MECH 370: Modelling, Simulation and Analysis of Physical Systems

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Course Goal

To introduce methods for predicting the dynamic behavior of physical systems used in engineering

As an introductory course for modeling, simulation and analysis of physical systems containing individual or mixed mechanical, electrical, thermal and fluid elements

You should after the course be able to

- build mathematical models of physical systems from first principles
- analyze behaviour of the system using built mathematical models
- use software tools (e.g. Matlab/Simulink) for modelling, simulation, and analysis

Textbook and References

- **Textbook:**
  - C. M. Close, D. K. Frederick and J. C. Newell, "***Modeling and Analysis of Dynamic Systems***", 3rd edition, John Wiley and Sons Inc., 2002, ISBN: 0-471-39442-4.
- **References:**
  - Lennart Ljung and Torkel Glad, "***Modeling of Dynamic Systems***", Prentice Hall, 1994, ISBN 0-13-597097-0.
  - Robert L. Woods and Kent L. Lawrence, "***Modeling and Simulation of Dynamic Systems***", 1st edition, Prentice Hall, 1997, ISBN: 0133373797.

Course Outline

1. Definition and classification of dynamic systems (chapter 1)
2. Translational mechanical systems (chapter 2)
3. Standard forms for system models (chapter 3)
4. Block diagrams and computer simulation with Matlab/Simulink (chapter 4)
5. Rotational mechanical systems (chapter 5)
6. Electrical systems (chapter 6)
7. Analysis and solution techniques for linear systems (chapters 7 and 8)
8. Developing a linear model (chapter 9)
9. Electromechanical systems (chapter 10)
10. Thermal and fluid systems (chapters 11, 12)

Modelling and Analysis of Physical Systems

Chapter 1

Introduction

- Definition of dynamic systems and models
- Classification of systems
- Ways to build mathematical models (physical and experimental modeling)
- General procedure of system modeling

Systems

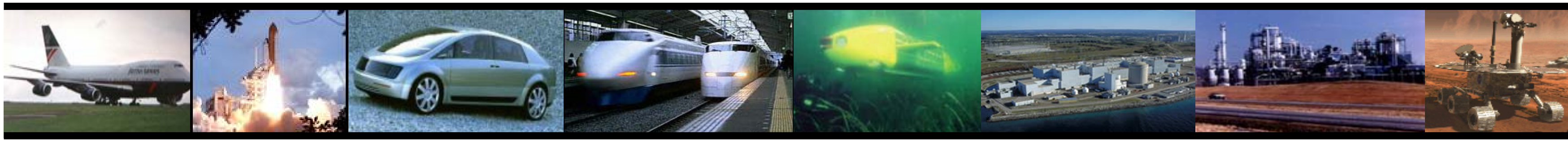
System: A collection of components which are coordinated together to perform a function

A system is a defined part of the real world. Interactions with the environment are described by *inputs*, *outputs*, and *disturbances*.

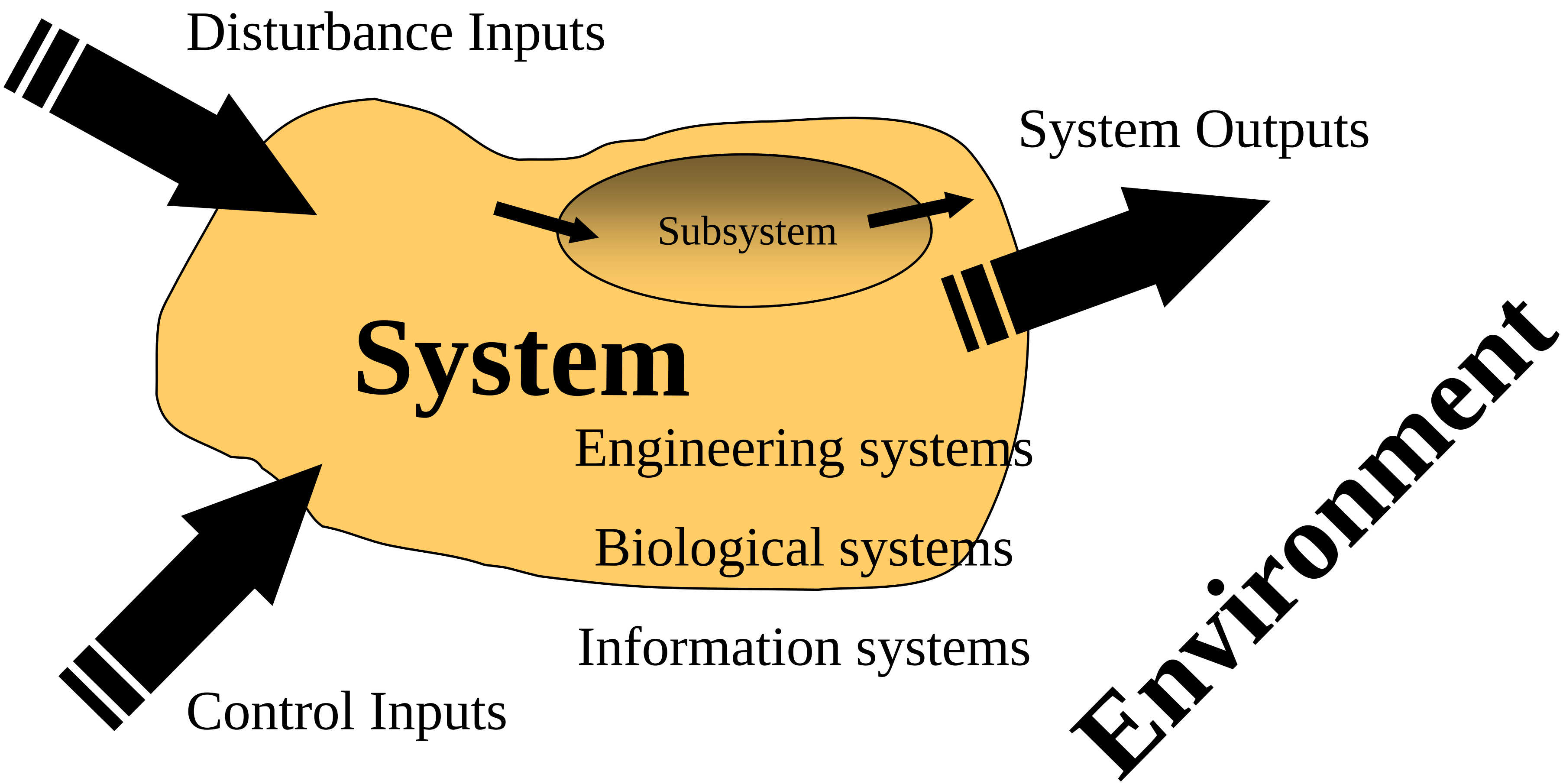
Dynamic system: A system with a memory, i.e., the input value at time *t* will influence the output at future instants (or a system that changes over time).

Examples of dynamic system:

- An aircraft
- An automobile/car
- A robot …



Systems



Subsystem: a component of a larger system

Classification of Dynamic Systems

Temporal	**Dynamic** / Static
Spatial	**Lumped** / Distributed
Linearity	**Linear** / **Nonlinear**
Continuity of time	**Continuous** / Discrete-time / Hybrid
Parameter variation	**Fixed** / **Time-varying**
Quantization of dependent variables	**Nonquantized (Analog)** / Quantized (Digital)
Determinism	**Deterministic**/ Nondeterministic

Classification of Variables

Input / output system model

Inputs, $\mathbf{u}$: External influences on the system (force, current, ...)

Outputs, $\mathbf{y}$: Variables of interest to be calculated or measured (position, velocity, ...)

State variables, $\mathbf{x}$: Represent the status or memory of the system

Initial states $\mathbf{x}(t_o)$ and inputs $\mathbf{u}(t)$ completely determine future outputs $\mathbf{y}(t)$ and states $\mathbf{x}(t)$, $t \geq t_o$ (cause and effect)

From B. Gordon (CU)

Classification of Systems

Temporal characteristics:

Static: Steady state system with no states, e.g. $\mathbf{y} = \mathbf{c}(\mathbf{u})$

Dynamic: Transient system that varies with time, e.g.

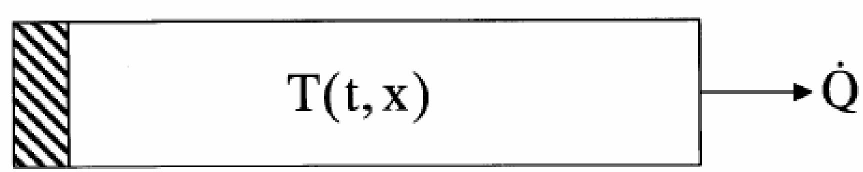
$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}, \mathbf{u}) \quad , \quad \mathbf{y} = \mathbf{c}(t, \mathbf{x}, \mathbf{u})$$

Spatial characteristics:

Lumped: Can be described by a finite number of state variables

Distributed: Cannot be described by a finite number of states, e.g.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



Classification of Systems

Linearity property:

Linear: Systems that have a special linearity property at rest,

$$u_1(t) \to x_1(t), y_1(t) \qquad \qquad u_1(t) + u_2(t) \to x_1(t) + x_2(t), y_1(t) + y_2(t)$$
$$u_2(t) \to x_2(t), y_2(t) \quad \Rightarrow \quad ku_1(t) \to kx_1(t), ky_1(t)$$

e.g.

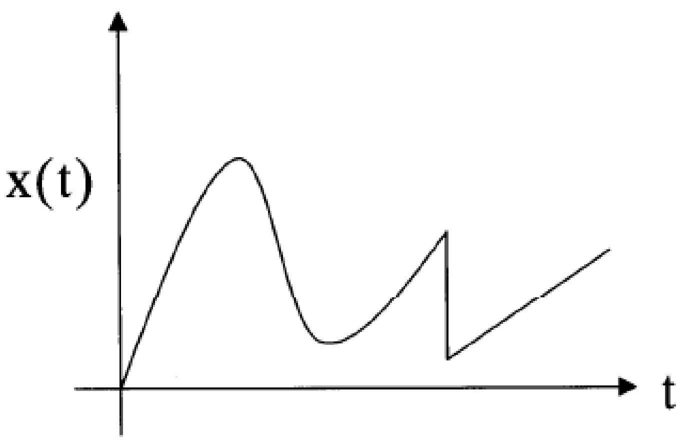
$$\dot{x} = a(t)x + u$$
$$y = x$$

Nonlinear: Superpostion property does not hold, e.g.

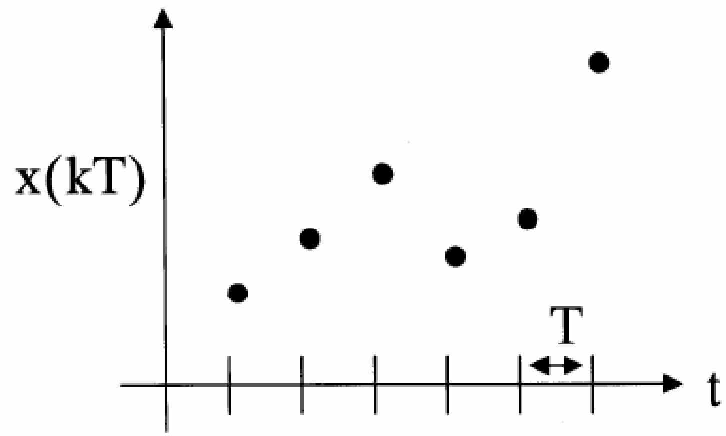
$$\dot{x} = a(t)x - x^3 + xu$$
$$y = x^2$$

Classification of Systems

Continuity of time variable:



Continuous
e.g. $\dot{x} = f(x)$



Discrete-time
e.g. $x[k+1] = f(x[k])$

Hybrid system: Contains both continuous and discrete subsystems

Classification of Systems

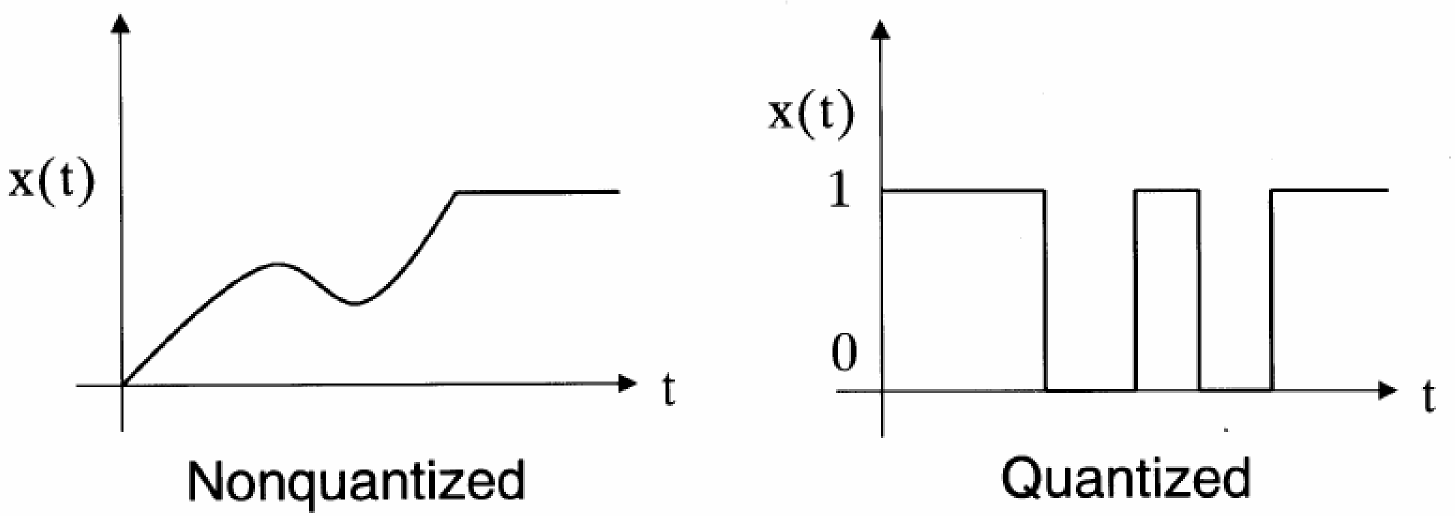
Parameter Variation:

Fixed: System parameters do not change over time, e.g. $\dot{x} = ax$

Time Varying: Parameters such as resistance vary with time, e.g.

$$\dot{x} = a(t)x$$

Quantization of the dependant variables:



Classification of Systems

Determinism:

Deterministic: The system changes in a predetermined manner

Nondeterministic:

The system changes in a random manner as a result of noise and other unpredictable factors

These systems often called stochastic problems are solved in terms of probability distributions, e.g.

$$\dot{x} = a(t)x + d \quad , \quad \bar{d} = 0 \quad , \quad \sigma_d = 1$$

Models

Model: *A description of the system*. The model should capture the essential information about the system.

Systems	**Models**
Complex	Approximate (However, model should capture the relevant information of the system)
Building/Examining systems is expensive, dangerous, time consuming, etc.	Models can answer many questions about the system.

Modelling: Development of a mathematical representation for a physical system.

Types of Models

- **Mental, intuitive or verbal models**
  - ➢ e.g., driving a car
- **Graphs and tables**
  - ➢ e.g., Bode plots and step responses
- **Mathematical models**
  - ➢ A class of model that the relationships between quantities (distances, currents, temperatures etc.) that can be observed in the system are described as mathematical relations
  - ➢ e.g., differential and difference equations, which are well-suited for modeling dynamic systems

Why Mathematical Models are Needed?

- Do not require a physical system
  - ➢ Can treat new designs/technologies without prototype
  - ➢ Do not disturb operation of existing system
- Easier to work with than real world
  - ➢ Easy to check many approaches, parameter values, ...
  - ➢ Flexible to time-scales
  - ➢ Can access un-measurable quantities
- Support safety
  - ➢ Experiments may be dangerous
  - ➢ Operators need to be trained for extreme situations
- Help to gain insight and better understanding

Why Mathematical Models are Needed?

- Analogous Systems
  - ➢ Can have the same mathematical model though different types of physical systems
  - ➢ Common analysis methods and tools can be used

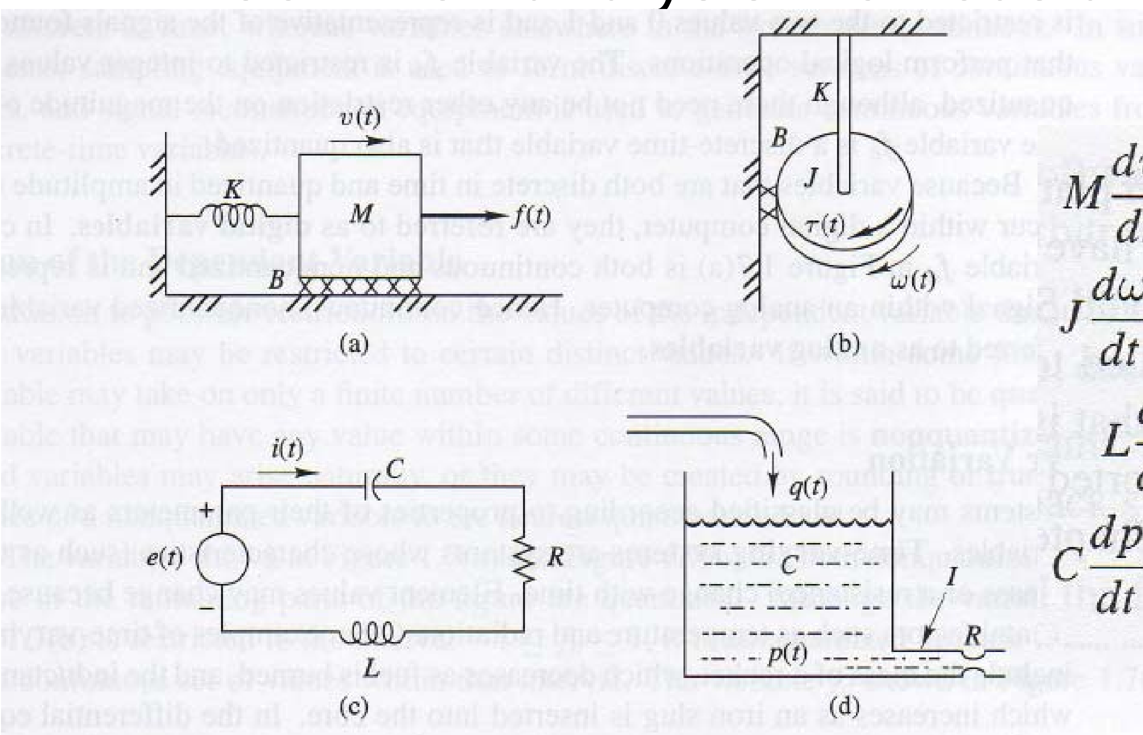


Figure 1.8 Analogous systems. (a) Translational mechanical. (b) Rotational mechanical. (c) Electrical. (d) Hydraulic.

$$
\begin{aligned}
M\frac{dv}{dt}+Bv(t)+K\int_0^t v(\lambda)d\lambda &= f(t)\\
J\frac{d\omega}{dt}+B\omega(t)+K\int_0^t \omega(\lambda)d\lambda &= \tau(t)\\
L\frac{di}{dt}+Ri(t)+\frac{1}{C}\int_0^t i(\lambda)d\lambda &= e(t)\\
C\frac{dp}{dt}+\frac{1}{R}p(t)+\frac{1}{I}\int_0^t p(\lambda)d\lambda &= q(t)
\end{aligned}
$$

How to Build Mathematical Models?

Two basic approaches:

- **Physical/Theoretical modeling** – main topic in this course
  - ❑ Use first principles, laws of nature, etc. to model components
  - ❑ Need to understand system and master relevant facts!

- **Experimental modeling – System identification** – not covered in this course
  - ❑ Use experiments and observations to deduce model
  - ❑ Need prototype or real system!

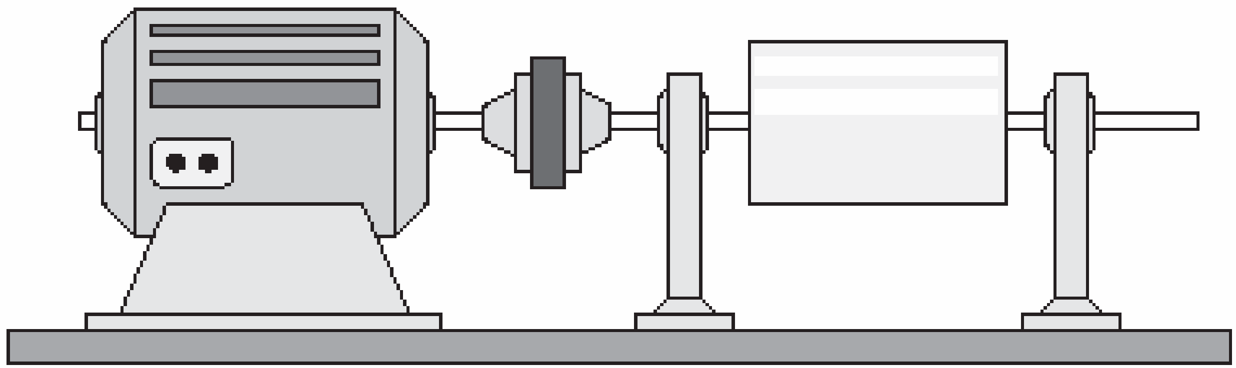
Principle of Physical Modeling

- **Basic idea**: use physics to model system dynamics
  - ➢ balance equations and constitutive relations
    - ✓ e.g. Newton's laws, Kirchhoff's laws etc.
  - ➢ requires detailed knowledge about physics, brings much insight
- Naturally done in continuous-time, leads to ODEs (Ordinary Differential Equations) or DAEs (Differential Algebraic Equations)

$$\text{ODEs}: \ \dot{x}(t) = f(t,x) \ \text{ or } \ \text{DAEs}: \ F(\dot{z}, z, t) = 0$$

Example – Physical Modeling

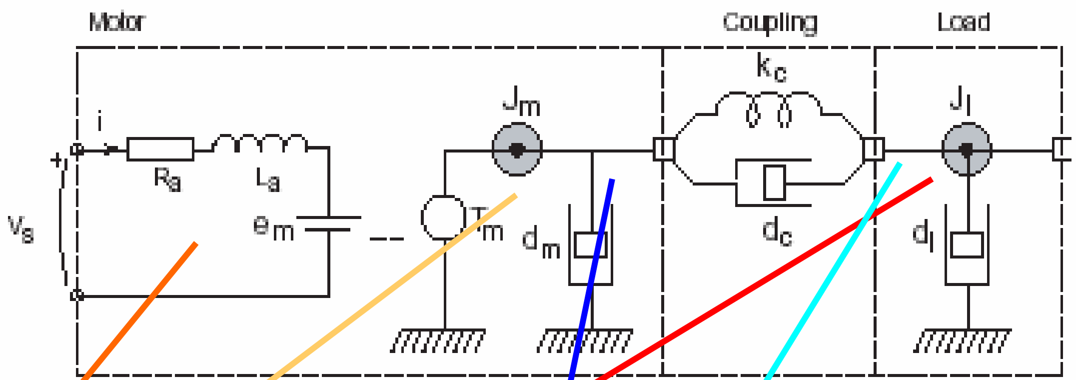
DC motor



A schematical illustration of the system structure

Example – Physical Modeling

More detailed schematic



State equations

$$L_a \frac{di}{dt} = V_s - R_a i - k_m \omega_m$$

$$\frac{d\theta_m}{dt} = \omega_m \qquad J_m \frac{d\omega_m}{dt} = k_m i - d_m \omega_m - k_c(\theta_m - \theta_l) - d_c(\omega_m - \omega_c)$$

$$\frac{d\theta_l}{dt} = \omega_l \qquad J_l \frac{d\omega_l}{dt} = -d_l \omega_l - k_c(\theta_l - \theta_m) - d_c(\omega_l - \omega_m)$$

These are ODEs. How about other forms of mathematical models?

Mathematical Models

Mathematical model descriptions

- Transfer functions
- State space
- Block diagrams

Notation for continuous-time and discrete-time models

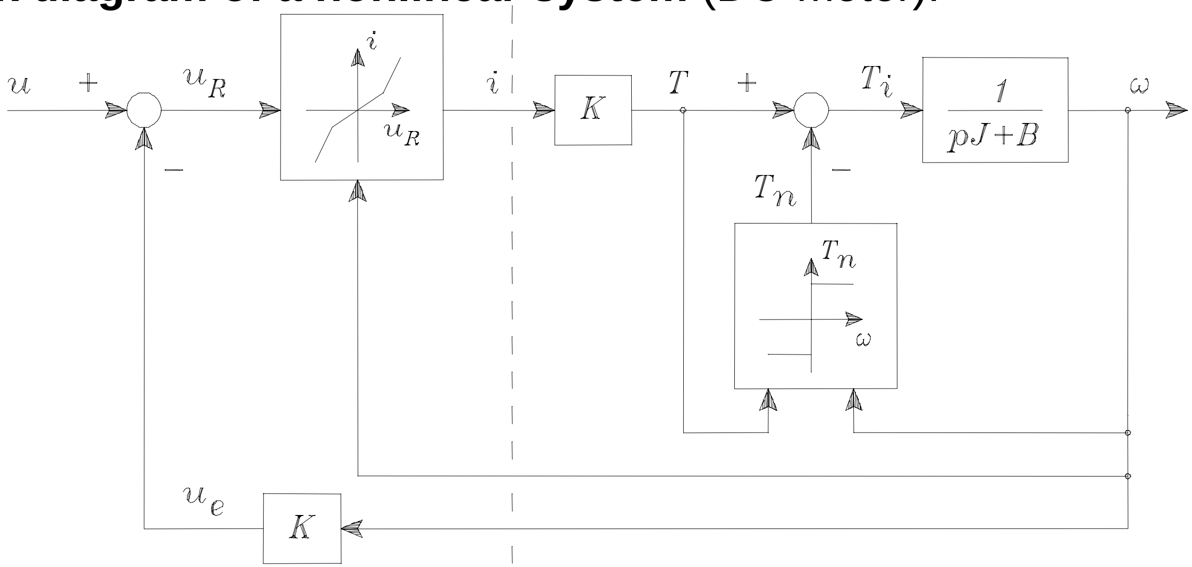
Complex Laplace transform variable s and differential operator p:

$$\dot{x}(t) = dx(t)/dt = px(t)$$

Complex z-transform variable z and shift operator q:

$$x(k+1) = qx(k)$$

Block diagram of a nonlinear system (DC-motor):



From M. Knudsen, AAU

Type of Models and System Modeling Approaches

Models:

mathematical – other

parametric – nonparametric

continuous-time – discrete-time

input/output – state-space

linear – nonlinear

dynamic – static

time-invariant – time-varying

SISO – MIMO

Modelling / System Identification:

physical (theoretical) – experimental

white-box – **grey-box** – black-box

structure determination – parameter estimation

time-domain – frequency-domain

Types of Mathematical Models

- Parametric and Non-parametric Models

Many approaches to system modelling, depending on model class

– linear/nonlinear

– parametric/nonparametric

<u>Non-parametric</u> methods try to estimate a generic model of a system

– step responses, impulse responses, frequency responses, etc.

<u>Parametric</u> methods estimate parameters in a user-specified model

– parameters in transfer functions, state-space matrices of a given order, etc.

Types of Mathematical Models

- Linear and Nonlinear Models

The system modelling methods are characterized by model type:

A. Linear model: Classical system identification

B. Neural network: Strongly non-linear systems with complicated structures – no relation to the actual physical structures/parameters (will not be covered)

C. General simulation model: Any mathematical model, that can be simulated e.g. with Matlab/Simulink. It requires a realistic physical model structure, typically developed by theoretical modelling

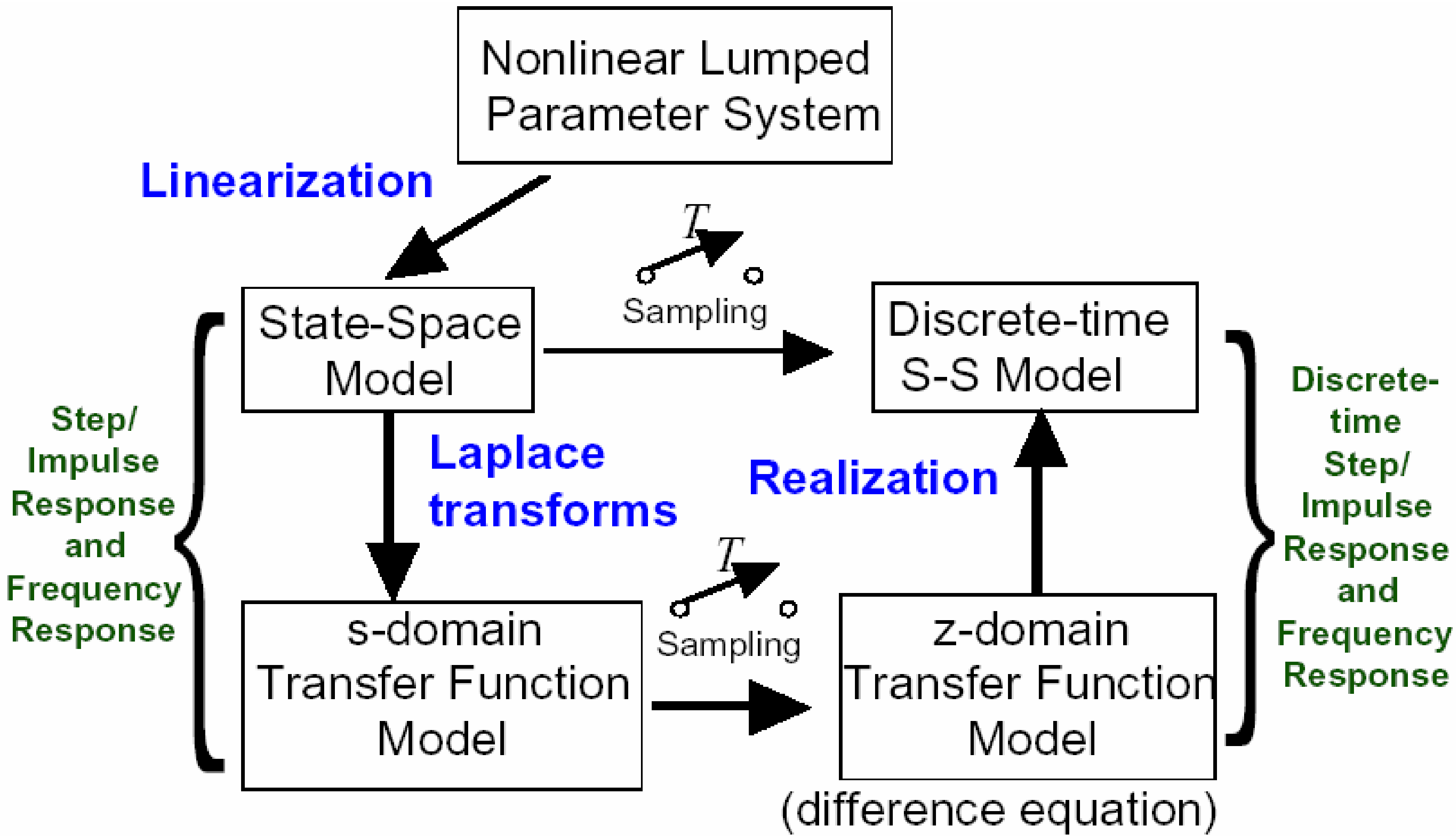
Types of Mathematical Models

- Purpose of Models

Models can also be classified according to purpose:

- **Models to assist plant design and operation**
  - ➢ Detailed, physically based, often non-dynamic models to assist in fixing plant dimensions and other basic parameters
  - ➢ Economic models allowing the size and product mix of a projected plant to be selected
  - ➢ Economic models to assist decisions on plant renovation
- **Models to assist control system design and operation**
  - ➢ Fairly complete dynamic model, valid over a wide range of process operation to assist detailed quantitative design of a control system
  - ➢ Simple models based on crude approximation to the plant, but including some economically quantifiable variables, to allow the scope and type of a proposed control system to be decided
  - ➢ Reduced dynamic models for use on-line as part of a control system

Systems/Models Representations



The Modeling Process

1. Define the purpose or objective of the model

   Identify system boundaries, functional blocks, interconnecting variables, inputs and outputs. Construct a functional block diagram.

2. Determine the model for each component or subsystem

   Apply known physical laws when possible, otherwise use experimental data to identify input-output relationships - system identification.

The Modeling Process

3. Integrate the subsystem models into an overall system model

   Combine equations, eliminate variables, check for sufficient equations to solve the system.

4. Verify the model validity and accuracy

   Implement a *simulation* of the model equations and compare with experimental data for the same conditions (Chapter 4).

The Modeling Process

5. Make simplifications to create an approximate model suitable for design
   - Linearization of model equations (Chapter 9)
   - Reduce the order of the model by eliminating unimportant dynamics

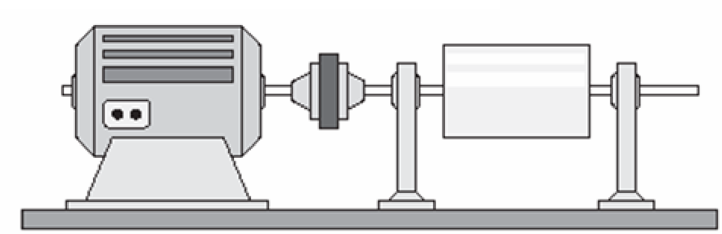
trade-off

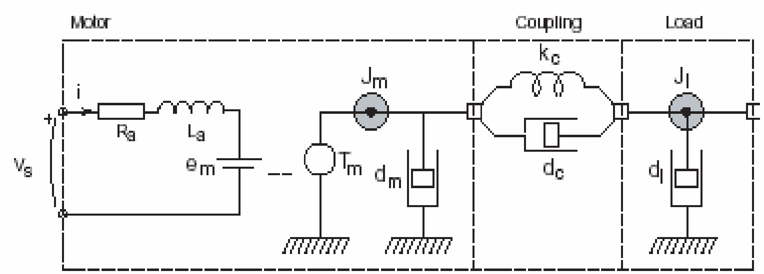
Model Complexity ⟷ Model Accuracy

Specified Procedure of System Modeling

- Divide the system into idealized components
- Apply physical laws to the elements
- Apply interconnection laws between elements
- Combine the equations to obtain the model

More detailed schematic





State equations

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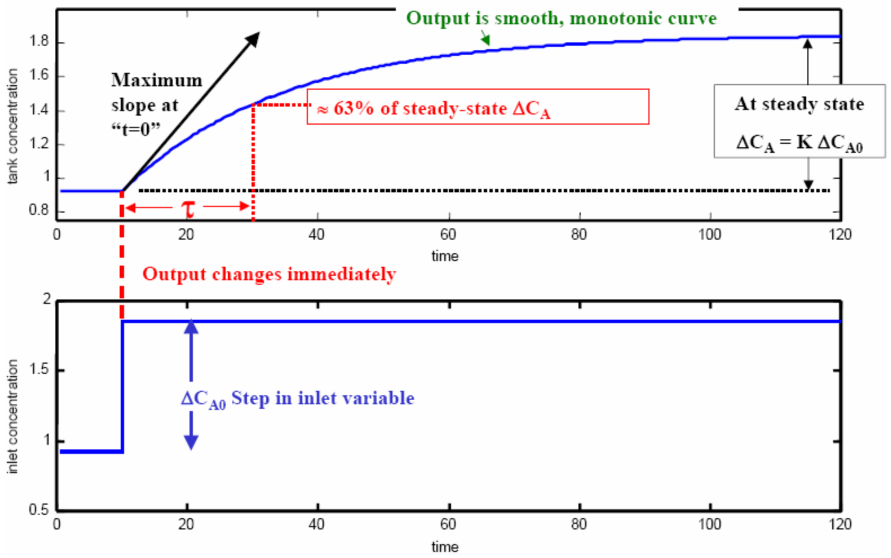
$$\frac{d\theta_l}{dt} = \omega_l \qquad J_l \frac{d\omega_l}{dt} = -d_l \omega_l - k_c(\theta_l - \theta_m) - d_c(\omega_l - \omega_m)$$

Analysis of Systems

- Dynamic models obtained from modelling step will involve differential/algebraic equations
- We can solve simple models analytically to provide information on relationship between process and dynamic response
- We can solve complex models numerically, e.g. using Euler or Runge-Kutta method with computer simulation – relevant to ENGR 391- numerical methods in engineering



Sample time response analysis of a system

Modelling, Simulation and Analysis of Physical Systems

Chapter 2

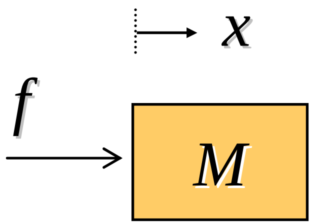
Modelling of System Components

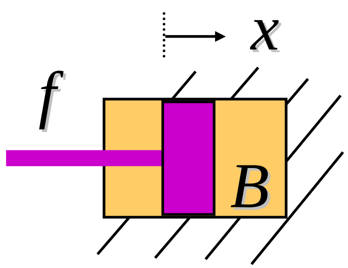
-- Translational Mechanical Systems

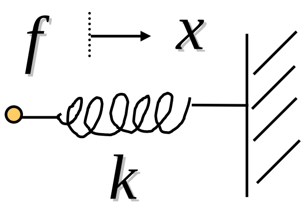
- Modelling process
- Overview of element models of various types of systems
- Modelling of translational mechanical systems

Overview of Element Models in Physical Systems

Mechanical Translational Models



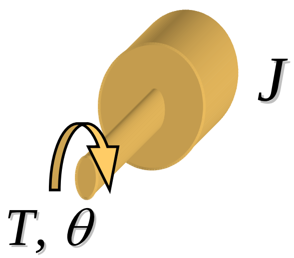
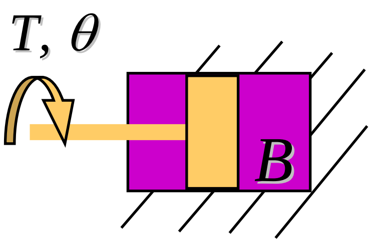
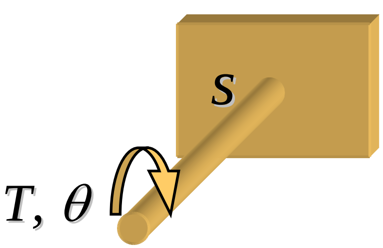




	force/velocity	force/position
Mass	$f = M\, dv/dt$	$f = M\, dx^2/dt^2$
Viscous friction	$f = B\, v$	$f = B\, dx/dt$
Spring	$f = k \int v\, dt$	$f = k\, x$

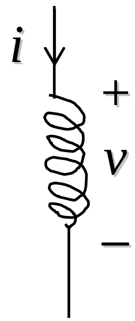
Overview of Element Models in Physical Systems

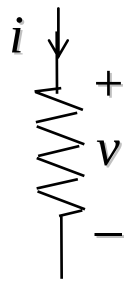
Mechanical Rotational Models

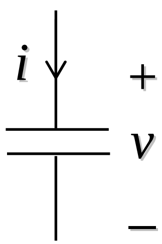
		torque/velocity	torque/position
	Inertia	$T = J\, d\omega/dt$	$T = J\, d\theta^2/dt^2$
	Viscous friction	$T = B\, \omega$	$T = B\, d\theta/dt$
	Stiffness	$T = s \int \omega\, dt$	$T = s\ \theta$

Overview of Element Models in Physical Systems

Electrical Component Models



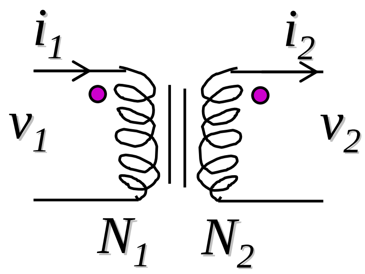




	voltage/current	voltage/charge
Inductance	$v = L\, di/dt$	$v = L\, dq^2/dt^2$
Resistance	$v = R\, i$	$v = R\, dq/dt$
Capacitance	$v = 1/C \int i\, dt$	$v = 1/C\, q$

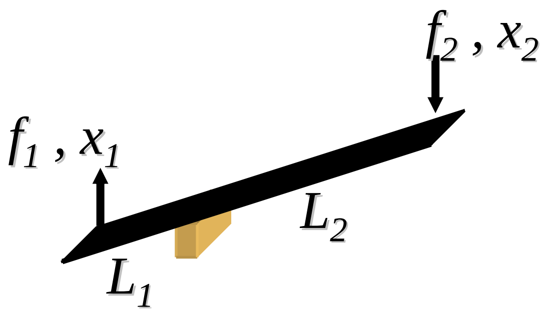
Overview of Element Models in Physical Systems

Transformation Models



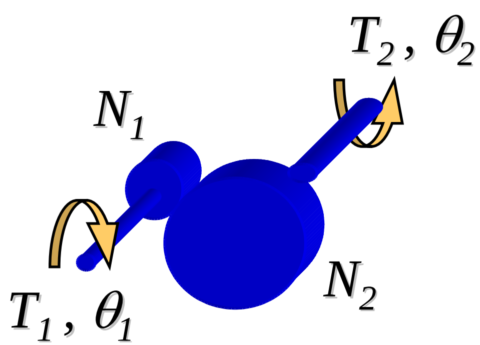
Transformer

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \qquad \frac{i_1}{i_2} = \frac{N_2}{N_1}$$



Lever

$$\frac{f_1}{f_2} = \frac{L_2}{L_1} \qquad \frac{x_1}{x_2} = \frac{L_1}{L_2}$$



Gears

$$\frac{T_1}{T_2} = \frac{N_1}{N_2} \qquad \frac{\theta_1}{\theta_2} = \frac{N_2}{N_1}$$

Mathematical Modelling of Mechanical Systems

Elementary parts

- A means for storing kinetic energy (mass or inertia)
- A means for storing potential energy (spring or elasticity)
- A means by which energy is gradually dissipated (damper)

Mathematical Modelling of Mechanical Systems

Motion in mechanical systems can be

- Translational
- Rotational, or
- Combination of above

Mechanical systems can be of two types

- Translational systems
- Rotational systems

Variables that describe motion

- Displacement, x
- Velocity, v
- Acceleration, a

Modeling of translational mechanical systems

Key concepts to remember

- **Three primary elements of interest**
  - – Mass (inertia) m
  - – Stiffness (spring) K
  - – Friction - Dissipation (damper) B
  - – Usually we deal with "equivalent" m, B, K
    - ➢ Distributed mass -> lumped mass
- **Lumped parameters**
  - – Mass maintains motion
  - – Stiffness restores motion
  - – Damping eliminates motion

Modeling of translational mechanical systems

Variables

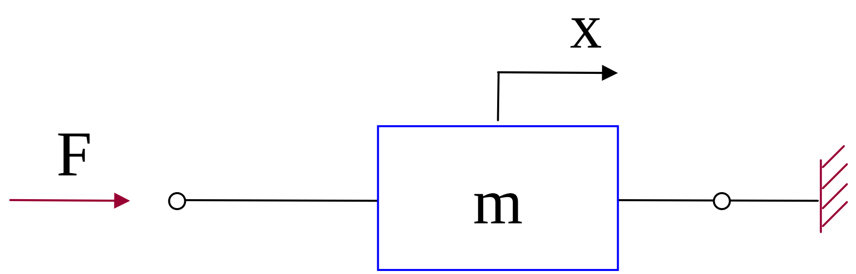
- x: displacement (m)
- v: velocity (m/sec)
- a: acceleration (m/sec^2)
- f: force (N)
- p: power (Nm/sec)
- w: work (energy) (Nm)

All these variables are functions of time, t

Element Laws

Mass

Mass: Property or means of kinetic energy is stored



F = Mass * Acceleration

$= m\ddot{x}$

$= m\dot{v}$

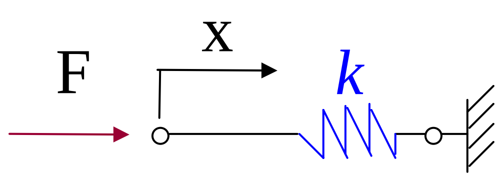
$= m\,a$

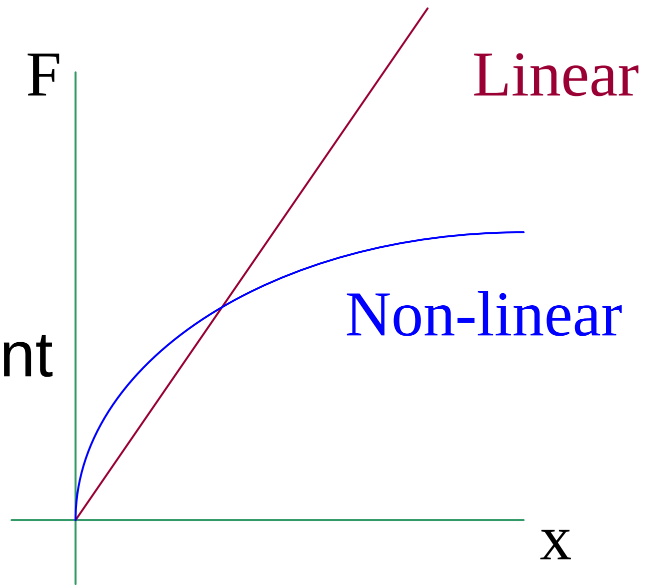
Element Laws
Stiffness

Stiffness is the resistance of an elastic body to deflection or deformation by an applied force

The most common stiffness element is the spring

Spring force is proportional to *displacement*





Spring force =Stiffness * Displacement

$$F_s = k\,x$$

Element Laws

Friction

Friction is the force that opposes the relative motion or tendency of such motion of two surfaces in contact

Exists in all systems and opposes the motion of the mass

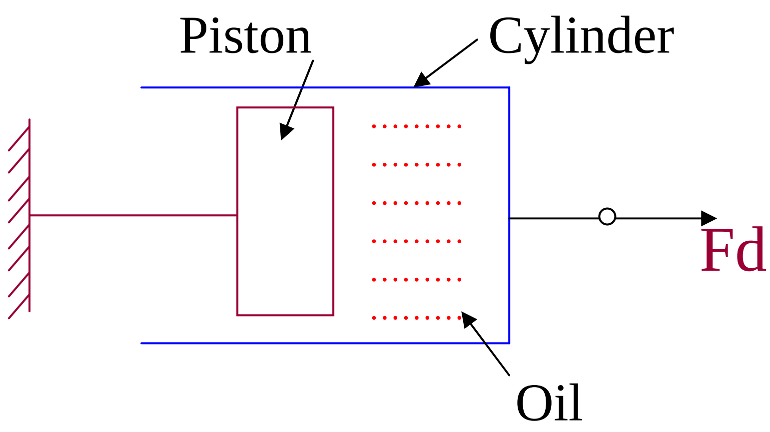
- Static friction: occurs when the two objects are moving relative to each other (like a book on a desk)
- Coulomb friction: the classical approximation of the force of friction is known as Coulomb friction (dry friction)
- Viscous friction: a mass sliding on an oil film is subject to viscous friction

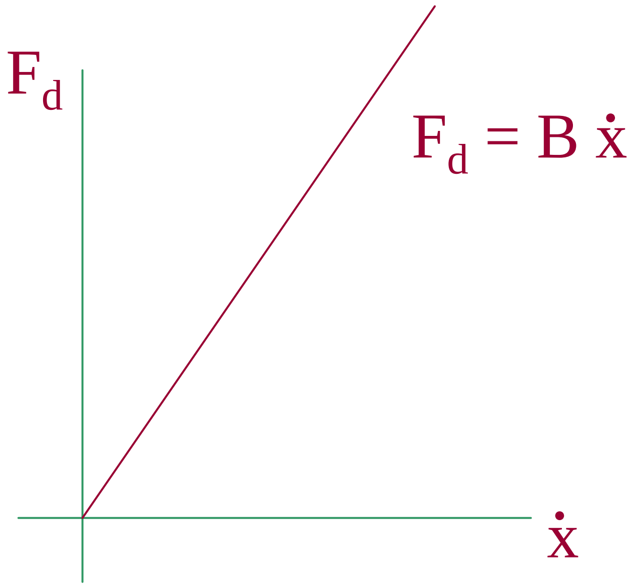
Element Laws
Friction (cont'd)

Viscous Friction (Damping)

Viscous Damping: Means by which energy is absorbed

Damping Force is proportional to *velocity*

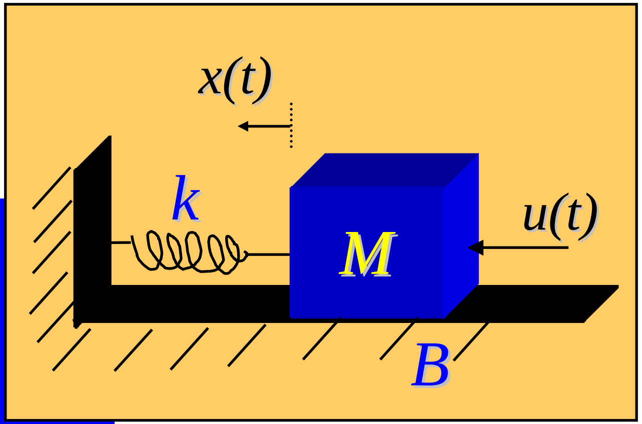




Damping Force = Damping Coefficient * Velocity

$$F_d = B\dot{x}$$

A Translational System Example



Stiffness

Friction

spring force $f_s = k\,x$

sliding force $f_b = B v = B \dfrac{dx}{dt}$

net force on mass $= u - f_s - f_b$, then

$$M\frac{d^2x}{dt^2} = u - f_s - f_b = u - k\,x - B\frac{dx}{dt}, \text{ or}$$

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + k\,x = u$$

Reading and Exercise

- **Reading**
  - ❑ Chapter 1 and Sections 2.1-2.2
- **Exercise**
  - ❑ No assignment today

Your any questions, suggestions or comments are welcome