ELEC353
Practice Problem Set #9

1. A cell phone antenna at 850 MHz behaves as a vertical dipole of length 0.65 wavelengths. The largest current on the dipole is 1 mA. The cell phone communicates with a base station that is at the top of a nearby building, which is 200 m tall. The cell phone is 50 m from the base of the building at street level.
   (i) What is the electric field strength at the location of the base station antenna atop the building?
   (ii) What is the received power density at the location of the base station?

2. A Bluetooth antenna at 2450 MHz radiates 10 mW. The antenna behaves as a vertical half-wave dipole.
   (i) What is the power density as a function of distance r from the antenna, in the azimuth plane?
   (ii) At what distance from the antenna is the power density 1.3×10^{-6} watts/meter^2, in the azimuth plane. This is approximately the smallest power density for adequate communication.
   (iii) At what distance from the antenna is the power density 1.3×10^{-6} watts/meter^2 at an elevation of 60 degrees above the azimuth plane?

3. A communications link operates at 6 GHz. The distance between the transmitter and the receiver is 20 km. The power density at the receiver must be -100 dB re: 1 Watt per square meter. The transmitted power is 20 mW. What is the gain required for the transmit antenna? If the antenna efficiency is 99.6%, what is the directivity of the antenna?

Solution to Practice Problem Set #9

1. A cell phone antenna at 850 MHz behaves as a vertical dipole of length 0.65 wavelengths. The largest current on the dipole is 1 mA. The cell phone communicates with a base station that is at the top of a nearby building, which is 200 m tall. The cell phone is 50 m from the base of the building at street level.
(i) What is the electric field strength at the location of the base station antenna atop the building?
(ii) What is the received power density at the location of the base station?

Solution: (thanks to Armin Parsa)

(i)

$$\beta h = \frac{2\pi}{\lambda} \times \frac{0.65\lambda}{2} = 0.65\pi$$

$$r = \sqrt{50^2 + 200^2} = 206.1553 \qquad \text{m}$$

$$\theta = \frac{\pi}{2} - \tan^{-1}\frac{200}{50} = 14.036°$$

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos(\beta h)}{\sin\theta} = \frac{\cos(0.65\pi \times \cos(23.7°)) - \cos(0.65\pi)}{\sin(23.7°)} = 0.2273$$

$$E_\theta = jI_0 \frac{\eta}{2\pi} F(\theta) = j \times 0.001 \times \frac{377}{2\pi} \times 0.2273 = 0.0136j$$

$$E = \hat{a}_\theta E_\theta \frac{e^{-j\beta r}}{r}$$

$$|E| = \frac{|E_\theta|}{r} = \frac{0.0136}{206.15} = 6.6157 \times 10^{-5} \qquad \text{V/m}$$

(ii)

$$S_{av} = \frac{|E_\theta|^2}{2\eta} = 5.8 \times 10^{-12} \qquad \text{W/m2}$$

2. A Bluetooth antenna at 2450 MHz radiates 10 mW. The antenna behaves as a vertical half-wave dipole.
(i) What is the power density as a function of distance r from the antenna, in the azimuth plane?
(ii) At what distance from the antenna is the power density $1.3\text{x}10^{-6}$ watts/meter^2, in the azimuth plane. This is approximately the smallest power density for adequate communication.
(iii) At what distance from the antenna is the power density $1.3\text{x}10^{-6}$ watts/meter^2 at an elevation of 60 degrees above the azimuth plane?

Solution of #9-2 (Thanks to Guilin Sun)

Known for the half-wave dipole Bluetooth antenna: f=2450 Mhz, P=10 mW

i) The power density, the field and the F factor for the far field

$$\overline{S_{av}} = \vec{a}_r \frac{1}{r^2}\left(\frac{|E_\theta|^2}{2\eta} + \frac{|E_\phi|^2}{2\eta}\right) \qquad E_\theta = jI_0 \frac{2\pi}{\eta} F(\theta) \quad E_\phi = 0 \qquad F(\theta) = \frac{\cos(\pi\cos(\theta)/2)}{\sin\theta}$$

in the azimuth plane, $\theta = 90°$, so $F(\theta) = 1$. To find I_0 use the relation between the power and I_0:

$P = 36.5|I_0|^2$, so

$$S_{av}\Big|_{\theta=90°} = \frac{1}{r^2}\left(\frac{|I_0|\eta}{2\pi}\right)^2 \frac{1}{2\eta} = \frac{1}{r^2}\frac{\eta}{8\pi}\frac{P}{36.5} = \frac{1}{r^2}\frac{377}{8\pi^2}\frac{0.01}{36.5} \approx \frac{1.3\cdot 10^{-3}}{r^2} \quad \text{W/m}^2$$

ii) let $S_{av} = 1.3\cdot 10^{-6}$ you can get r=31.62 m

iii) The definition of "elevation angle" is the angle above the horizon, that is, above the xy plane. So an elevation angle of zero degrees corresponds to θ=90 degrees, and an elevation angle of 90 degrees is straight upward, correspond to θ=0 degrees. In general, θ=90-(elevation angle), so for elevation angle 60 degrees, θ=90-60=30 degrees.

Then $F(\theta) = \dfrac{\cos(\sqrt{3}\pi/4)}{\sin 30°} = 0.418$

$$S_{av}\Big|_{\theta=30°} = S_{av}\Big|_{\theta=90°} F^2(\theta = 30°) = \frac{1.3\times .418^2\cdot 10^{-3}}{r^2} = \frac{0.227\cdot 10^{-3}}{r^2}$$

So the distance will be r=13.21 m.

You can also use the directivity and the radiated power for the calculation.

For half dipole, the directivity and directivity gain are $D_{\max} = 1.64$, $D(\theta) = D_{\max}F^2(\theta)$.

And use the relation $S_{av} = \dfrac{PD(\theta)}{4\pi R^2}$

You can write $R = \sqrt{\dfrac{PD_{\max}}{4\pi S_{av}}}F(\theta)$

In the azimuth plane, θ=90 degrees, $F(\theta = 90) = 1$ and

$$R\Big|_{\theta=90°} = \sqrt{\frac{PD_{\max}}{4\pi S_{av}}} = \sqrt{\frac{0.01*1.64}{4\pi*1.3\cdot 10^{-6}}} = 31.68 \text{ m}$$

Then for elevation angle 60 degrees, corresponding to θ=30 degrees, we have

$$R\Big|_{\theta=30°} = \sqrt{\frac{PD_{\max}}{4\pi S_{av}}}F(\theta = 30°) = R\Big|_{\theta=90°}F(\theta = 30°) = 31.68 * 0.418 = 13.24 \text{ m}$$

3. A communications link operates at 6 GHz. The distance between the transmitter and the receiver is 20 km. The power density at the receiver must be -100 dB re: 1 Watt per square meter. The transmitted power is 20 mW. What is the gain required for the transmit antenna? If the antenna efficiency is 99.6%, what is the directivity of the antenna?

Solution: (Thanks to Ibrahem Abdalla)

The power density is -100 dB relative to 1 Watt per square meter. Convert to a linear scale to get $S_{av} = 10^{=100/10} = 10^{-10}$ watts per square meter.
The power density S_{av} is related to the input power $P_{in} = 20$ mW and the gain G by

$$S_{av} = \frac{P_{in}}{4\pi r^2} G$$

So the gain is

$$G = 4\pi r^2 S_{av} / P_{in} = 25.13$$

Since the gain is the efficiency time the directivity, $G = eD$, we have the directivity given by

$$D = G / e = 25.234$$