

ELEC353 Lecture Notes Set 2

The homework assignments are posted on the course web site.

Homework #1: Do this assignment by January 14th.

Homework #2: Do this assignment by January 21th.

The course web site is:

www.ece.concordia.ca/~trueman/web_page_353.htm

- The course outline
- The lecture notes
- The homework assignment each week
- A set of practice problems with solutions
- Software: BOUNCE, TRLINE, WAVES

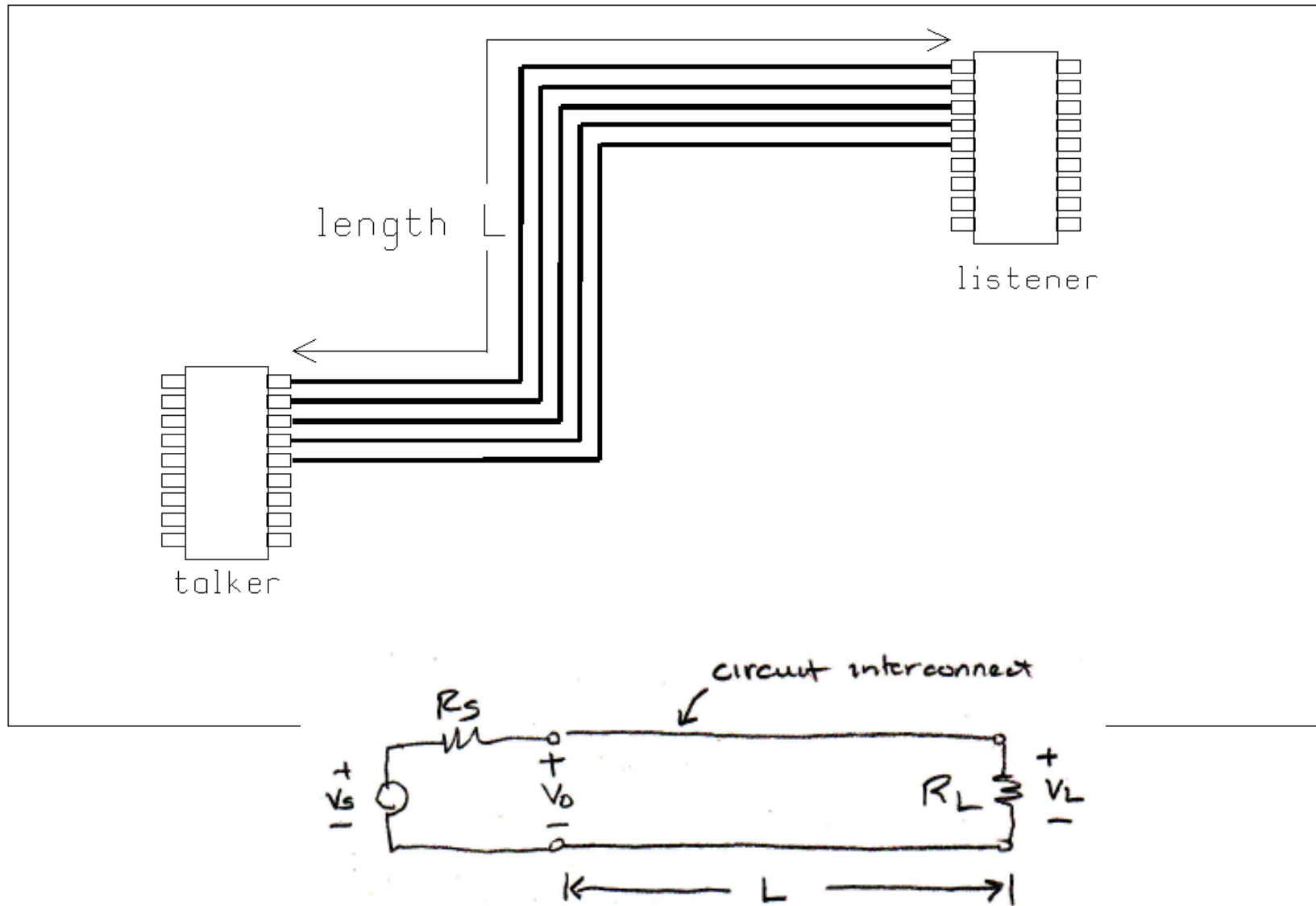
Reading Assignment

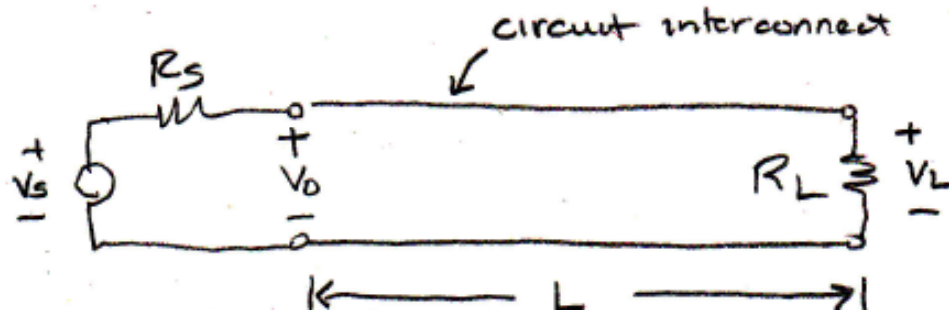
- Read Inan and Inan Chapter 1 – when is “distributed” circuit analysis needed?

All about Transmission Lines

- Read Inan and Inan Chapter 2

Equivalent Circuit for an Interconnection





Example:

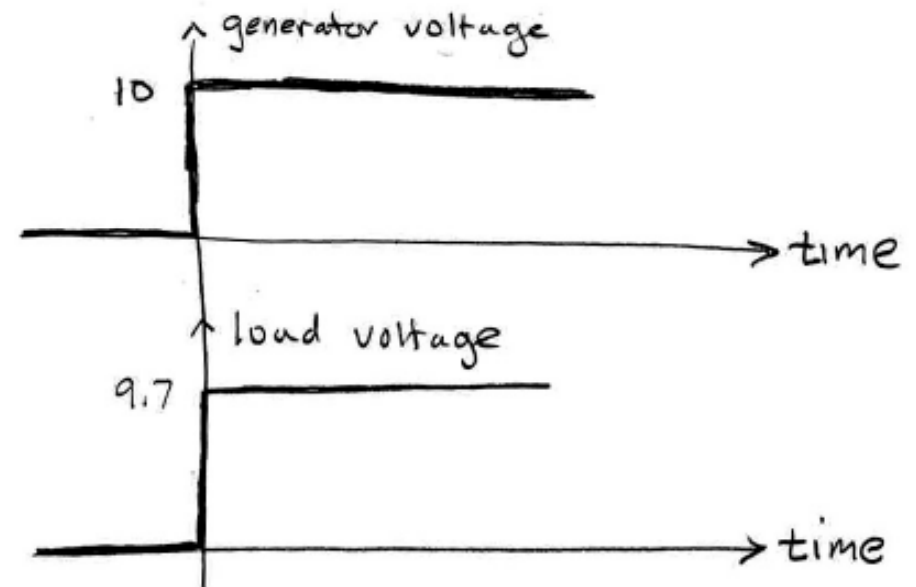
Suppose $V_s = 10$ volts, and is a step function generator. Suppose $R_s = 30$ ohms and $R_L = 1000$ ohms.

If the circuit board interconnection behaves as an **ideal short circuit**, then what is the output voltage?

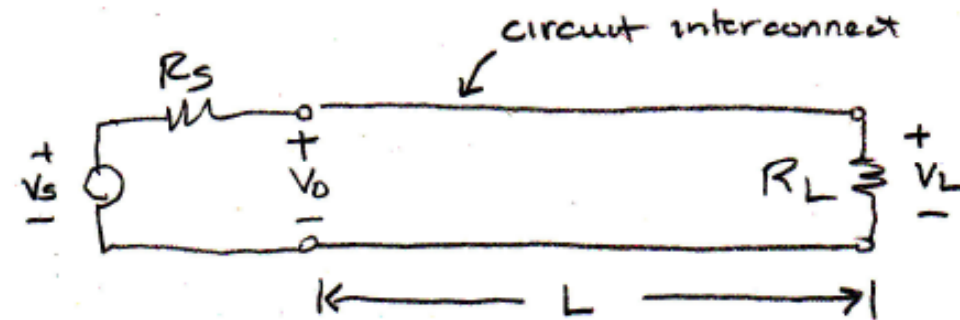
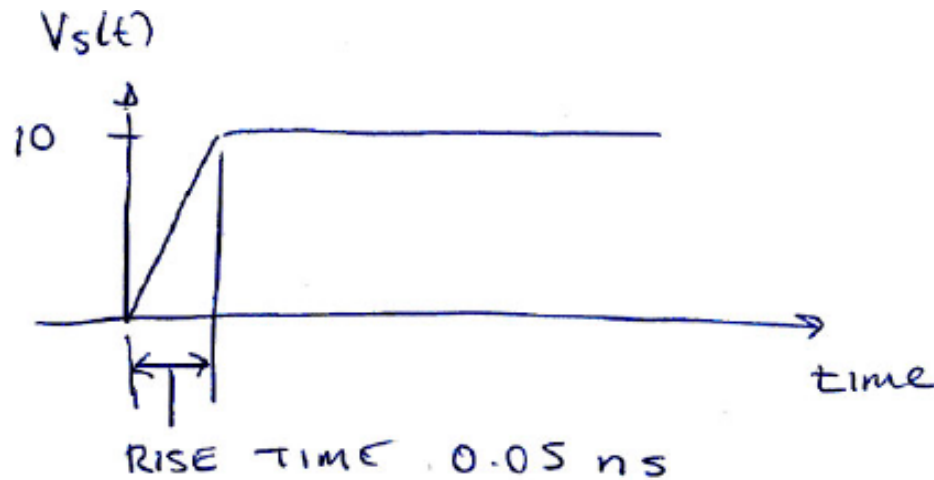
- What is a short circuit?
 - A short-circuit path behaves as a resistor with zero resistance.
 - A short-circuit path has zero TIME DELAY to go from the input across the distance L to the output.
 - This is also called an "ideal" short circuit.

Then the output voltage is

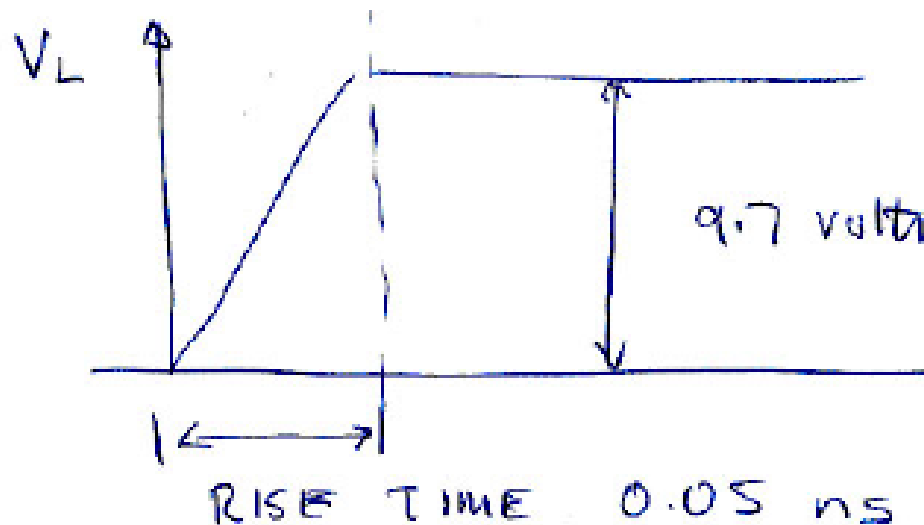
$$V_1 = \frac{10 \times 1000}{1000 + 30} = 9.7 \text{ volts}$$



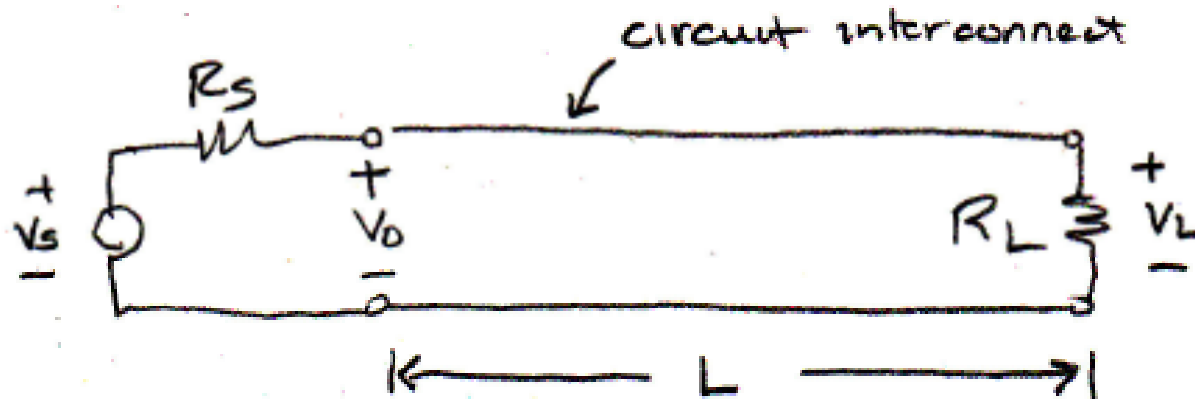
Accounting for RISE TIME



- Suppose the clock speed is 2 GHz so the period is 0.5 ns.
- Suppose the rise time is 1/10 of a period, or 0.05 ns.
- If the interconnection path in the circuit board behaves as an *ideal short circuit*, then the output is an exact copy of the input, except that the amplitude is 9.7 volts instead of 10 volts:



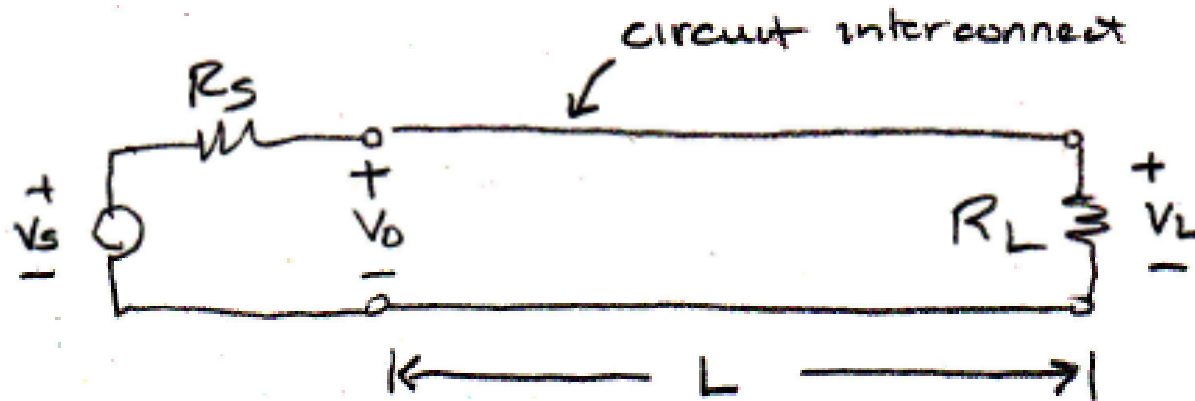
Accounting for TIME DELAY



- Suppose the speed of travel on the interconnection path is $u = 20$ cm/ns:
 - This is a typical figure for real circuit boards
 - The speed-of-light in space is $c = 30$ cm/ns.
- We will see later that we need to know the “characteristic resistance” of the interconnection path, so assume it is $R_c = 50$ ohms.

Three cases:

- Short interconnection path
- Intermediate length
- Long interconnection path



Case 1: Short interconnection path of $L = 1 \text{ mm} = 0.1 \text{ cm}$.

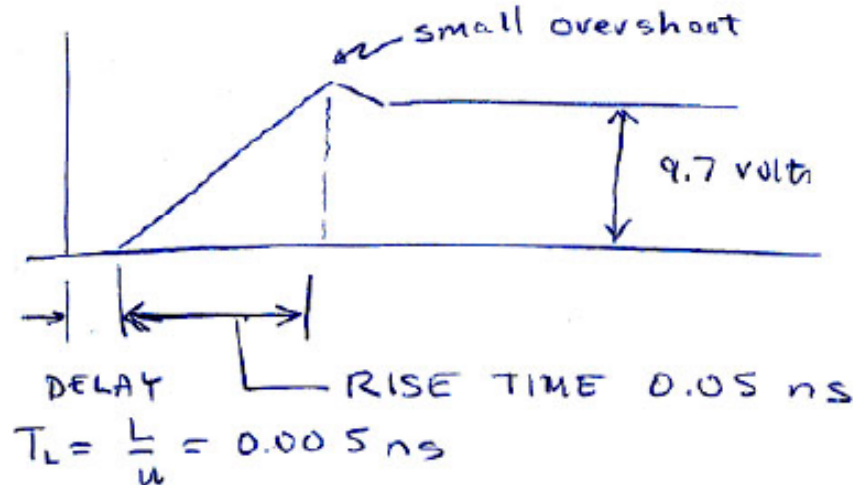
- Then the time delay is $T_d = \frac{L}{u} = \frac{0.1}{20} = 0.005 \text{ ns}$, which is much less than the rise time of 0.05 ns .
- The ratio of the rise time to the time delay is $\frac{T_r}{T_d} = 10$ and is **LARGE** because the time delay is **SHORT**.

The "length of the leading edge" or "rise length" is defined as $L_r = uT_r$

The rise length is $L_r = uT_r = 20 \times 0.05 = 1 \text{ cm}$ so $L_r \gg L$

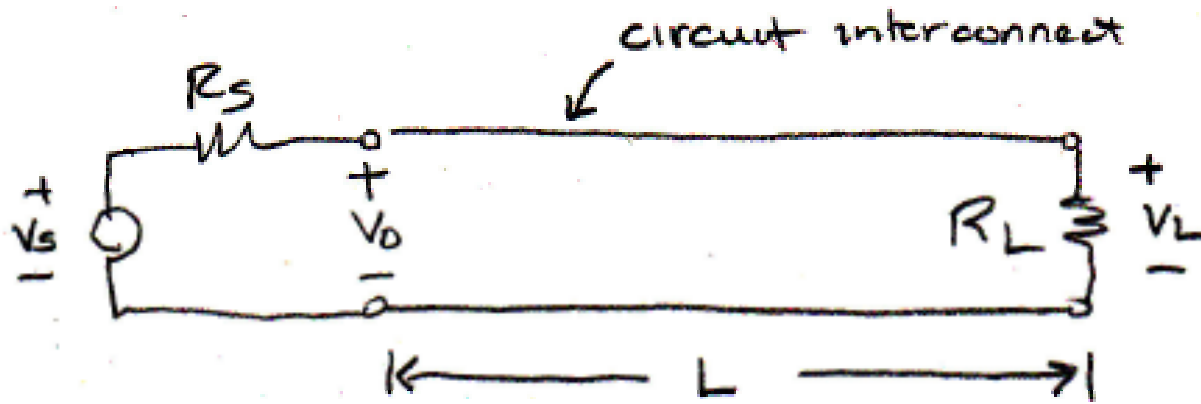
Rise length \gg path length

SHORT LINE $L = 1 \text{ mm}$:



The output is *almost* a copy of the input, with some small differences:

- there is an initial time delay of 0.005 ns, which is the time taken to propagate along the circuit path.
 - there is a small overshoot before the voltage settles down to 9.7 volts.
-
- The rise time is much much longer than the delay time.
 - The “rise length” (1 cm) is much much longer than the length of the circuit path (0.1 cm).
 - Use “lumped” circuit analysis.



Case 2: Intermediate-length interconnection path of $L=1$ cm.

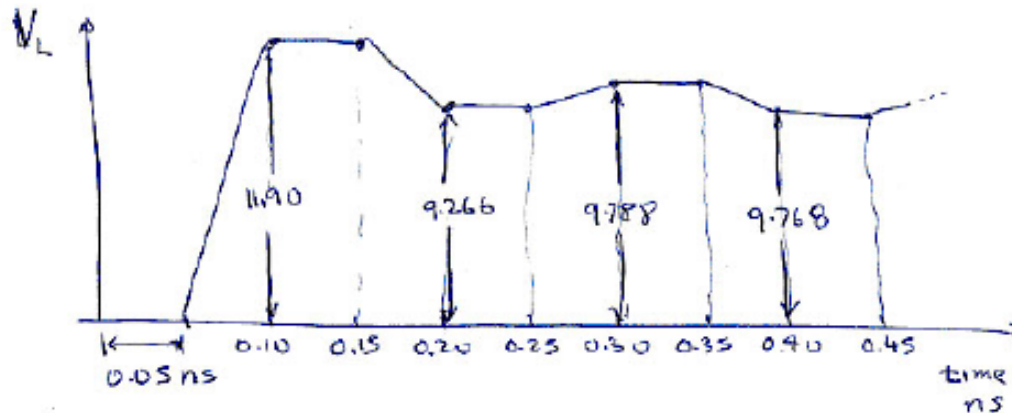
- Then the time delay is $T_d = \frac{L}{u} = \frac{1}{20} = 0.05$ ns, which is equal to the rise time of 0.05 ns.
- The ratio of the rise time to the delay time is $\frac{T_r}{T_d} = 1$, hence the time delay is EQUAL to the rise time.

Remark:

1. The RISE TIME of $T_r=0.05$ ns is equal to the DELAY TIME of $T_d = 0.05$ ns.
2. The RISE TIME of $T_r=0.05$ ns represents a distance of $L_r = uT_r = 1$ cm, equal to the 1 cm length of the interconnection path.

The rise length is $L_r = uT_r = 20 \times 0.05 = 1$ cm so $L_r = L$

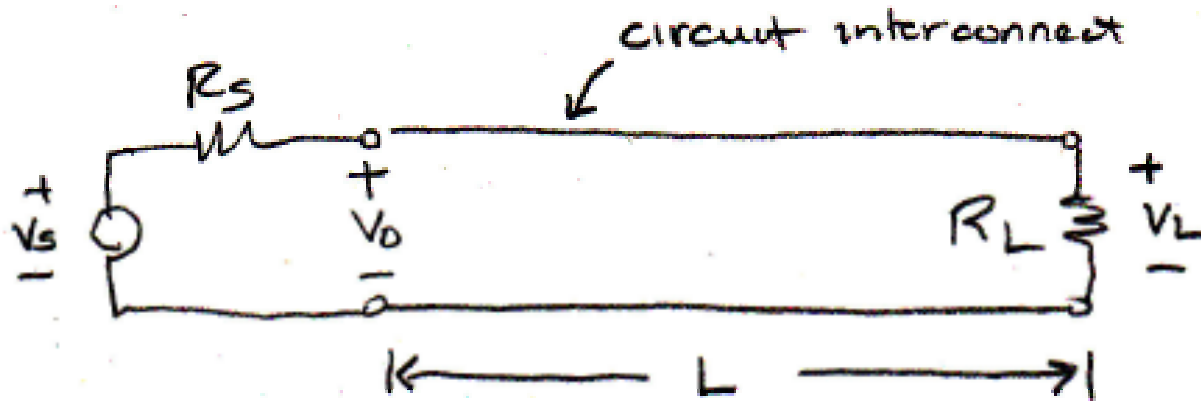
Rise length comparable to path length



The output is **NOT** a copy of the input. It is much more complex

Why is the “output” waveform so different from the generator waveform?

- Answer: the generator’s step-function voltage behaves as a “traveling wave” which bounces back and forth on the “circuit interconnect” path between the generator and the load.
- The circuit is quite easy to analyze and we will learn how in the following lectures.
- The rise time is equal to the delay time.
- The “rise length” (1 cm) is equal to the length of the circuit path (1 cm).
- Cannot use “lumped” circuit analysis!

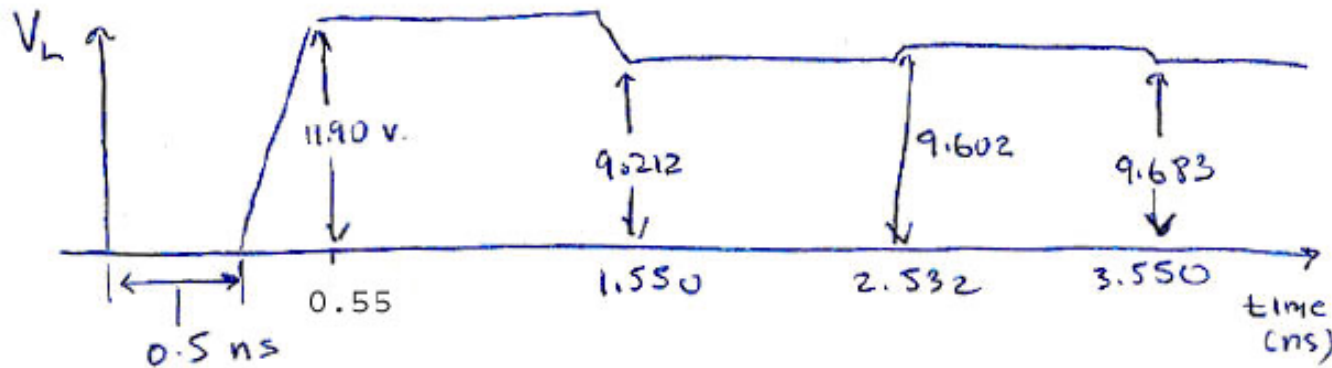


Case 3: Long interconnection path: $L = 10$ cm.

- The delay time is $T_d = \frac{L}{u} = 0.5$ ns, and is much longer than the rise time of 0.05 ns.

The rise length is $L_r = uT_r = 20 \times 0.05 = 1$ cm so $L_r \ll L$

Rise length \ll path length



- The rise time is much much shorter than the delay time.
- The "rise length" (1 cm) is much much shorter than the length of the circuit path (10 cm).

RULE OF THUMB: (Inan and Inan page 7)

- If the rise time is more than 6 times the delay time, then use LUMPED circuit analysis.
- If the rise time is less than 2.5 times the delay time, then use DISTRIBUTED circuit analysis.

If $\frac{T_r}{T_d} > 6$, use lumped circuit analysis. (Short time delay case.)

If $\frac{T_r}{T_d} < 2.5$, use distributed circuit analysis. (Long time delay case.)

We usually know:

- (i) the rise time T_r for our logic family;
- (ii) the length of the interconnection, L ; and
- (iii) the speed of propagation, u .

So the **rule-of-thumb** is often stated in terms of the “length of the leading edge”, or “rise length”, $L_r = uT_r$:

$L_r > 6L$ Rise length much longer than line length:
Use lumped circuit analysis.

$L_r < 2.5L$ Rise length shorter than line length:
Use distributed circuit analysis.

Types of Transmission Lines

Inan and Inan Chapter 2

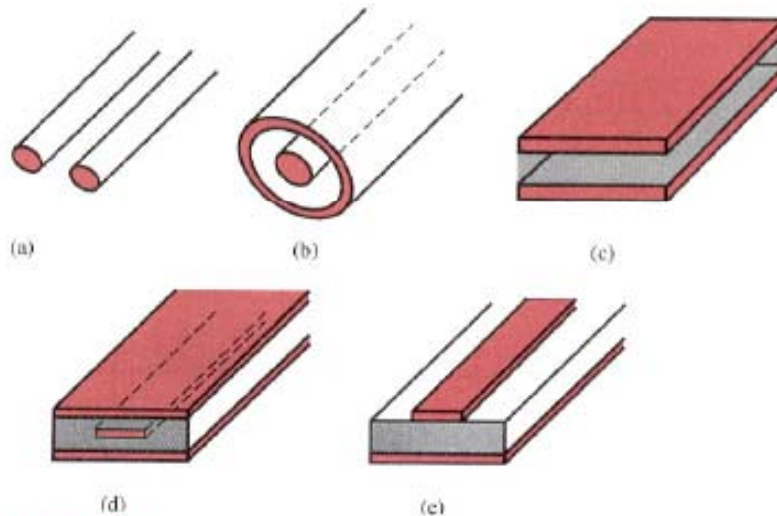


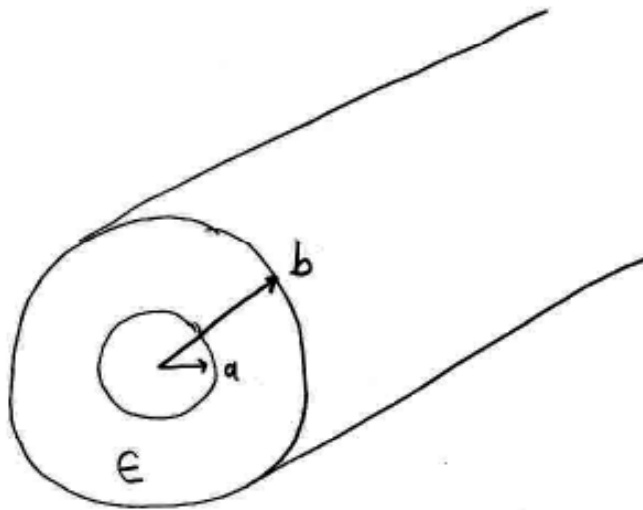
FIGURE 2.1. Different types of uniform transmission lines: (a) parallel two-wire; (b) coaxial; (c) parallel-plate; (d) stripline; (e) microstrip.

Inan and Inan Fig. 2.1

Transmission Line Properties

- Every transmission line has:
 - inductance-per-unit-length ℓ Henries/meter or H/m
 - capacitance-per-unit-length c Farads/meter or F/m
- Often ℓ and c are found by measurement in the lab.
- We can find formulas for ℓ and c for various transmission-line geometries in textbooks and handbooks-see Inan and Inan Section 2.7.

Coaxial Cable



- The inner conductor is a cylindrical metal wire of radius a
- The outer conductor is a hollow metal cylinder of inner radius b
- The outer conductor is usually made of braided wire so that the cable will be flexible
- The conductors are separated by an insulator or “dielectric” of permittivity ϵ
- Dielectrics are usually characterized by the “relative” permittivity, defined as $\epsilon_r = \frac{\epsilon}{\epsilon_0}$, where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m is the permittivity of empty space.
- A typical dielectric material is polyethylene, which has $\epsilon_r = 2.26$

Transmission line properties:

The capacitance-per-unit-length is

$$c = \frac{2\pi\epsilon}{\ln(b/a)} \text{ F/m}$$

The inductance per unit length is

$$\ell = \frac{\mu}{2\pi} \ln(b/a) \text{ H/m}$$

Hence, the speed-of-propagation is

$$u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

and the characteristic resistance is

$$R_c = \sqrt{\frac{\ell}{c}} = \frac{\ln(b/a)}{2\pi} \sqrt{\frac{\mu}{\epsilon}}$$

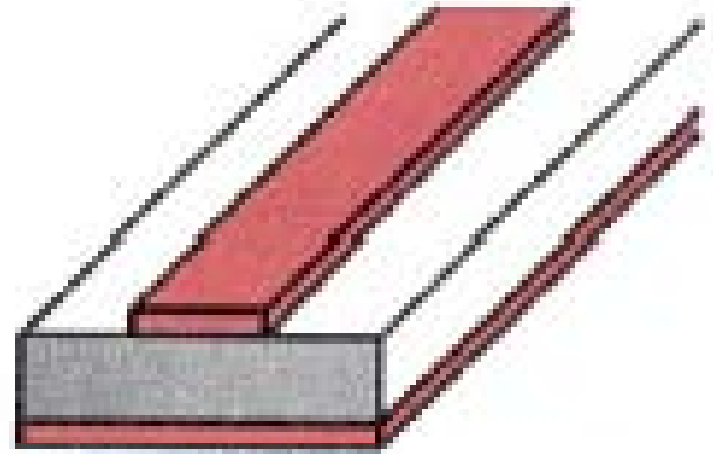
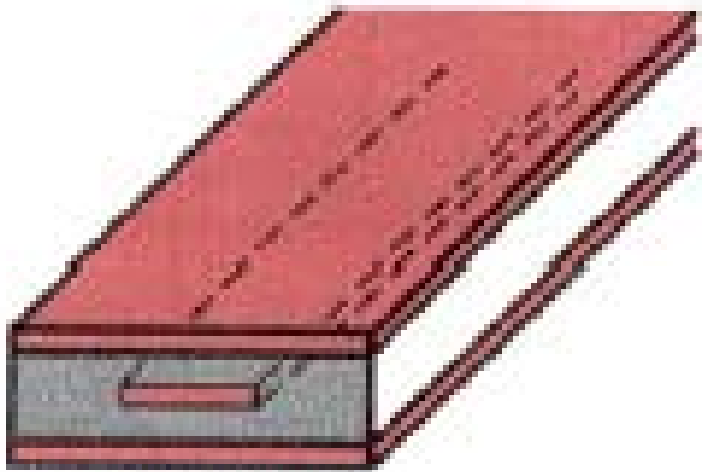
where

μ = permeability of the dielectric material

ϵ = permittivity of the dielectric material

Stripline and Microstrip

Inan and Inan
Figure 2.1



Stripline

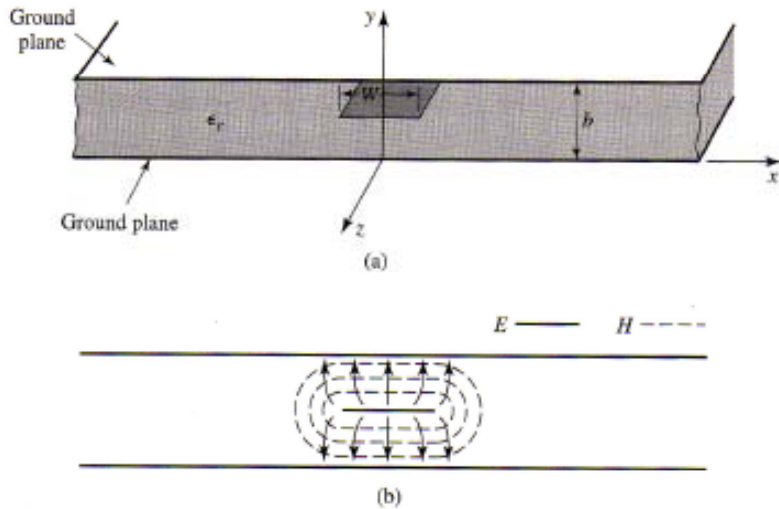
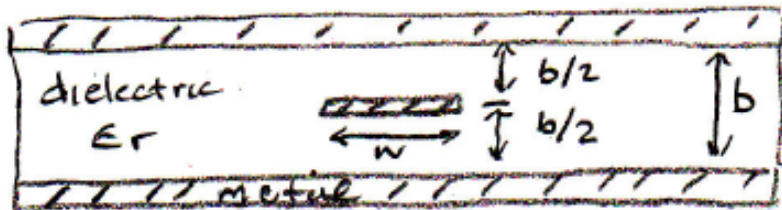


FIGURE 3.22 Stripline transmission line. (a) Geometry. (b) Electric and magnetic field lines.

From Pozar, "Microwave Engineering", Wiley, Fig. 3.22.



ground layer
ground layer

The speed of propagation is

$$u = \frac{c}{\sqrt{\epsilon_r}}$$

The characteristic resistance is

$$R_c \approx \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}$$

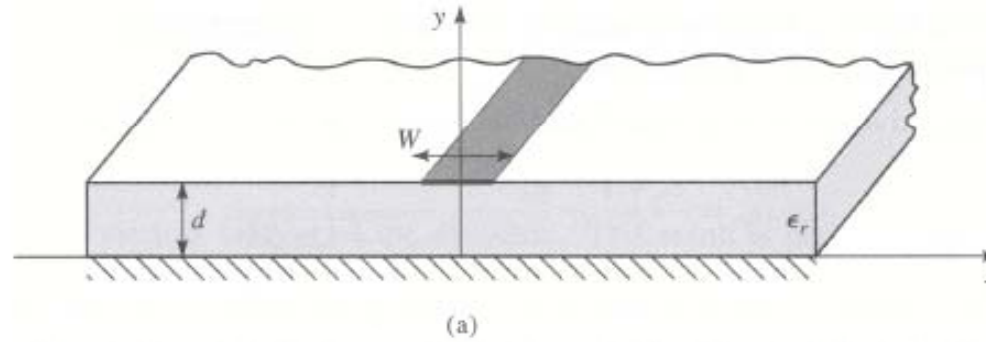
where

$$W_e = \begin{cases} W & \text{for } \frac{W}{b} > 0.35 \\ b \left(0.35 - \frac{W}{b} \right)^2 & \text{for } \frac{W}{b} < 0.35 \end{cases}$$

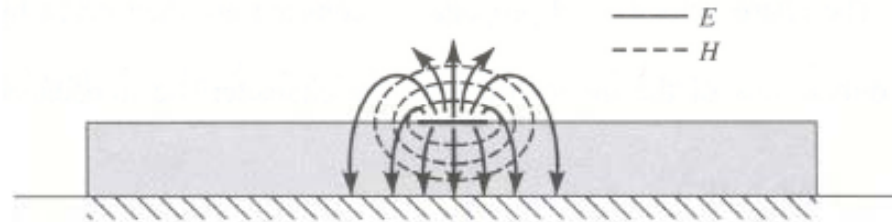
and W is the width of the circuit interconnect path

Pozar in "Microwave Engineering" (Wiley, 2004)

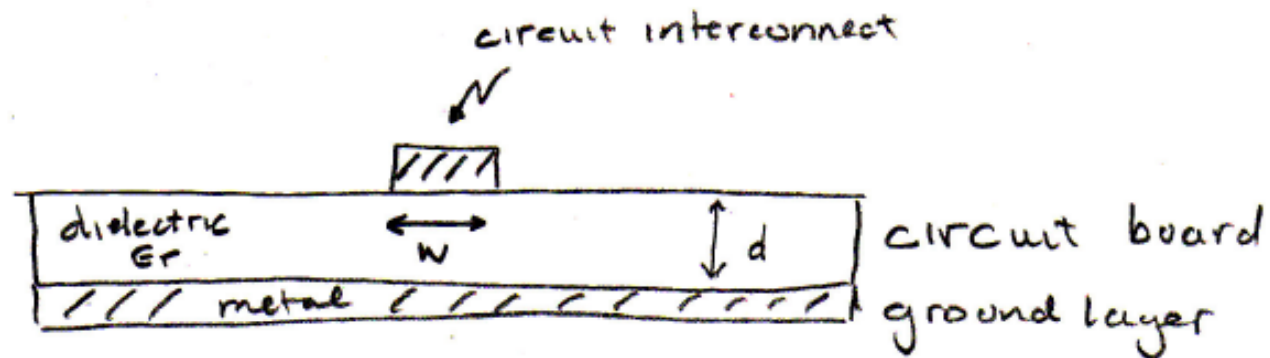
Microstrip



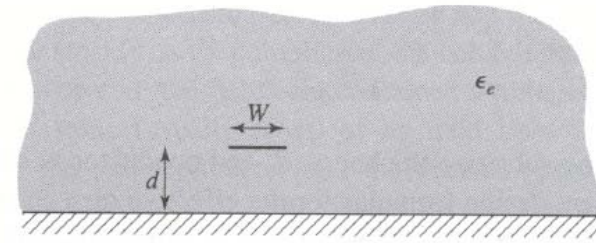
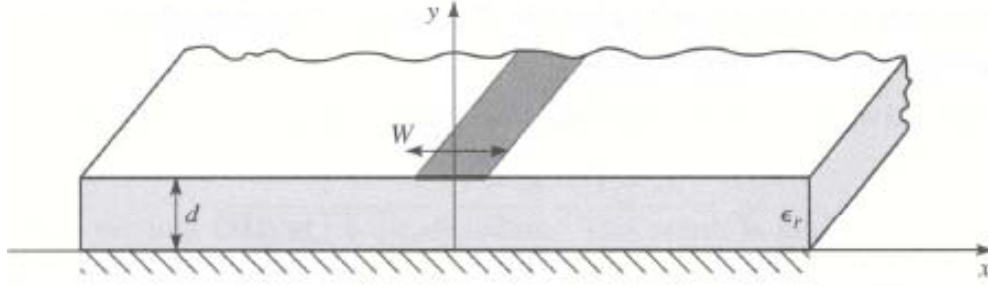
Pozar Fig. 3.25



D.M. Pozar, "Microwave Engineering", 2nd edition, Wiley, 1998.



Formulas for Microstrip



Pozar Fig. 3.26

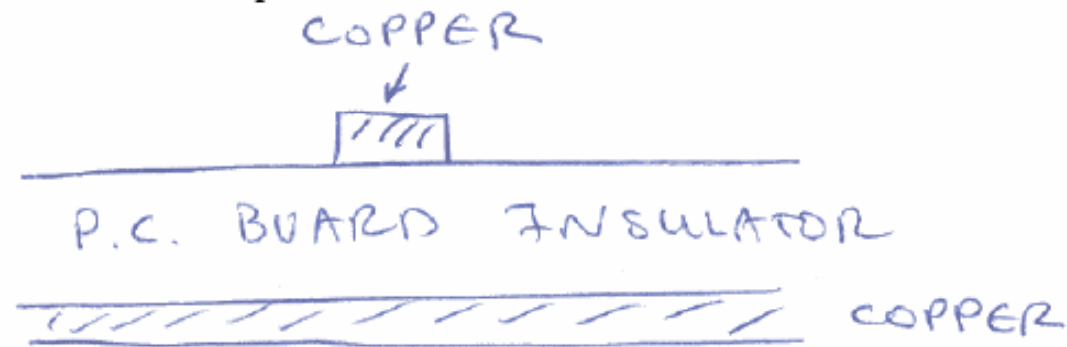
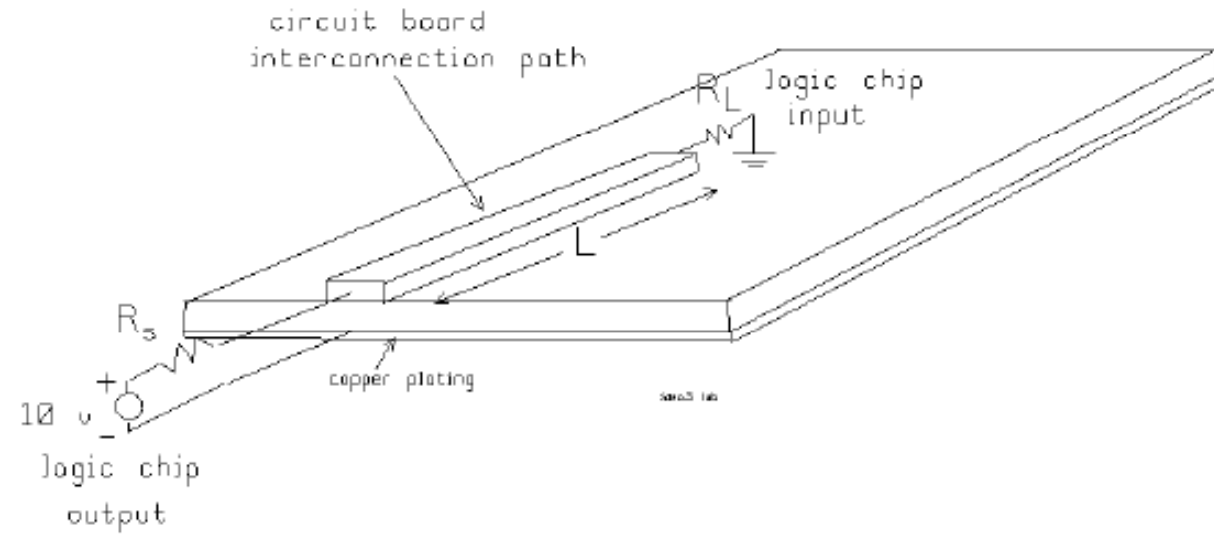
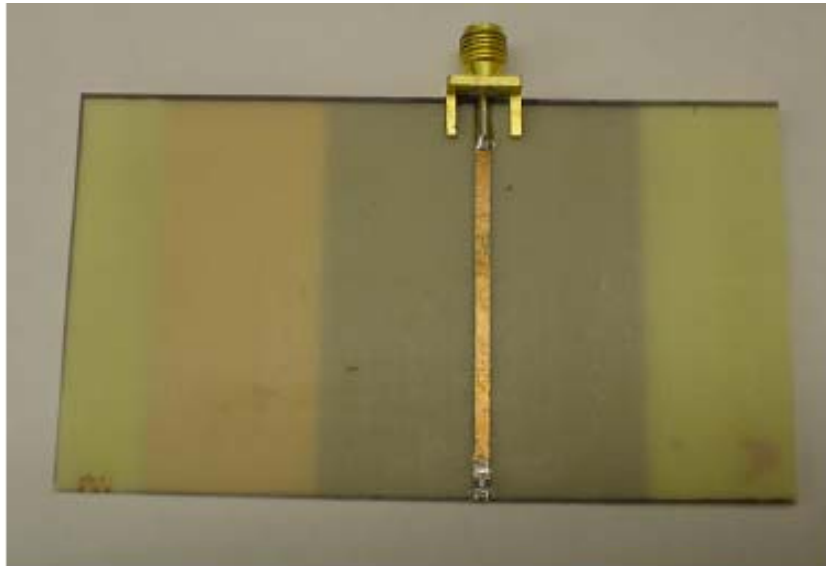
$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + \frac{12d}{W}}}$$

$$u = \frac{c}{\sqrt{\epsilon_e}}$$

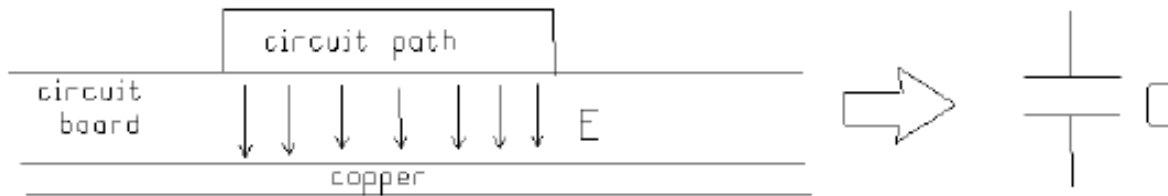
$$R_c = \frac{60}{\sqrt{\epsilon_e}} \ln\left(\frac{8d}{W} + \frac{W}{4d}\right) \quad \text{for } \frac{W}{d} \leq 1$$

$$R_c = \frac{120\pi}{\sqrt{\epsilon_e} \left[\frac{W}{d} + 1.393 + 0.667 \ln\left(\frac{W}{d} + 1.444\right) \right]} \quad \text{for } \frac{W}{d} \geq 1$$

Microstrip Transmission Line



Capacitance and Inductance of a Transmission Line



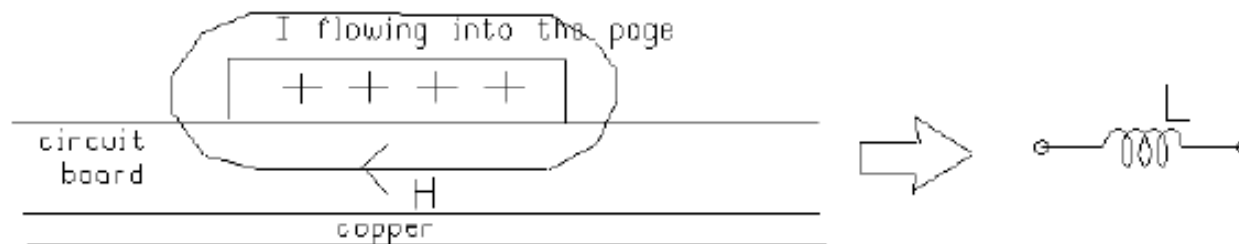
The capacitance is given by

$$C = cp \text{ Farads}$$

where

c = the capacitance-per-unit-length of the microstrip

p = the length of the interconnection path



The inductance of the interconnection path is given by

$$L = \ell p \text{ Farads}$$

where

ℓ = the inductance-per-unit-length of the microstrip

p = the length of the interconnection path

An interconnection path having capacitance and inductance is called a “transmission line”.