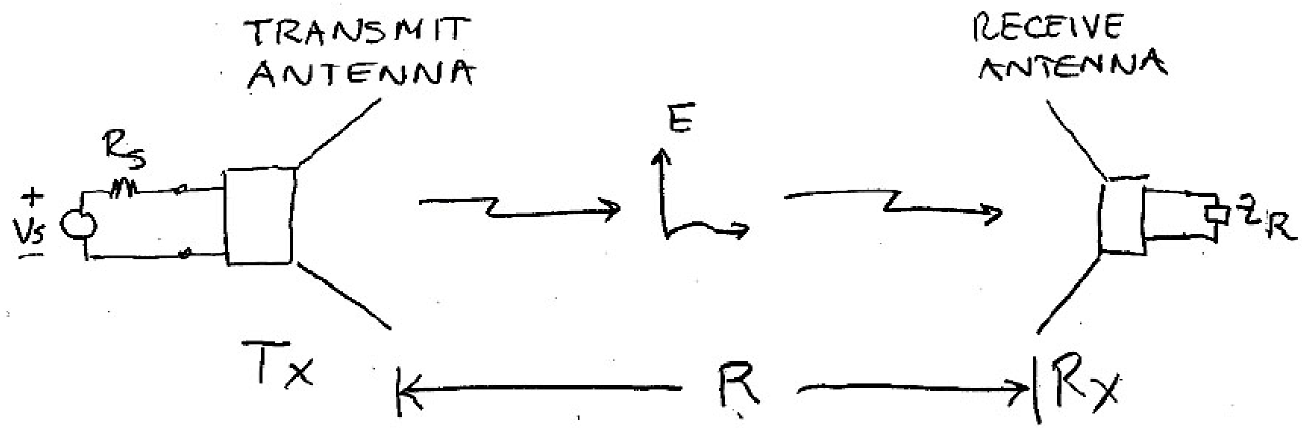
- The radiated power is $P_{rad} = eP_{in}$ where e is the efficiency of the antenna.
- The isotropic power density at a distance R from the transmitter is

$$S_{iso} = \frac{P_{rad}}{4\pi R^2} = \frac{eP_{in}}{4\pi R^2}$$

- The actual power density S_{av} is the directivity times the isotropic power density,

$$S_{av} = D_T S_{iso} = D_T \frac{eP_{in}}{4\pi R^2} = eD_T \frac{P_{in}}{4\pi R^2}$$

where D_T is the directivity of the transmit antenna.



$$S_{av} = eD_T \frac{P_{in}}{4\pi R^2}$$

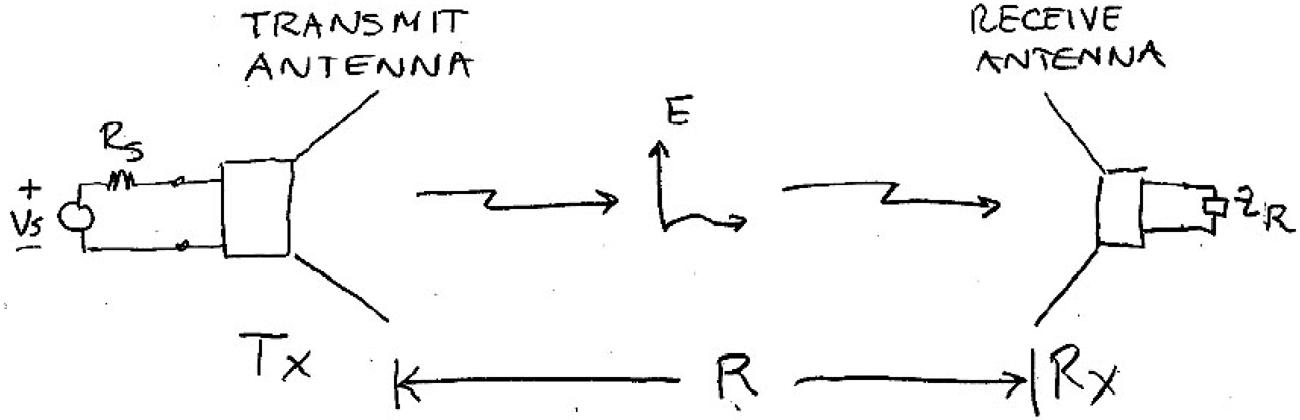
- Recognize eD_T as the gain of the antenna $G_T = eD_T$ so we can write that the power density at the location of the receive antenna is

$$S_{av} = G_T \frac{P_{in}}{4\pi R^2}$$

- The receive antenna is terminated in a matched load so the power delivered to the matched load is

$$P_R = A_e S_{av}$$

where A_e is the "effective area" of the receive antenna.



$$S_{av} = G_T \frac{P_{in}}{4\pi R^2}$$

$$P_R = A_e S_{av}$$

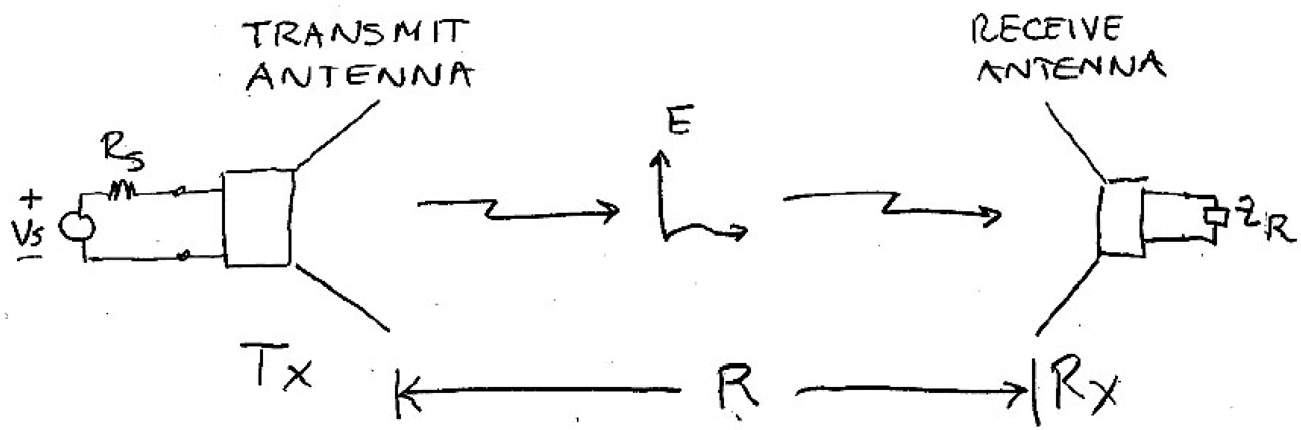
The effective area is related to the gain by

$$A_e = \frac{\lambda^2}{4\pi} G_R$$

so the received power is

$$P_R = \frac{\lambda^2}{4\pi} G_R S_{av}$$

$$P_R = \frac{\lambda^2}{4\pi} G_R \, G_T \frac{P_{in}}{4\pi R^2}$$



$$P_R = \frac{\lambda^2}{4\pi} G_R \, G_T \, \frac{P_{in}}{4\pi R^2}$$

- Neaten this equation by writing it as

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in} \quad \text{watts}$$

- This is called the "**Friis Transmission Equation**".
- It is the "design equation" used for wireless links.
- The factor $\left(\frac{\lambda}{4\pi R}\right)^2$ is sometimes called the "spreading factor" or "spatial loss factor".

$$L_d = \left(\frac{\lambda}{4\pi R}\right)^2$$

Exam Question on Antennas

A 20 Watt transmitter operates at a frequency of 147.06 MHz. It is connected to an antenna that has 4 dB of gain. A receiving antenna is 40 km away and has 2 dB of gain.

(1) The received power is

(a) $2.08x10^{-9}$ W
(b) $1.31x10^{-9}$ W
(c) $4.04x10^{-9}$ W
(d) $9.42x10^{-10}$ W
(e) none of these

(2) The aperture of the receive antenna is

(a) 0. 402 m^2
(b) 0. 118 m^2
(c) 0. 589 m^2
(d) 0. 527 m^2
(e) none of these

(3) The incident power density at the receive antenna is

(a) $1.11x10^{-9}$ W/ m^2
(b) $3.55x10^{-9}$ W/ m^2
(c) $6.10x10^{-9}$ W/ m^2
(d) $2.49x10^{-9}$ W/ m^2
(e) none of these

(4) The RMS electric field strength at the receive antenna is

(a) 552 $\mu V/m$
(b) 968 $\mu V/m$
(c) 777 $\mu V/m$
(d) 440 $\mu V/m$
(e) none of these

Solution

A 20 Watt transmitter operates at a frequency of 147.06 MHz. It is connected to an antenna that has 4 dB of gain. A receiving antenna is 40 km away and has 2 dB of gain.

$$P_{in} = 20 \text{ watts}$$
$$f = 147.06 \text{ MHz}$$
$$G_{T,dB} = 4 \text{ dB}$$
$$R = 40 \text{ km}$$
$$G_{R,dB} = 2 \text{ dB}$$

(1) The received power is?

Evaluate the Friis Transmission Equation

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

where

$$f = 147.06 \text{ MHz so } \lambda = \frac{c}{f} = \frac{3x10^8}{147.06x10^6} = 2.04 \text{ m}$$

Recall that gain in dB is 10 log (gain on a linear scale) so

$$G_{T,dB} = 4 \text{ dB so on a linear scale } G_T = 10^{4/10} = 2.51$$

$$G_{R,dB} = 2 \text{ dB so on a linear scale } G_R = 10^{2/10} = 1.58$$

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in}$$

$\lambda = 2.04$ m

$G_T = 2.51$

$G_R = 1.58$

$R = 40$ km

Spatial loss or "spreading factor":

$$L = \left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{2.04}{4\pi x 40{,}000}\right)^2 = 1.65x10^{-11}$$

Friis Transmission Equation

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in} = 1.65x10^{-11} x 2.51 x 1.58 x 20 = 1.31x10^{-9} \text{ watts}$$

(a) $2.08x10^{-9}$ W

(b) $1.31x10^{-9}$ W >>> correct answer

(c) $4.04x10^{-9}$ W

(d) $9.42x10^{-10}$ W

(e) none of these

(2) The aperture of the receive antenna is?

$$A_e = \frac{\lambda^2}{4\pi} G$$

$$A_e = \frac{\lambda^2}{4\pi} G_R = \frac{2.04^2}{4\pi} x1.58 = 0.523m^2$$

(a) 0. 402 m^2
(b) 0. 118 m^2
(c) 0. 589 m^2
(d) 0. 527 m^2 >>> correct answer (within 3%)
(e) none of these

(3) The incident power density at the receive antenna is?

The power density is the Poynting Vector and is related to the gain of the transmitter by

$$S_{iso} = \frac{P_{rad}}{4\pi R^2} \qquad \text{Lossless antenna:} \quad P_{rad} = P_{in}$$

$$D_T = \frac{S_{av}}{S_{iso}} \qquad S_{av} = D_T S_{iso}$$

and for a lossless antenna, the gain is equal to the directivity so $D_T = G_T$.

$$S_{av} = G_T S_{iso} = G_T \frac{P_{in}}{4\pi R^2} = 2.51 \frac{20}{4\pi(40,000)^2} = 2.50x10^{-9} \quad \text{W/m}^2$$

(a) $1.11x10^{-9}$ W/ m^2

(b) $3.55x10^{-9}$ W/ m^2

(c) $6.10x10^{-9}$ W/ m^2

(d) $2.49x10^{-9}$ W/ m^2 >>> Correct answer

(e) none of these

(4) The RMS electric field at the antenna is?

The incoming wave behaves as a plane wave so

$$S_{av} = \frac{E_{rms}^2}{\eta}$$

$$E_{rms} = \sqrt{\eta S_{av}} = \sqrt{377x2.50x10^{-9}} = 9.71x10^{-4} \text{ V/m} = 971x10^{-6} \text{ V/m} = 971 \ \mu V/\text{m}$$

(a) 552 $\mu V/\text{m}$

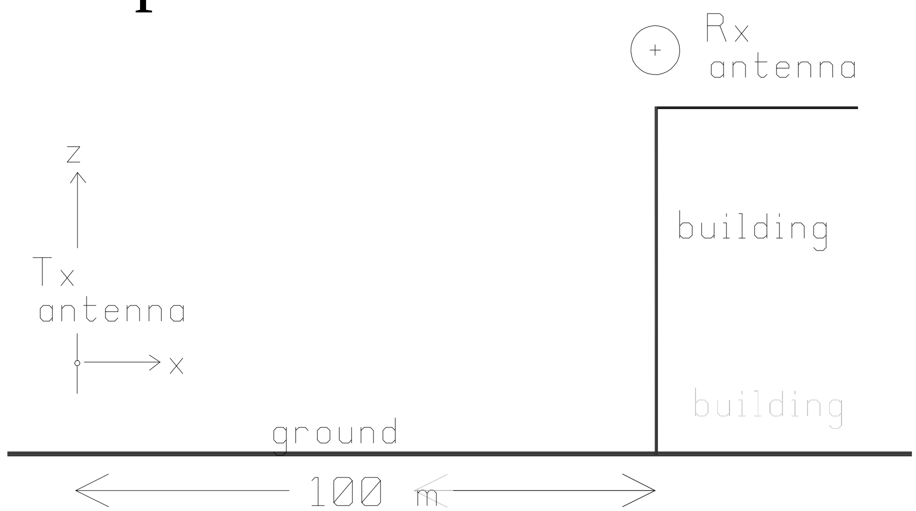
(b) 968 $\mu V/\text{m}$ >>> correct answer (close enough)

(c) 777 $\mu V/\text{m}$

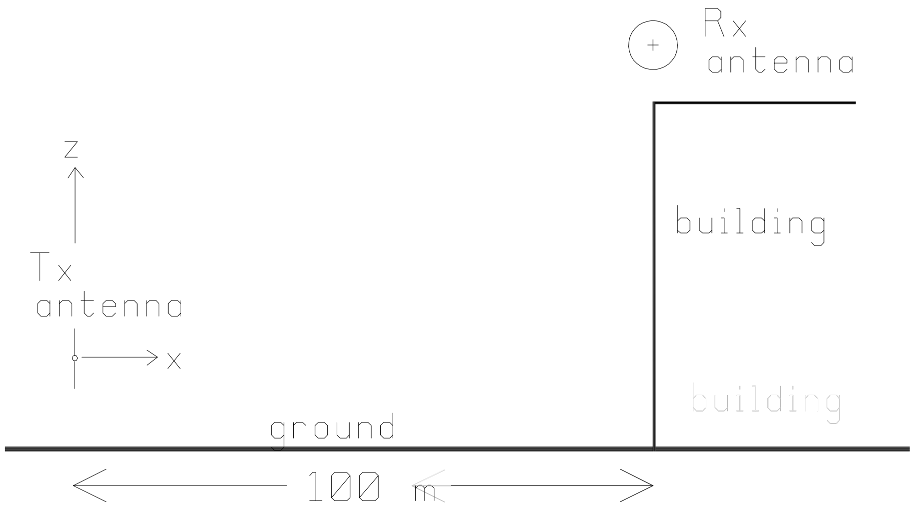
(d) 440 $\mu V/\text{m}$

(e) none of these

Cell Phone Example



A cell phone user at Tx communicates with a base station antenna at Rx on top of a nearby building. The transmit antenna at Tx behaves as a lossless, half-wave dipole antenna oriented vertically, or in the z direction. The center of the antenna is 1.5 m above the ground and the antenna is 100 m from the base of the building. The Rx antenna is omnidirectional and is lossless. The center of the antenna lies in the plane of the face of the building at an elevation of 65 m above the ground. The operating frequency is 1.9 GHz. The transmitted power is 125 mW.



Half-wave dipole formulas:

$$E_\theta(\theta) = \frac{jI_0\eta_0}{2\pi} F(\theta) \frac{e^{-j\beta r}}{r}$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$P_{rad} = 36.5 I_0^2$$

1)What is the value of angle θ for the path from the transmitter to the receiver?

2)What is the value of $F(\theta)$ for the path from the transmitter to the receiver?

3) What is the current flowing on the dipole antenna?

4)What is the electric field strength of the dipole antenna at the location of the receiver?

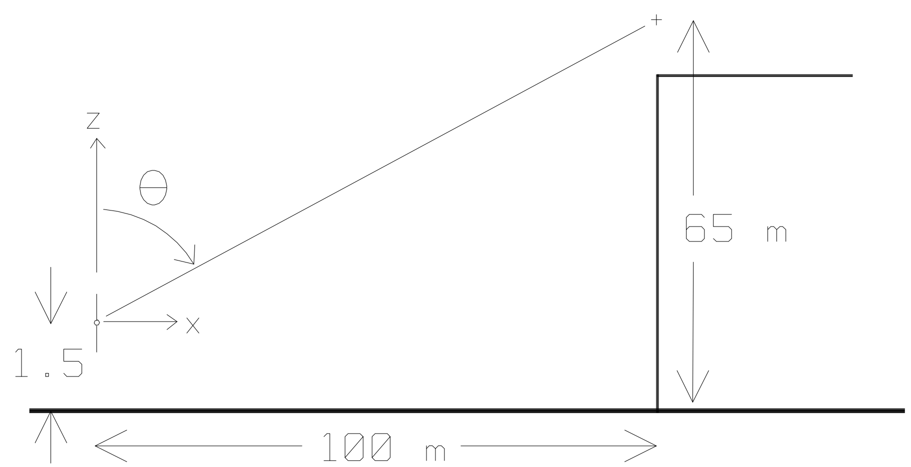
5)What is the power density due to the transmitter at the location of the receive antenna?

6)What is the effective area of the receive antenna?

7)What is the power received by the Rx antenna into a matched load?

Solution

1.What is the value of angle θ for the path from the transmitter to the receiver?



$$\tan\theta = \frac{100}{(65-1.5)}$$

$$\theta \quad = 56.976 \text{ degrees}$$

$$r = \sqrt{100^2 + (65-1.5)^2} = 118.46$$

2.What is the value of $F(\theta)$ for the path from the transmitter to the receiver?

$$F(\theta)=\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

θ =56.976 degrees

$$F(\theta)=\frac{\cos(90x\cos 56.976)}{\sin 56.976}$$ =0.7817

3.What is the current flowing on the dipole antenna?

$$P_{rad} = 36.5 I_0^2$$

$$P_{rad} = 125 \quad \text{mW}$$

$$I_0 = \sqrt{\frac{P_{rad}}{36.5}} = \sqrt{\frac{0.125}{36.5}} = 0.05852 \quad \text{amps}$$

4.What is the electric field strength of the dipole antenna at the location of the receiver?

$$E_\theta(\theta) = \frac{jI_0\eta_0}{2\pi} F(\theta)\frac{e^{-j\beta r}}{r}$$

$$|E_\theta| = \frac{I_0\eta_0}{2\pi r} F(\theta)$$

$F(\theta)$ =0.7817

$I_0 = 0.05852$ amps

$r = \sqrt{100^2 + (65-1.5)^2} = 118.46$ m

$$|E_\theta| = \frac{I_0\eta_0}{2\pi r} F(\theta) = \frac{0.05852 x 376.73}{2 x \pi x 118.46} * 0.7817 = 0.02315 \quad \text{V/m}$$

5.What is the power density due to the transmitter at the location of the receive antenna?

$$S_{av} = \frac{E^2}{2\eta} = \frac{0.02961^2}{2x376.73} = 1.163x10^{-6} = 1.163$$ Microwatts per square meter

6.What is the effective area of the receive antenna?.

$$A_e = \frac{\lambda^2}{4\pi} G$$

$$\lambda = \frac{c}{f} = 300/1900 = 0.15789$$ m

The gain of an “isotropic” antenna is unity.

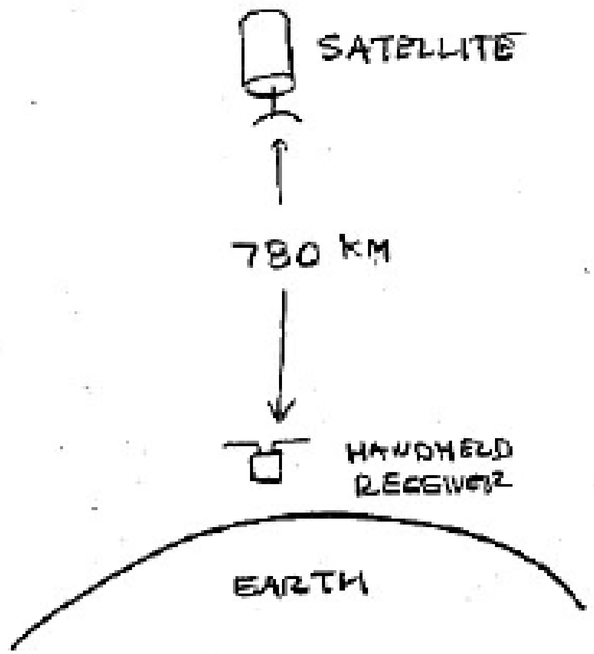
$$A_e = \frac{0.15789^2}{4\pi} x1 = 1.984x10^{-3}$$ square meters

7.What is the power received by the Rx antenna into a matched load?

.

$$P_r = S_{av} A_e = 0.7116x10^{-6} x1.984x10^{-3} = 1.4097x10^{-9}$$ watts

Example – Satellite Link



A satellite in low-earth orbit (LEO) must communicate with a handheld "satellite telephone". The satellite is 780 km above the ground. The operating frequency is 1.65 GHz. The satellite transmits 14 watts of power using an antenna with a gain of 6 dB. The handheld receiver uses a half-wave dipole antenna to receive the signal. What is the received power into a matched load?

Solution

The gain of the transmitter is 6 dB so $6 = 10\log G_T$ and $G_T = 3.98$. The gain of the receiver is $G_R = 1.64$ for a half-wave dipole. The spatial loss factor for a distance of $r = 780$ km at 1.65 GHz is

$$L_d = \left(\frac{\lambda}{4\pi r}\right)^2 = 3.44x10^{-16}$$

The Friis Transmission Equation reads

$$P_R = \left(\frac{\lambda}{4\pi R}\right)^2 G_R G_T P_{in} = L_d G_R G_T P_{in}$$

so we can calculate the received power as

$$P_R = (3.44x10^{-16})(1.65)(3.98)(14) = \;\; 3.16x10^{-14} \;\; \text{watts}$$

Remarks:

- To "design" the satellite link we need to know the minimum *signal-to-noise ratio* that must be maintained at the receiver.

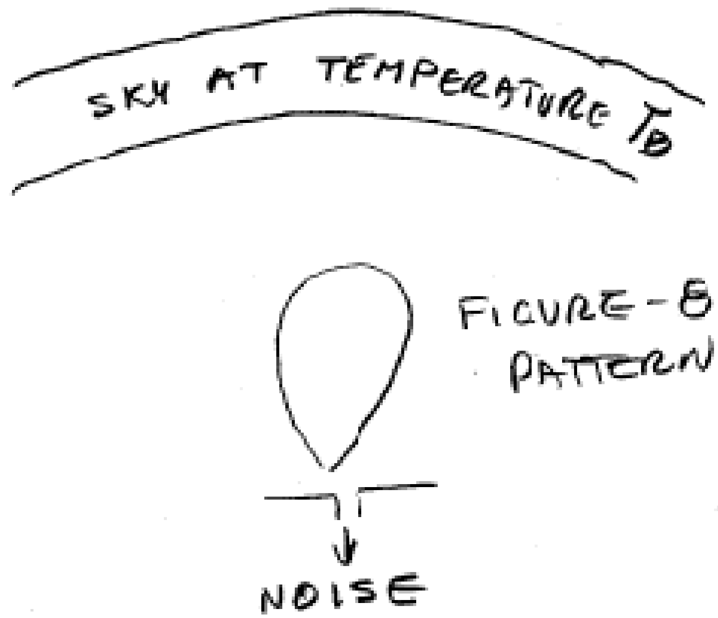
The noise power is given by

$$N = kT_r B$$

where B Hz is the channel bandwidth

k is Boltzmann's Constant =1.38x10^{-23} Joules per Kelvin degree

T_r is the "brightness temperature" of the receive antenna in degrees



$$T_b = \frac{\iint_{4\pi} T_B(\theta,\phi) D(\theta,\phi) \sin\theta d\theta d\phi}{\iint_{4\pi} D(\theta,\phi) \sin\theta d\theta d\phi}$$

Sky: $T_B = 0$ K

Ground: $T_B = 273$ K