MECH 6091 – Flight Control System

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Contents

- Building the Model
- Control Mechanisms
- PID and Gain Scheduling
- LQR
- Trajectory Tracking and Disturbance Recovery
Introduction
Building the Height Model

\[ M \ddot{Z} = 4F \cos(r) \cos(p) - Mg \]

\[ \ddot{Z} = \frac{4F}{M} \cos(r) \cos(p) - g \]

\[ Z = \int \int \frac{4F}{M} \cos(r) \cos(p) - g \]
Building X and Pitch Model

\[ M \ddot{X} = 4 F \sin(p) \]

- The motion along X is caused by changing pitch angle.
- The command to change Pitch is increasing or decreasing the rear propeller speed and decreasing or increasing the front propeller.

\[ J \ddot{\theta} = \Delta F L \]
\[ J = J_{\text{roll}} = J_{\text{pitch}} \]
Building Y and Roll Model

\[ M \ddot{Y} = -4F \sin(r) \]

- The motion along \( Y \) is caused by changing Roll angle.
- The command to change Roll is increasing or decreasing the left propeller speed and decreasing or increasing the right propeller.

\[ J \ddot{\theta} = \Delta F L \]

\[ J = J_{\text{roll}} = J_{\text{pitch}} \]
Control Mechanisms

PID

LQR
<table>
<thead>
<tr>
<th>Term</th>
<th>Math Function</th>
<th>Effect on Control System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td>$KP \times \text{Verror}$</td>
<td>Typically the main drive in a control loop, KP reduces a large part of the overall error.</td>
</tr>
<tr>
<td><strong>I</strong></td>
<td>$KI \times \int \text{Verror} , dt$</td>
<td>Reduces the final error in a system. Summing even a small error over time produces a drive signal large enough to move the system toward a smaller error.</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>$KD \times \frac{d\text{Verror}}{dt}$</td>
<td>Counteracts the KP and KI terms when the output changes quickly. This helps reduce overshoot and ringing. It has no effect on final error.</td>
</tr>
</tbody>
</table>
PID – Height Model

PID Controller (mask) (link)
Enter expressions for proportional, integral, and derivative terms.
F=1/s+Ds
Parameters
Proportional:
85
Integral:
20
Derivative:
45

Z Response

PID Output

13.7N

Trial
**PID – Height Model**

**Desired Z=5**

<table>
<thead>
<tr>
<th>Term</th>
<th>1st Trial</th>
<th>Final Trial</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>85</td>
<td>20</td>
</tr>
<tr>
<td>I</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>45</td>
<td>10</td>
</tr>
</tbody>
</table>

![Graph showing PID output with desired Z=5](image)

![Graph showing height over time](image)
**PID Height Model**

**Desired Z=10**

<table>
<thead>
<tr>
<th>Term</th>
<th>1st Trial</th>
<th>Final Trial</th>
<th>New Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>85</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>I</td>
<td>20</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>45</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>
PID – Gain Scheduling

%function out = GAIN(Zd)
if Zd == 1
    Kp = 85
    Ki = 20
    Kd = 45
elseif (Zd >= 2 && Zd <= 5)
    Kp = 70
    Ki = 8
    Kd = 10
elseif (Zd >= 6 && Zd <= 10)
    Kp = 10
    Ki = 3
    Kd = 7
end
out(1) = Kp
out(2) = Ki
out(3) = Kd
**PID Other Elements**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proportional</th>
<th>Integral</th>
<th>Derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>X position</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Y position</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>10</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Roll angle</td>
<td>10</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Yaw angle</td>
<td>0.2</td>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>
LQR

- A system can be defined in state space form as:

\[ \dot{X} = Ax + Bu \]
\[ Y = Cx \]

- The feedback control law is to determine the gain \( K \) to stabilize and improve the performance of the system with the state-variable feedback

\[ U = -Kx \]

- The new controlled dynamics of the system becomes

\[ \dot{X} = (A - BK)x \]
The selection of the feedback gains $K$ is made by LQR (Linear Quadratic Regulator). This method is based in the minimized of the cost function $J$

$$J = \int_0^\infty (x^TQx + u^TRu)dt$$

The Matlab Function ‘$K=lqr(A,B,Q,R)$’ is used to find the values for $K$.

Where $Q$ is a positive-define matrix and $R$ is positive-define matrix, both are symmetric.

LQR methodology attempts to balance between a faster response and a low control effort.
For tracking a reference input we implement a LQR + INTEGRAL FORWARD CONSTANT (Ki).
LQR

LQR MATLAB

- Controllable and Observable.
- Matlab function ‘ctrb’ and ‘obsv’

<table>
<thead>
<tr>
<th>MODEL</th>
<th>CTR_RANK</th>
<th>OBSV_RANK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>X &amp; Y</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Θ &amp; φ</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Yaw</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
The last row is a forth state to facilitate the use of an integrator for tracking input controller. The values for $K$, $M$ and $w$ is taken from Lab manual ($K= 120N$, $M=1.4Kg$ & $w=15 \text{ rad/sec}$)
We use the LQR matlab function to calculate K. We selected Q as a diagonal matrix of 1 (4x4) and R is 1.

\[ K = \text{lqr}(A_z, B_z, Q, 1) \]

\[ K = \begin{bmatrix} 1.7522 & 1.0351 & 6.0227 & 1 \end{bmatrix} \]

The first 3 terms are the LQR feedback controller and the last term is the Ki (forward integral constant).
LQR

HEIGHT SIMULINK MODEL
These equations were implemented on simulink. We use the ‘linmod’ matlab function to get the matrix A,B,C and D for X & Y.
\[ [Axlin, Bxlin, Cxlin, Dxlin] = \text{linmod('NONLIN\_X1')} \]

\[ Axlin = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad Bxlin = \begin{bmatrix} 0 \\ 9.2857 \end{bmatrix} \quad Cxlin = [1 \ 0] \quad Dxlin = 0 \]

- The lineal and non lineal were compared.
The K gain matrix is calculated with ‘lqr’ matlab function and the Q matrix is a diagonal matrix of 1 (3X3) and R is 1.

\[ K = lqr(Axlin1, Bxlin, Q, 1) \]

\[ K = \begin{bmatrix} 1.8336 & 1.1811 & 1.0000 \end{bmatrix} \]
The values for K, L, w & J are taken from lab manual (K=120N, L=0.2m, J=0.03Kgm² & w=15 rad/sec)

The A, B, C and D matrix are:

\[
\begin{bmatrix}
    \dot{\theta} \\
    \dot{\phi} \\
    \dot{\psi} \\
    \dot{\xi}
\end{bmatrix}
= \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & \frac{KL}{J} & 0 \\
    0 & 0 & -w & 0 \\
    1 & 0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    w
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \dot{\theta} \\
    \dot{\phi} \\
    \dot{\psi} \\
    \dot{\xi}
\end{bmatrix}
= \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & 800 & 0 \\
    0 & 0 & -15 & 0 \\
    1 & 0 & 0 & 0
\end{bmatrix}
+ \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    15
\end{bmatrix}
\]

Aₚ = \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & 800 & 0 \\
    0 & 0 & -15 & 0 \\
    1 & 0 & 0 & 0
\end{bmatrix}

Bₚ = \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    15
\end{bmatrix}

Cₚ = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}

Dₚ = \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix}
The first design was with an identity matrix 4x4 and R=1.

The response in pitch is not stable. We need to play with matrix Q and R to improve the response. The final values for Q and R:

\[
Q = \begin{bmatrix}
10 & 0 & 0 & 0 \\
0 & 0.05 & 0 & 0 \\
0 & 0 & 0.05 & 0 \\
0 & 0 & 0 & 20000
\end{bmatrix}, \quad R = 0.05
\]
LQR

- The final response on pith and simulink model:
LQR

**YAW**

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{Ky}{Jyaw}
\end{bmatrix}
\]

- Ky and Jyaw is taken from Lab manual (Ky=4Nm & Jyaw=0.4 Kg m\(^2\))

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
Ayaw =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix} \\
Byaw =
\begin{bmatrix}
0 \\
100
\end{bmatrix} \\
Cyaw =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \\
Dyaw =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
The K gain matrix is calculated with ‘lqr’ matlab function and the Q matrix is a diagonal matrix of 1 (2X2) and R is 1.

\[ K = \begin{bmatrix} 1.0000 & 1.0100 \end{bmatrix} \]
LQR

LQR OUTPUTS RESULTS

- For $Z_d = 1$  $X_d = 1$  $Y_d = 1$  $\gamma_d = 1$
## Compare Control Methods

<table>
<thead>
<tr>
<th>PID</th>
<th>LQR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with Guess</td>
<td>Modeled to System Dynamics</td>
</tr>
<tr>
<td>Range Limit</td>
<td>No Range Limit</td>
</tr>
<tr>
<td>Controllability not known</td>
<td>Controllability Known</td>
</tr>
</tbody>
</table>

![PID Control Method Graph](image1)

![LQR Control Method Graph](image2)
Trajectory Tracking

No Noise

0.3 Noise

1 Noise
Trajectory Tracking

Top View

Front View
Disturbance

Negative Pulse
Disturbance
3D Build

VR Sink

VR Signal Expander

VR Sink

VR Sink

VR Sink

World properties
- Source file: Ballard
- Open VRML Viewer automatically
- Allow viewing from the Internet

Description:

Block properties
- Sample time (s) for display: 0.1
- Show video output port
- Video output signal dimensions

VRML Tree

- ROOT
  - (Viewpoint)
  - (SpotLight)
  - (PointLight)
  - (ScaleOrientation)
  - (Translation)
  - (BoxCenter Xyz)
  - (BoxSize Xyz)
  - children (MNode)
  - (Background)
  - Qball (Transform)
    - addChildren (MNode)
    - removeChildren (MNode)
    - rotation (SFVec3f)
    - scale (SFVec3f)
    - scaleOrientation (SFMatrix3f)
    - translation (SFVec3f)
    - boxCenter (SFVec3f)
    - boxSize (SFVec3f)
    - children (MNode)
    - (Wall (Transform))
    - (Column_2 (Transform))
    - (Column_1 (Transform))
    - (Column_3 (Transform))
Conclusion

Design Conditions

Qball Control

System Dynamics
Control
Response
Trajectory
Questions?