Fault Tolerant Flight Control System Design by Dual-Loop Control Strategy

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This paper describes a dual-loop control scheme for fault tolerant flight control system design. The dual-loop controller consists of an outer loop controller—so-called adaptive neural sliding mode control (ANSC) and an inner loop controller designed by using nonlinear dynamic inversion (NDI) technique. The merits of adaptive neural network and sliding mode control scheme are that 1) the ability of adaptive neural network control to deal with unstructured uncertainty and 2) the ability of sliding mode control to guarantee transient response. Using timescale separation principal, the aircraft dynamics can be decomposed into fast and slow dynamics and the decomposed dynamics are inverted for NDI controllers. For real-time pilot simulation, one-stage inverse dynamics is used and the pilot inputs are translated to roll, pitch and yaw rate commands. For cascade NDI, two-stage dynamic inversion is used. The stability analysis of the proposed controller is performed using Lyapunov theory. To verify the effectiveness of the proposed control scheme, numerical simulation is performed for six degree-of-freedom nonlinear aircraft model while a failure occurs in longitudinal control surface. Simulation results demonstrate that closed-loop system has good performance while encountering lock-in-place, partial destruction and floating actuator failures.

Nomenclature

\( \delta_{\text{long, axi}}, \delta_{\text{lat, axi}}, \delta_{\text{dir, axi}} \) = pilot inputs for longitudinal, lateral and directional command 

\( K_{\text{lat}}, K_{\text{lon}}, K_{\text{dir}} \) = sticks and pedal gains 

\( p_{\text{ref}}, q_{\text{ref}}, r_{\text{ref}} \) = reference model rate commands 

\( n_{\text{r,cmd}}, n_{\text{f,cmd}} \) = normal and lateral acceleration command 

\( \phi, \theta, \psi \) = Euler angles 

\( p, q, r \) = perturbed angular velocities 

\( U, V, W \) = aircraft velocity components vector 

\( P, Q, R \) = aircraft angular velocity components vector 

\( \alpha, \beta \) = angle of attack and side slip 

\( u_{\text{cs}} \) = aircraft control surfaces vector \(( [\delta_{\text{i}} \ldots \delta_{\text{n}}] )\) 

\( A_{\text{fast}} , B_{\text{fast}} \) = aircraft rotational dynamic 

\( A_{\text{slow}} , B_{\text{slow}} \) = aircraft angular dynamic 

\( \delta_x, \delta_y, \delta_r \) = aircraft virtual inputs (aileron, elevator, rudder) 

\( T_{CA} \) = control allocation matrix

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VER the last three decades, the growing demand for reliability, maintainability, and survivability in flight control system has drawn significant attention of the researchers in fault tolerant flight control systems (FTFCS). FTFCS are control systems that possess the ability to accommodate system component failures automatically. They are capable of maintaining overall system stability and acceptable performance when the system component failures occur. Generally, FTFCS can be classified into two types: passive and active. In passive FTFCS, controllers are designed to deal with a class of presumed faults as uncertainties bound. Discussions on passive method are beyond the scope of this paper and interested readers are referred to references. In contrast to passive method, active strategy reacts to the system component failures actively by reconfiguring control system, so the stability and acceptable performance of the entire system can be maintained. Traditional approaches to flight control reconfiguration can address two major problems: a) Failure detection and isolation (FDI), b) Reconfiguration mechanism (RM). There are a number of important issues in designing active FTFCS and the most significant one is the integration between the FDI and RM. Imprecise information from the FDI that is incorrectly interpreted by the RM scheme might lead to a complete loss of stability of the system, therefore it is highly desirable to develop techniques that can integrate the FDI and RM in coherent fashion without any pre-assumption on the knowledge of the post-fault system model.

Conventional flight control systems require extensive gain scheduling for a large number of operating points within the aircraft flight envelope. When such a controller must be extended to account for anticipated failures, this leads to a very large scheduling table, making it difficult for design and real time implementation. In addition, a truly FTFCS must also be able to accommodate un-anticipated failures.

Recently control researchers have developed an advanced flight control method called nonlinear dynamic inversion (NDI) to avoid extensive gain scheduling, which is time consuming, costly, and iterative. In addition, NDI technique offers the following advantages: reusability across different airframe; flexibility against the model change and specific state variable command.

A problem with NDI is that the mathematical model of a system is often very complex, which means that the controller will also be complex. Hence by using time-scale separation principle a cascade NDI can be applied. For aircrafts under NDI, modeling mismatch and component failures lead to unpredictable behaviors, so robust inverse dynamics estimation (RIDE) has been developed such as Salford singular perturbation method and Pseudo-derivative feedback method. This research will use active one-step approach and an additional outer loop is designed to enhance robustness to NDI. The outer loop is a combination of sliding mode control (SMC) and neural network (NN) adaptive control - called Adaptive Neural Sliding Mode Control (ANSC) in this paper.

The ability of neural network as universal approximator makes it very desirable to deal with inversion errors. Thus NN-based adaptive control can be introduced to compensate inversion errors as outer loop controller. This methodology has more flexibility and greater potential than conventional adaptive design. If the inversion error variation is slow, NN adaptive control can have great success. However in the case of rapidly changing of inversion error, there is no guarantee of transient response that system will behave as desired. To ensure the desirable transient response, SMC can be applied to control law design. SMC provides a systematic approach to closed-loop
stability. Pure SMC technique has some drawbacks such as large control authority and chattering\textsuperscript{12}. However, combining SMC with NN adaptive term can overcome these drawbacks. In this research, the stability of ANSC is investigated by using Lyapunov stability theorem. Numerical simulations are performed using a six-degree of freedom nonlinear F-16 aircraft model to demonstrate the effectiveness of ANSC within the setting of NDI in the presence of partial and complete loss of longitudinal control surface.

II. Outer Loop Control Strategy

A. Feedback Linearization

One of the common methods for controlling nonlinear system is dynamic inversion strategy which is a form of feedback linearization that can be employed to render the dynamics of system. The main drawback of feedback linearization might be that the modes become unobservable under linearization or decoupling constraints which can be insurmountable in case they are unstable. Thus preliminary physical considerations are necessary to obtain a good design.

The essentials of the approach are most easily understood in case of dealing with SISO system and for MIMO case it is qualitatively similar to SISO. To simplify the discussion, we assume a class of nonlinear affine system which can be represented by the following differential equation:

\begin{equation}
    \dot{x} = f(x) + g(x)u \quad x \in \mathbb{R}^{n_x}, \quad u \in \mathbb{R}
\end{equation}

where \(f, g\) are smooth vector fields defining dynamics of plant with state vector \(x\), output vector \(y\) and input vector \(u\). Such system is feedback linearizable of relative degree of \(r\), if there exist difomorphism and input transformations\textsuperscript{18}

\begin{equation}
    z = \phi(x)
\end{equation}

\begin{equation}
    u = \alpha(x) + \beta(x)v
\end{equation}

The transformation changes nonlinear dynamics (1) into the following controllable linear system

\begin{equation}
    \dot{z} = A_z z + B_z v
\end{equation}

where \(A_z, B_z\) determine the sliding surface characteristic. To obtain the transformation, we time-differentiate (1) and continue in this way until a nonzero coefficient appears. This process can be succinctly described by introducing some conventional notation of differential geometry. The Lie derivative of the scalar function \(h\) with respect to the vector field \(f\) is defined as

\begin{equation}
    L_f h(x) = \frac{\partial h}{\partial x} f(x)
\end{equation}

Higher order derivatives may be successively defined by

\begin{equation}
    L_f^k h(x) = L_f \left( L_f^{k-1} h(x) \right)
\end{equation}

If \(L_f^k h(x) = 0\) for \(k = 1, \ldots, r-1\), and \(L_f^r h(x) \neq 0\), the process of differentiating ends with
\[
y^{(r)} = L_f^r h(x) + L_g L_f^{r-1} h(x) u
\] (7)

The number \( r \) is called the relative degree of (1). Now we define the following transformation

\[
z_k = \phi_k(x) = L_f^{k-1} h(x) \quad k = 1, \ldots, r
\] (8)

We get the \( r \)-dimensional completely controllable and observable form in (4) with following control

\[
u = - \frac{L_f^r h(x)}{L_g L_f^{r-1} h(x)} + \frac{1}{L_g L_f^{r-1} h(x)} V
\] (9)

By assigning the pseudo-control \( V \) with outer loop, the desired behavior is achieved. Modeling errors and component failures lead to the departure of model from nominal behavior. Thus the system is represented as follows:

\[
y' = v + \Delta(t, x, v)
\] (10)

where \( \Delta(t, x, v) \) is the modeling error or component failures. Hence, the total pseudo-control is designed to reduce the inversion error as following part.

**B. Adaptive Neural Sliding Control (ANSC)**

In this study, the combination of sliding mode control (SMC) and adaptive radial base function neural network (ARBFNN) is used to compensate the inversion error and this combination strategy is called Adaptive Neural Sliding Control (ANSC). The SMC emphasizes the rate of convergence and ensures the stability as well as the transient performance and the ARBFNN can learn the structure of uncertainty in the system and guarantee the steady state tracking performance. The proposed ANSC provides the bridge between SMC and adaptive strategy. The backbone of the SMC methodology is the sliding surface. The sliding surface is defined by a stable differential equation with the tracking error \( e = x - x_d \) and it's up to \( n-1 \) derivatives.

\[
s = \left( \frac{d}{dt} + \lambda \right)^{n-1} e
\] (11)

where \( \lambda \) is positive gain, \( n \) is the order of system, \( x \) is dynamic output and \( x_d \) is desired output. Implementing this control in practice relies on the availability of derivatives. This requires taking numerical derivatives of signals which result in large amount of noise. Using the following filter implementation prevents the drawback of the system noise

\[
\dot{x}_f = A_f x_f + B_f e \\
s = C_f x_f + e = y_f + e
\] (12)

where \( (A_f, B_f, C_f) \) describes the filter dynamics and \( x_f \) refers to filter states. A control law that can achieve stability and performance is given below:

\[
v = v_f + v_c + v_s + v_{nn}
\] (13)
where \( V_{ff} \) is a feedforward signal, \( V_{lc} \) is linear compensator signal, \( V_s \) is SMC signal and \( V_{nn} \) is adaptive neural network signal. The value of feedforward term, \( V_{ff} \), is defined by

\[
V_{ff} = \dot{y}_s
\]

(14)

where \( y_s = y_r - y_f \) and \( y_r \) is referred as to reference command. The value of \( V_{lc} \) is

\[
V_{lc} = -Ks
\]

(15)

where \( K \) represents the behavior of linearized system. The value of \( V_s \) is

\[
V_s = -\mu(h,s)
\]

(16)

The function \( \mu \) is the effective sliding mode portion of the control, where in standard SMC, this function is \textit{signum}. Discontinuous control law by \textit{signum} function causes chattering, so a continuous approximation is used as \( \mu(h,s) = h \cdot \text{sat}(s/\varepsilon) \).

where \( \varepsilon \) represents the value of boundary layer and \( h \) is a bounded function for exponential stability guarantee, derived as outlined in the Appendix. The adaptive neural network control term is defined as

\[
V_{nn} = -\hat{w}^T \xi
\]

(17)

where \( \xi \) is the radial base function of neural network and \( \hat{w} \) is its weighting vector. The update law for the weights can be derived as outlined in the Appendix

\[
\dot{\hat{w}} = -\gamma \xi \dot{s}
\]

(18)

where \( \gamma > 0 \) is the learning rate of network. To ensure the robustness of weighting vector a standard \textit{e-modification} is also used as follows

\[
\dot{\hat{w}} = -\gamma(\xi s + \eta|s|\hat{w}) \xi
\]

(19)

where \( \eta > 0 \) is \textit{e-modification} factor, which does not influence the stability. Figure 1 should illustrate the control strategy for a general nonlinear system. The implementation of control law on F-16 will be mentioned in Section III.

For the stability and transient response analysis of proposed ANSC control system, two theorems have been investigated. The first theorem shows the asymptotically stability of error and the second theorem describes exponential convergence of tracking error toward boundary layer, and guarantees the transient performance.

\textbf{Theorem 1}: For the system, Eq.(10), the ANSC controller ,Eq. (13), guarantees that the tracking error converges asymptotically and weights of NN are bounded.

Proof: To assess the stability of ANSC, Lyapunov function is used. Substituting the control law Eq. (13) into
feedback linearized system dynamics Eq. (10), we obtain the following sliding surface dynamic as

\[ \dot{s} = -Ks - h \text{sat}(s) - \dot{w}^T \zeta + \Delta \]  

(20)

where \( \Delta^* \) represents optimized estimation with neural networks structure. Under the existence assumption of fixed point for weighting vector \( w^* \)

\[ \Delta = w^T \zeta \]  

(21)

Therefore an equivalent expression for sliding surface dynamic is

\[ \dot{s} = -Ks - h \text{sat}(s) + \tilde{w}^T \zeta + \Delta^* - \Delta \]  

(22)

where \( \tilde{w} = \dot{w} - w^* \). The following positive definite function is used as Lyapunov candidate function

\[ L = \frac{1}{2} s^T P s + \frac{1}{2\kappa} \tilde{w}^T \tilde{w} \]  

(23)

where \( \kappa > 0 \) and \( A = -K \). For \( K > 0 \), \( A \) in Eq. 24 is stable, and for all \( Q > 0 \) the solution of

\[ A^T P + PA = -Q \]  

(24)

is unique and positive definite. Sliding surface \( s \) is scalar so \( P \) becomes \( 1 \times 1 \) matrix. Differentiating Lyapunov function and substituting Eq. (22), we obtain:

\[ \dot{L} = -\frac{1}{2} s^T Q s + s^T P (\Delta^* - \Delta) - s^T P h \text{sat} \left( \frac{s}{\varepsilon} \right) + s^T P \tilde{w}^T \zeta + \frac{1}{\kappa} \tilde{w}^T \tilde{w} \]  

(25)

Substituting the update law without e-modification term and assuming \( \tilde{w} = \dot{w} \) yield the following:

\[ \dot{L} = -\frac{1}{2} s^T Q s + s^T P (\Delta^* - \Delta) - s^T P h \text{sat} \left( \frac{s}{\varepsilon} \right) + s^T P \tilde{w}^T \zeta - \frac{\gamma}{\kappa} s^T \tilde{w}^T \zeta \]  

(26)

By choosing \( \kappa = \gamma P^{-1} \),

\[ \dot{L} = -\frac{1}{2} s^T Q s + s^T P (\Delta^* - \Delta) - s^T P h \text{sat} \left( \frac{s}{\varepsilon} \right) \]  

(27)

For the adaptation without e-modification term, and setting \( Q = I \), Eq. (27) reduces to

\[ \dot{L} = -\frac{1}{2} s^T s + s^T P (\Delta^* - \Delta) - s^T P h \text{sat} \left( \frac{s}{\varepsilon} \right) \leq -\frac{1}{2} s^T s + s^T P \left( \delta - h \text{sat} \left( \frac{s}{\varepsilon} \right) \right) \]  

(28)
As it will be shown in theorem II, the bound on $h$ will guarantee that $\dot{L}$ will be strictly negative semi definite. This is sufficient to show, with the Barbalat’s Lemma\textsuperscript{17} that $s$ and $\hat{w}$ remain bounded. Also asymptotic stability is proved. Furthermore, without the sliding mode term $\dot{L}$ will become as

$$\dot{L} \leq -\frac{1}{2} s^T s + |s|^T P |\delta|$$

which is negative semi definite for all $|s| \geq P \delta$ the tracking errors converge to a residual set with size proportional to $\delta$, with asymptotic tracking achieved in the limiting case when $\delta = 0$. The NN has the capability to approximate nonlinear functions, if no NN approximation error ($\delta = 0$) then asymptotic stability will be guaranteed.

**Theorem II**: If the ANSC control law, Eq.(13), is applied to the system, Eq.(10), the tracking error converges exponentially toward boundary layer of $z = 0$ and transient performance is guaranteed.

Define the following positive definite Lyapunov function

$$L = \frac{1}{2} s^T s \quad (30)$$

The time derivative of this function is given by

$$\dot{L} = -s^T K s + s^T \left[ (\Delta^* - \Delta) - h \text{sat} \left( \frac{s}{\epsilon} \right) - \hat{w}^T \xi \right] \leq -s^T K s + |s|^T |\delta - \hat{w}^T \xi| - s^T h \text{sat} \left( \frac{s}{\epsilon} \right) \quad (31)$$

The $h$ function is a bounded function that must meet the following requirement for strictly negative of time derivative of Lyapunov function

$$h \geq |\delta - \hat{w}^T \xi| \quad (32)$$

Choose $h$ which will ensure the above bound is met:

$$h = |\delta - \hat{w}^T \xi| \quad (33)$$

It follows that

$$\dot{L} \leq -s^T K s + \frac{h \epsilon}{4} \quad (34)$$

If we use $-s^T K s \leq -\lambda_{\text{min}} (K) s^T s$ where $\lambda_{\text{min}}$ the smallest eigenvalue of matrix is $K$, the above expression is simplified to the following:

$$\dot{L} \leq -2 \lambda_{\text{min}} L + \frac{h \epsilon}{4} \quad (35)$$

We can integrate this expression to get the following result:
This implies that the system converges exponentially to the boundary layer region where \( s^T s \leq \frac{h\epsilon}{4\lambda_{\min}(K)} \). Two theorems show that the tracking error will not only decay to some boundary layer exponentially, but also will asymptotically decay to zero inside boundary layer.

### III. Aircraft Nonlinear Dynamic Inversion (NDI)

Dynamic inversion is a control law methodology that can be developed into desired generic set of control laws. In this paper we design nonlinear dynamic inversion controller as an inner loop controller based on time-scale separation assumption. The dynamics of aircraft is inverted in several stages. We have designed two NDI controllers, one for real-time pilot simulation and the other one for autopilot application which is named Cascade NDI, each controller is described below.

#### A. Real-Time Pilot Simulation

In real-time piloted simulation, flight commands are generated by the pilot through longitudinal and lateral stick \((\bar{\delta}_{\text{long}} , \bar{\delta}_{\text{lat}})\) and directional pedals \((\delta_{\text{dir}})\). These displacement commands are converted into roll rate \((P_{\text{cmd}})\), aerodynamic normal acceleration \((n_{\text{cmd}})\) and lateral acceleration commands \((n_{y,\text{cmd}})\) through stick and pedal gains \((K_{\text{lat}} , K_{\text{long}} , K_{\text{dir}})\):

\[
P_{\text{cmd}} = K_{\text{lat}} \delta_{\text{lat,stick}}
\]

\[
n_{z,\text{cmd}} = K_{\text{long}} \delta_{\text{long,stick}}
\]

\[
n_{y,\text{cmd}} = K_{\text{dir}} \delta_{\text{dir,pedal}}
\]

These commands are then transformed into corresponding roll rate, pitch rate, and yaw rate commands:

\[
P_c = P_{\text{cmd}}
\]

\[
q_c = \frac{g}{V_t} n_{z,\text{cmd}}
\]

\[
r_c = \frac{g}{V_t} \left(n_{y,\text{cmd}} + \sin \phi \right)
\]

The reference model provides filtered rate commands \((P_{\text{ref}}, q_{\text{ref}}, r_{\text{ref}})\). For this purpose, first order roll rate and second order pitch and yaw rate transfer function are used. Generally the dynamics of angular velocity variables \(x_{fd} = [p \quad q \quad r]\) are fast in comparison with the other flight state variables. Therefore by using time-scale separation the inverse dynamics can be obtained from rotational equation of aircraft,

\[
\dot{x}_{fd} = A_{\text{fast}} + B_{\text{fast}} u_{es}, \quad A_{\text{fast}} \in \mathbb{R}^{3\times1}, B_{\text{fast}} \in \mathbb{R}^{3\times n}, u_{es} \in \mathbb{R}^{n\times1}
\]
The values of $A_{\text{fast}}$ and $B_{\text{fast}}$ are derived in appendix. To compute the inverse dynamics, it is necessary to use control allocation\textsuperscript{23} since the control surfaces are more than three. By using control allocation rule ($T_{CA}$) the effect of all control surfaces is transformed to three virtual control surfaces ($\delta_a, \delta_e, \delta_r$) for roll, pitch and yaw channels.

$$u_{ea} = T_{CA} \begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix}$$ \hspace{1cm} (44)

It is clear that the rotational dynamic relative degree is one, so the inverse dynamic can be derived by inverting the equation. Therefore the inverse dynamic for finding pseudo-controls is illustrated in the following equation

$$\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} = (\dot{x}_{\text{ Idol}} - A_{\text{fast}} B_{\text{fast}} T_{CA})^{-1}$$ \hspace{1cm} (45)

That can be simplified to the following relation,

$$\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} = \begin{bmatrix} b_1 & 0 & b_3 \\ 0 & b_5 & 0 \\ b_7 & 0 & b_9 \end{bmatrix}^{-1} \begin{bmatrix} v_p \\ v_q \\ v_r \end{bmatrix} - A_{\text{fast}} \begin{bmatrix} b_{v1} \\ \vdots \end{bmatrix}$$ \hspace{1cm} (46)

where $V_p$, $V_q$ and $V_r$ are pseudo-controls for roll, pitch and yaw channels. The inverse of virtual control matrix ($B_{v1}$) elements ($b_1, b_3, b_5, b_7, b_9$) are functions of inertial terms, geometric terms, aerodynamic derivatives and control derivatives. Virtual control matrix is nonsingular for the general aircraft flight envelopes\textsuperscript{7,20,21,22}, therefore $B_{v1}$ is invertible, Figure 2 shows the schematic block diagram of pseudo-controls.

B. Cascade NDI

Cascade NDI approach for inverse dynamic has been developed to design a flight control system that regulates angular variables ($\phi, \alpha, \beta$). Generally the dynamics of angular velocity variables are faster than that of angular variables, therefore the time scale separation can be used and the inversion is carried out in two stages. Control variables for the second stage of inversion (DI-II) are angular velocities and the control variables for the first stage of inversion (DI-I) are the virtual control surfaces displacements. For DI-I modeling, Eq. (43) is used and for the DI-II the following relation is used,

$$\begin{bmatrix} \dot{\phi} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = A_{\text{slow}} + B_{\text{slow}} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$ \hspace{1cm} (47)

The values of $A_{\text{slow}}$ and $B_{\text{slow}}$ are derived in appendix. To obtain pseudo-controls the following relation is used,
The DI-II dynamics is defined by the Eq. (48), where $V_\alpha$, $V_\beta$ and $V_\phi$ are pseudo-controls for angle of attack, side slip angle and roll angle. Control matrix $B_{\text{slow}}$ for DI-II for majority of flight envelopes is nonsingular, just only for $\alpha = 90^\circ$ or $\beta > 70^\circ$.

IV. Aircraft and Failure Modeling-Simulation

A. Aircraft Modeling

The model is derived from data of a high performance military aircraft distributed by NASA. Aerodynamic and thrust characteristics are provided through a number of look-up tables as function of angle of attack, sideslip and control surface deflections. Nonlinear dynamic model of aircraft is derived, the equations of motion are written as a system of nine scalar first-order differential equations:

**Force equations:**

\[
\begin{align*}
\dot{U} &= -g \sin \theta + VR - WQ + \frac{F_{Ax} + F_{Ar}}{m} \\
\dot{V} &= g \sin \phi \cos \theta - UR + WP + \frac{F_{Ay} + F_{Ay}}{m} \\
\dot{W} &= g \cos \phi \cos \theta + UQ - VP + \frac{F_{Az} + F_{Ar}}{m}
\end{align*}
\]

**Moment equations:**

\[
\begin{align*}
\dot{P} &= -\frac{1}{I_{xx}} \left( I_{xx} N + I_{zz} L + I_{zx} (I_{xx} + I_{zz} - I_{yy}) PQ + \left( I_{yy} - I_{zx} \right) QR \right) \\
\dot{Q} &= -\frac{1}{I_{yy}} \left( M + (I_{xx} - I_{zz}) PR + I_{zz} \left( R^2 - P^2 \right) \right) \\
\dot{R} &= -\frac{1}{I_{zz}} \left( L + I_{xx} N + I_{zx} (I_{yy} - I_{xx} - I_{zz}) QR + \left( I_{xx} - I_{zy} \right) PQ \right)
\end{align*}
\]

**Kinematic equations:**

\[
\begin{align*}
\dot{\phi} &= P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta \\
\dot{\theta} &= Q \cos \phi - R \sin \phi
\end{align*}
\]
\[ \psi = (Q \sin \phi + R \cos \phi) \sec \theta \]  

Aerodynamic forces and moments are modeled as follows:

\[ F_{A_x} = qS \left[ C_x(\alpha, \beta, \delta_1) + C_{q\alpha}(\alpha) \frac{C}{2V_T} q \right] \]  

\[ F_{A_y} = qS \left[ C_y(\beta, \delta_1) + C_{y\beta}(\alpha) \frac{b}{2V_T} p + C_{p\beta}(\alpha) \frac{b}{2V_T} r \right] \]  

\[ F_{A_z} = qS \left[ C_z(\alpha, \beta, \delta_1) + C_{z\delta}(\alpha) \frac{C}{2V_T} q \right] \]  

\[ L = \left[ C_i(\alpha, \beta) + \sum_{i=1}^{n} \left( C_{i\delta}(\alpha, \beta) \delta_i + C_{i\beta}(\alpha, \beta) \delta_i \right) + \frac{bp}{2V} C_{ip}(\alpha) + \frac{br}{2V} C_{ir}(\alpha) \right] qSb \]  

\[ M = \left[ C_m(\alpha) + \sum_{i=1}^{n} C_{m\delta}(\alpha) \delta_i + \frac{cq}{2V} C_{mq}(\alpha) + \frac{cr}{2V} C_{rq}(\alpha) \right] qSc \]  

\[ N = \left[ C_n(\alpha, \beta) + \sum_{i=1}^{n} \left( C_{n\delta}(\alpha, \beta) \delta_i + C_{n\beta}(\alpha, \beta) \delta_i \right) + \frac{bp}{2V} C_{np}(\alpha) + \frac{br}{2V} C_{nr}(\alpha) \right] qSb \]  

**B. Failure Modeling**

A failure modeling strategy has been applied for longitudinal control surface such as blockage, partial destruction and floating actuator. The methodology is based on the assumption that when a control device failure occurs, an alteration of the aerodynamic force and moment will be induced, which is equivalent to a loss of “aerodynamic efficiency”.

A failure involving a blockage of the control surface at a fixed deflection value does not alter the aerodynamic properties of the control surface. Therefore, all the formulation to compute the forces and moments remains unchanged. The only real modeling issue is that each surface in a pair (left & right) will have different deflections and needs to be considered individually.

In the case of physical destruction of longitudinal control surface, the efficiency parameters are selected to be the normal force derivatives with respect to left and right control surface deflection, respectively \( c_{z\delta_{li}} \) and \( c_{z\delta_{ri}} \). This failure typically induces substantial alterations of the rolling moment coefficient and some mild alteration on the lift and pitching moment coefficients. If the linear aerodynamic theory is used for force and moment, the contribution of the horizontal tail can be isolated from longitudinal control surface based on relationships of the following form:

\[ C_{z_{\alpha}} = C_{z_{\alpha H}} + \eta_H \frac{S_H}{S} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) C_{z_{\delta_l}} \]  

Consider the following

\[ C_{z_{\alpha}} = \tau_{e_1} C_{z_{\alpha l}} + \tau_{e_2} C_{z_{\alpha r}} = \eta_H \frac{S_H}{S} C_{z_{\delta_l}} \]  

\[ \tau_{e_1} C_{z_{\alpha l}} + \tau_{e_2} C_{z_{\alpha r}} = \eta_H \frac{S_H}{S} C_{z_{\delta_l}} \]
where \( \tau \) is aerodynamic efficiency factor, expressed as the ratio between the efficiency parameter before and after the failure. Therefore the normal force in terms of the efficiency parameters can be expressed using the relationship:

\[
C_{Z\alpha} = C_{Z\alpha_0} + \tau_{e_0} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) C_{Z\alpha_0} + \tau_{e_1} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) C_{Z\alpha_1}
\]  \( (66) \)

In a similar manner, the other stability derivatives can be derived as following:

\[
C_{\alpha} = C_{\alpha_0} + \tau_{e_0} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_0} C_{\alpha_0} - \tau_{e_1} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_1} C_{\alpha_1}
\]  \( (67) \)

\[
C_{\alpha} = C_{\alpha_0} - 2 \tau_{e_0} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_0} C_{\alpha_0} - 2 \tau_{e_1} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_1} C_{\alpha_1}
\]  \( (68) \)

\[
C_{\alpha} = C_{\alpha_0} - 2.2 \tau_{e_0} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_0} C_{\alpha_0} - 2.2 \tau_{e_1} \left( 1 - \frac{\partial \epsilon}{\partial \alpha} \right) X_{\alpha_1} C_{\alpha_1}
\]  \( (69) \)

Finally, the coupling rolling moment induced by failure is modeled by

\[
C_{\dot{\psi}} = \frac{N_{\alpha_0}}{b} \left( \tau_{e_0} C_{\dot{Z}_{\alpha_0}} + \tau_{e_1} C_{\dot{Z}_{\alpha_1}} \right)
\]  \( (70) \)

A floating actuator failure occurs when control of an actuator is lost and it begins to move based on parameters other than pilot command. For this case, not only control authority over actuator is lost, but a disturbance is induced based on some other variable. In this research, the modeling of this failure will be accomplished by commanding the lost actuator with the local angle of attack.

Although experimental data were not available for validation of longitudinal control surface failures modeling but simulations show that results are qualitatively correct and the aircraft response is demonstrated as expected.

### V. Numerical Simulation

Presented here is simulated F-16 aircraft response after occurrence of numerous types of failures. The aircraft maneuver begins in trimmed flight at altitude 2000 ft and velocity equals 600 ft/sec. It is assumed that the thrust is constant and actuator dynamics is also considered including nonlinear effect of saturation and rate limit. The actuator parameters are shown in Table 1. The simulation is performed for real-time pilot simulation and cascade NDI. The performance of NDI and NDI+ANSC has been investigated while lock in place, partial destruction and floating actuator failures happened in longitudinal control surfaces.

In the nonlinear case, it is difficult to test the unobservable zero dynamics. However, one may linearize the model and use linear technique. The

| Table 1. Control surface actuator modes\(^{25}\) |
|-----------------|-----------------|
| **Deflection Lim (Deg)** | **Rate Lim (Deg/S)** |
| Aileron | ±21.5 | 80 |
| Elevator | ±30 | 120 |
| Rudder | ±25 | 60 |

| Table 2. Neural network input elements |
|-----------------|-----------------|-----------------|
| **Roll axis NN** | **Pitch axis NN** | **Yaw axis NN** |
| \( \alpha \) | \( \beta \) | \( p \) |
| \( q \) | \( q \) | \( q \) |
| \( r \) | \( r \) | \( r \) |
| \( \dot{p}_r \) | \( \dot{q}_r \) | \( \dot{p}_r \) |
| \( \dot{q}_r \) | 1 | \( \dot{q}_r \) |
| \( \dot{r}_r \) | \( \dot{r}_r \) | 1 |

| Table 3. ANSC parameters |
|-----------------|-----------------|-----------------|
| **\( \gamma \)** | **\( \eta \)** | **\( \varepsilon \)** |
| Roll axis | 50 | 2 | 0.3 |
| Pitch axis | 10 | 1 | 0.3 |
| Yaw axis | 8 | 1 | 0.3 |

| Table 4 Maneuver severity for three channels |
|-----------------|-----------------|
| **Maneuver** | **Command (Deg/Sec)** |
| Low | \( \delta_{\dot{p}}\text{Max}=20 \) | \( \delta_{\dot{q}}\text{Max}=10 \) | \( \delta_{\dot{r}}\text{Max}=5 \) |
| Med | \( \delta_{\dot{p}}\text{Max}=40 \) | \( \delta_{\dot{q}}\text{Max}=20 \) | \( \delta_{\dot{r}}\text{Max}=10 \) |
| High | \( \delta_{\dot{p}}\text{Max}=60 \) | \( \delta_{\dot{q}}\text{Max}=30 \) | \( \delta_{\dot{r}}\text{Max}=15 \) |
linearized models for entire flight envelope of this research are minimum phase so NDI for aircraft does not contain any unobservable zero dynamics. Thus, we can track an arbitrary trajectory and guarantee that the model will be stable with NDI controller.

A. Real-Time Pilot Simulation

To evaluate the performance of NDI and outer loop controller while longitudinal control surface failure happens, three maneuvers with different severity are used as shown in Figure 4 and Table 4. To show the influence of ANSC as outer loop, the following error functions have been investigated

\[ E_{std} = \sqrt{\frac{\sum_{l=1}^{N} e_{kl}^2}{N}} \quad k = p, q, r \] (71)

\[ E_{Cstd} = \sqrt{\frac{\sum_{l \neq k} \left( \frac{\sum_{i=1}^{N} e_{i,l} \delta_{ip}}{\delta_{i,\text{Max}}} \right)^2}{N}} \quad l, k = p, q, r \] (72)

where \( E_{std} \) shows the standard tracking error and \( E_{Cstd} \) is the standard cross coupling error, \( E_{std} \) value can be used for assessing qualitatively pilot handling quality and \( E_{Cstd} \) represents the pilot work load for one maneuver.

The model reference compensator is designed based on flying quality of piloted vehicle\textsuperscript{28}. For longitudinal reference filter, aircraft short period mode is used. There are several flying quality criteria; however, control anticipation parameter (CAP) is used widely. CAP is defined as follow

\[ CAP = \frac{\omega_q^2}{n_{\alpha}} \] (73)

where \( n_{\alpha} \) is aerodynamic load factor. In Figure 6 the desired dynamics is specified by CAP and short period damping ratio. Therefore longitudinal reference filter has \( \zeta_q = 0.8 \), \( \omega_q = 5 \text{ rad/ sec} \). For directional reference filter, aircraft dutch roll mode is used. In Figure 7 the desired dynamics is specified. Therefore directional reference filter has \( \zeta_r = 0.8 \), \( \omega_r = 5 \text{ rad/ sec} \). For roll reference filter, a first order filter with \( \tau < 1 \text{ sec} \) is used.

\[ \text{Figure 4. Pilot inputs to reference model.} \]
With piloted aircraft, the neural network must be capable of adapting quickly enough to assist pilots in controlling a damaged aircraft. However, the neural network should avoid transients that could overload the structure and interfere with pilot’s ability to control the aircraft. As a result of these constraints, input selection becomes a critical factor for successful implementation of neural network-based outer loop design. The input signals used in this study are aircraft state, bias and cross-coupling terms. The estimated angle of attack and sideslip inputs were included to capture modeling effects. However, the sideslip input effects for longitudinal channel did not appear to be significant, thus it was removed. The angle of attack for two other channels is similar to the sideslip for longitudinal channel. The purpose of bias term is to compensate for out of trim condition caused by failures. It drives the integrated error back down to zero under out of trim conditions. The purpose of cross coupling inputs is to allow the NN to learn how one axis may affect another axis. Although one of the functions of NDI is to decouple the axes, coupling can be reintroduced by events such as control surface failures. The adaptive NN implemented for aircraft is divided into three separate networks, one each for the pitch, roll and yaw axes respectively. Table 2 shows the input parameters for neural networks and also eight, six and eight neurons are used for roll, pitch and yaw axis for simulation.

With piloted aircraft, the neural network must be capable of adapting quickly enough to assist pilots in controlling a damaged aircraft. However, the neural network should avoid transients that could overload the structure and interfere with pilot’s ability to control the aircraft. As a result of these constraints, input selection becomes a critical factor for successful implementation of neural network-based outer loop design. The input signals used in this study are aircraft state, bias and cross-coupling terms. The estimated angle of attack and sideslip inputs were included to capture modeling effects. However, the sideslip input effects for longitudinal channel did not appear to be significant, thus it was removed. The angle of attack for two other channels is similar to the sideslip for longitudinal channel. The purpose of bias term is to compensate for out of trim condition caused by failures. It drives the integrated error back down to zero under out of trim conditions. The purpose of cross coupling inputs is to allow the NN to learn how one axis may affect another axis. Although one of the functions of NDI is to decouple the axes, coupling can be reintroduced by events such as control surface failures. The adaptive NN implemented for aircraft is divided into three separate networks, one each for the pitch, roll and yaw axes respectively. Table 2 shows the input parameters for neural networks and also eight, six and eight neurons are used for roll, pitch and yaw axis for simulation.

Figure 5. Lock in place of left elevator occurs at +10 deg from trim at 0.1 sec. a) roll, pitch and yaw response b) longitudinal and lateral states c) accelerations and controls d) neural network weights history
The ANSC parameters are selected as shown in Table 3. The learning rate and boundary layer parameter are selected in a way that actuator rate saturation and oscillation are avoided for different types of maneuver. Numerous failures have been investigated such as lock in place, partial destruction and floating actuator.

1. **Lock in place failure**

   Numerous simulations for lock in place has been investigated but the only failure in which the left elevator is locked at +10 degrees at 0.1 sec is shown here. Figure 5 illustrates time histories of command state of failed case with ANSC and the failed case without outer loop. At first glance, ANSC improves tracking results and reduces cross coupling between command channels, and also show more effect of cross coupling due to failure, with no ANSC as outer loop. Longitudinal maneuver started at 6 sec, induced high roll and sideslip angle which might increase the danger while the pilot is in close formation with other aircrafts or in a landing condition. In the failed case, the longitudinal and lateral state responses with ANSC are very similar to normal case. The history of control effectors and acceleration state that elevator moves to its failure position and stick and this leaves only one pitch actuator to deals with both steady state pitch moments and to track pitch commands. The stuck elevator also induces roll which is countered by the ailerons. Also Figure 5 illustrates several weight histories in each channel for failed case. All weights histories are well behaved bounded values. In Table 5 and Table 6, the effects of ANSC in standard error and standard cross coupling error Vs. the sensitivity of ANSC to maneuver severity is illustrated, it clearly indicates that the average of error with ANSC is near to healthy aircraft. Relation between maximum standard error and handling quality level is shown in Table 5, and the result of the ANSC influence is shown in Table 6. Figure 8 shows the sensitivity of ANSC to the failure amplitude and maneuver severity. Errors are increased by failure amplitude and maneuver severity, as the failure amplitude gets large value the errors grow very quickly because of limitation for control surface position. In most cases the ANSC outer loop improves the aircraft performance to Level 1 of handling quality.

2. **Lock in Place and Learning**

   Due to learning ability of neural network, the outer loop controller can learn the structure of inversion error, so

![Figure 6. CAP flying quality analysis](image)

![Figure 7. Dutch roll desired dynamic](image)

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>HQ. Level 1</th>
<th>HQ. Level 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>4.22 2.7 1.8 4.74 3.6 2.2</td>
<td></td>
</tr>
<tr>
<td>Med</td>
<td>8.4 5.4 3.7 9.4 7.3 4.4</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>12.6 8.2 5.6 14.2 10.9 6.6</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 8. performance as function of failures amplitude, lock in place occurs at 0.1 sec a) standard errors, b) Standard cross coupling](image)
the performance of NDI controller is enhanced. To investigate the learning behavior of ANSC a multiple maneuver in pitch channel is assumed. Left locked elevator lock in place at +10 degree at 6 sec and the simulation result is shown in Figure 9. During the first longitudinal command no failure occurs and the tracking is good for NDI controller with or without ANSC. However, the failure causes pitch channel error and cross coupling errors in roll and yaw channels, initially cross coupling reduces when the ANSC are active, compared to the inactive ANSC. Due to multiple longitudinal maneuvers the NN continues to adapt and show improvement. The errors are reduced gradually as shown in Figure 10. The NN weights learn the inversion structure and converge to real values. Note that the roll angle and sideslip angle excursion are lower when the ANSC is active.

3. Partial destruction of longitudinal control surface
A partial destruction occurs when part of a control surface is lost. A mismatch in dynamic inversion can result because the moments provided by surface have lessened. If the damage is asymmetric, this may result in different trim point position and symmetric command in longitudinal control surface may result in induced roll and yaw. This type of failure is much less demanding than lock in place failures because some control is still retained and the change in system dynamics is less drastic. In Figure 11 the sensitivity of ANSC to failure amplitude and maneuver severity is shown. Moreover ANSC reduces the sensitivities of standard error and standard cross coupling error to the amplitude of failure.

4. Floating Actuator
This type of failure is the most difficult for the control system to handle, because this not only involves a large change in system dynamics, but also a disturbance based on the angle of attack. The lock in place and partial destruction are involved the same change in dynamics, but result in disturbance which is not state dependant. Figure 12 illustrates the performance of outer loop controller. Although the performance of aircraft with ANSC is not good, it reduces the pilot effort to control aircraft. Also this figure shows the behavior of longitudinal and lateral states, while the left elevator is lost and the angle of attack decreases suddenly. ANSC suppresses the failure by commanding the right active elevator and aileron as shown. In Table 9 and Table 10 the sensitivity of aircraft behavior to maneuver severity is shown, as the severity of maneuver is increased the error and cross coupling error is increased. However, ANSC reduces the failure effect strikingly.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Maneuver</th>
<th>Handling Quality Level</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1 1 1</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>1 1 1</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>1 1 1</td>
</tr>
<tr>
<td>With Fault DI</td>
<td>Low</td>
<td>3 3 1</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>3 3 1</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3 3 1</td>
</tr>
<tr>
<td>With Fault DI=ANSC</td>
<td>Low</td>
<td>1 1 1</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>1 1 1</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>1 1 1</td>
</tr>
</tbody>
</table>

Figure 9. Multiple longitudinal maneuvers, while lock in place of left elevator occurs at +10 deg from trim at 6 sec.

Figure 10. Error reduction due to multiple longitudinal maneuvers, while lock in place of left elevator occurs at +10 deg from trim at 6 sec.
B. Cascade NDI Simulation

As it is shown in previous part, control surface failures have great influence on the moment derivatives of aircraft and small effect on force derivatives. Hence if the aircraft autopilot is based on dual loop NDI controller, while control surface failures occur, the first stage of NDI will be changed and the second doesn’t have noticeable variation. Therefore the ANSC as outer loop controller for first stage NDI is used to compensate the inversion error. Figure 13 shows the structure of two-stage NDI. A lock in place of left elevator at +10 deg at 6 sec is assumed to express the performance enhancement by ANSC as shown in Figure 14 and angular velocity and control inputs of aircraft are depicted.

VI. Conclusions

A dual loop control strategy is developed for the fault tolerant flight control system design. ANSC is implemented as outer loop for fast dynamics to compensate the inversion error and nonlinear dynamic inversion controller is designed to invert the aircraft dynamics in several stages. The control strategy is demonstrated in controlling F-16 aircraft in the presence of longitudinal control surfaces failures. Extensive simulations show that the proposed dual loop controller maintains good tracking accuracy during maneuvers with various severities. This is accomplished without requiring any failure detection and identification algorithms.

Acknowledgments

This work was supported in part by Ministère du Développement économique, de l’innovation et de l’exportation (MDEIE), Quebec, Canada and Natural Sciences and Engineering Research Council of Canada (NSERC).

<table>
<thead>
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<th>Condition</th>
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<th>$E_{\text{std}}$</th>
<th>$E_{\text{std}}$</th>
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</tr>
<tr>
<td>DI</td>
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<td>High</td>
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<td>0.78</td>
<td>0.68</td>
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<tr>
<td>Avg.</td>
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<td>0.45</td>
<td>0.35</td>
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</tr>
<tr>
<td>With Fault</td>
<td>Low</td>
<td>10.8</td>
<td>2.82</td>
<td>0.83</td>
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<tr>
<td>DI</td>
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<td>1.03</td>
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<td>1.33</td>
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</tr>
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<td>1.07</td>
<td></td>
</tr>
<tr>
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<td>3.31</td>
<td>1.91</td>
<td>0.44</td>
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<tr>
<td>DI+ANSC</td>
<td>Med</td>
<td>3.64</td>
<td>2.48</td>
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</tr>
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<td>High</td>
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<td>3.21</td>
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<td>2.53</td>
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<th>$E_{\text{std}}$</th>
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<td>0.97</td>
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</tr>
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<td>DI</td>
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<td>0.82</td>
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<td>DI+ANSC</td>
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<tr>
<td>Avg.</td>
<td>2.18</td>
<td>0.79</td>
<td>0.14</td>
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Figure 11. Performance as function of failures amplitude, while a partial destruction of left elevator occurs at 0.1 sec. a) standard error, b) standard cross coupling error
Table 9 Standard error vs. ANSC effect for three maneuvers with a left elevator floating actuator failure

<table>
<thead>
<tr>
<th>Condition</th>
<th>Manuever</th>
<th>( E_{\text{rel}} )</th>
<th>( E_{\text{col}} )</th>
<th>( E_{\text{rol}} )</th>
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<tr>
<td>Without Fault DI</td>
<td>Low</td>
<td>0.38</td>
<td>0.22</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>Med</td>
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<tr>
<td></td>
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<tr>
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<td>1.07</td>
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<tr>
<td>With Fault DI+ANSC</td>
<td>Low</td>
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<td></td>
<td>Avg.</td>
<td>3.66</td>
<td>2.53</td>
<td>0.60</td>
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Table 10 Standard cross coupling error vs. ANSC effect for three maneuvers with a left elevator floating actuator failure

<table>
<thead>
<tr>
<th>Condition</th>
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<th>( E_{\text{col} \rightarrow \text{rol}} )</th>
<th>( E_{\text{rol} \rightarrow \text{col}} )</th>
<th>( E_{\text{rol} \rightarrow \text{col}} )</th>
</tr>
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<tr>
<td>Without Fault DI</td>
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<td>0.007</td>
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<tr>
<td></td>
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<td></td>
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<td>1.75</td>
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<td>With Fault DI</td>
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Figure 12. Floating of left elevator occurs at 0.1 sec. a) roll, pitch and yaw response b) longitudinal and lateral states c) accelerations and controls
References


