“Control Theory” - Exercise #3

E5.2, E5.4, E5.5, E5.7, E5.8, P5.4

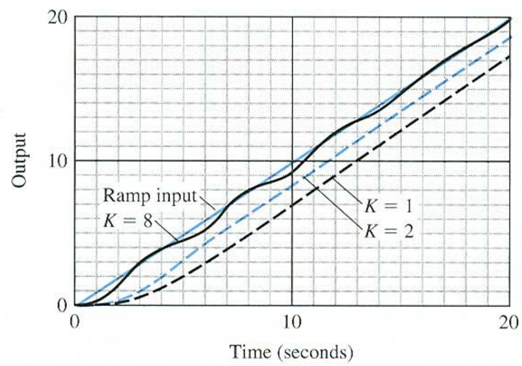


FIGURE 5.46 The response of a feedback system to a ramp input with $K = 1$, 2, and 8 when $G(s) = K/[s(s+1)(s+3)]$. The steady-state error is reduced as K is increased, but the response becomes oscillatory at $K = 8$.

5.14 SUMMARY

In this chapter we have considered the definition and measurement of the performance of a feedback control system. The concept of a performance measure or index was discussed, and the usefulness of standard test signals was outlined. Then several performance measures for a standard step input test signal were delineated. For example, the overshoot, peak time, and settling time of the response of the system under test for a step input signal were considered. The fact that often the specifications on the desired response are contradictory was noted, and the concept of a design compromise was proposed. The relationship between the location of the s-plane root of the system transfer function and the system response was discussed. A most important measure of system performance is the steady-state error for specific test input signals. Thus the relationship of the steady-state error of a system in terms of the system parameters was developed by utilizing the final-value theorem. The capability of a feedback control system is demonstrated in Fig. 5.46. Finally the utility of an integral performance index was outlined, and several examples of design that minimized a system's performance index were completed. Thus we have been concerned with the definition and usefulness of quantitative measures of the performance of feedback control systems.

EXERCISES

E5.1 A motor control system for a computer disk drive must reduce the effect of disturbances and parameter variations, as well as reduce the steady-state error. We desire to have no steady-state error for the head-positioning control system, which is of the form shown in Fig. 5.18, where $H(s) = 1$. (a) What type number is required? (How many integrations?) (b) If the input is a ramp signal, then to achieve a zero steady-state error, what type number is required?

E5.2 The engine, body, and tires of a racing vehicle affect the acceleration and speed attainable [11]. The speed control of the car is represented by the model shown in Fig. E5.2. (a) Calculate the steady-state error of the car to a step command in speed. (b) Calculate overshoot of the speed to a step command.

Answer: (a) $e_{ss} = A/11$; (b) $P.O. = 20.8\%$

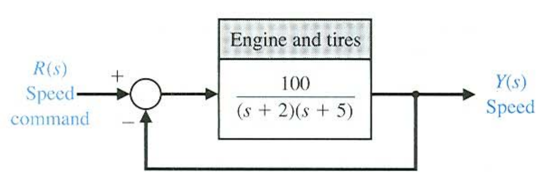


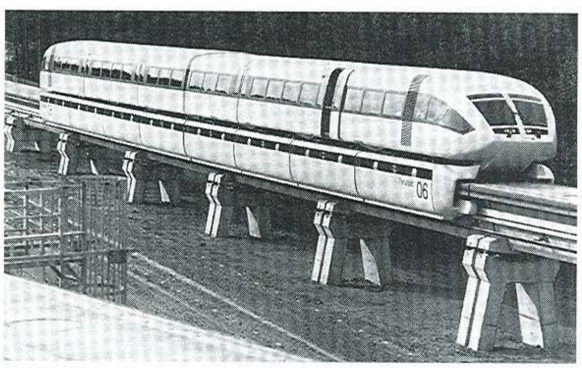
FIGURE E5.2 Racing car speed control.

E5.3 For years, Amtrak has struggled to attract passengers on its routes in the Midwest, using technology developed decades ago. During the same time, foreign railroads were developing new passenger rail systems that could profitably compete with air travel. Two of these systems, the French TGV and the Japanese Shinkansen, reach speeds of 160 mph [20]. The Transrapid-06, a U.S. experimental magnetic levitation train, is shown in Fig. E5.3(a).

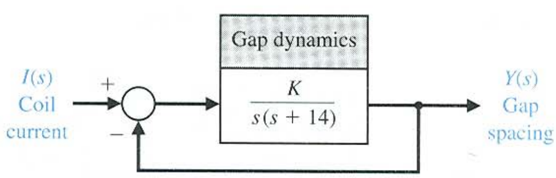
The use of magnetic levitation and electromagnetic propulsion to provide contactless vehicle movement makes the Transrapid-06 technology radically different from the existing Metroliner. The underside of the TR-06 carriage (where the wheel trucks would be on a conventional car) wraps around a guideway. Magnets on the bottom of the guideway attract electromagnets on the "wraparound," pulling it up toward the guideway. This suspends the vehicles about one centimeter above the guideway. (See Problem 2.27.)

The levitation control is represented by Fig. E5.3(b). (a) Using Table 5.6 for a step input, select K so that the system provides an optimum ITAE response. (b) Using Fig. 5.8, determine the expected overshoot to a step input of $I(s)$.

Answers: $K = 100$; 4.6%



(a)



(b)

FIGURE E5.3 Levitated train control.

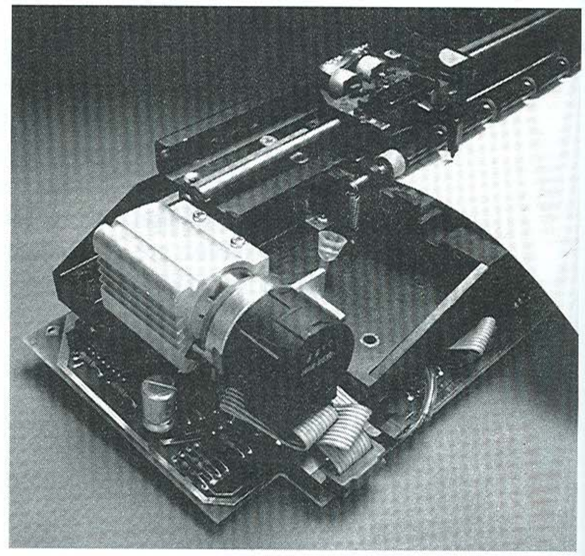
E5.4 A feedback system with negative unity feedback has a plant

$$G(s) = \frac{2(s+8)}{s(s+4)}.$$

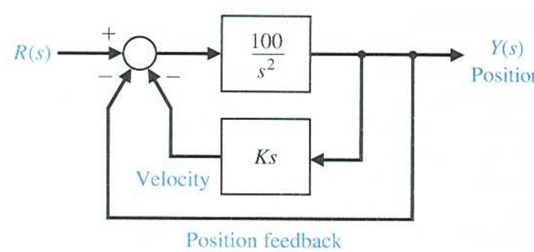
(a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$. (b) Find the time response $y(t)$ for a step input $r(t) = A$ for $t > 0$. (c) Using Fig. 5.13(a), determine the overshoot of the response. (d) Using the final-value theorem, determine the steady-state value of $y(t)$.

Answer: (b) $y(t) = 1 - 1.07e^{-3t}\sin(\sqrt{7}t + 1.2)$

E5.5 A low-inertia plotter is shown in Fig. E5.5(a). This system may be represented by the block diagram shown in Fig. E5.5(b) [18]. (a) Calculate the steady-state error for a ramp input. (b) Select a value of K that will result



(a)



(b)

FIGURE E5.5 (a) The Hewlett-Packard *x-y*-plotter. (Courtesy of Hewlett-Packard Co.) (b) Block diagram of plotter.

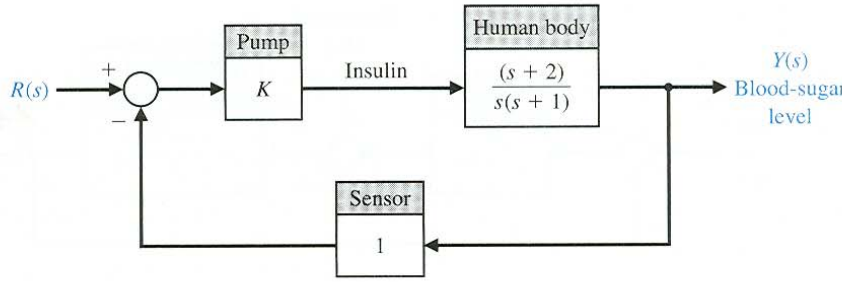


FIGURE E5.6 Blood-sugar level control.

in zero overshoot to a step input but as rapid response as is attainable.

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?

E5.6 Effective control of insulin injections can result in better lives for diabetic persons. Automatically controlled insulin injection by means of a pump and a sensor that measures blood sugar can be very effective. A pump and injection system has a feedback control as shown in Fig. E5.6. Calculate the suitable gain K so that the overshoot of the step response due to the drug injection is approximately 7%. $R(s)$ is the desired blood-sugar level and $Y(s)$ is the actual blood-sugar level. (*Hint:* Use Fig. 5.13a.)

Answer: $K = 1.67$

E5.7 A control system for positioning the head of a floppy disk drive has the closed-loop transfer function

$$T(s) = \frac{0.313(s + 0.8)}{(s + 0.6)(s^2 + 4s + 5)}.$$

Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?

E5.8 A unity negative feedback control system has the plant

$$G(s) = \frac{K}{s(s + \sqrt{2K})}.$$

(a) Determine the percent overshoot and settling time (using a 2% settling criterion) due to a unit step input.

(b) For what range of K is the settling time less than 1 second?

E5.9 A second-order control system has the closed-loop transfer function $T(s) = Y(s)/R(s)$. The system specifications for a step input follow:

(1) Percent overshoot $\leq 5\%$.
(2) Settling time < 4 seconds.
(3) Peak time $T_p < 1$ second.

Show the permissible area for the poles of $T(s)$ in order to achieve the desired response. Use a 2% settling criterion to determine settling time.

E5.10 A system with unity feedback is shown in Fig. E5.10. Determine the steady-state error for a step and a ramp input when

$$G(s) = \frac{10(s + 4)}{s(s + 1)(s + 3)(s + 8)}.$$

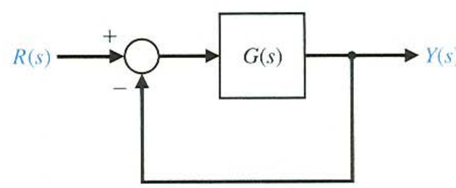


FIGURE E5.10 Unity feedback system.

E5.11 We are all familiar with the Ferris wheel featured at state fairs and carnivals. George Ferris, born in Galesburg, Illinois, in 1859, later moved to Nevada and then graduated from Rensselaer Polytechnic Institute in 1881. By 1891, Ferris had considerable experience with iron, steel, and bridge construction. He conceived and constructed his famous wheel for the 1893 Columbian Exposition in Chicago [9]. To avoid upsetting passengers, let us set a requirement that the steady-state speed be controlled to within 5% of the desired speed for the system shown in Fig. E5.11.

(a) Determine the required gain K to achieve the steady-state requirement.

PROBLEMS

P5.1 An important problem for television systems is the jumping or wobbling of the picture due to the movement of the camera. This effect occurs when the camera is mounted in a moving truck or airplane. The Dynalens system has been designed to reduce the effect of rapid scanning motion; see Fig. P5.1. A maximum scanning motion of 25°/s is expected. Let $K_g = K_t = 1$ and assume that τ_g is negligible. (a) Determine the error of the system $E(s)$. (b) Determine the necessary loop gain, $K_a K_m K_t$, when a 1°/s steady-state error is allowable. (c) The motor time constant is 0.40 s. Determine the necessary loop gain so that the settling time to within 2% of the final value of v_b is less than or equal to 0.03 s.

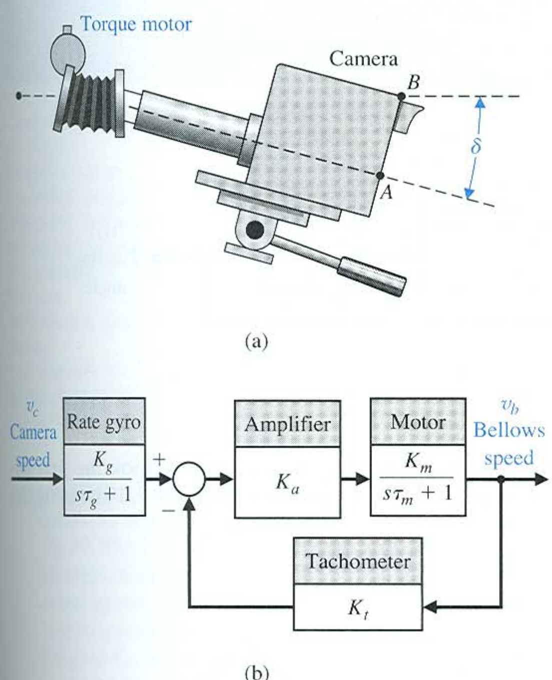


FIGURE P5.1 Camera wobble control.

P5.2 A specific closed-loop control system is to be designed for an underdamped response to a step input. The specifications for the system are as follows:

$$20\% > \text{percent overshoot} > 10\%,$$

$$\text{Settling time} < 0.6 \text{ s}.$$

(a) Identify the desired area for the dominant roots of the system. (b) Determine the smallest value of a third root, r_3, if the complex conjugate roots are to represent the dominant response. (c) The closed-loop system transfer function $T(s)$ is third-order, and the feedback has a unity gain. Determine the forward transfer function $G(s) = Y(s)/E(s)$ when the settling time to within 2% of the final value is 0.6 s and the percent overshoot is 20%.

P5.3 A laser beam can be used to weld, drill, etch, cut, and mark metals, as shown in Fig. P5.3(a) [16]. Assume we have a work requirement for an accurate laser to mark a parabolic path with a closed-loop control system, as shown in Fig. P5.3(b). Calculate the necessary gain to result in a steady-state error of 5 mm for $r(t) = t^2$ cm.

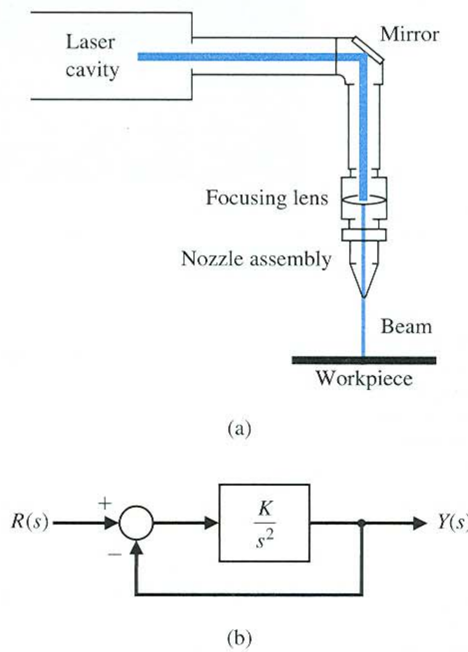


FIGURE P5.3 Laser beam control.

P5.4 The open-loop transfer function of a unity negative feedback system is

$$G(s) = \frac{K}{s(s+2)}.$$

A system response to a step input is specified as follows:

$$\text{peak time } T_p = 1.1 \text{ s},$$

$$\text{percent overshoot} = 5\%.$$

(a) Determine whether both specifications can be met simultaneously. (b) If the specifications cannot be met simultaneously, determine a compromise value for K so that the peak time and percent overshoot specifications are relaxed the same percentage.