ELEC353 Lecture Notes Set 9

Mid-term Test: Friday February 17, 2012

The mid-term test covers homework assignments 1, 2, 3, 4 and 5.
The mid-term test covers the "self-learning" topic of A.C. Circuit Analysis.

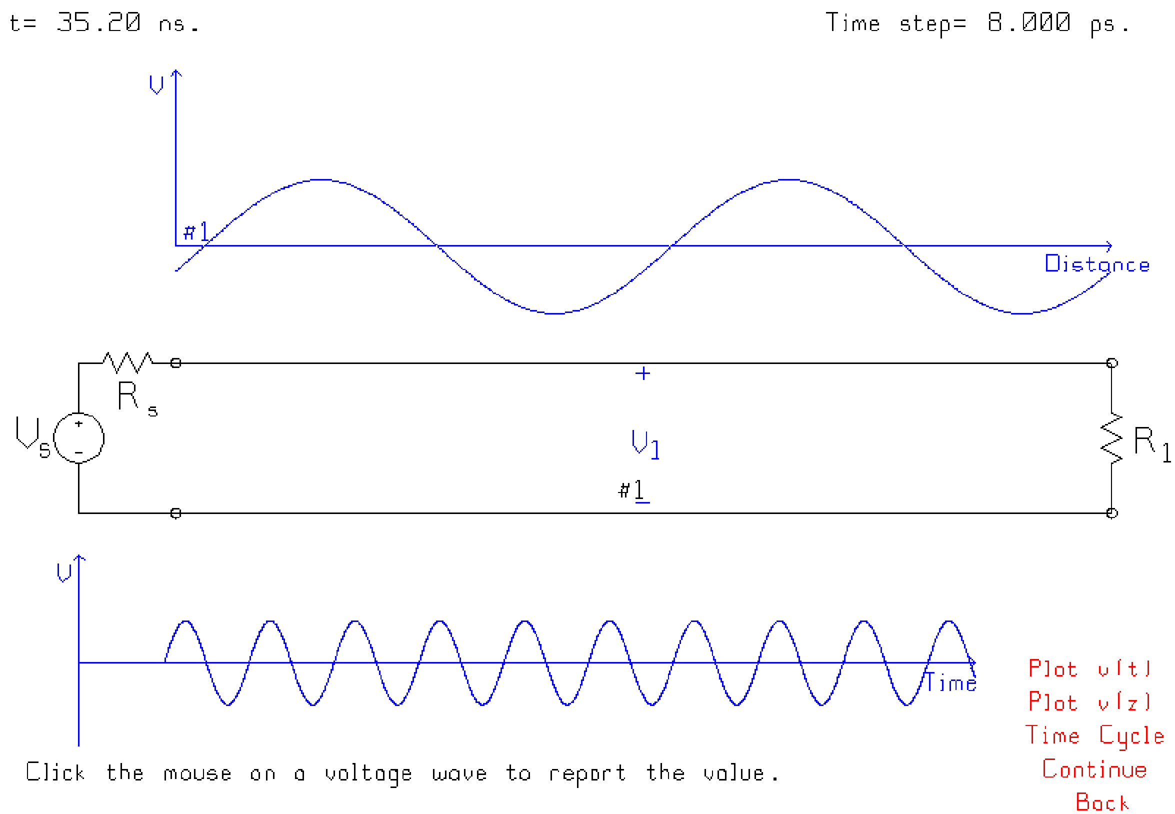
The homework assignments are posted on the course web site.
Homework #4: You should finish this assignment by February 4.
Homework #5: This assignment has **practice problems** for the midterm test. Do this assignment before February 11.

Previous mid-term tests with solutions are available from the course web site.

About the Mid-Term Test

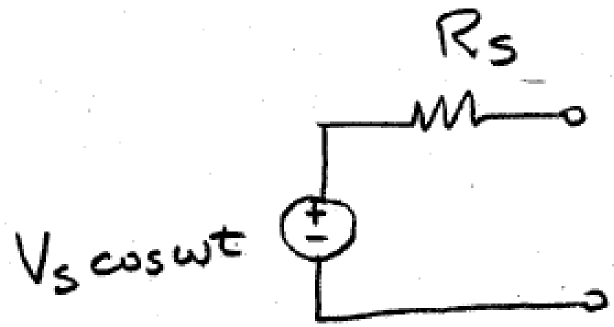
The midterm test is a multiple-choice test:
•Each question has four numerical answers plus "none of these."
•Circle the correct numerical answer on the exam paper.
•The test is closed book, so no textbook or notes are allowed.
•A sheet of formulas is attached to the test paper.
•You can get the formula sheet from the course web site.
•The mid-term test is "practice" for the final examination.
•The mid-term test will include a question on the "self-learning" topic of A.C. Circuit Analysis.

Transition to the Sinusoidal Steady State



- Travelling wave
- Reflection from an unmatched load
- Standing wave

A.C. Circuit Analysis



$$v(t) = V_s \cos \omega t$$

- The "amplitude" of the voltage is V_s .
- The "RMS value" of the voltage is $\frac{V_s}{\sqrt{2}}$.
- The frequency of operation is f Hertz, and the "radian frequency" is $\omega = 2\pi f$.

- In an LTI system, when the generator is sinusoidal, then at "steady state" all the voltages are sinusoidal *at the same frequency as the generator* and have the form

$$v(t) = A\cos(\omega t + \phi)$$

where

A is the *amplitude* of the voltage
ϕ is the *phase* of the voltage

Phasor Representation

$$v(t) = A\cos(\omega t + \phi) \leftrightarrow \bar{V} = Ae^{j\phi}$$

The **magnitude** of the phasor is the **amplitude** of the cosine.
The **angle** of the phasor is the **phase angle** of the cosine.

We can “recover” the cosine wave $v(t)$ from the phasor $\bar{V}$ with the formula

$$v(t) = \mathrm{Re}(\bar{V}e^{j\omega t})$$

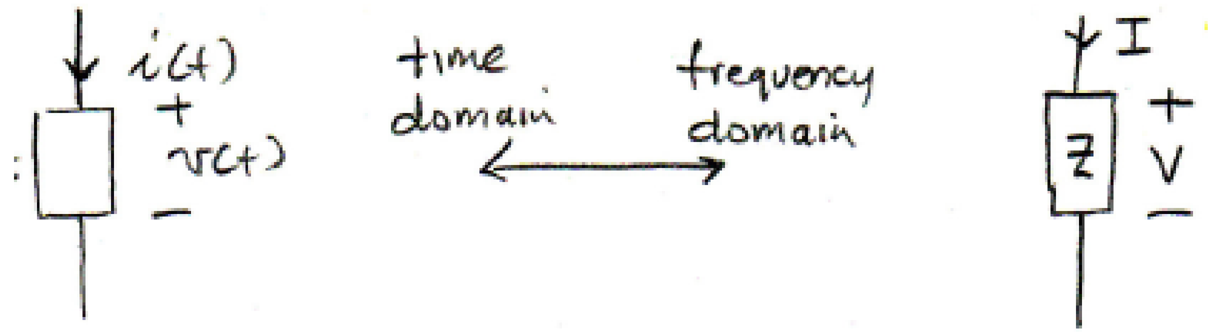
where the function $\mathrm{Re}(\ldots)$ means “take the real part of”, so

$$v(t) = \mathrm{Re}(Ae^{j\phi}e^{j\omega t}) = \mathrm{Re}(Ae^{j(\omega t+\phi)}) = \mathrm{Re}[A\cos(\omega t + \phi) + jA\sin(\omega t + \phi)]$$

and taking the real part

$$v(t) = A\cos(\omega t + \phi)$$

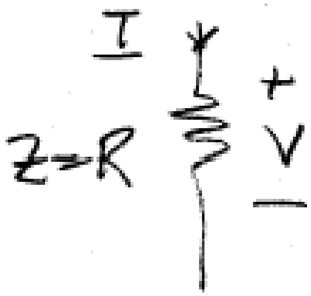
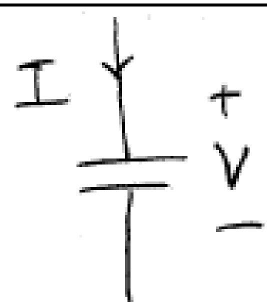
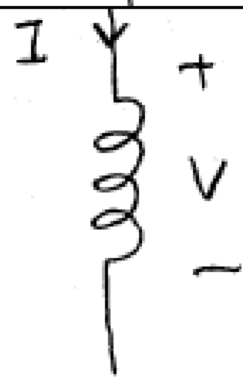
Impedance



- Let the voltage across a component be $v(t) = V_o \cos(\omega t + \theta)$ so that the phasor representing the voltage is $V = V_o e^{j\theta}$.
- Let the current flowing through the component be $i(t) = I_o \cos(\omega t + \phi)$ so that the phasor representing the current $I = I_o e^{j\phi}$.
- Then the "impedance" of the component is defined as the ratio of the voltage phasor to the current phasor:

$$Z = \frac{V}{I}$$

Impedance for R, L and C Elements

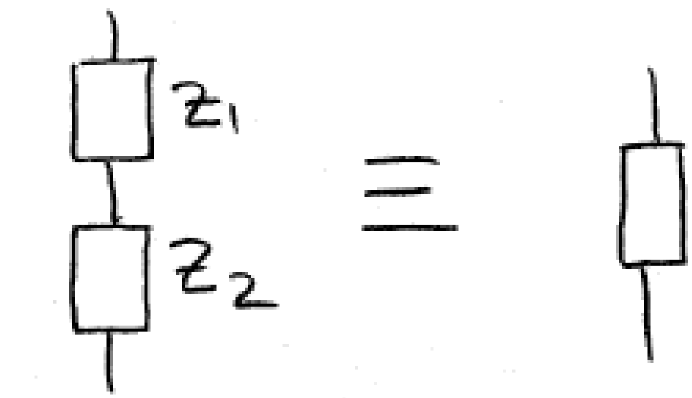
Component		Time Domain	Frequency Domain "Impedance"
Resistor		$v(t) = Ri(t)$	$Z = R$
Capacitance		$i(t) = C\frac{d}{dt}v(t)$	$Z = \frac{1}{j\omega C}$
Inductance		$v(t) = L\frac{d}{dt}i(t)$	$Z = j\omega L$

Homework:

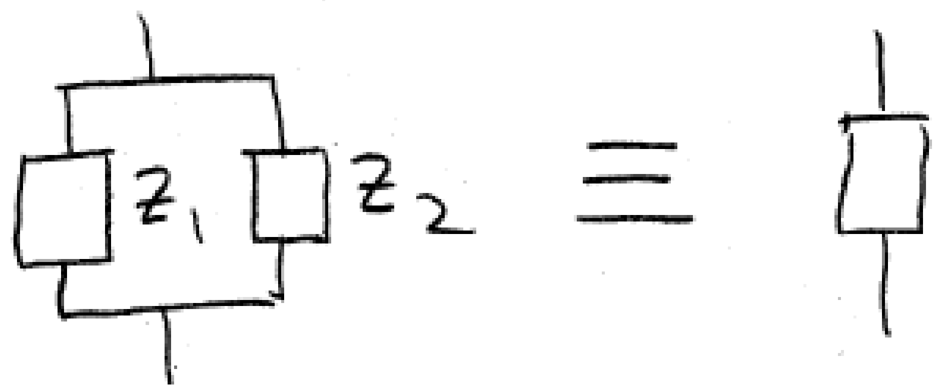
Prove that if $i(t) = C\frac{d}{dt}v(t)$ then $Z = \frac{1}{j\omega C}$.

Prove that if $v(t) = L\frac{d}{dt}i(t)$ then $Z = j\omega L$.

Series and Parallel Impedances

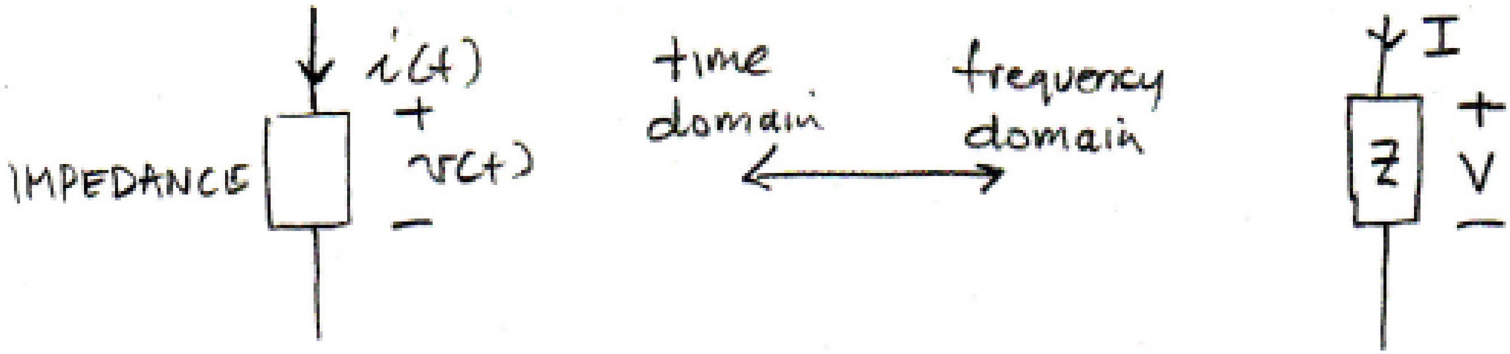


$$Z = Z_1 + Z_2$$



$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Power in A.C. Circuits



- The "instantaneous power" is

$$p(t) = v(t)i(t) = V_0 \cos(\omega t + \phi) I_0 \cos(\omega t + \theta)$$

- The "average power" is the instantaneous power averaged over one A.C. cycle:

$$P_{av} = \frac{1}{T}\int_0^T p(t)dt$$

- You can evaluate the integral with trig identities to show that the average power is

$$P_{av} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos(\phi - \theta)$$

RMS Value

The "RMS" value of a voltage $v(t)$ is given by

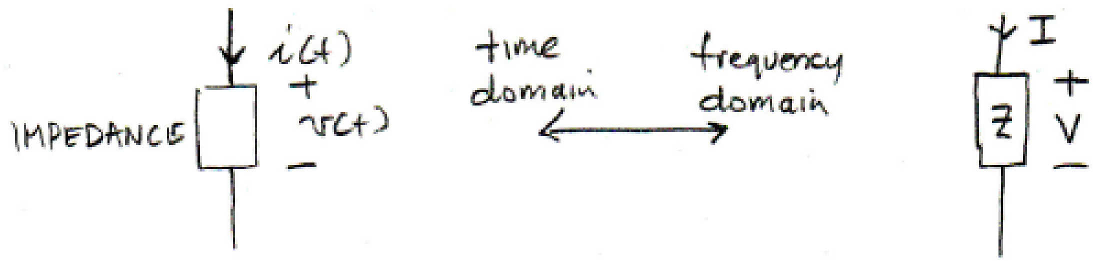
$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

You can prove that if $v(t) = V_0 \cos(\omega t + \theta)$, then the RMS value is

$$V_{RMS} = \frac{V_0}{\sqrt{2}}$$

$$P_{av} = V_{RMS} I_{RMS} \cos(\phi - \theta)$$

Complex Power



The phasor for $v(t) = V_0 \cos(\omega t + \phi)$ is $V = V_0 e^{j\phi}$

The phasor for $i(t) = I_0 \cos(\omega t + \theta)$ is $I = I_0 e^{j\theta}$

The "complex power" is defined as

$$S = \frac{1}{2} V I^*$$

where I^* is the complex conjugate of the current phasor, $I^* = I_0 e^{-j\theta}$.
(To get the complex conjugate, replace j by $-j$.)

$$S = \frac{1}{2} V I^*$$

$$S = P_{av} + jQ$$

The real part P_{av} is the average power:

$$P_{av} = \mathrm{Re}(S) = \frac{1}{2}\mathrm{Re}(VI^*)$$

The imaginary part Q is called the "reactive power".

The average formula is the same as the one we wrote before:

$$P_{av} = \frac{1}{2}\mathrm{Re}(VI^*)$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left(V_0 e^{j\phi} I_0 e^{-j\theta}\right) = \frac{1}{2}\mathrm{Re}\left(V_0 I_0 e^{j(\phi-\theta)}\right)$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left[V_0 I_0 (\cos(\phi-\theta) + j\sin(\phi-\theta))\right]$$

$$P_{av} = \frac{1}{2} V_0 I_0 \cos(\phi-\theta)$$

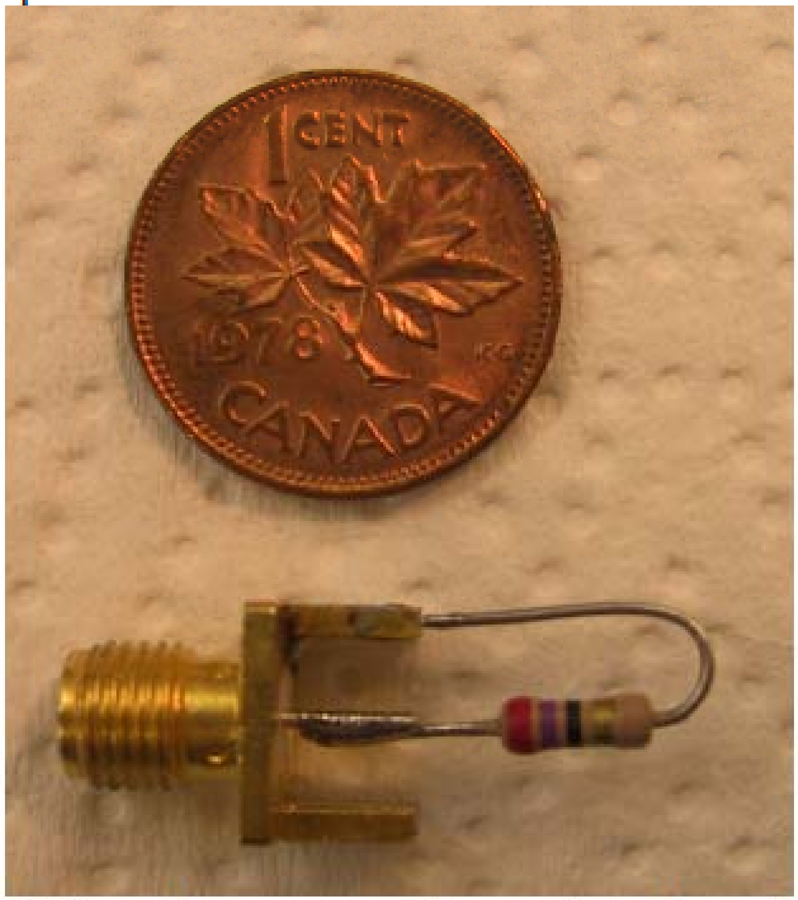
High Frequency Circuits

Objective:

- Learn to find the transfer function $H(j\omega)$ of an interconnection on a high-speed logic board.
- Learn to use the transfer function to identify shortcomings in the interconnection or "communication path".
- Learn to "fix" the circuit interconnection so that it provides acceptable performance.
- Learn some design principles for radio frequency circuits that apply equally well to logic circuits and to communication systems.

Resistors at Radio Frequencies

- Devices behave in unexpected ways at "radio" frequencies!
- What is the impedance of a 27 ohm resistor at 2 GHz?

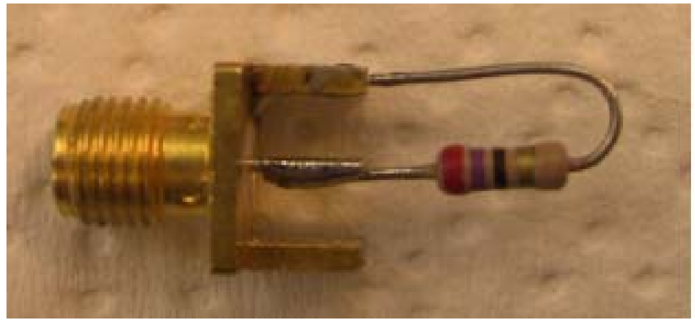


  - At D.C. this carbon-composition resistor has a "nominal" value of 27 ohms.
  - The resistor is shown here mounted on an "SMA" connector.
  - The SMA connector allows the resistor to be connected to a coaxial-cable "transmission line" of characteristic impedance $R_c = 50$ ohms.
  - What is the impedance of the resistor at 2 GHz?


HP8720 Network Analyzer

- A "network analyzer" can be used to measure the impedance of the resistor at 2 GHz.
- The 27-Ω resistor is used as the load on a 50-Ω coaxial "transmission line".
- The network analyzer measures the magnitude and phase of the **reflection coefficient** Γ of the 27-Ω resistor at the end of a 50-ohm cable.
- Then we can calculate the impedance using $\Gamma = \dfrac{Z_L - R_c}{Z_L + R_c}$, hence

$$Z_L = R_c \frac{1+\Gamma}{1-\Gamma}$$



The "27-Ω" resistor mounted on an SMA connector is measured to have an "impedance" of Z_L =100+j200 ohms at 2 GHz.

So R=100 ohms and $\omega L = 200$ ohms.

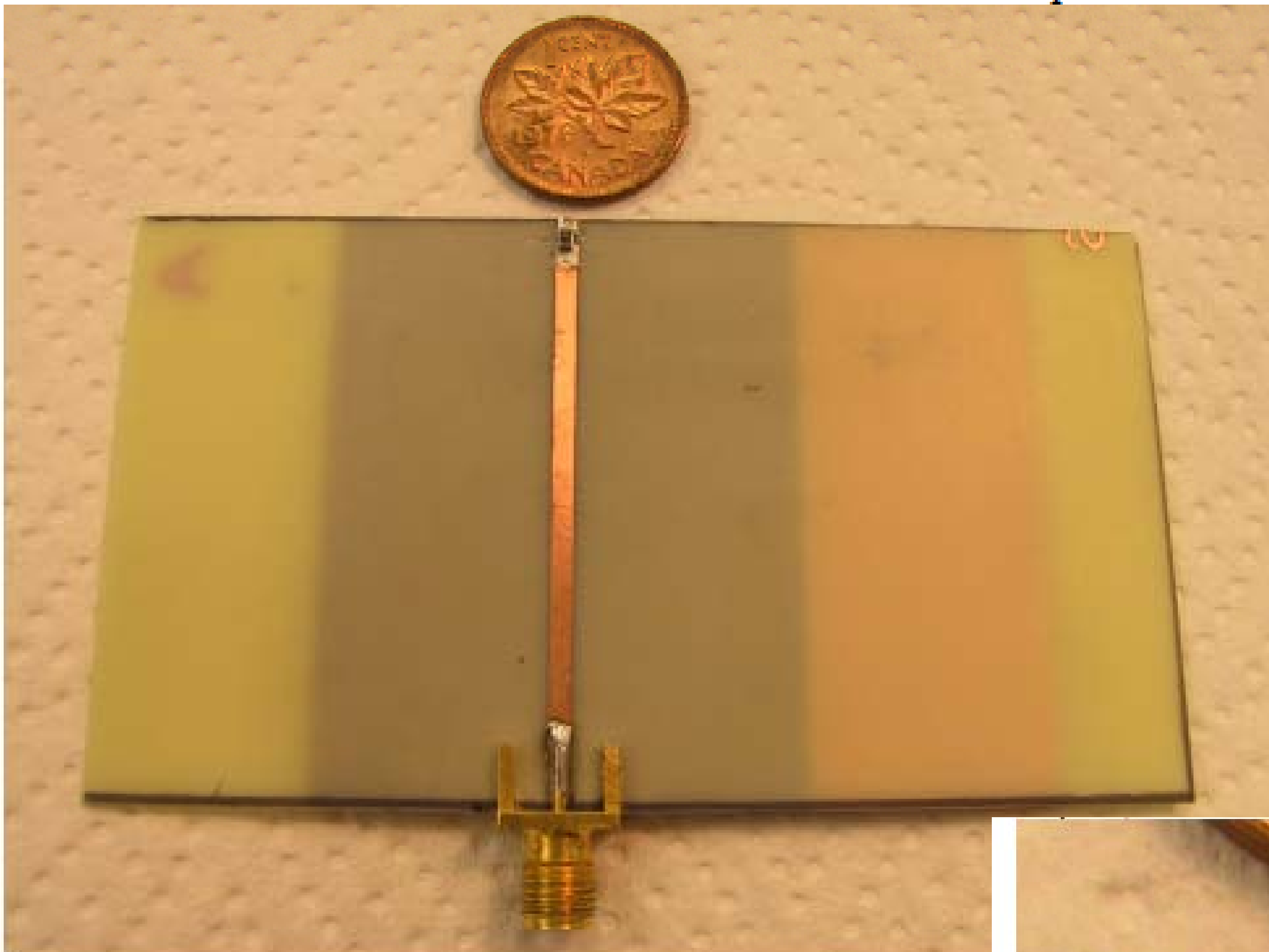
The resistance is much larger than 27 ohms-why?

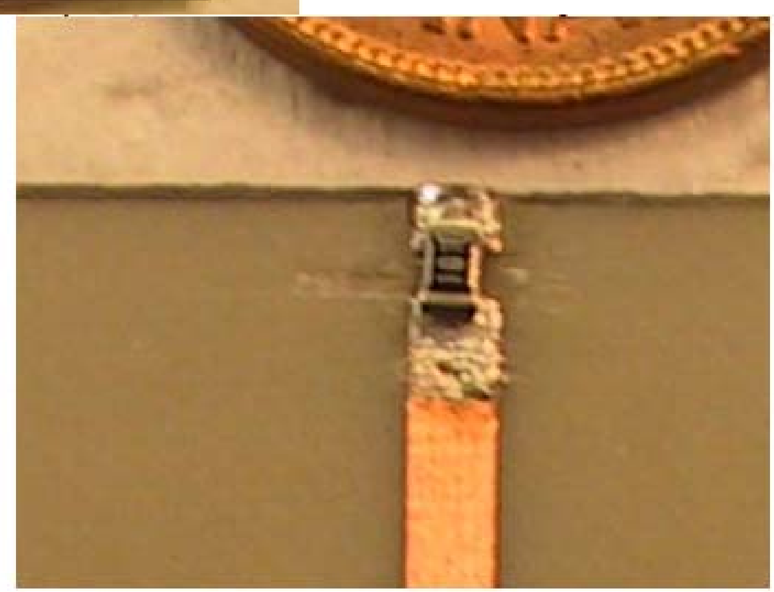
- Answer: the current flows in a thin layer at the surface called the "skin depth". So the cross-sectional area A of the resistor used for current flow is much smaller and the resistance $R = \dfrac{\rho L}{A}$ is higher.

There is an inductance of $j\omega L = j200$ ohms-why?

- Answer: at "radio" frequencies, the small one-turn loop formed with the lead wires of the resistor has inductance!

What is the resistance of a surface-mount 100 ohm “chip resistor” at 2 GHz?

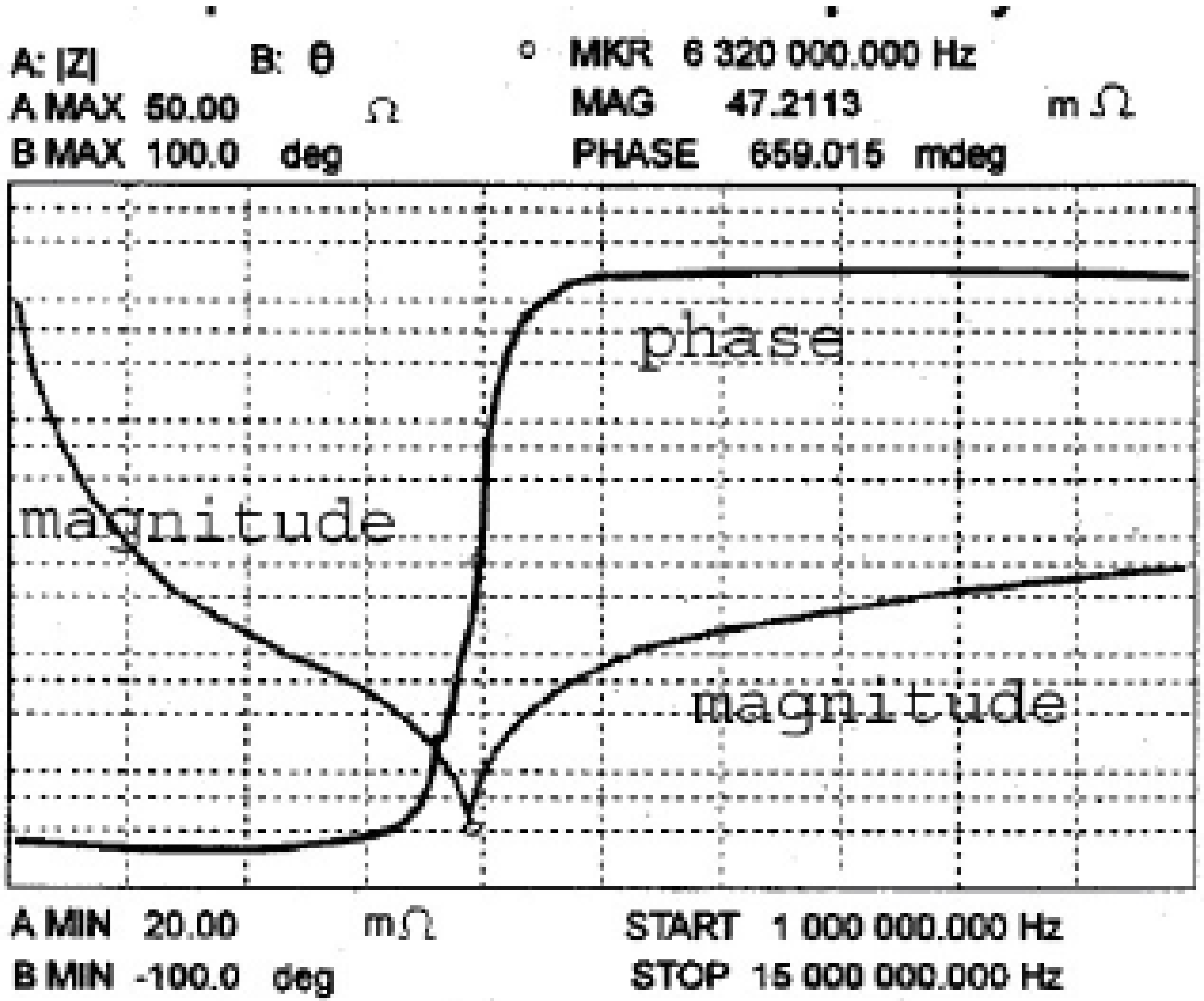




$$Z_L = 90 - j10 \text{ ohms}$$

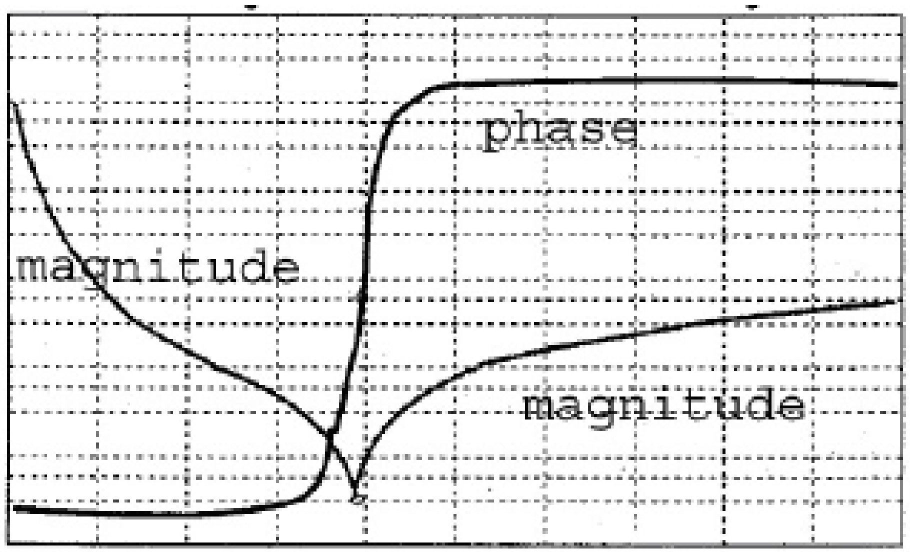
- The resistance of 90 ohms is much closer to the "nominal" value of 100 ohms because this resistor is "designed" for use at 2 GHz.
- There is some capacitance, hence a reactance of -j10 ohms.
- This is associated with the construction of the "chip resistor".
- There will also be some "lead inductance" which is in series with the capacitance. But it is not the dominant effect at 2 GHz.

Capacitance and Inductance as a function of Frequency



Horizontal: 1 MHz to 15 MHz
Vertical: log scale, 20 milliohms to 50 ohms
-100 deg to 100 deg

Capacitor: $$Z = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle -90^\circ$$



Inductor: $$Z = j\omega L = \omega L \angle +90^\circ$$