ELEC 353 – Solution to Assignment #12

1.A wireless link is set up at 14 GHz between a transmit antenna of gain $G_T = 14$ dB, and a receive antenna of gain $G_R = 10$ dB. The antennas are 20 km apart. The input power to the transmitter is 1 kW. The antennas are lossless.

$$f = 14 \quad GHz$$

$$\lambda = \frac{300}{14000} = 0.02143 \quad m$$

$$r = 20000 \quad m$$

$$P_{in} = P_{rad} = 1 * 10^3$$

$$G_T = 14 \quad dB \Rightarrow 10^{\frac{14}{10}} = 25.12$$

$$G_R = 10 \; dB \Rightarrow 10^{\frac{10}{10}} = 10$$

(i) What is the power flow density S_{av} in microwatts per square meter at the location of the receiver?

$$S_{av} = \frac{P_{in}}{4\pi r^2} G_T \approx 5 \quad \mu W / m^2$$

(ii) What is the effective area of the receive antenna?

$$A_e = \frac{\lambda^2}{4\pi} G_R \approx 3.65 * 10^{-4} m^2$$

(iii) The receive antenna is terminated in a matched load. What is the power delivered to the matched load?

$$P_R = A_e S_{av} = 1.83 \;\; nW$$

2.A half-wave dipole has directivity 1.64. It radiates 100 mW of power in free space at 2450 MHz. The dipole is located a distance of $R = 5$ m from a wall of thickness $d = 20.6$ cm which is made of a lossless, non-magnetic material with relative permittivity $\varepsilon_r = 5$.

$$\varepsilon_{rw} = 5$$

$$f = 2450 \;\; MHZ$$

$$d = 0.206 \;\; m$$

(i) What is the magnitude of the transmission coefficient for a plane wave normally incident on the wall?

$$\eta_w = \frac{\eta_o}{\sqrt{\varepsilon_{rw}}} = \frac{377}{\sqrt{5}} = 168.6$$

$$\beta_w = \frac{2\pi}{\lambda_w}$$

$$\lambda_w = \frac{\lambda_o}{\sqrt{\varepsilon_{rw}}} = \frac{300}{2450 * \sqrt{5}} = 0.055m$$

$$\beta_w = \frac{2\pi}{0.055} * \frac{180}{\pi} = 6545.5° / m$$

$$\beta_w d = 268.4°$$

$$\tau = \frac{4\eta_o \eta_w e^{-j\beta_w d}}{(\eta_o + \eta_w)^2 - (\eta_o - \eta_w)^2 e^{-j2\beta_w d}}$$

$$\tau = \frac{4*377*168.6*1\angle -268.4°}{(377+168.6)^2 - (377-168.6)^2 \angle -176.73} = 0.75\angle 91.2°$$

(ii) If the dipole radiates in free space, then what is the power density at distance R and angle θ=90 degrees?

$$S_{av} = \frac{P_{rad}}{4\pi r^2} D_{\max} \approx 5.22*10^{-4} \quad W/m^2$$

(iii) What is the electric field amplitude of the dipole antenna at distance R and angle θ=90 degrees?

$$E_i = \frac{9.92\sqrt{P_{rad}}}{r} = 0.63 \quad V/m$$

(iv) Assuming that the plane wave transmission coefficient of part (i) can be used with the dipole's field, what is the amplitude of the electric field E_t transmitted through the wall?

$$E_t = |\tau| E_i = 0.473 \quad V/m$$

3.A Bluetooth link operates at 2450 MHz. The transmitter behaves as a lossless vertical half-wave dipole antenna and radiates 10 mW. The receive antenna also behaves as a lossless vertical half-wave dipole and is terminated with a matched load.

$$f = 2450 \quad MHz$$

$$\lambda = \frac{300}{2450} = 0.1224 \quad m$$

$$r = 10 \quad m$$

(i) If the receiver is 10 m from the transmitter, what is the received power?

$$P_R = (\frac{\lambda}{4\pi r})^2 G_T G_R P_{in} = 2.621*10^{-5} \quad W$$

(ii) If the minimum field power density at the receiver required for satisfactory communication is 1.30x10^{-6} watts/meter^2 then how far can the receiver be from the transmitter?

$$R_{\max} = \sqrt{\frac{P_{rad} D_{\max}}{4\pi * S_{av_{\min}}}} = 31.623 \quad m$$

4.A cell phone antenna at 1900 MHz behaves as a vertical dipole of length one-half wavelength. The cell phone is 1.6 m above the street. The largest current on the dipole is 10 mA. The cell phone communicates with a base station that is 600 m away along the street and 150 m above the level of the street, on top of a building.

(i) What is the distance from the cell phone antenna to the base station antenna? What is the elevation angle of the base station antenna seen from the cell phone antenna? "Elevation angle" is measured from the horizontal plane or "azimuth plane". An object directly overhead has an elevation angle of 90 degrees. Conversely, the angle θ in spherical coordinates is zero degrees directly overhead and 90 degrees in the azimuth plane. What is the value of θ for the base station seen from the cell phone?

$$r = \sqrt{600^2 + (150-1.6)^2} = 618.13 \quad m$$

Elevation angle = $\tan^{-1}$ (Height difference/separation distance) = $\tan^{-1}$ ((150-1.6)/600)=13.89 deg

(ii) What is the electric field strength at the location of the base station antenna atop the building?

$$\theta = 90 - 13.89 = 76.11$$

$$F(\theta) = \frac{\cos(\pi \cos \theta)}{\sin \theta} = 0.751$$

$$E_\theta = jI_o \frac{\eta}{2\pi} F(\theta) = j0.451$$

$$\overline{E} = E_\theta \frac{e^{-j\beta r}}{r} a_\theta$$

$$\left|\overline{E}\right| = \frac{E_\theta}{r} = 7.3 * 10^{-4} \quad V/m$$

(iii) What is the received power density at the location of the base station?

$$S_{av} = \frac{|E|^2}{2\eta r^2} = 1.85 * 10^{-15} \text{ W}$$

(iv) If the base station antenna has a gain of 6 dB, what is the power received into a matched load?

$$A_e = \frac{\lambda^2}{4\pi} G = 7.9 * 10^{-3} m^2$$

$$P_R = A_e S_{av} = 1.46 * 10^{-17} \quad W$$

5. A communications link operates at 12 GHz. The distance between the transmitter and the receiver is 10 km. The power density at the receiver must be -100 dB re: 1 Watt per square meter. The transmitted power is 100 mW.

(i) What is the gain required for the transmit antenna?

(ii) If the antenna efficiency is 99.6%, what is the directivity of the antenna?

$$f = 12 \quad GHz$$

$$r = 10000 \quad m$$

$$P_{in} = P_{rad} = 100 * 10^{-3} = 0.1$$

$$S_{av} = -100 \quad dB$$

$$S_{av} = 10^{\frac{-100}{10}} * 10^{-3} = 10^{-13}$$

$$S_{av} = \frac{P_{in}}{4\pi r^2} G$$

$$G = 4\pi r^2 S_{av} / P_{in}$$

$$G = 1.26$$

$$D = G / e = 1.262$$

6. Consider a microwave link at 6 GHz. The transmit and receive antennas are located 25 km apart. The transmitted power is 12 dBm. The received power into a matched load must be -90 dBm. The transmit antenna has a gain of 10 dB. What is the gain required for the receive antenna?

("dBm is decibels relative to 1 millWatt, dBm=$10\log\left(\frac{P}{0.001}\right)$).

$$P_R = \left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T P_{in}$$

$$P_R / 0.001 = \left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T \, P_{in} / 0.001$$

$$10\log\left(P_R / 0.001\right) = 10\log\left(\left(\frac{\lambda}{4\pi r}\right)^2 G_R G_T \, P_{in} / 0.001\right)$$

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$$-90 = 20\log\left(\frac{0.05}{4\pi * 25 * 10^3}\right) + G_R + 10 + 12$$

$$G_R = 23.96 \ \ dB$$