ELEC353
Practice Problem Set #10

1. Design a microwave link at 4 GHz. The transmit and receive antennas are identical and are located 36 km apart. The transmitted power is 43 dBm. The received power into a matched load must be -90 dBm. What is the gain required for each antenna?
(“dBm is decibels relative to 1 millWatt, dBm=$10\log\left(\frac{P}{0.001}\right)$).

2. A cell phone operates at 850 MHz. The cell phone’s antenna behaves as a vertical, half-wave dipole. The cell phone is operated in a room in a building and radiates 600 mW. The cell phone’s field passes perpendicularly through the wall of the building to a base station antenna 1 km distant. The building’s wall is made of limestone with relative permittivity ε_r=7.5, and for this problem the conductivity will be taken as σ=0. The wall is 20 cm in thickness.
(i) What is the transmission coefficient for the limestone wall?
(ii) What is the electric field strength at the location of the base station?

3. A Bluetooth link operates at 2450 MHz. The transmitter behaves as a vertical half-wave dipole antenna and radiates 1 mW. The transmitter communicates with a receiver in an adjacent room. The receiver is 20 m from the transmitter. The wall separating the two rooms can be represented as a concrete layer 5 cm thick, with ε_r = 6.1 and σ=0. The receive antenna also behaves as a vertical half-wave dipole and is terminated with a matched load.
(i) What is the transmission coefficient through the wall?
(ii) What is the power density at the receive antenna including the “transmission loss” of the wall?
(iv) How much power is delivered to the receiver’s matched load?

Solution to Practice Problem Set #10

1. Design a microwave link at 4 GHz. The transmit and receive antennas are identical and are located 36 km apart. The transmitted power is 43 dBm. The received power into a matched load must be -90 dBm. What is the gain required for each antenna?
   ("dBm is decibels relative to 1 millWatt, dBm=$10\log\left(\frac{P}{0.001}\right)$).

Solution: (thanks to Armin Parsa)

$$P_R = \left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T P_{in}$$

$$P_R/0.001 = \left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T \; P_{in}/0.001$$

$$10\log\left(P_R/0.001\right) = 10\log\left(\left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T \; P_{in}/0.001\right)$$

$$10\log\left(P_R/0.001\right) = 10\log\left(\left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T \; P_{in}/0.001\right)$$

$$-90 = 20\log\left(\frac{\lambda}{4\pi r}\right) + 10\log(G_R G_T) + 43$$

$$-90 = -135.6 + 10\log(G_R G_T) + 43$$

$$10\log(G_R G_T) = 2.609$$

$$G_R = G_T = 1.3504$$

2. A cell phone operates at 850 MHz. The cell phone's antenna behaves as a vertical, half-wave dipole. The cell phone is operated in a room in a building and radiates 600 mW. The cell phone's field passes perpendicularly through the wall of the building to a base station antenna 1 km distant. The building's wall is made of limestone with relative permittivity ε_r=7.5, and for this problem the conductivity will be taken as σ=0. The wall is 20 cm in thickness.
   (i) What is the transmission coefficient for the limestone wall?
   (ii) What is the electric field strength at the location of the base station?

Solution of #9-2 (Thanks to Guilin Sun)
Known f=850 MHz, half-dipole, P=600mW, d=1km,wall $\varepsilon_r = 7.5, \sigma = 0$
i) The transmission coefficient is

$$T = \frac{4\eta_0\eta_w e^{-j\beta d}}{(\eta_0+\eta_w)^2 - (\eta_0-\eta_w)^2 e^{-j2\beta d}} = \frac{4\sqrt{\varepsilon_r}\,e^{-j\beta d}}{(1+\sqrt{\varepsilon_r})^2 - (1_0-\sqrt{\varepsilon_r})^2 e^{-j2\beta d}}$$

Note we used the relation in the dielectrics $\eta_w = \eta_0/\sqrt{\varepsilon_r}$. To avoid large computation error, the phase is calculated by the fraction of the "electronic length"

$\frac{d}{\lambda} = \frac{d * f}{c} = \frac{1000 * 850 \cdot 10^6}{3 \cdot 10^8} = 28333.33$. The fraction is 1/3.

So the phase $[\beta d] = 360° * (1/3) = 120°$

$$T = \frac{4\sqrt{7.5}e^{-j120°}}{(1+\sqrt{7.5})^2 - (1-\sqrt{7.5})^2 e^{-j240°}} = \frac{4\sqrt{7.5}e^{-j120°}}{13.98 - 3.02e^{-j240°}} = \frac{4\sqrt{7.5}e^{-j120°}}{15.49 + j2.6} = 0.6974e^{-j129.5°}$$

ii) The location of the base station is considered in the azimuth plane, $\theta = 90°$, so for the dipole $F(\theta) = 1$.

Since $P = 36.5|I_0|^2$ so you can get $|I_0| = \sqrt{0.6/36.5} = 0.1282$ A. Therefore $|E_\theta| = \frac{|I_0|\eta}{2\pi} = \frac{0.1282 * 376.7}{2\pi} = 7.686$

The field strength at the base station is

$|E| = |T||E_\theta|\frac{1}{d} = \frac{0.6974 * 7.686}{1000} = 6.36 \cdot 10^{-3}$ V/m or 6.36 mV/m.

Note if the loss of the wall is considered, the above result should be multiplied by the attenuation $e^{-\alpha d}$. Since d=1000m is quite large, if the loss coefficient is only one-thousandth, the field strength will be attenuated to 0.3679*6.36=2.33 mV/m.

3. A Bluetooth link operates at 2450 MHz. The transmitter behaves as a vertical half-wave dipole antenna and radiates 1 mW. The transmitter communicates with a receiver in an adjacent room. The receiver is 20 m from the transmitter. The wall separating the two rooms can be represented as a concrete layer 5 cm thick, with ε_r = 6.1 and σ =0. The receive antenna also behaves as a vertical half-wave dipole and is terminated with a matched load.
(i) What is the transmission coefficient through the wall?
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Solution: (Thanks to Ibrahem Abdalla)

$$\varepsilon_{rw} = 6.1$$

$$f = 2450 \ \ MHZ$$

$$d = 0.05 \ \ m$$

$$\eta_w = \frac{\eta_o}{\sqrt{\varepsilon_{rw}}} = \frac{377}{\sqrt{6.1}} = 152.64$$

$$\beta_w = \frac{2\pi}{\lambda_w}$$

$$\lambda_w = \frac{\lambda_o}{\sqrt{\varepsilon_{rw}}} = \frac{3*10^8}{2450*10^6*\sqrt{6.1}} = 0.05m$$

$$\beta_w = \frac{2\pi}{0.05} = 7261°/m$$

$$\beta_w d = 363.05 \Rightarrow 3.05°$$

$$\tau = \frac{4\eta_o\eta_w e^{-j\beta_w d}}{(\eta_o+\eta_w)^2 - (\eta_o-\eta_w)^2 e^{-j2\beta_w d}}$$

$$\tau = \frac{4*377*152.64*1\angle -3.05}{(377+152.64)^2 - (377-152.64)^2\angle -6.1} = 9.411*10^{-4}\angle -3.0513°$$

$$r = 20 \quad m$$

$$P_{rad} = 1*10^{-3}$$

$$S_{av} = \frac{P_{rad}}{4\pi r^2} D_{\max} |\tau|^2$$

$$S_{av} = \frac{1*10^{-3}*1.64*(9.411*10^{-4})^2}{4*\pi*(20)^2} = 2.9*10^{-13} \quad w/m^2$$

$$P_{rec} = S_{av} A_e$$

$$A_e = \frac{\lambda^2}{4\pi} D_{\max} = 1.96*10^{-3}$$

$$P_{rec} = 5.675*10^{-16} \quad w$$