**F7-1: System Identification**

Youmin Zhang

Phone: 7912 7741   Office Location: FUV 1.6  
Email: ymzhang@cs.aue.dk  

---

**Course Outline**

1. Introduction and overview on system identification
2. Non-recursive (off-line) identification methods
3. Non-recursive and recursive (on-line) identification methods
4. Recursive identification methods
5. Practical aspects and applications of system identification

---

**Associated Reading in the Textbook**

1. Introduction and overview on system identification (Ch. 1; 4.1-4.3; Ch. 6)
2. Non-recursive (off-line) identification methods (Ch. 7)
3. Non-recursive and recursive (on-line) identification methods (Ch. 10; Ch. 11)
4. Recursive identification methods (Ch. 11)
5. Practical aspects and applications of system identification (Ch. 13, 14, 16, 17)

---

**General Course Information**

- **Lectures:**  
  Location: Wednesdays (B201); Time: 8:15-12:00
- **Instructor:**  
  Youmin Zhang, FUV 1.6, Email: ymzhang@cs.aue.dk
- **Textbook:**  
- **Reference books:**
- **Course web-page:**
F7-1: System Identification

Lecture 1
Introduction and Overview

• What is System Identification (SI)?
• Introduction to systems and models
• Procedure of system identification
• Methods of system identification
• Review on topics covered in course “Modeling and Simulation”
• Examples of system identification

System Identification

“Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent.”
- L. Zadeh, (1962)

Disturbances $v(t)$

Inputs $u(t)$

System

Outputs $y(t)$

System identification is the field of modeling dynamic systems from experimental data.

Systems

System: A collection of components which are coordinated together to perform a function.

A system is a defined part of the real world. Interactions with the environment are described by inputs, outputs, and disturbances.

Dynamic system: A system with a memory, i.e., the input value at time $t$ will influence the output at future instants.

Examples of dynamic system: (pp. 2-6, textbook)

• Example 1.1 A Solar-Heated House
• Example 1.2 A Military Aircraft
• Example 1.3 Speech

Models

Model: A description of the system. The model should capture the essential information about the system.

<table>
<thead>
<tr>
<th>Systems</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex</td>
<td>Approximative (However, model should capture the relevant information of the system)</td>
</tr>
<tr>
<td>Building/Examining systems is expensive, dangerous, time consuming, etc.</td>
<td>Models can answer many questions about the system.</td>
</tr>
</tbody>
</table>

From E.K. Larsson, UU
Types of Models

- Mental, intuitive or verbal models
  - e.g., driving a car
- Graphs and tables
  - e.g., Bode plots and step responses
- Mathematical models
  - e.g., differential and difference equations, which are well-suited for modeling dynamic systems

Mathematical Models and Benefits

- Do not require a physical system
  - Can treat new designs/technologies without prototype
  - Do not disturb operation of existing system
- Easier to work with than real world
  - Easy to check many approaches, parameter values, ...
  - Flexible to time-scales
  - Can access un-measurable quantities
- Support safety
  - Experiments may be dangerous
  - Operators need to be trained for extreme situations
- Help to gain insight and better understanding

Mathematical Models

- Model descriptions
  - Transfer functions
  - State-space models
  - Block diagrams
- Notation for continuous-time and discrete-time models
  - Complex Laplace variable $s$ and differential operator $p$:
    $$ x(t) = \frac{\partial x(t)}{\partial t} = px(t) $$
  - Complex z-transform variable $z$ and shift operator $q$:
    $$ x(k+1) = qx(k) $$

Block diagram of a nonlinear system (DC-motor):

Type of Models and System Modeling

- Models
  - mathematical – other
  - parametric – nonparametric
  - continuous-time – discrete-time
  - input/output – state-space
  - linear – nonlinear
  - dynamic – static
  - time-invariant – time-varying
  - SISO – MIMO
- Modeling/System Identification
  - theoretical (physical) – experimental
  - white-box – grey-box – black-box
  - structure determination – parameter estimation
  - time-domain – frequency-domain
  - direct – indirect
Types of Models
- Parametric and Non-parametric Models

Many approaches to system identification, depending on model class
- linear/nonlinear
- parametric/nonparametric

Non-parametric methods try to estimate a generic model of a signal or system.
- step responses, impulse responses, frequency responses, etc.

Parametric methods estimate parameters in a user-specified model
- parameters in transfer functions, state-space matrices of given order, etc.

Types of Models – Cont’d

Models can also be classified according to purpose:
• Models to assist plant design and operation
  - Detailed, physically based, often non-dynamic models to assist in fixing plant dimensions and other basic parameters
  - Economic models allowing the size and product mix of a projected plant to be selected
  - Economic models to assist decisions on plant renovation
• Models to assist control system design and operation
  - Fairly complete dynamic model, valid over a wide range of process operation to assist detailed quantitative design of a control system
  - Simple models based on crude approximation to the plant, but including some economically quantifiable variables, to allow the scope and type of a proposed control system to be decided
  - Reduced dynamic models for use on-line as part of a control system

Systems/Models Representations

The system identification methods are characterized by model type:
A. Linear discrete-time model: Classical system identification
B. Neural network: Strongly non-linear systems with complicated structures – no relation to the actual physical structures/parameters (will not be covered)
C. General simulation model: Any mathematical model, that can be simulated e.g. with Matlab/Simulink. It requires a realistic physical model structure, typically developed by theoretical modelling
How to Build Mathematical Models?

Two basic approaches:

- **Physical modeling**
  - Use first principles, laws of nature, etc. to model components
  - Need to understand system and master relevant facts!

- **System identification** - Experimental modeling
  - Use experiments and observations to deduce model
  - Need prototype or real system!

Principle of System Identification

**Basic Idea**: estimate system from measurement of $u(t)$ and $y(t)$

**Issues:**
- Choice of sampling frequency, input signal (experimental conditions)
- What class of models – how to model disturbances?
- Estimating model parameters from sampled, finite and noisy data

Procedure of System Identification

- Experiment design and data collection
- Data preprocessing
- Model structure selection
- Parameter estimation
- Model validation

Procedure of System Identification – I

- Experimental design and execution
- Data preprocessing
- Model structure determination
- Parameter estimation
- Model validation

An iterative procedure!
**Experiments and Data Collection**

Often good to use a two-stage approach

1. **Preliminary experiments**
   - step/impulse response tests to get basic understanding of system dynamics
   - linearity, static gains, time delays, time constants, sampling interval

2. **Data collection for model estimation**
   - carefully designed experiment to enable good model fit
   - operating point, input signal type, number of data points to collect, etc.

**Preliminary Experiments: Step Response Experiment**

Useful for obtaining qualitative information about system

- Indicates dead-times, static gain, time constants and resonance frequency etc.
- Aids sampling time selection (rule-of-thumb: 4-10 sampling points over the rise time)

**Designing Experiment for Model Estimation**

Input signal should excite all relevant frequencies

- estimated model are more accurate in frequency ranges where input has high energy
- a good choice is often a binary sequence with random “hold times” (e.g., PRBS – Pseudo-Random Binary Sequence)

**Trade-off in selection of signal amplitude**

- large amplitude gives high signal-to-noise ratio (SNR), low parameter variance
- most systems are nonlinear for large input amplitudes

**Many pitfalls if estimating a model of a system under closed-loop control!**
Data Collection

Sampling time selection and anti-alias filtering are central!

Prefiltering of Data

Remove
- transients needed to reach desired operating point
- mean values of input and output signals, i.e., work with
  \[ \Delta u[t] = u[t] - \frac{1}{N} \sum_{i=1}^{N} u[i] \]
  \[ \Delta y[t] = y[t] - \frac{1}{N} \sum_{i=1}^{N} y[i] \]
- trends (use `detrend` in MATLAB)
- outliers ("obviously erroneous data points")

Procedure of System Identification

An iterative procedure!
Model Structures

Model structures commonly used (BJ includes all others as special cases)

**ARMAX (autoregressive moving average exogenous input)**

\[ y[k] = \frac{B(q)}{A(q)} u[k] + \frac{C(q)}{A(q)} e[k] \]

**BJ (Box-Jenkins)**

\[ y[k] = \frac{B(q)}{A(q)} u[k] + \frac{C(q)}{A(q)} e[k] \]

**ARX (autoregressive with exogeneous input)**

\[ y[k] = \frac{B(q)}{A(q)} y[k-1] + \frac{1}{A(q)} e[k] \]

**Output Error (OE) (output error)**

\[ y[k] = \frac{B(q)}{F(q)} \theta D(q) \]


Choice of Model Structure

1. Start with non-parametric estimates (correlation analysis, spectral estimation)
   – give information about model order and important frequency regions
2. Prefilter input/output data to emphasize important frequency ranges
3. Begin with ARX (AutoRegressive with eXogeneous input) models
4. Select model orders via
   – cross-validation (simulate model and compare with new data)
   – Akaike’s Information Criterion (AIC), i.e., pick the model that minimizes
     \[
     (1 + 2\frac{d}{N}) \sum_{i=1}^{N} |e[i; \theta]|^2
     \]
     (where \( d \) is the number of estimated parameters in the model)


Procedure of System Identification

An iterative procedure!
Nonparametric Estimation Methods

Nonparametric methods
- Transient response
- Correlation analysis
- Frequency responses analysis and Fourier analysis
- Spectral analysis

- Discussed in the “Modelling and Simulation” course, will not elaborated further in this course

Parametric Estimation Methods

- Non-recursive/Batch (off-line) methods
  - Linear regression and (block) least squares methods
  - Prediction error methods
  - Instrumental variable methods
  - Subspace methods (if possible)

- Recursive (on-line) methods
  - Recursive Least Squares (RLS) methods
  - Recursive prediction error methods
  - Recursive instrumental variable methods
  - Forgetting factor techniques and time-varying systems identification methods
  - Blockwise sliding window least squares methods

Procedure of System Identification

An iterative procedure!

Model Validation

A critical evaluation: “is model good enough”?
- typically depends on the purpose of the model

Example

\[ G(s) = \frac{1}{(s + 1)(s + a)} \]

Open- and closed-loop responses for \( a = -0.01, 0, 0.01 \)

Insufficient for open-loop prediction, good enough for closed-loop control.
Model Validation – cont’d

- Bode diagrams reveal why model is good enough for closed-loop control

![Bode diagram example](image)

- Different low-frequency behavior, similar responses around cross-over frequency

Principle of Model Validation

1. Compare model simulation/prediction with real data – in time domain
2. Compare estimated model’s frequency response and spectral analysis result – in frequency domain
3. Perform statistical tests on prediction errors

Validation: simulation and prediction

- Split data into two parts: one for estimation and one for validation
- Apply input signal in validation data set to estimated model
- Compare simulated output with output stored in validation data set

Statistical Model Validation

If we fit the parameters of the model

\[ y[t] = G(q; \theta)u[t] + H(q; \theta)e[t] \]

to data, the residuals

\[ e[t] = H(q; \theta)^{-1} \{ y[t] - G(q; \theta)u[t] \} \]

represent a disturbance that explains mismatch between model and observed data.

If the model is correct, the residuals should be
- white, and
- uncorrelated with \( u \).
Statistical Model Validation – cont’d

To test if the residuals $\epsilon[t]$ are **white**, we compute the auto-
covariance function

$$
\hat{R}_e(\tau) = \frac{1}{N} \sum_{t=1}^{N} \epsilon[t]\epsilon[t+\tau]
$$

and verify that its components lie within a 95% confidence region around zero.

- large components indicate un-modelled dynamics

**Independence** tested by verifying that cross-correlation function

$$
\hat{R}_{\epsilon u}(\tau) = \frac{1}{N} \sum_{t=1}^{N} \epsilon[t+\tau]u[t]
$$

lie within a 95% confidence region around zero.

- large components indicate un-modelled dynamics,
- $\hat{R}_{\epsilon u}(\tau)$ nonzero for $\tau < 0$ (non-causality) indicate the presence of feedback

Software Tools

- MATLAB Toolbox: System Identification

```matlab
>> help ident
System Identification Toolbox.
Version 5.0.1 (R12.1)  18-May-2001

Simulation and prediction.
predict - M-step ahead prediction.
pe      - Compute prediction errors.
sim     - Simulate a given system.

Data manipulation.
iddata  - Construct a data object.
detrend - Remove trends from data sets.
idfilt  - Filter data through Butterworth filters.
idinput - Generates input signals for identification.
merge   - Merge several experiments.
misdata - Estimate and replace missing input and output data.
resample - Resamples data by decimation and interpolation.
```

Software Tools

- MATLAB Toolbox: System Identification – cont’d

Nonparametric estimation.
covf    - Covariance function estimate for a data matrix.
cra     - Correlation analysis.
etfe    - Empirical Transfer Function Estimate and Periodogram.
impulse - Direct estimation of impulse response.
spa     - Spectral analysis.
step    - Direct estimation of step response.

Parameter estimation.
ar      - AR-models of signals using various approaches.
arx     - LS-estimate of ARX-models.
armax   - Prediction error estimate of an ARMAX model.
bj      - Prediction error estimate of a Box-Jenkins model.
ivar    - IV-estimates for the AR-part of a scalar time series.
iv4     - Approximately optimal IV-estimates for ARX-models.
n4sid   - State-space model estimation using a sub-space method.
oe      - Prediction error estimate of an output-error model.
pem     - Prediction error estimate of a general linear model.
Software Tools
- MATLAB Toolbox: System Identification – cont’d

Model structure creation.
idpoly - Construct a model object from given polynomials.
ids - Construct a state space model object.
idarx - Construct a multivariable ARX model object.
idgrey - Construct a user-parameterized model object.

Model conversions.
arxdata - Convert a model to its ARX-matrices (if applicable).
polydata - Polynomials associated with a given model.
ssdata - IDMODEL conversion to state-space.
tfdata - IDMODEL conversion to transfer function.
zpkdata - Zeros, poles, static gains and their standard deviations.
idfrd - Model's frequency function, along with its covariance.
idmodred - Reduce a model to lower order.
c2d, d2c - Continuous/discrete transformations.
ss, tf, zpk, frd - Transformations to the LTI-objects of the CSTB. Most CSTB conversion routines also apply to the model objects of the Identification Toolbox.

Model presentation.
bode - Bode diagram of a transfer function or spectrum (with uncertainty regions).
ffplot - Frequency functions (with uncertainty regions).
plot - Input-output data for data objects.
present - Display the model with uncertainties.
pzmap - Zeros and poles (with uncertainty regions).
nyquist - Nyquist diagram of a transfer function (with uncertainty regions).
view - The LTI viewer (with the Control Systems Toolbox for model objects).

Model validation.
compare - Compare the simulated/predicted output with the measured output.
pe - Prediction errors.
predict - M-step ahead prediction.
resid - Compute and test the residuals associated with a model.
sim - Simulate a given system (with uncertainty).

Model structure selection.
aic, fpe - Compute Akaike's information and final prediction criteria.
arxstruc - Loss functions for families of ARX-models.
setstruc - Select model structures according to various criteria.
struc - Typical structure matrices for ARXSTRUC.

Software Tools
- MATLAB Toolbox: System Identification – cont’d

Practice yourself using Matlab System Identification toolbox demonstrations: “iddemo”
>> iddemo

The SYSTEM IDENTIFICATION TOOLBOX is an analysis module that contains tools for building mathematical models of dynamical systems, based upon observed input-output data. The toolbox contains both PARAMETRIC and NON-PARAMETRIC MODELING methods.

Identification Toolbox demonstrations:
1) The Graphical User Interface (ident): A guided Tour.
2) Build simple models from real laboratory process data.
3) Compare different identification methods.
4) Data and model objects in the Toolbox.
5) Dealing with multivariable systems.
6) Building structured and user-defined models.
7) Model structure determination case study.
8) How to deal with multiple experiments.
9) Spectrum estimation (Marple’s test case).
10) Adaptive/Recursive algorithms.
11) Use of SIMULINK and continuous time models.
12) Case studies.

Software Tools
- MATLAB Toolbox: System Identification – cont’d

A System Identification Example: Hairdryer

"Hairdryer" process: input is the voltage over the heating device; output is outlet temperature
Matlab: “iddemo” (demonstration 2)
Main Focus in This Course

- Experiment design
- Data collection
- Data prefiltering
- Model structure selection
- Parameter estimation
- Model validation

An iterative procedure!

Reading and Exercise

- **Reading**: Textbook, Chapter 1; Sections 4.1-4.3
- **Further Reading** (optional):
- **Exercise**: None

F7-1: System Identification

*Any question, comment or suggestion?*