

Program WAVES Version 1D

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Program WAVES animates plane wave incidence on a dielectric half space. These notes describe how to use the program and outline a series of wave demonstrations that can be done with WAVES. The WAVES program is intended to be very easy to use. Students should download the program and run the animation associated with each figure in these notes. This User's Guide has been updated to WAVES Version 1D.

You can download the program by clicking this button: [Get WAVES](#). You can download CPLOT.EXE by clicking this button: [Get CPLOT](#). You can download RPLOT.EXE by clicking this button: [Get RPLOT](#).

Program WAVES graphs the instantaneous value of the electric field as a function of time t and of position (x,y) for a plane wave incident on an interface between two dielectrics. In material #1, the total field is graphed, consisting of the incident electric field plus the reflected electric field. In material #2 the transmitted field is graphed. The program uses the reflection and transmission coefficients given in Reference [1].

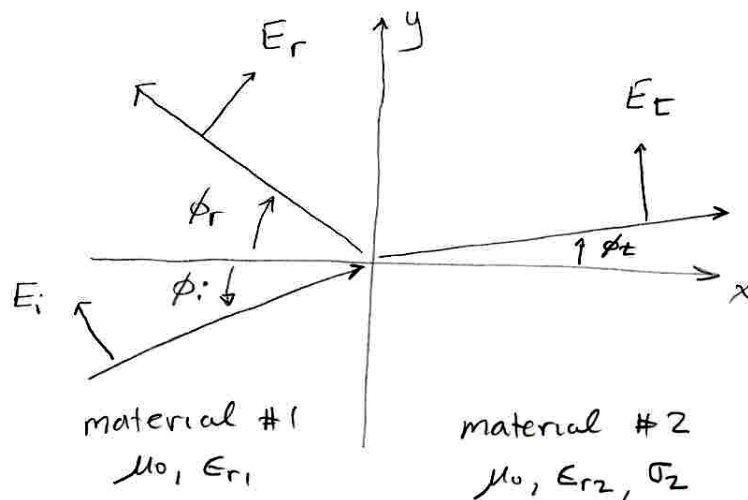


Fig. 1 Geometry used in WAVES.

The geometry of the problem is shown in Fig. 1. The boundary between material #1 and material #2 is the $x=0$ plane, with unit normal vector in the x direction. The “incident” plane wave travels in the xy plane in material #1, for $x < 0$. On the far side of the boundary, for $x > 0$, we have material #2. The wave is partially reflected back into material #1, so the electric field in material #1 is the sum of the electric field of the incident wave plus the electric field of the reflected wave. The wave is partially transmitted into material #2, and the electric field in material #2 is called the transmitted field. At time instant t , WAVES calculates the *instantaneous* value of the electric field in the xy plane on both sides of the boundary between the two materials. The field at time t is drawn as a color map in the xy plane. Then time is advanced by one time step Δt . WAVES computes the instantaneous value of the electric field in the xy plane at time $t + \Delta t$, and then re-draws the color map at time $t + \Delta t$. By re-drawing the color map at successive time instants, the WAVES program animates the movement of the plane wave. The user sees the field advance smoothly across the computer screen.

WAVES is useful for understanding the distinction between the *instantaneous* value of the electric field, $E_z(x, y, t) = A(x, y)\cos(\omega t + \theta(x, y))$, which varies sinusoidally with time at any location xy , and the *amplitude* of the electric field, $A(x, y)$, which is constant with time. WAVES can draw a color map of the amplitude of the field as a function of position xy over the xy plane, for comparison with the animated display. Understanding the amplitude maps in relation to the time animation is a useful exercise.

Running WAVES

Download WAVES.EXE, CPLOT.EXE and RPLOT.EXE. WAVES runs CPLOT, which in turn runs RPLOT, so you need all three programs. Make a directory for WAVES called “WAVES”, and put the three “exe” files into the directory. The best way to run WAVES is to open a DOS window, use “cd” to change directories to the WAVES directory, then type WAVES to start the program. Then WAVES can find CPLOT.EXE when it is needed.

Another way to organize your work is to put the three “exe” files into a directory that is in your “path”. You can start WAVES by double-clicking the mouse on the WAVES icon. Or you can open a DOS window and type WAVES, and because the directory in which WAVES.EXE is stored is part of the path, the operating system will find the WAVES program and run it. When WAVES tries to start CPLOT, the CPLOT.EXE file will be found because the directory in which it is saved is part of the “path”.

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*** WAVES      ** VERSION 1D      **** March 19, 2004 ****
----- Program WAVES ----- Plane Waves at a Dielectric Interface -----
Frequency:      300.0 MHz.
Angle of Incidence: 30.00 degrees.
Polarization:  PAR PERpendicular or PARallel.

Electrical properties of the dielectrics:
Medium 1, Relative Permittivity: 1.0
Medium 2, Relative Permittivity: 3.0
Medium 2, Conductivity: 0.000 mS/m

F3 = Run the simulation.
F4 = Graph the amplitude of the field with CPLOT.
F5 = Critical angle and Brewster Angle.
F6 = Intrinsic impedance/reflection coefficient.
F7 = Area of the graph.
F8 = Time cycle.
F10= Exit from the program.

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Fig. 2 The WAVES main menu.

The WAVES Main Menu

Start WAVES, and you will see the main menu of Figure 2. There are various boxes in which you can enter the parameters of your problem, and the function keys are used to make the program do stuff.

The first data box asks for the frequency in MHz. The second asks for the angle of incidence ϕ_i in Fig. 1, relative to the unit normal vector to the surface of the dielectric, which is the x axis. If the angle is zero degrees, the wave travels parallel to the x -axis, which is horizontally across the computer screen, and is normally incident on the boundary between the two media. If the angle of incidence is $\phi_i=45$ degrees, the wave travels upward and to the right, and is obliquely incident on the surface of the dielectric. If the angle is 90 degrees, the wave travels parallel to the y -axis, which is vertically up the screen. Then the wave is traveling parallel to the interface between the two media.

Specify either “perpendicular” or “parallel” polarization as defined in Ref. [1]. Figure 1 shows the electric field vector parallel to the “plane of incidence”, hence Fig. 1 shows the parallel polarization.

Specify the relative permittivity of material #1, which is the material to the left of the interface, by typing the value into the menu box on the screen. Material #1 is always lossless, that is, with zero conductivity. Both material #1 and material #2 are “non-magnetic”, meaning that the permeability is μ_0 , the permeability of free space. Specify the relative permittivity and conductivity for material #2. Note that the units for the conductivity are milliSiemens per meter.

WAVES uses the function keys to control the program. The function keys are used to choose various actions:

F3=show the *instantaneous* value of the electric field as a function of time in the xy plane.

F4=draw a color map of the amplitude of the electric field as a function of x and y .

F5=report the value of the “critical angle” for total reflection and the “Brewster angle” for total transmission.

F6=report the intrinsic impedance of each material, the propagation constant, and the reflection and transmission coefficients (useful for checking homework answers!)

F7=define the area in the xy plane where the field is plotted.

F8=define the “time cycle”, meaning the number of periods of the sine wave for drawing the animation.

F10=terminate the program and return to the operating system.

The action obtained with each function key is explained in the following.

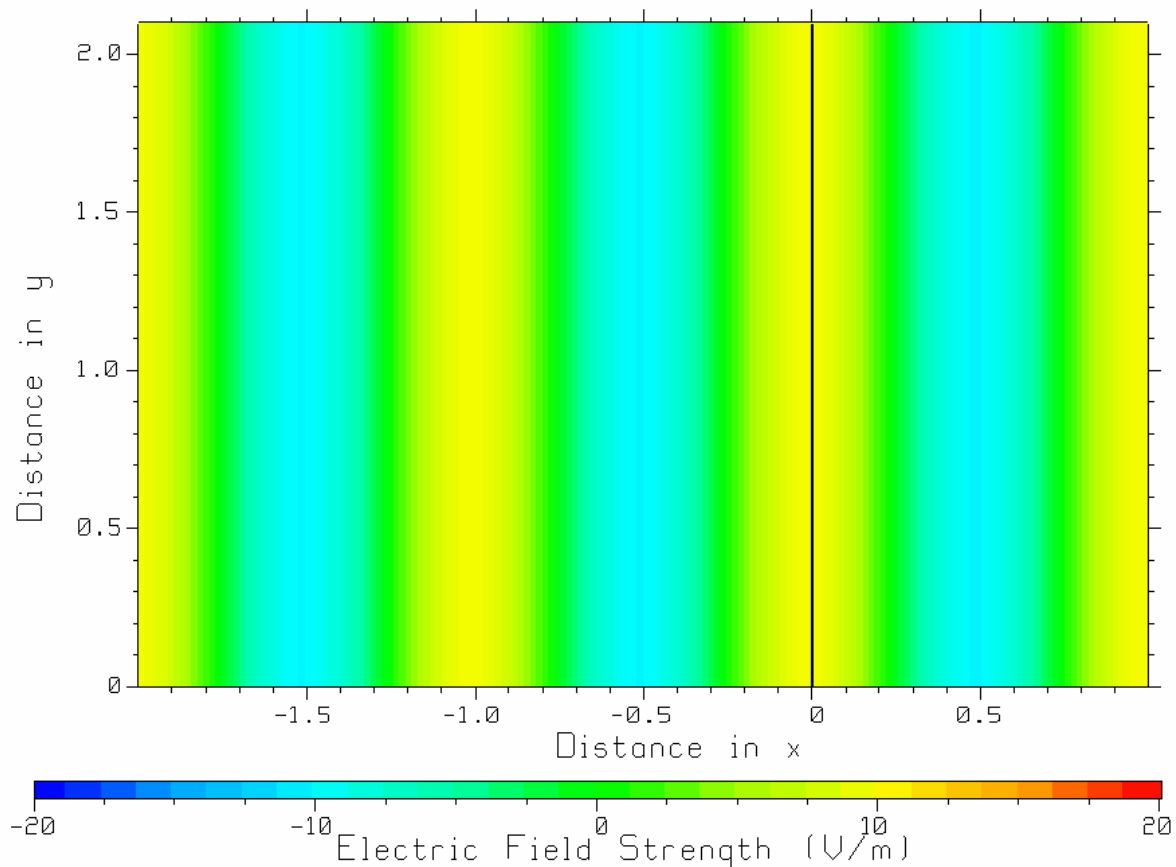


Fig. 3 The instantaneous value of the field of a plane wave in free space.

WAVES Animation

The purpose of WAVES is to draw the instantaneous value of the electric field as a function of time throughout the AC cycle for the plane wave problem of Fig. 1. Specify the frequency, the direction of incidence, parallel or perpendicular polarization, the permittivity of material #1 and the permittivity and conductivity of material #2 in the menu boxes, and then type the F3 key to animate the plane wave in the xy plane.

To illustrate the animation, let's start with a really simple problem. Suppose the plane wave traveling parallel to the x axis in free space and there is no second material hence no reflection. Set the relative permittivity of both materials to unity, $\epsilon_{r1} = 1$ and $\epsilon_{r2} = 1$, and the conductivity of material #2 to zero, $\sigma_2 = 0$. Set the angle of incidence to zero degrees so that the wave travels parallel to the x axis. Since both materials are free space there is no reflected wave and the incident wave is totally transmitted into material #2. Type F3 and watch the wave travel across the computer screen in the x direction from left to right. Push the space bar to stop the animation. Fig. 3 shows the instantaneous value of the electric field at a time instant when the largest electric field coincides with the surface of the dielectric at $x=0$.

WAVES graphs the electric field in the region of the xy plane from $x = -2$ to $x = 1$ m, and from $y = 0$ to $y \approx 2$ m. (You can control the size of the region by invoking the "area of the graph" menu with F7 in the main menu.) The instantaneous value of the electric field is shown on a false-color scale from -20 V/m to 20 V/m. The amplitude of the incident wave is 10 V/m; the range of the field strength scale allows standing-waves with a reflection coefficient of 1 to be graphed. WAVES divides the region of the xy plane into boxes, and in each box uses a color to depict the strength of the field. The number of boxes also controls the speed of the animation: use more boxes and the computer computes more field values at each time step, resulting in slower movement of the wave across the screen.

The false-color contour map is more informally called a "color map". It takes some experience to learn to read color maps skillfully. In the WAVES display shown in Fig. 3, negative field strengths of -10 V/m are represented with the color cyan; zero field strength is represented in green, and $+10$ V/m is yellow. Fig. 3 shows a plane wave in free space at 300 MHz traveling in the $+x$ direction. The wave has a peak value of 10 V/m, represented in yellow. Figure 3 shows that the yellow regions are separated by 1 meter, and as expected this is 1 wavelength at the operating frequency of 300 MHz. The field strength is represented in green where the wave has zero-crossings, and the distance between zero-crossings is $\frac{1}{2}$ wavelength or 0.5 m. Finally the wave has "troughs" of -10 V/m field strength, represented in cyan. As time advances in the WAVES display the peaks advance across the screen at a constant speed, equal to the speed of light.

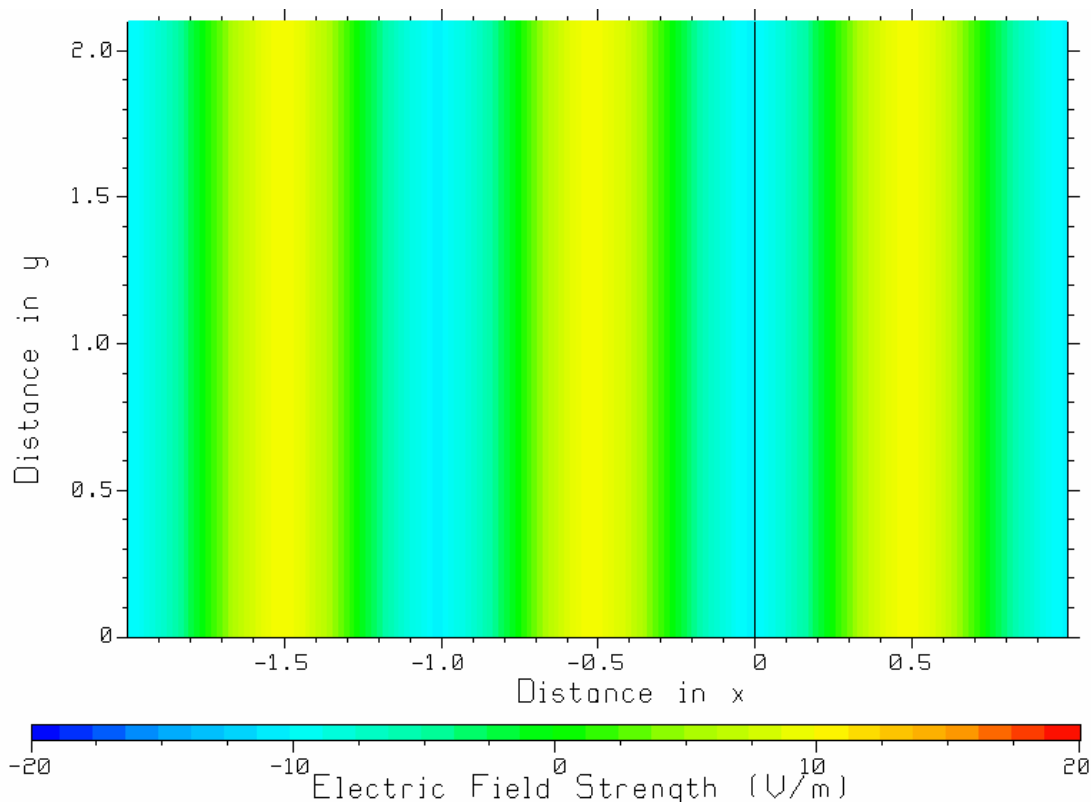


Fig. 4 The field of Fig. 3, half an AC cycle later.

Re-start the animation by returning to the main menu (push the “return” key) and typing F3 once again. As time advances, the wave travels smoothly across the screen. Half a period after the field shown in Fig. 3, the wave has advanced half a wavelength to the right (in the positive x direction) and the instantaneous electric field strength looks like Fig. 4. There is now a “trough” at $x = 0$, shown with the color cyan. To stop the animation, push any key. To copy the screen to the Windows buffer, use Edit then Copy in the upper left corner of the window. You can then paste the display into a document.

At each instant of time, WAVES computes the instantaneous value of the electric field over a region of the xy plane, and then draws a “color map” of the instantaneous value. At each point in space x, y the electric field varies sinusoidally with time, $E_z(x, y, t) = A(x, y)\cos(\omega t + \theta(x, y))$, with amplitude $A(x, y)$ and phase angle $\theta(x, y)$. At some instant of time t , WAVES evaluates the instantaneous electric field $E_z(x_j, y_k, t)$ over a grid of points $\{x_i, y_j\}$ and graphs the values as a color map. Then time is advanced by one time step, Δt , and WAVES computes the new instantaneous field $E_z(x_i, y_j, t + \Delta t) = A(x_i, y_j)\cos(\omega(t + \Delta t) + \theta(x_i, y_j))$ at each point x_i, y_j , and graphs the new field values with a new color map. Since Δt is much shorter than the period $T = \frac{2\pi}{\omega}$ of the sine wave, the change in the field is small from one time step to the next, and the field appears to advance smoothly across the computer screen.

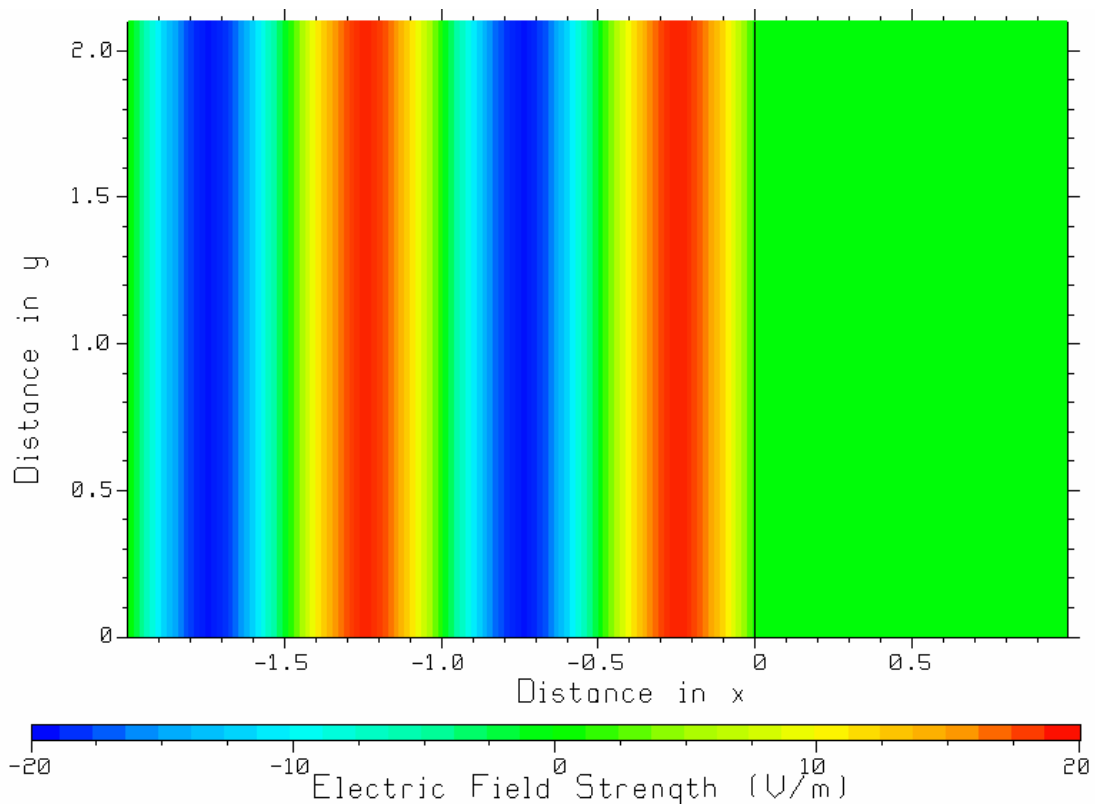


Fig. 5 The instantaneous value of the field when material #2 is a good conductor.

Reflection from a Good Conductor

If material #2 is changed to a very good conductor with relative permittivity 1 and conductivity say 1,000,000,000 mS/m, then the reflection coefficient is (almost) -1 and the reflected wave is (almost) equal in magnitude to the incident wave. Type F3 to see WAVES animate the standing wave in material #1. In Fig. 5 time has been frozen when there is a peak of 20 V/m (red) at a distance of $\frac{1}{4}$ of a wavelength from the surface of the conductor at $x=0$, and a trough of -20 V/m (blue) at $\frac{3}{4}$ of a wavelength from the interface. Fig. 5 depicts a standing wave at an instant of time when the field strength $\frac{1}{4}$ of a wavelength from the surface has a maximum value.

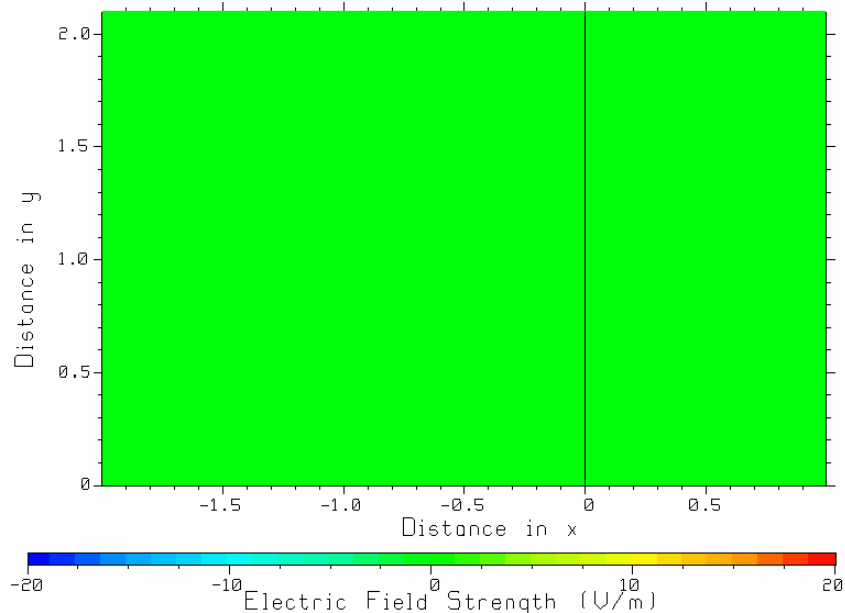


Fig. 6 The instantaneous value of the field $\frac{1}{4}$ of a period after that of Fig. 5.

If time is allowed to advance by one-quarter of the period, then the instantaneous field is zero everywhere, as shown in Fig. 6. The field is zero for only one instant of time; then time advances and the field becomes positive at some locations and negative at others. But notice there are certain locations where the field is *always* zero, at integer multiples of the half-wavelength from the surface of the good conductor.

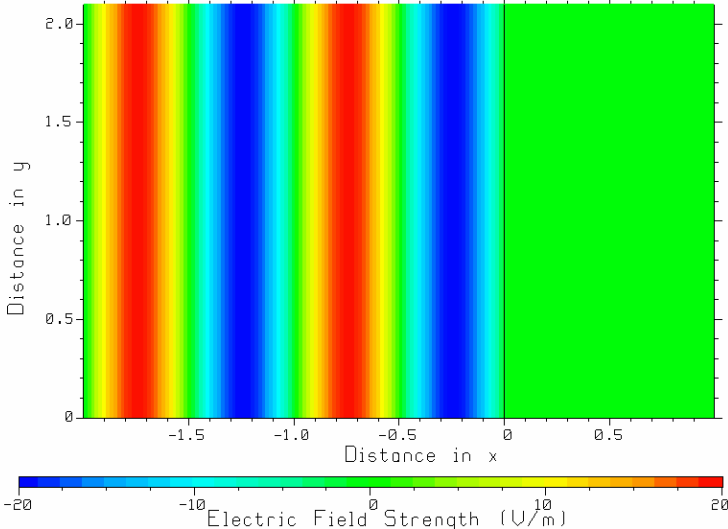


Fig. 7 The instantaneous electric field one-half of the period after that of Fig. 5.

If time is allowed to advance a further half-period then the instantaneous field is that of Fig. 7. There is now a trough (blue) a quarter-wavelength from the surface of the conductor and a peak (red) half a wavelength from the surface.

The animation associated with a plane wave in free space of Figs. 3 and 4 shows fundamentally different behavior from that of the plane wave reflected from the surface of a good conductor in Figs. 5, 6, and 7. Figs. 3 and 4 show a “pure” traveling wave, moving across the screen from left to right at constant wave speed. Figs. 5, 6 and 7 show a “pure” standing wave, which marches in place as time advances. In class we have shown that a “pure” standing wave is the sum of a wave traveling to the right plus a wave of equal amplitude traveling to the left.

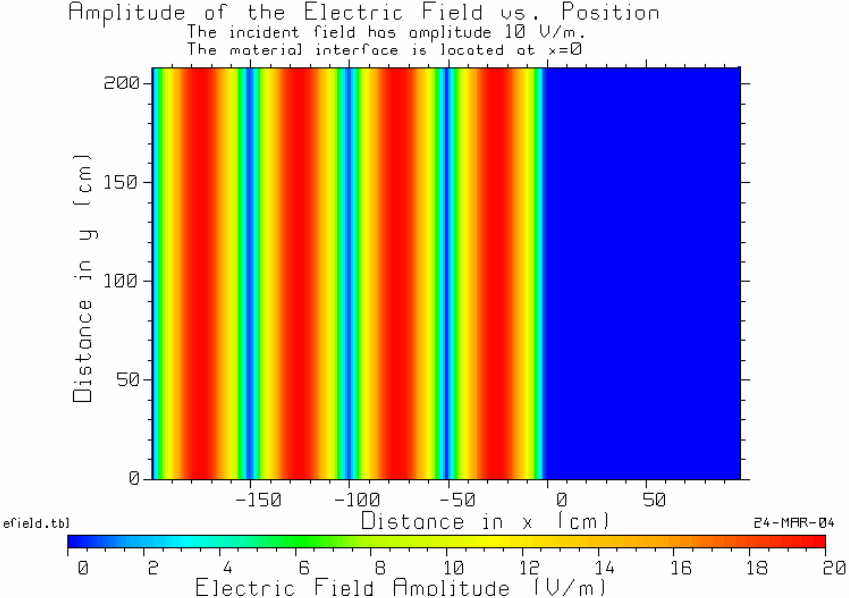


Fig. 8 The amplitude of the electric field in material #1 and #2 when material #2 is a good conductor.

Graphing the Amplitude of the Field

It is instructive to compare the color maps of the *instantaneous value* of the field in Figs. 5, 6 and 7 with a color map of the *amplitude* of the field. The amplitude of the electric field time function $E_z(x, y, t) = A(x, y) \cos(\omega t + \theta(x, y))$ is $A(x, y)$, and is *not* a function of time.

Suppose we want to study the standing-wave pattern associated with the instantaneous fields of Figs. 5, 6 and 7. Run WAVES and set up the problem (300 MHz, 0 degree incidence, material #1 with relative permittivity 1, material 2 with relative permittivity 1 and conductivity 1,000,000,000 mS/m). Type F3 to draw the animation, then the space bar to stop the animation and the space bar again to return to the main menu. (You must draw the animation to force WAVES to calculate the field amplitude function.) Then type F4 to start CPLOT and draw the amplitude of the field, shown in Fig. 8.

Fig. 8 depicts the amplitude on a false-color scale, but note that the correspondence of colors to field strengths is quite different than in WAVES. Since the amplitude is, by definition, *positive* in value, there is no reason to “waste” colors from dark blue to green depicting negative values. In CPLOT’s display in Fig. 8, dark blue is zero field, cyan is field strength about 4 V/m, green about 6 C/m, yellow about 10 V/m, orange about 15 V/m, and red 20 V/m. Since the incident field is 10 V/m field strength and the reflection coefficient has magnitude unity, the reflected field is also 10 V/m amplitude. At the maxima in the standing-wave pattern the field strength is the sum of the incident wave plus the reflected wave, or 20 V/m, which is depicted in dark red. Note that there are maxima in the standing-wave pattern located $\frac{1}{4}$ wavelength from the interface, $\frac{3}{4}$ wavelengths, $1\frac{1}{4}$ wavelengths, and so forth. The zeros in the standing-wave pattern occur where the incident wave and the reflected wave are exactly out of phase with one another. There is a null in the electric field at the surface of the conductor, and at half-wavelength intervals from the surface.

When F4 is typed from the main menu of WAVES to graph the amplitude as a function of position xy , WAVES creates the graph by writing a “table” file called “efield.tbl”, containing the value of the amplitude function $A(x, y)$ at the same grid of points $\{x_i, y_j\}$ used for the animation. Then WAVES starts program CPLOT.EXE to actually draw the color map. CPLOT is a general-purpose color contour plotter. You can format the graph in CPLOT according to your tastes and then copy the color map into a homework assignment solution, for example. CPLOT gives considerable flexibility for plotting the data on a linear scale or decibel scale, and for controlling the correspondence of field strength values to colors. Also, note that in CPLOT you can click the mouse on a point in the color map, and the program “reads back” the position of the mouse, x and y , and the value of the field strength, which is called the “ z ” coordinate in CPLOT.

It is useful to study the standing-wave pattern by graphing the amplitude of the electric field as a function of distance in the x direction for a constant value of y , say $y = 0$. CPLOT can draw a “cut” through the contour map, graphing the field strength as a function of x for constant y . In CPLOT, to draw a “cut” of $A(x, y)$ as a function of x for $y=0$, in the main menu type F8 to invoke the “cut” menu, then type F5 to change to a y -cut, then type F3 to draw the cut. CPLOT creates a file called “cplot.rpl”, then runs the program RPLOT.EXE to actually graph the data. RPLOT is a general-purpose rectangular plotting program. An example is shown in the following.

The standing-wave pattern is a graph of the amplitude of the electric field amplitude $A(x, y)$ as a function of x for constant y , say $y = 0$. Using the F8 “cut” menu in CPLOT, then type F5 to change from a constant- x cut to a constant- y cut, and type F3 to graph the field. CPLOT starts the general-purpose rectangular graphing program RPLOT, which creates the graph shown in Fig. 9. In RPLOT if you click the mouse on the curve, RPLOT reports the nearest data point in the input data file.

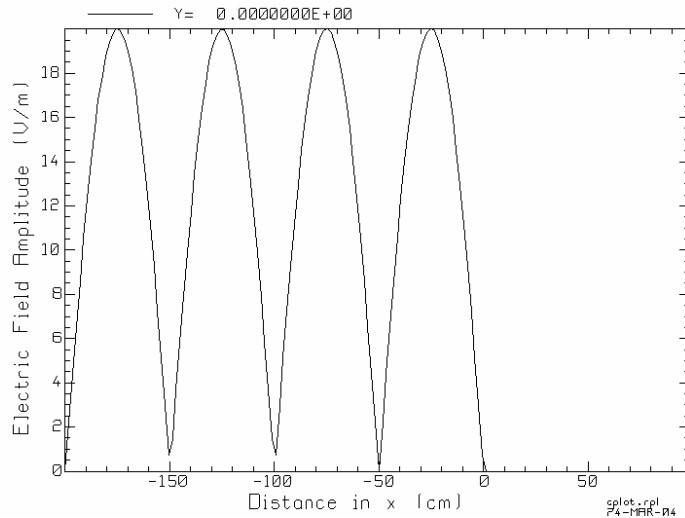


Fig. 9 The electric field amplitude as a function of x for $y = 0$.

Fig. 9 shows that the electric field is zero in the perfect conductor. In material 1, the electric field has maxima of 20 V/m at -25 cm, -75 cm, -125 cm, and so forth, and (approximate) nulls at -50 cm, -100 cm, -150 cm, as expected.

Intrinsic Impedance, Propagation Constant, Reflection and Transmission Coefficients

Suppose we set up a problem where material #1 is free space with $\epsilon_{r1}=1$, and material #2 is a dielectric with $\epsilon_{rs} = 4$ and conductivity $\sigma_2 = 10$ mS/m, at 300 MHz. We expect to see a standing wave in material #1, and a wave transmitted into material #2. Homework problems often ask for the computation of the propagation constant in each material, of the intrinsic impedance, and of the reflection and transmission coefficient. Type F6 in the WAVES main menu, and WAVES reports the propagation constant, the intrinsic impedance, and the reflection and transmission coefficients to the screen, as shown in Fig. 10.

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*** WAVES    ** VERSION 1D    **** March 19, 2004 ****

Medium #1 (lossless)
  Intrinsic Impedance:  376.7303 ohms.
  Propagation Constant:  0.0000 + 6.288 j
  Wavelength:          0.9993 meters.

Medium #2
  Intrinsic Impedance:  186.8057 + 13.9134j ohms.
  Magnitude:          0.1873231E+03 Phase:  4.259571 degrees.
  Propagation Constant:0.9392 + 12.61 j
  Wavelength:          0.4983 meters.
  Skin Depth:          1.065 meters.

Reflection and transmission coefficients:

Reflection coefficient: -.3362 + 0.3299E-01j
Reflection coefficient magnitude:0.3378

Transmission coefficient: 0.6638 + 0.3299E-01j
Transmission coefficient magnitude:0.6646

Press any key to continue...

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Fig. 10 Type F6 to calculate the intrinsic impedance, propagation constant, and reflection and transmission coefficients.

For the example given above, Fig. 10 shows that WAVES calculates the intrinsic impedance for free space as 376.73 ohms, as expected. The propagation constant is $\gamma = \alpha + j\beta$. In material #1 the attenuation constant is $\alpha = 0$ Np/m (because by definition material #1 is lossless) and the phase constant is $\beta = \omega\sqrt{\mu_o\epsilon_{r1}\epsilon_o} = 6.288$ radians/meter for a 300-MHz wave in free space. The program reports that the wavelength is $\lambda = \frac{2\pi}{\beta} = 0.9993$ meters. Material #2 is a lossy dielectric so $\gamma = \sqrt{j\omega\mu_o(\sigma_2 + j\omega\epsilon_{rs}\epsilon_o)}$ $= \alpha_2 + j\beta_2$. The program reports that with $\epsilon_{r2} = 4$ and $\sigma_2 = 10$ mS/m, the material has $\alpha_2 = 0.9392$ Np/m and $\beta_2 = 12.61$ radians/meter. The reader should verify that these figures are correct.

The program computes and reports the transmission coefficient and the reflection coefficient. These values are dependent on the direction of incidence of the plane wave and on the polarization. For normal incidence, Fig. 10 shows that the reflection coefficient is $\Gamma = -0.3362 + j0.03299$, and the transmission coefficient is $T = 0.6638 + j0.03299$. The reader should verify by hand-calculation that these values are correct. For incidence at angles other than 0 degrees, the reflection and transmission coefficients are dependent on the polarization and the WAVES program uses the appropriate formula according to the user's choice of parallel or perpendicular polarization.

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*** WAVES    ** VERSION 1D    ****  March 19, 2004  ****
Frequency:           300.00 MHz.
Angle of incidence:   10.00 degrees.
Angle of transmission: 31.40

Brewster angle:      18.43
(Total transmission for the parallel polarization.)

Critical angle for total reflection:
Critical angle:      19.47

Press any key to continue...

```

Fig. 11 Type F5 to find the Brewster Angle and the Critical Angle.

Critical Angle and Brewster Angle

If the polarization is “parallel” then if the angle of incidence is equal to the Brewster Angle, there will be total transmission from material #1 into material #2. Also, material #1 is “more dense” than material #2, $\epsilon_{r1} > \epsilon_{r2}$, then if the angle of incidence is greater than the Critical Angle, there is total reflection of the wave back into material #1. WAVES evaluates the angle of transmission, the Brewster Angle and the Critical Angle and reports them to the screen when the F5 key is typed. For example, if $\epsilon_{r1} = 9$, $\epsilon_{r2} = 1$ and material #2 is lossless, and the angle of incidence is 10 degrees from the normal, then typing F5 obtains the report shown in Fig. 11. WAVES evaluates Snell’s Law and reports that the angle of transmission is 31.4 degrees. The Brewster Angle is reported as 18.43 degrees, and the critical angle is 19.47 degrees. These angles are demonstrated with WAVES below.

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*** WAVES      ** VERSION 1D      **** March 19, 2004 ****
----- Graph Size Menu -----
Use this menu to set the size of the region in the xy plane where
the where the field will be graphed.

The region size is set in terms of the free-space wavelength.
The interface between material #1 and material #2 is at x=0.
The region starts at x=XMIN (negative) and ends at x=XMAX (positive).
The region size in the y-direction is chosen by the program so that
the graph fits the size of the screen.

Material #1: From XMIN= -2.000 free-space wavelengths (<0)
Material #2: To XMAX= 1.000 free-space wavelengths (>=0)

The region is divided into cells and the field is represented by a color
which fills each cell. The user sets the number of cells cross the
screen and the program chooses the number of cells up the screen in proportion.

Number of cells across the screen: 170

XY axis labels? YES (Change with F1.)

F1 = Toggle the xy axis labels on or off.

F10= Return to the main menu.

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Fig.12 Type F7 to set the size of the patch in the xy plane used by the WAVES program.

The Area Covered by the Color Map

WAVES scales the area of the xy plane over which the field is graphed according to the frequency. When you change the frequency, the program re-calculates the size of the “patch” in the xy plane used by the WAVES program. The user can control the size of the patch with the F7 menu. The user specifies the size of the “patch” in the horizontal direction in free-space wavelengths. Thus, if the frequency is changed the size in meters scales according to the wavelength. The program uses the size of the region in the x direction to compute a corresponding size in the y direction so that the area covered fits conveniently on the computer screen.

WAVES works by dividing the interval between the minimum x -value and the maximum x -value into N_x cells, and divides the y -interval into N_y cells. The user can specify the number of cells in the x -direction, with the default choice being $N_x=170$ cells. The program chooses N_y in proportion. The program calculates the centers of the cells, $\{x_i, y_j : i = 1, \dots, N_x; j = 1, \dots, N_y\}$, and then evaluates the electric field strength at the cell centers to create the color map of the field. If a large value of N_x is specified, then fine detail is seen in the color map, but the program must do a lot of computation at each time step and so the animation is slow. If a small value of N_x is chosen, the resolution of the graph is poor but the animation goes quickly.

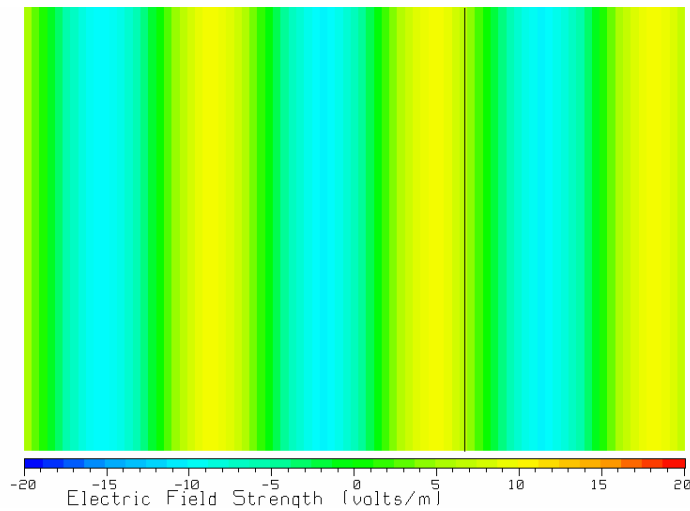


Fig. 13 The x and y axis labels can be suppressed to obtain a larger display.

The F7 menu provides a toggle to let the user turn off the labeling of the x -axis and y -axis, as shown in Fig. 13. Once it is understood that distance across the screen is the x coordinate and up the screen is the y coordinate, the axes can be suppressed to obtain a larger display of the “patch” of xy plane on the computer screen.

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*** WAVES    ** VERSION 1D    **** March 19, 2004 ****
----- Time Cycle Menu -----
The program graphs the field as a function of time from t=0 until
t=N*T, where T is the period of the wave, T=1/f and f is the frequency.

Use this menu to choose the number of periods N in the time cycle.

Number of periods: 4.000
F10= Return to the main menu.

```

Fig. 14 Type F8 to obtain the time cycle menu.

The Time Cycle

WAVES chops the period of the AC field, T , into time steps of $\Delta t = \frac{T}{N_t}$, with N_t time steps per AC cycle. WAVES is “hard-wired” to use $N_t = 50$ time steps per cycle. When the user types F3, the program time-steps through N_p periods, for a maximum time of $t_{\max} = N_p T$ seconds, called the “time cycle”. Sometimes it is convenient to allow the animation to continue for a long time, and then the number of periods can be set to a large value with the “time cycle” menu shown in Fig. 14. Thus, if a lecturer is describing standing-wave behavior in some detail, a long time-cycle can be used to allow the animation to run during the whole presentation. Note that typing any key freezes the animation, and typing any key once again returns to the WAVES main menu.

Demonstrations with WAVES

This section describes demonstrations that can be done with the WAVES program.

A “Traveling Wave”

We can demonstrate a simple “traveling wave” by making both materials the same, say $\epsilon_{r1} = 1$ and $\epsilon_{r2} = 1$, with zero conductivity for material #2. Set the frequency to 300 MHz and the angle of incidence to zero. Run the simulation and see a plane wave traveling from left to right across the screen, as shown in Figures 3 and 4 at two separate instants of time. This display shows the fundamental “traveling wave”, which is the basis of our understanding of wave phenomena. Verify that the wavelength corresponds to the frequency and the permittivity of free space.

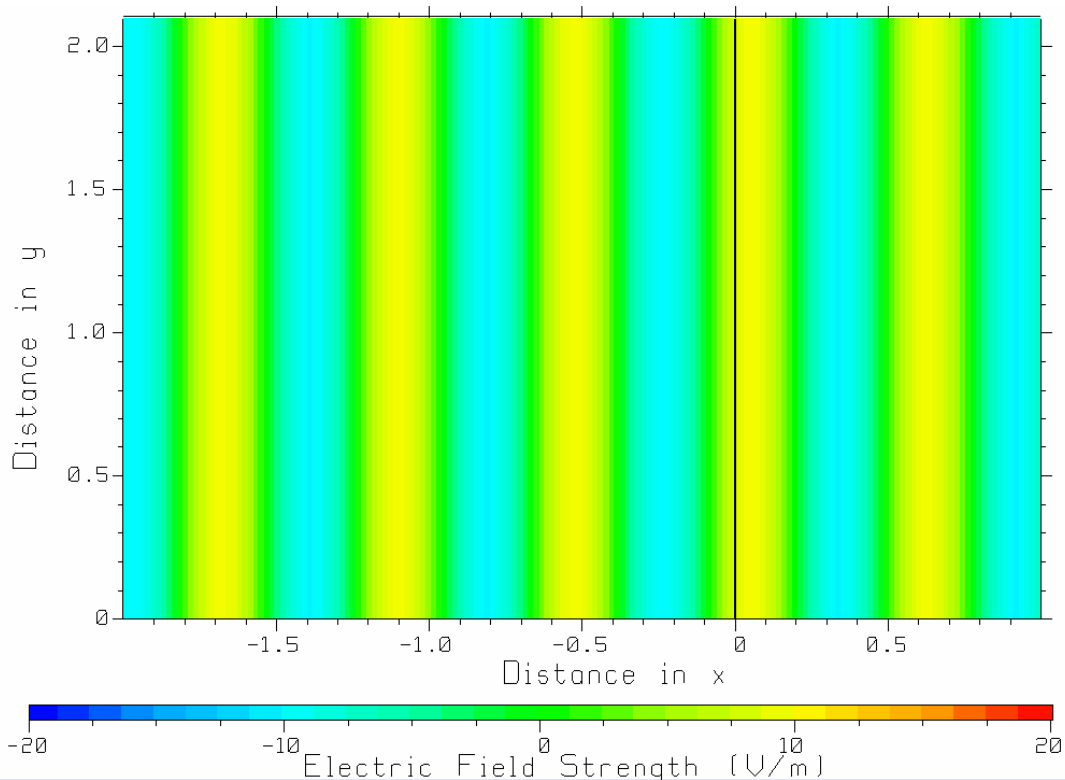


Fig. 15 The plane wave in a dielectric of relative permittivity 3 has a shorter wavelength.

Traveling Wave in a Dielectric

Change the relative permittivity of both media to 3 and re-run the simulation to obtain Fig. 15. Compare the wave in Fig. 15 with that in Fig. 4. It is clear that the wavelength with $\epsilon_r = 3$ is shorter than in free space. The student should read the wavelength from Fig. 15, and verify that it is the expected value for $\epsilon_r = 3$. Compare the speed of travel of the wave with $\epsilon_r = 1$ with the speed of travel of the wave for $\epsilon_r = 3$. If you watch the animation carefully, the wave with $\epsilon_r = 3$ travels more slowly than the wave in free space.

Total Reflection and Standing Waves

The waves in Fig. 3 and Fig. 15 are “traveling waves” and are the simplest kind of wave. When material #1 and material #2 have different permittivities then there is a reflected wave from the boundary between the two materials. Then in material #1, the field is more complex, being the superposition of a wave traveling to the right plus a wave traveling to the left.

The easiest way to obtain almost complete reflection at the boundary between the two materials is to make material #2 a very good conductor. Set the permittivity of material #2 to unity and the conductivity to, say, 1,000,000,000 mS/m. The reflection coefficient is then -0.9998 , close to the value of -1 for a perfect conductor. Fig. 5 shows the resulting field.

Fig. 5 is quite different from Fig. 3 or Fig. 4. In the animated display, the peaks and troughs for Fig. 5 do not move across the screen as they do for the traveling wave of Fig. 4. The field oscillates in place and the wave is called a “standing wave”.

Fig.4 shows the field in material #1 at an instant of time where there are large maxima(red) and deep troughs(blue). The field in material #2, the good conductor, is of course (almost) equal to zero. As time progresses the wave does not travel across the screen. Instead the green bars of zero field strength remain in place and the peaks and troughs oscillate up and down without moving. This is a standing wave. A pure standing wave is composed of equal-amplitude waves traveling in opposite directions. The incident wave travels to the right and the reflected wave travels to the left.

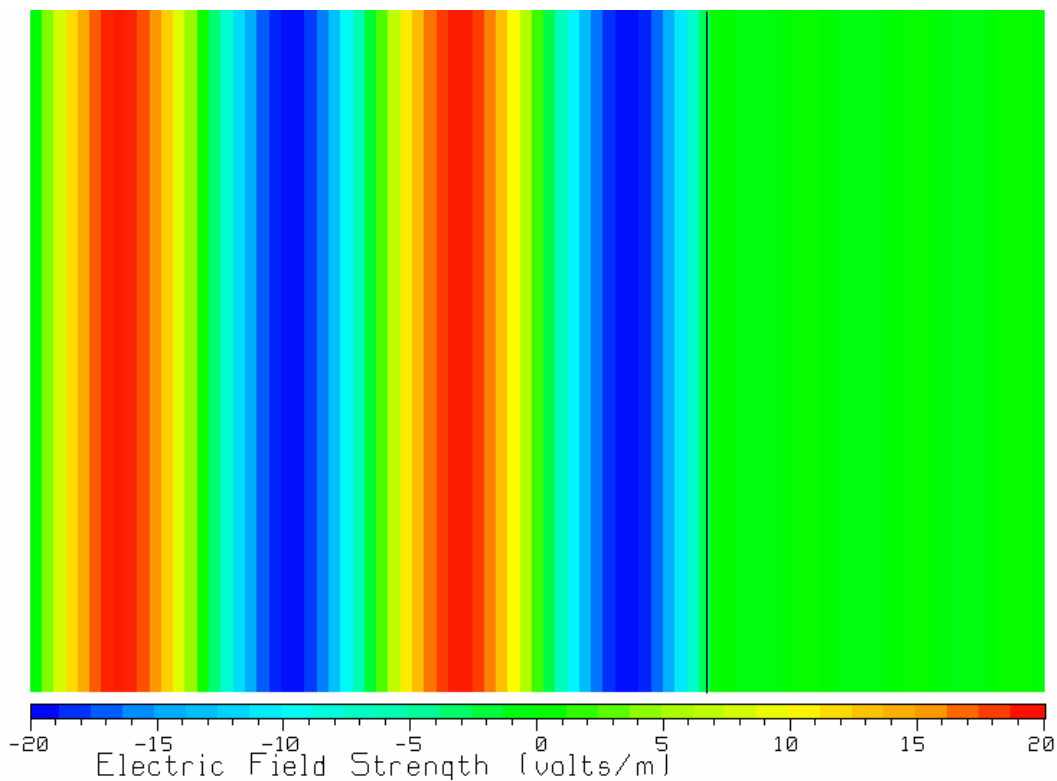


Fig. 16 A standing wave in material #1 when material #2 is a lossless material with a high relative permittivity of 1000.

Another way to obtain a standing wave is make material #2 lossless (zero conductivity), but with a high relative permittivity, say equal to 1000. This leads to a large impedance mismatch between the two regions, so little field is transmitted into the second material. The reflection coefficient is -0.94 , and the transmission coefficient 0.06 . The wave is almost fully reflected back into material #1. Fig. 16 shows that the standing wave in this case is very similar to that for the perfect conductor in Fig. 6. Note that for Fig. 16, the labels for the x axis and the y axis have been suppressed with F1 on the F7 menu in WAVES.

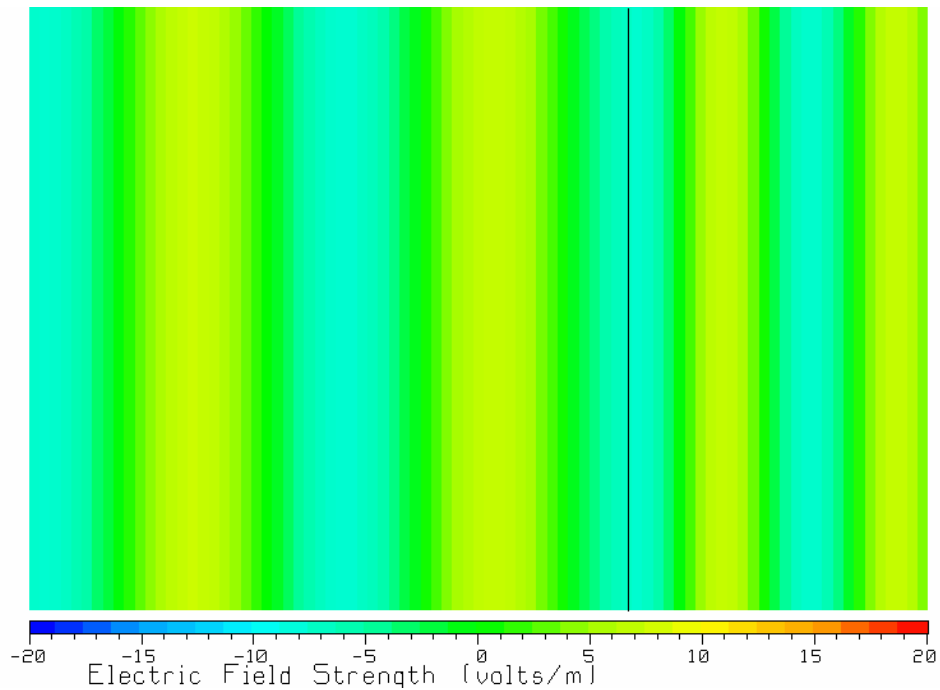


Fig. 17 Partial reflection, with a reflection coefficient of -0.27 .

Partial Reflection and Partial Transmission

Make the relative permittivity of region #1 equal to one and that of region #2 equal to 3, with zero conductivity. Then we have partial reflection. The reflection coefficient is -0.27 and the transmission coefficient is 0.73 . Most of the wave is transmitted into material #2. Fig. 17 shows the instantaneous value of the electric field strength, and is a “snapshot” of the animation in the WAVES program. In material #2 we see the field traveling away to the right with the shorter wavelength characteristic of a wave in a dielectric. In material #1, the wave looks very much like a traveling wave, but if you watch the animation closely the phase fronts do not progress uniformly across the screen. This behavior is enhanced if the relative permittivity of material #2 is increased.

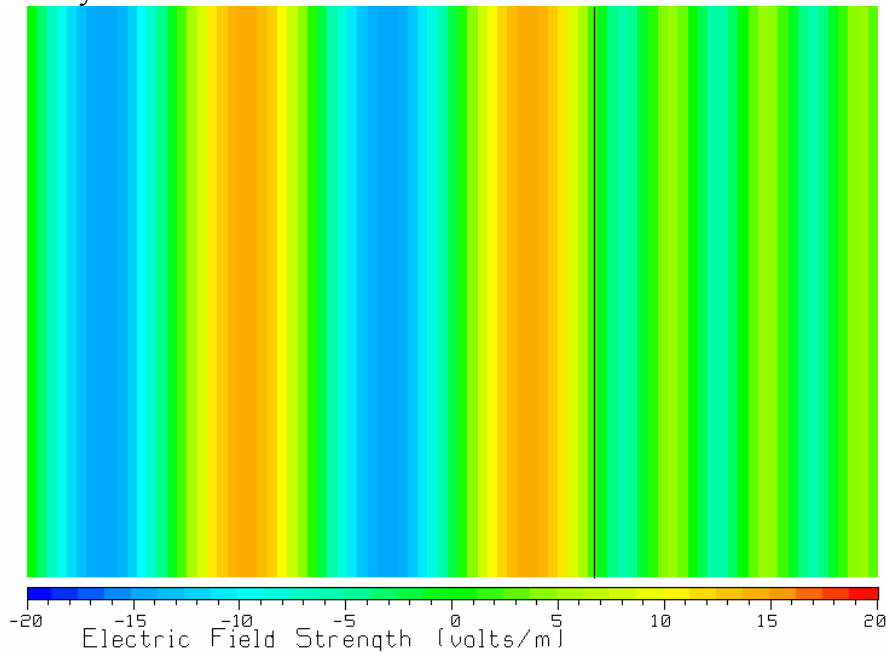


Fig. 18 Partial reflection with a reflection coefficient of -0.5 .

Fig. 18 shows the field at one instant of time with a relative permittivity for material #2 of $\epsilon_r=9$, hence a reflection coefficient of -0.5 . The field to the left of the boundary has troughs of -15 V/m, which shows up as a blue region, and peaks of 15 V/m, graphed as orange. As time progresses the peak of the wave does not progress uniformly across the screen. If the relative permittivity of material #2 is further increased, the behavior becomes more and more like the standing wave of Fig. 5.

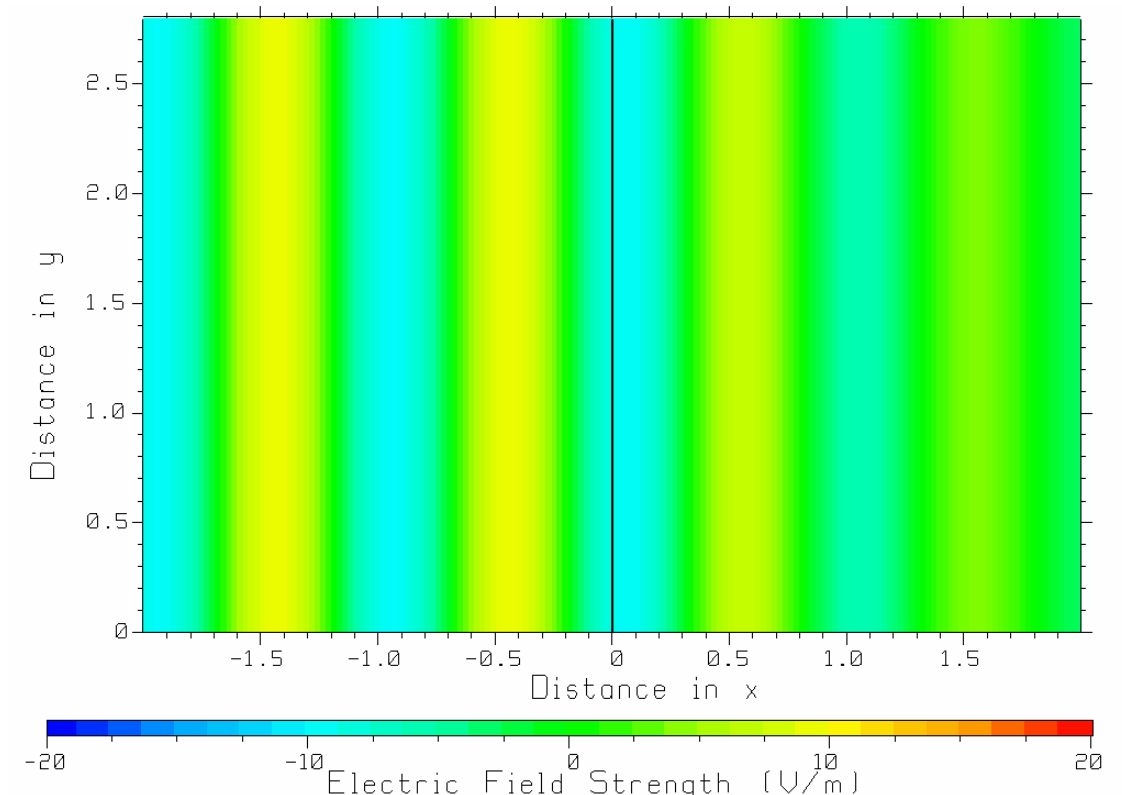


Fig. 19 Transmission from a lossless material at left into a lossy material at right.

Transmission into “Lossy Free Space”

If material #2 is an insulator or “dielectric” that has a small conductivity, it is called a “lossy dielectric”. It absorbs energy. Then the wave transmitted into material #2 decreases in amplitude as it travels into material #2. We can demonstrate this by setting the relative permittivity of material #1 to 1 for free space, and making material #2 “lossy free space”, with relative permittivity 1 and conductivity, say, 3 mS/m. With this small conductivity the reflection coefficient magnitude is only 0.04467 , and the transmission coefficient 0.997 , so most of the wave is transmitted into the material #2. So we expect that the standing-wave ratio in material #1 is $(1 + 0.04467)/(1 - 0.04467) = 1.09$. Set the right-hand boundary of the space to 2 wavelengths to see farther into material #2 (use the F7 “area of the graph” menu) and run the simulation.

Fig. 19 shows the wave progressing from free space at left into the lossy region at right. In material #1 for $x=0$ the amplitude of the wave does not change as the wave travels from left to right across the screen. The two yellow bars at left are of the same intensity, representing electric field strength equal to about 10 V/m. But after the wave crosses the black line representing the boundary, the amplitude of the wave in material #2 gradually diminishes. Thus the yellow maximum just to the right of the interface in Fig.10 is almost as large as the yellow maxima to the left. But moving a wavelength deeper into the 2nd material, the yellow maximum at right is much less intense, showing a greatly decreased amplitude as the wave propagates into material #2.

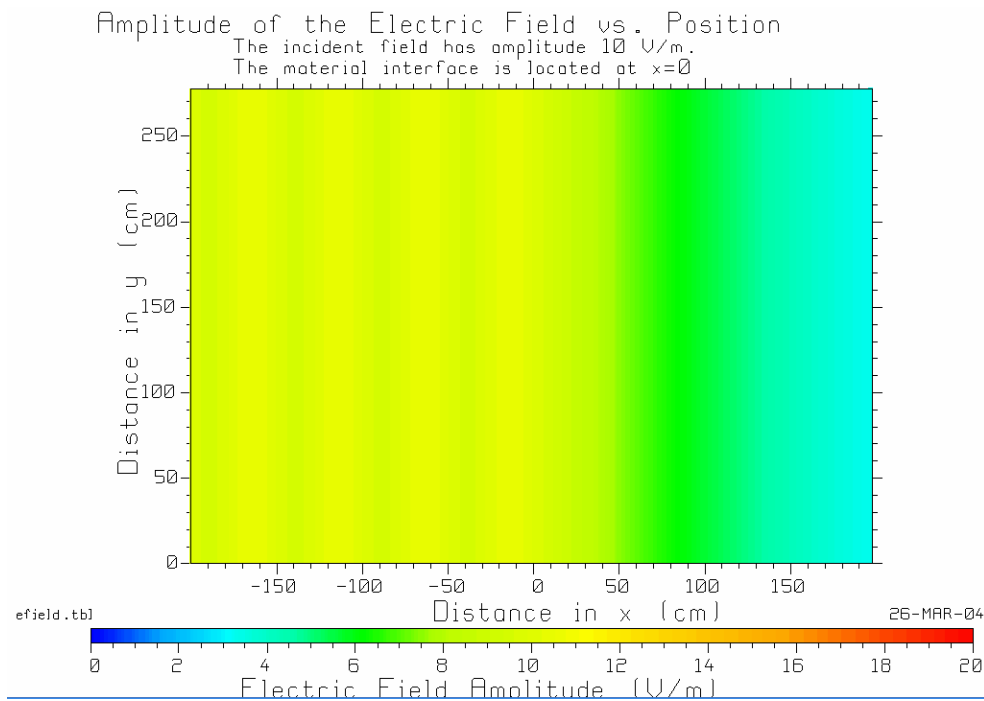


Fig. 20 The amplitude of the wave of Fig. 19.

Use the F4 function key to plot the amplitude of the wave with CPLOT. The amplitude is (almost) constant in material #1, for $x < 0$, but decreases rapidly in material #2 for $x > 0$. To see this more quantitatively, use the “cut” menu in CPLOT to make a y-cut for $y=0$, shown in Fig. 21.

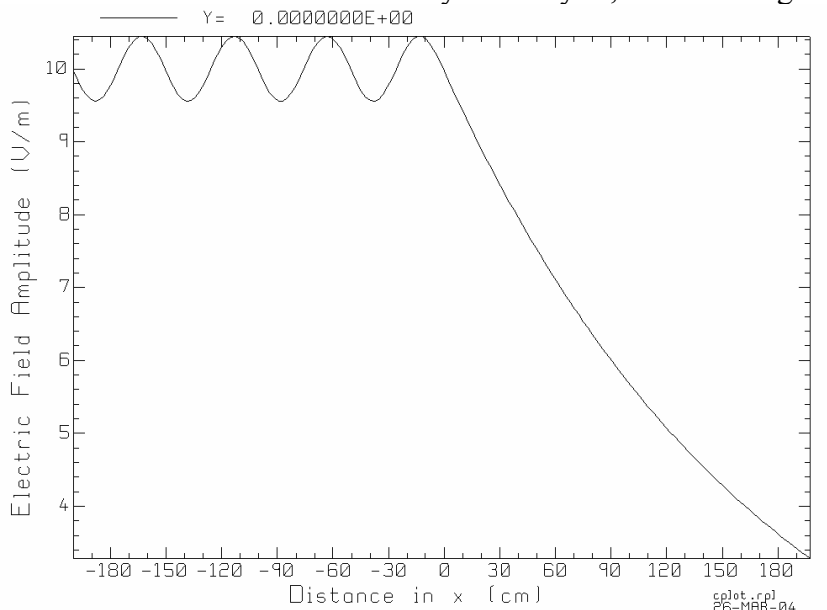


Fig. 21 The amplitude as a function of x in Figure. 19.

Fig. 21 shows the amplitude of the wave as a function of distance. In material #1, for $x < 0$, we see a standing-wave with maxima of about 10.45 V/m, and minima of about 9.55 volts, so the SWR is 1.09 as expected. So the field is “almost” constant in material #1. But in the lossy dielectric region, the amplitude of the field diminishes rapidly with distance. The material in to the right of the boundary in Fig. 21 behaves like microwave absorber material. Its permittivity is the same as free space, and its small conductivity absorbs the microwave energy. Adding conductivity gradually as the wave progresses deeper

into material #2 can reduce the reflection coefficient below 0.04467, but we can't simulate this with WAVES!

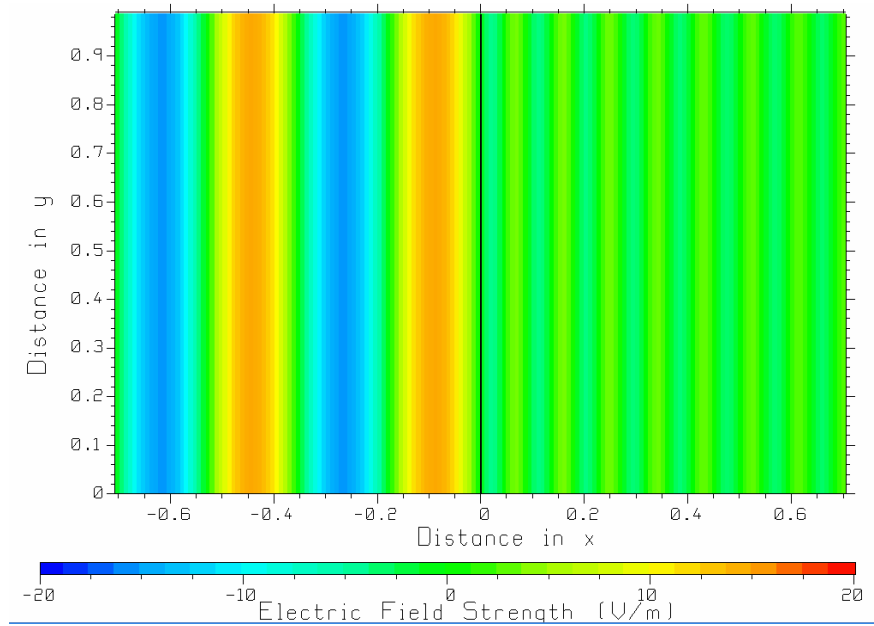


Fig. 22 A plane wave in air incident on a material with the parameters of real ground, at 850 MHz.

Transmission of a Wave into the Ground

Real ground has a relative permittivity of about 15 and a conductivity that varies with moisture content, with 10 mS/m being typical for wet ground. Fig. 22 shows a plane wave of amplitude 10 V/m at the analog cell phone frequency of 850 MHz propagating from free space into the ground. There is an impedance mismatch at the surface of the ground. The intrinsic impedance of air is about 377 ohms; for ground at 850 MHz, the intrinsic impedance is $97.3 + j0.686 \approx 97$ ohms, so the reflection coefficient magnitude is $0.5896 \approx 0.6$. The animation associated with the snapshot in Fig. 22 shows a pattern in material #1 which looks very much like a standing wave, and the standing-wave ratio is $SWR \approx (1 + 0.6)/(1 - 0.6) = 4$. With a transmission coefficient of 0.4, the wave transmitted into material #2 has only 4 V/m amplitude at the surface. The amplitude of the wave transmitted into the ground decreases rapidly with distance into material #2. Thus the yellow bars in material #2 become fainter as we get further from the surface.

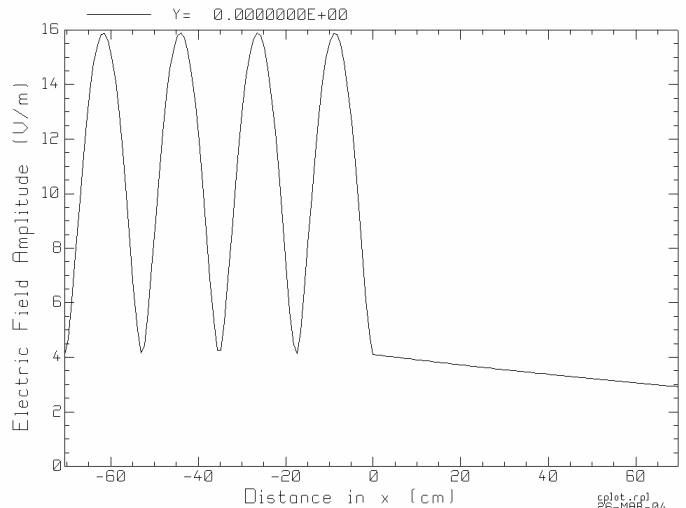


Fig. 23 Amplitude as a function of distance for propagation into the ground.

Use the F4 function in WAVES to graph the amplitude with CPLOT and then draw the “cut” of the amplitude as a function of distance x to obtain Fig. 23. We see the standing-wave in the air quite clearly; real ground at 850 MHz is quite a good reflector. We see the decrease of the amplitude of the transmitted wave with distance into the ground. WAVES reports that the “skin depth” is about 2 m, so the 70 cm distance into material #2 is only a fraction of a skin depth.

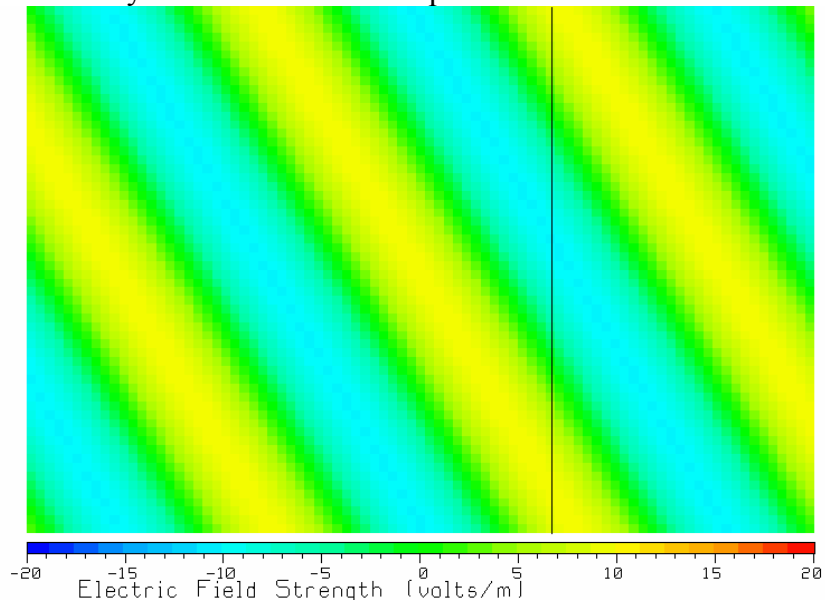


Fig. 24 A plane wave in free space traveling at 30 degrees to the “normal” to the interface, that is, at 30 degrees to the horizontal axis..

Waves Traveling Obliquely

To demonstrate waves that travel obliquely to the surface of the dielectric, set both material #1 and material #2 to “free space”, and choose an incidence angle of 30 degrees, at 300 MHz. Fig. 24 is a snapshot of the WAVES animation of the wave traveling at 30 degrees to the surface. The wave looks much the same as it did traveling in the x direction; except that it travels at 30 degrees to the x -axis. Note also that it does not matter whether you graph the “parallel” or the “perpendicular” polarization. WAVES plots the value of electric field, which is the same for both polarizations when both materials are free space.

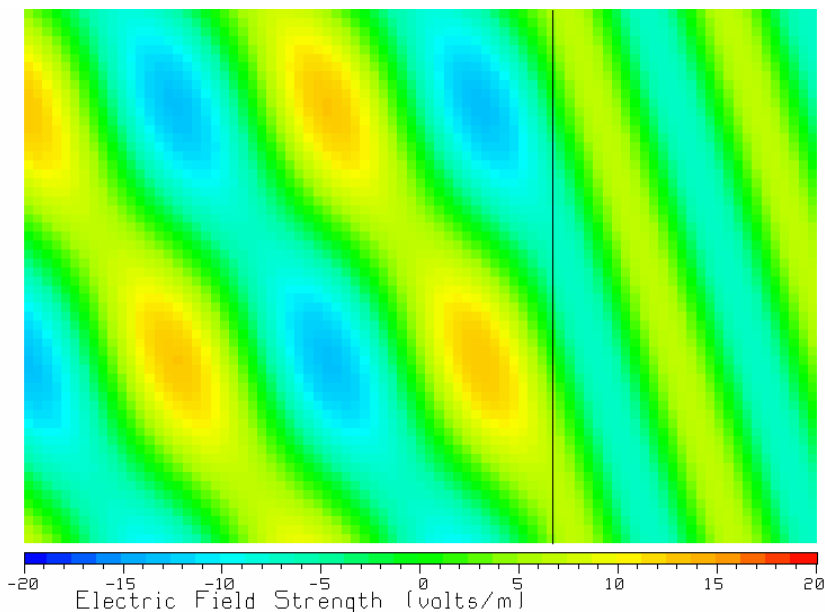


Fig. 25 Incidence at 30 degrees for the perpendicular polarization.

Oblique Incidence – Perpendicular Polarization

Keep the relative permittivity of region #1 equal to unity, but change the relative permittivity of region #2 to $\epsilon_{r2} = 3$, and set the angle of incidence to 30 degrees. The frequency is 300 MHz. Use the perpendicular polarization, which makes the electric field vector perpendicular to the “plane of incidence”, which is the computer screen. Thus the direction of the electric field vector is parallel to the planar interface between the two media. That is, the electric field is perpendicular to the computer screen. The reflection coefficient is -0.31 . Fig. 13 shows a snapshot of the electric field strength. In the associated animation, the plane wave travels diagonally, upwards-and-to-the-right, in material #1. The wave fronts are not straight lines, however, because there is a wave reflected from the interface, traveling upward-and-to-the-left. The pattern has orange regions of high field strength in the maxima, and blue regions of low field in the minima, reminiscent of a standing-wave pattern. Note that as we move from one side of the interface to the other, the colors are continuous. This demonstrates that the electric field satisfies the boundary condition that the tangential electric field must be continuous across the interface. In material #2 a plane wave moves away from the interface, propagating at a different angle, according to Snell’s Law of Refraction

$$\sin \theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} \sin \theta_i$$

For the present problem the angle of transmission is $\theta_t = 16.78$ degrees. Note also that the wavelength is shorter in the dielectric, as expected.

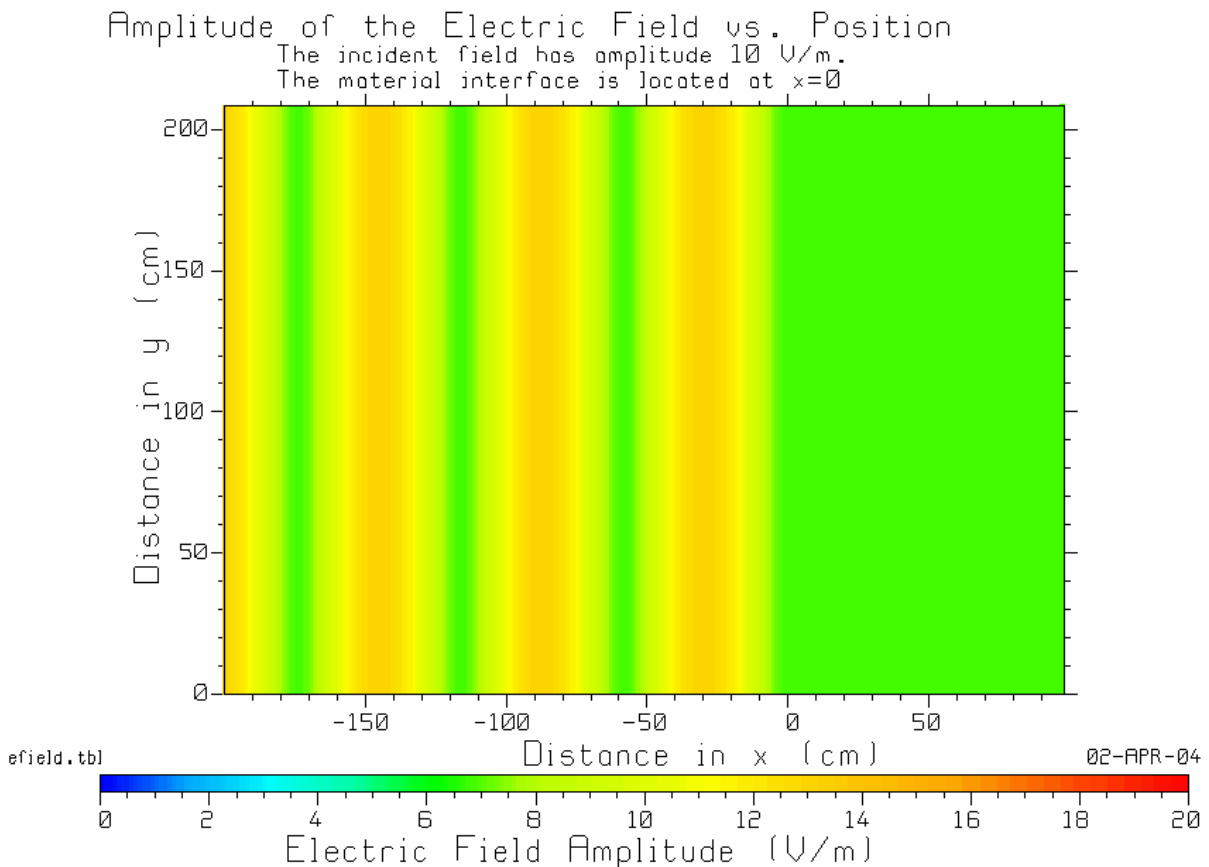


Fig. 26 The standing-wave pattern for the problem of Figure 25.

Fig. 26 shows the standing-wave pattern associated with the animation snapshot in Fig. 25. There are maxima and minima at fixed distances from the interface. As the standing-wave pattern in the

animation proceeds, the maxima (reddish splotches) and minima (blue splotches) in Fig. 25 move parallel to the interface, leading to the standing-wave pattern in Fig. 26.

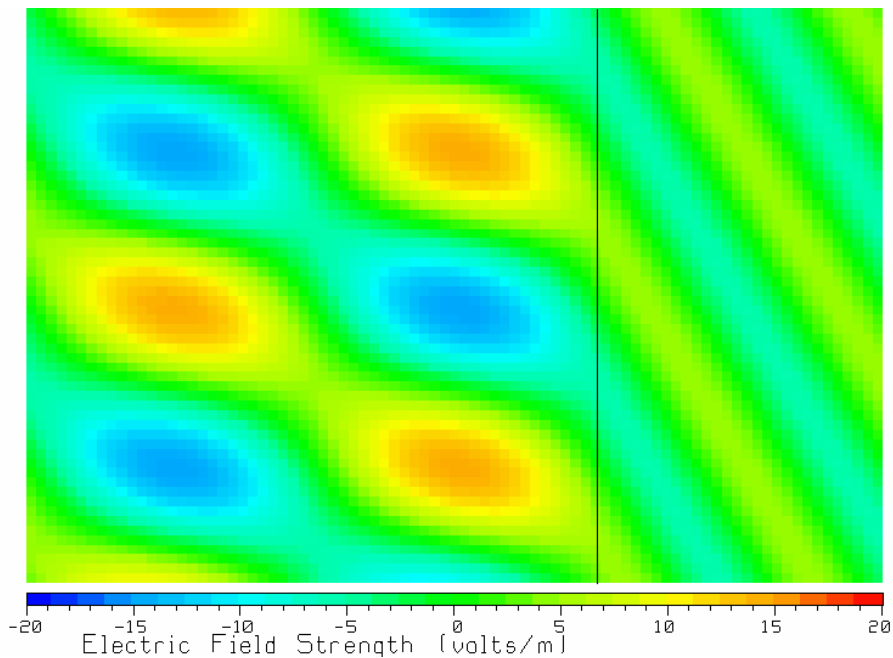


Fig. 27 Incidence at 60 degrees for perpendicular polarization.

Change the angle of incidence to 60 degrees. The reflection coefficient is now -0.5, and the standing-wave pattern in material #1 is much more pronounced, as shown in the snapshot of Fig. 27. As we increase the angle of incidence the reflection coefficient rises in value and the standing-wave pattern becomes more and more clearly pronounced.

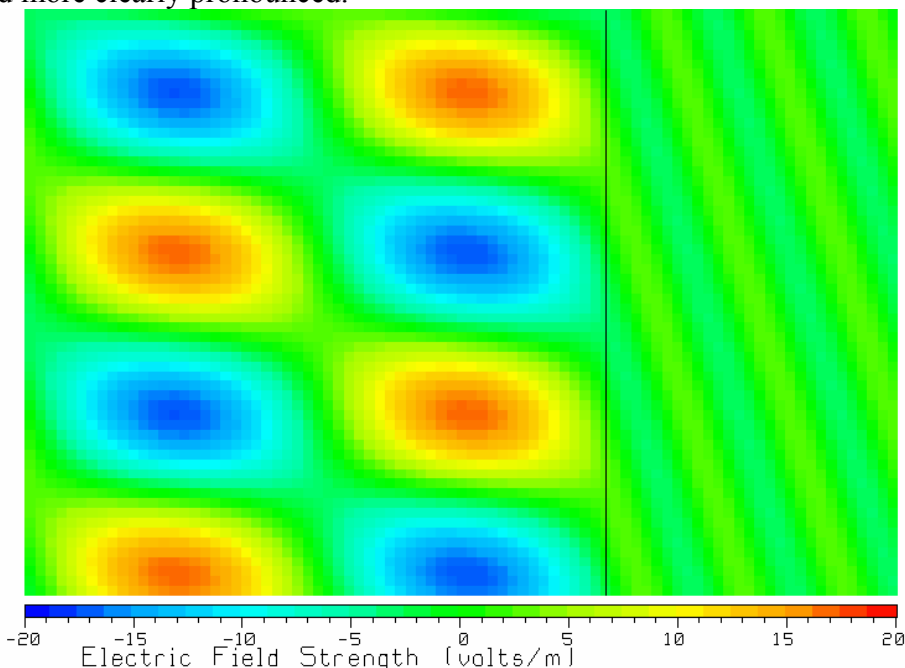


Fig. 28 The field for incidence at 60 degrees on a material with relative permittivity 9, for the perpendicular polarization.

Change the relative permittivity of material #2 to $\epsilon_{r2} = 9$, and run the animation. Fig. 28 is a snapshot of the field. As the relative permittivity of material #2 increases, the reflection coefficient rises in

value and the standing wave pattern becomes more pronounced. There are clear troughs(blue) and peaks(red). As time progresses, the pattern moves upward on the screen, moving parallel to the interface between the two materials. This behavior is a standing wave in the direction perpendicular to the interface (x direction) and a traveling wave along the interface (y direction). We encounter similar behavior in waveguides[1].

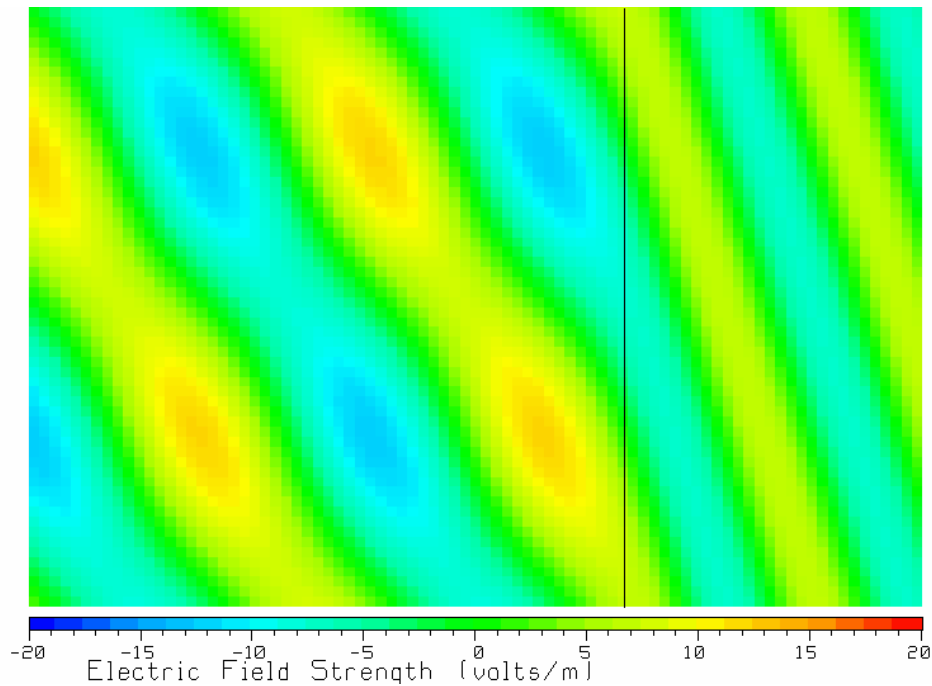


Fig. 29 Incidence at 30 degrees for the parallel polarization.

Parallel Polarization

Reset the relative permittivity of material #2 to $\epsilon_{r,2} = 3$, and the incidence angle to 30 degrees. Set the polarization to parallel, meaning that the electric field vector lies in the plane of the computer screen. Fig. 29 is a snapshot of the time-varying field. Note that since the electric field is also parallel to the wave front of each wave, the field is not parallel to the interface between the two materials. Program WAVES graphs the value of the electric field vector instant by instant as time advances. But since the electric field is not parallel to the *interface* in the parallel polarization, we do not expect the colors representing the electric field to be continuous across the interface.

Fig. 29 shows the field in the parallel polarization, where the reflection coefficient from the interface is -0.22 . Compare this with the corresponding picture for perpendicular polarization, Fig. 25. The reflection coefficient is smaller and the standing-wave pattern is less pronounced. As we increase the angle of incidence the reflection coefficient decreases. At 60 degrees incidence angle, the reflection coefficient is zero and there is total transmission into material #2. This angle is called the “Brewster angle”.

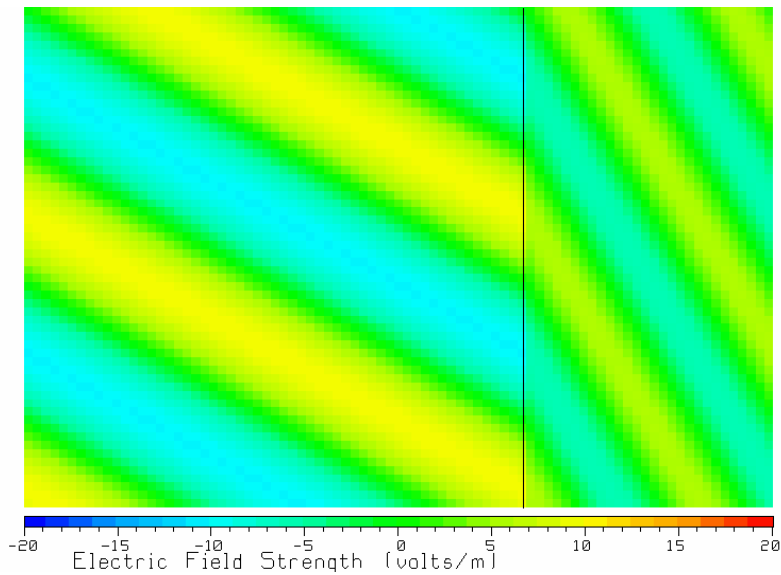


Fig. 30 Incidence at the Brewster angle, parallel polarization.

Fig. 30 shows the field when the angle of incidence equals the Brewster Angle. Note that there is no standing wave in material #1. There is no reflection from the surface, hence there is no standing wave. The wave is a pure traveling wave with straight wavefronts. In material #2 the wave travels away from the interface with a smaller amplitude than in material #1, because the transmission coefficient is 0.58. If we have total transmission, why is the transmission coefficient not equal to unity? The answer is that the power density is conserved, so the power traveling towards the interface is equal to the power traveling away from the interface. In the higher-permittivity material #2, a lower-amplitude field carries the same power.

Note in Fig. 30 that the colors representing the field are not continuous across the interface. This is because the program graphs the instantaneous value of the electric field, irrespective of its vector direction. In the parallel polarization the field vectors are parallel to the plane of the screen, not parallel to the interface between the two media, and so the field is not continuous across the boundary.

Change the polarization to perpendicular, and look at the field with the angle of incidence equal to the Brewster angle, shown in Fig. 14. The reflection coefficient is -0.5 and there is a pronounced standing wave in material #1.

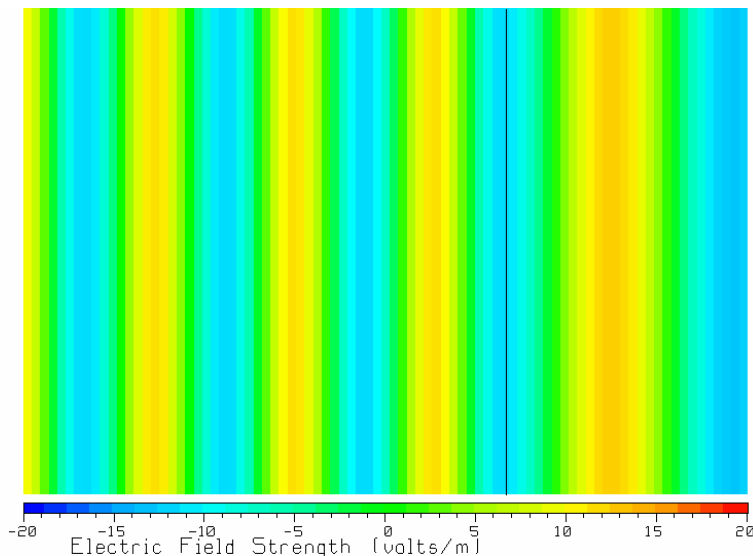


Fig. 31 Transmission from a dielectric into air.

Transmission from a Dielectric into Air

Consider a wave in material #1 with relative permittivity $\epsilon_{r1}=3$, traveling into material #2 which is free space with relative permittivity $\epsilon_{r2}=1$. Choose the perpendicular polarization. Fig. 31 shows a snapshot of the time-varying field for normal incidence. The reflection coefficient is +0.27. In Fig. 31, we see that the wavelength in the dielectric is shorter than that in air, as expected. Since the reflection coefficient is small, the wave in the dielectric is largely a traveling wave. The transmission coefficient is about 1.3, so the wave transmitted into the air has a larger amplitude than the wave incident on the interface in the dielectric.

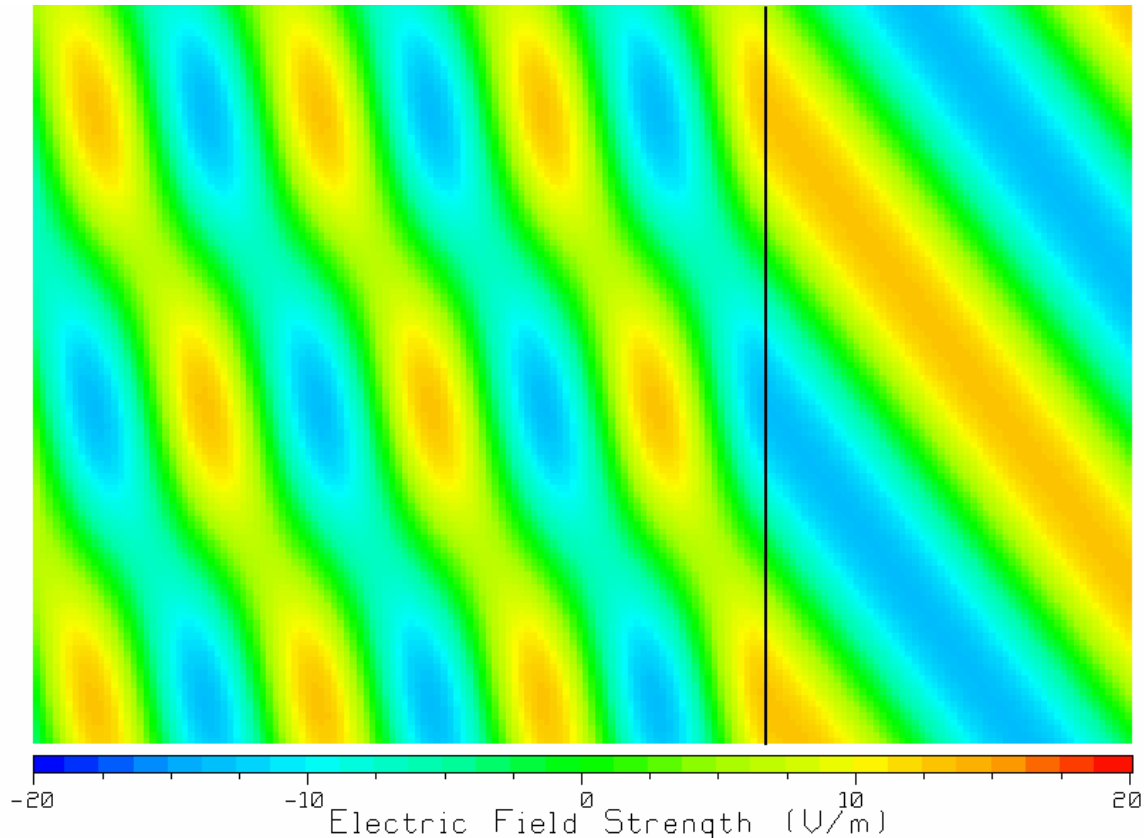


Fig. 32 Incidence on the interface at 20 degrees.

For an angle of incidence of 20 degrees, the reflection coefficient decreases to 0.20. Fig. 32 shows a snapshot of the field. The transmission angle is 36.33 degrees; when the permittivity of material #1 is greater than the permittivity of material #2, Snell's Law tells us that the transmission angle is greater than the incidence angle. As the angle of incidence increases, the angle of transmission increases more rapidly. Thus for incidence at 30 degrees, the transmission angle is 60 degrees. For incidence at 35 degrees, the transmission angle is 83.45 degrees. For incidence at 35.26 degrees, the angle of transmission increases to 90 degrees and the transmitted wave travels parallel to the interface. The smallest angle of incidence that makes the angle of transmission equal to 90 degrees is called the *critical angle*.

As the angle of incidence increases, the value of the reflection coefficient also increases. For 20 degree incidence, the magnitude of the reflection coefficient is 0.3378; for 30 degrees, 0.50; and for 35 degree incidence, the reflection coefficient magnitude is 0.8511. As the angle of incidence approaches the critical angle, the magnitude of the reflection coefficient approaches unity.

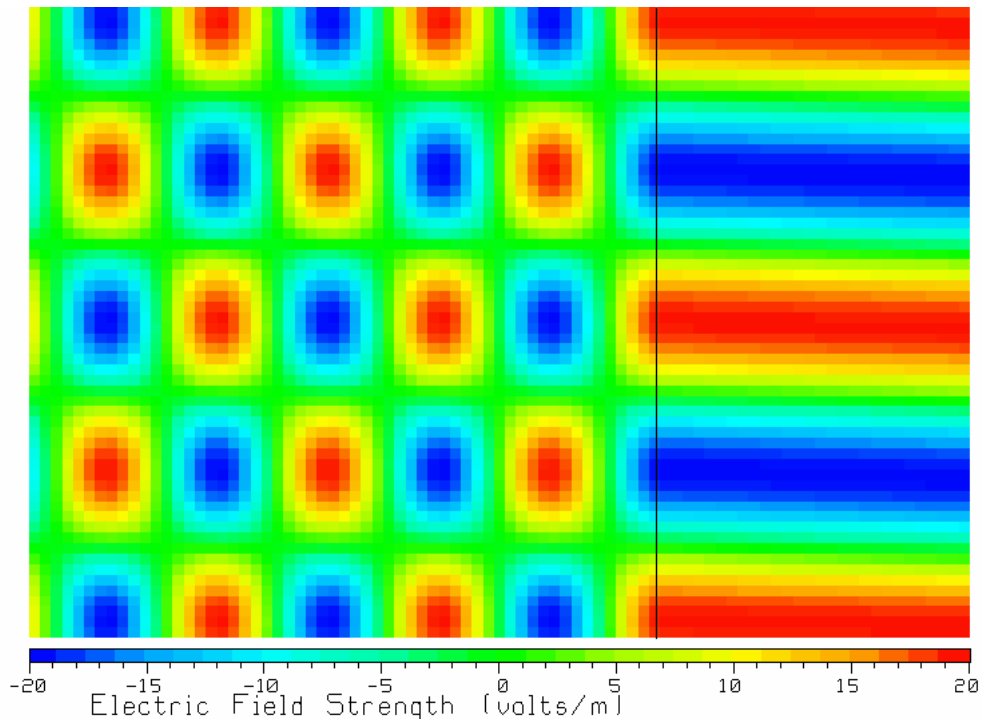


Fig. 33 The field for a wave incident at 35.25 degrees, very close to the critical angle.

Set the incidence angle to 35.25 degrees, just less than the critical angle. The reflection coefficient is 0.96. Fig. 33 is a snapshot of the field. In material #1 we have a standing wave pattern in the direction perpendicular to the interface. As time progresses the pattern travels up the screen parallel to the interface. In material #2 we have a wave traveling almost parallel to the interface, traveling away from the interface at a transmission angle of 88.47 degrees to the normal.

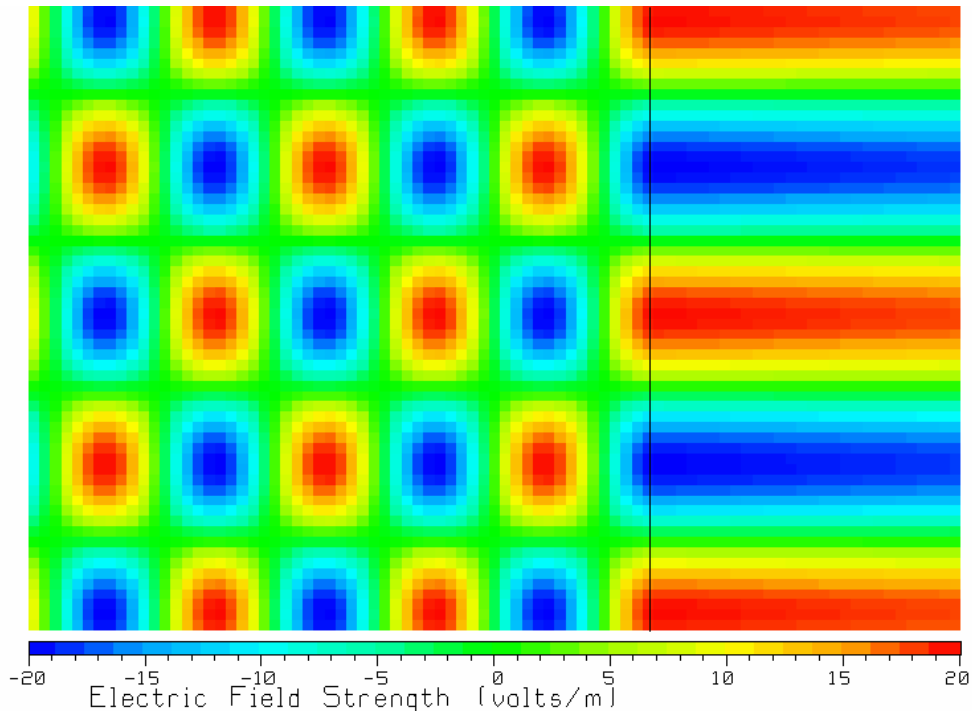


Fig. 34 The angle of incidence is 35.27 degrees, just larger than the critical angle.

If we change the angle of incidence to 35.27 degrees, a little larger than the critical angle, then the field is that in Fig. 34. Fig. 34 is superficially similar to Fig. 33, but there are subtle, important differences. The reflection coefficient magnitude is now exactly equal to unity, and the incident wave is fully reflected. Because the transmission angle is 90 degrees, the wave in material #2 travels parallel to the y axis, vertically up the screen. In the x direction the wave attenuates exponentially with distance away from the surface, but so slowly that we do not see much change in the field strength in Fig. 34 as we go farther from the boundary.

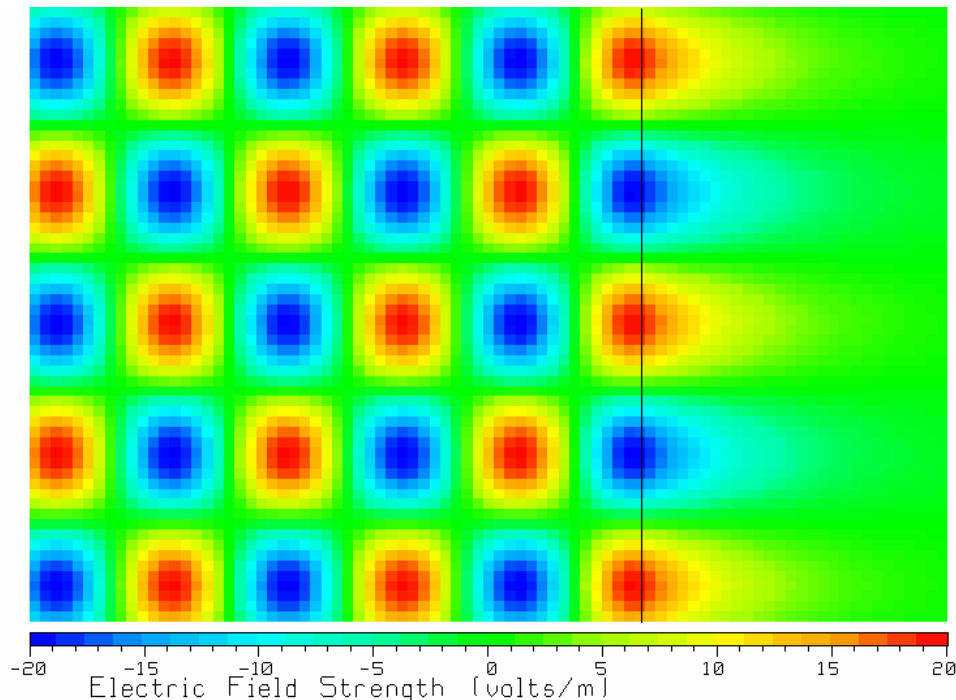


Fig. 35 The field with incidence angle 40 degrees.

When the angle of incidence is 40 degrees, considerably larger than the critical angle, the field is that shown in the snapshot of Fig. 35. Now the field decays rapidly as we go into material #2, and is negligible in value at the right hand side of the screen. The wave in material #2 is called a surface wave. It travels along the surface, up the screen, and decays rapidly with distance away from the surface. Surface waves are frequently encountered in electromagnetics. Fiber optic cables carry surface waves at interfaces where the permittivity changes. At HF frequencies (2-30 MHz), the surface of the earth carries a surface wave that can propagate the field over the horizon.

Conclusion

These notes have described a variety of demonstrations that can be done with WAVES. The program can be used to demonstrate the basic concepts of “traveling wave” and “standing wave”. The program demonstrates the shortening of the wavelength as the permittivity increases. The attenuation of waves with distance traveled in lossy materials can be demonstrated. The difference between “instantaneous value” and “amplitude” can be demonstrated. The program can be used to demonstrate the Brewster Angle and the Critical Angle. Each figure in these notes is one frame or “snapshot” of the animation of the wave behavior as a function of time. With a little experience in interpreting the animated color contour map of the field strength as time advances, many useful insights into wave interactions at dielectric surfaces can be gained. Students are encouraged to download the program, watch the animation, and experiment with various angles of incidence, polarizations, and material properties.

Reference

[1] B.S. Singh Guru and H.R. Hiziroglu, “Electromagnetic Field Theory Fundamentals”, PWS Publishing Company, Boston, 1998