## ELEC 353 – Solution to Assignment #9

2. A plane wave travels in the $z$ direction in a material at 850 MHz. The relative permittivity of the material is $\varepsilon_r = 9$ and the loss tangent is 0.15. The electric field is oriented parallel to the $x$ axis. The amplitude of the electric field at $z = 0$ is 5 volts/meter.

$\omega = 2 * \pi * f =$ 5.3407x$10^9$ rad/sec

(i) What is the value of the conductivity?

$\sigma = \omega\varepsilon * \tan\delta = 0.0638$ => 63.8 mS/m

(ii) What is the value of the propagation constant? What is the attenuation constant? What is the phase constant?

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ = 3.9971 + j 53.5927

$\alpha = 3.9971$ Np/m

$\beta = 190.64$ deg./m

(iii) What is the penetration depth?

$\delta = \dfrac{1}{\alpha} = 0.2502$ m => 25.02 cm

(iv) What is the intrinsic impedance?

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = 124.54 + \text{j}9.29 = 124.9\angle 4.3°$$

(v) Write the vector-phasor for the electric field.

$$\overline{E} = E_0 e^{-\alpha Z} e^{-j\beta Z} a_x = 5e^{-3.9971Z} e^{-j190.64Z} a_x \text{ V/m}$$

(vi) Write the vector-phasor for the magnetic field.

$$\overline{H} = \frac{E_0}{\eta} e^{-\alpha Z} e^{-j\beta Z} a_y = 0.04e^{-3.9971Z} e^{-j190.64Z} e^{-j4.3} a_y$$

(vii) Write the electric field vector and the magnetic field vector in the time domain.

$$\overline{E}(z,t) = 5e^{-3.9971Z} \cos(\omega t - 190.64Z) a_x$$

$$\overline{H}(z,t) = 0.04e^{-3.9971Z} \cos(\omega t - 190.64Z - 4.3°) a_y$$

(viii) What is the amplitude of the electric field at $z = 2$ m?

$$\left|\overline{E(z=2)}\right| = 5e^{-3.9971*2} = 1.7 \text{ mV/m}$$

(ix) What is the power density at $z = 0$ m? At $z = 2$? (Hint: evaluate the $\overline{E}$ and the $\overline{H}$ phasors at $z = 0$ and then evaluate the Poynting Vector using $\overline{S} = \frac{1}{2}\text{Re}\left(\overline{E} \times \overline{H}^*\right)$. Then repeat with the $\overline{E}$ and $\overline{H}$ phasors evaluated at $z = 2$.)

Using the expression for both E and H we Have

$$\left|\overline{S}_{av}\right| = \frac{|E_0|^2}{|\eta|} e^{-2\alpha Z} \cos\theta_\eta \text{ W/m2}$$

For Z = 0

$$\left|\overline{S}_{av}\right| = \frac{|5|^2}{|124.9|}\cos 4.3 = 99.8 \text{ mW/m2}$$

For Z = 2 m

$$\left|\overline{S}_{av}\right| = \frac{|5|^2}{|124.9|} e^{-2*3.9971*2}\cos 4.3 = 11.364 \text{ nW/m2}$$

3. A plane wave traveling in a lossless material at 2450 MHz can be written as

$$\overline{E} = \hat{a}_x A e^{-j\beta z}$$

where $A$ is a complex-valued constant. The relative permittivity is $\varepsilon_r = 2.45$. The amplitude of the wave at $z = 15$ cm is 2 mV/m and the phase angle is -26 degrees. The material is non-magnetic.

(i) What is the value of the phase constant $\beta$?

$\beta = \omega\sqrt{\mu\varepsilon}$ = 80.3718 rad/m => 285 deg/m

(ii) What is the value of the constant $A$?

$\overline{E} = |A| e^{-j\beta z} e^{j\phi_A} \hat{a}_x$

since the material is lossless Then the amplitude of fields do not change with distance

thus $\left|\overline{E}\right| = |A|$ = 2 mV/m

The phase angle = $-\beta Z + \phi_A$

For Z = 0.15 m we have

-26 = $-285 * 0.15 + \phi_A$ => $\phi_A = 16.75°$

(iii) What is the value of the intrinsic impedance?

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.45}} = 240.86$$

(iv) Use Maxwell's Equations to find the vector-phasor for the magnetic field. Give numerical values for all the constants.

$$\overline{H} = \frac{-1}{j\omega\mu}\nabla x\overline{E} = \frac{\beta}{\omega\mu} A e^{-j\beta Z} a_y = 8.3 e^{-j285°*Z} e^{j16.75°} \ \mu A/m$$

(v) What is the amplitude and phase of the magnetic field at $z = 1.12$ m?

Amplitude = 8.3 $\mu A/m$

Phase = -285*1.12 + 16.75 = -302.45 deg