"Control Theory" - Solution #2

E2.8, E2.9, E2.10, E2.14, E2.15, P2.18, P2.32, P2.34

E2.8 The block diagram is shown in Figure E2.8.

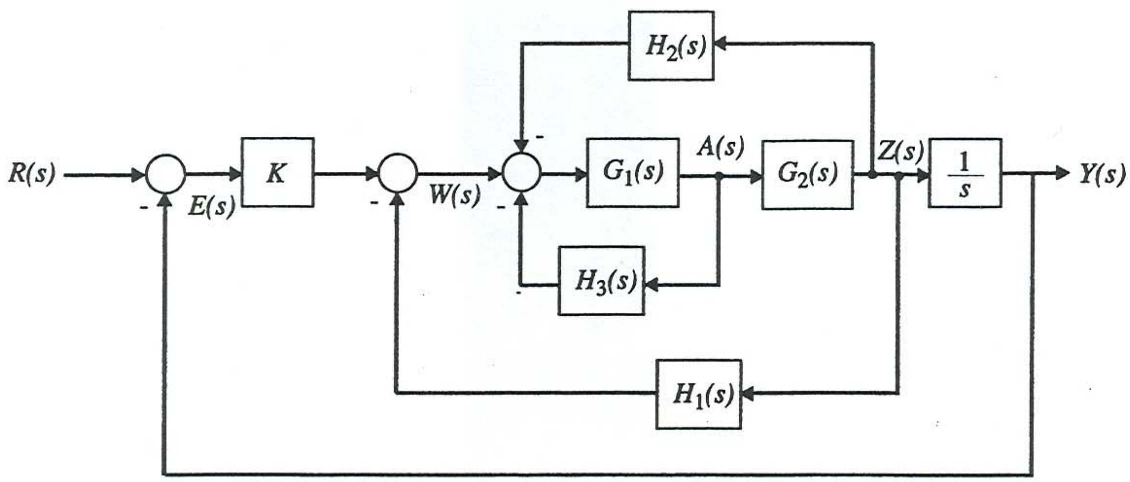

FIGURE E2.8
Block diagram model.

Starting at the output we obtain

$$Y(s) = \frac{1}{s}Z(s) = \frac{1}{s}G_2(s)A(s).$$

But $A(s) = G_1(s)\,[-H_2(s)Z(s) - H_3(s)A(s) + W(s)]$ and $Z(s) = sY(s)$, so

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)W(s).$$

Substituting $W(s) = KE(s) - H_1(s)Z(s)$ into the above equation yields

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) - G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)\,[KE(s) - H_1(s)Z(s)]$$

and with $E(s) = R(s) - Y(s)$ and $Z(s) = sY(s)$ this reduces to

$$Y(s) = [-G_1(s)G_2(s)\,(H_2(s) + H_1(s)) - G_1(s)H_3(s) - \frac{1}{s}G_1(s)G_2(s)K]Y(s) + \frac{1}{s}G_1(s)G_2(s)KR(s).$$

Solving for $Y(s)$ yields the transfer function

$$Y(s) = T(s)R(s),$$

where

$$T(s) = \frac{KG_1(s)G_2(s)/s}{1 + G_1(s)G_2(s)\,[(H_2(s) + H_1(s)] + G_1(s)H_3(s) + KG_1(s)G_2(s)/s}.$$

E2.9 The transfer function is

$$\frac{F_f(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 - L_1(s) - L_2(s)}$$

where

$$L_1(s) = -G_1(s)G_2(s)H_1(s) \quad (loop1)$$
$$L_2(s) = -G_1(s)G_3(s)H_2(s) \quad (loop2)\,.$$

E2.10 The shock absorber block diagram is shown in Figure E2.10. The closed-loop transfer function model is

$$T(s) = \frac{G_c(s)G_p(s)G(s)}{1 + H(s)G_c(s)G_p(s)G(s)}\,.$$

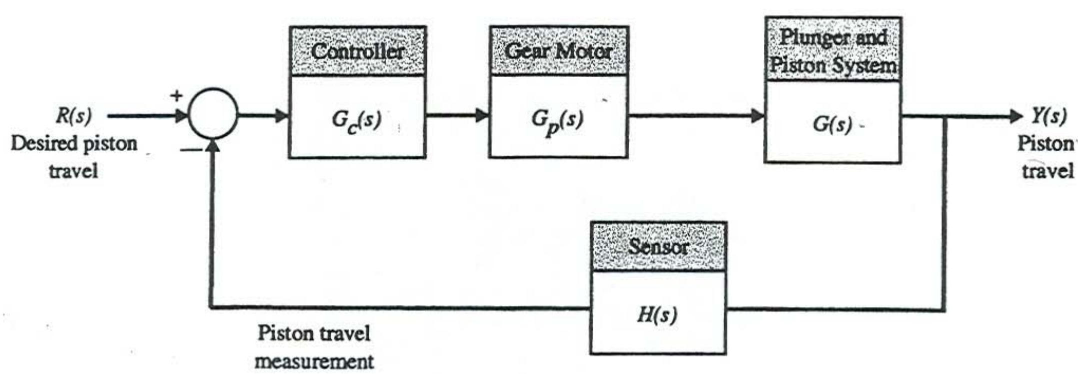

FIGURE E2.10
Shock absorber block diagram.

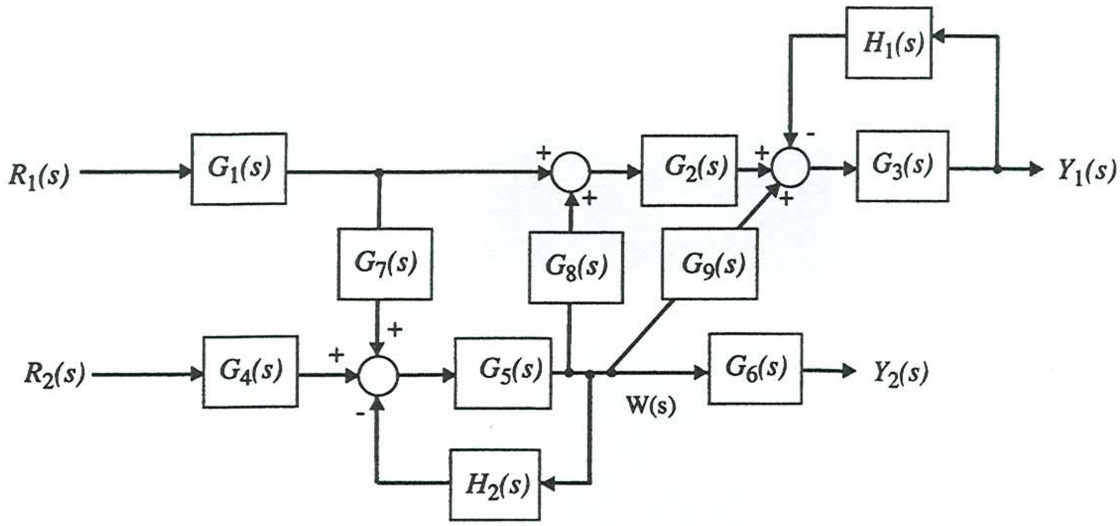

FIGURE E2.13
Block diagram model.

or

$$[1 + G_5(s)H_2(s)]\, W(s) = G_5(s)G_4(s)R_2(s).$$

Substituting the expression for $W(s)$ into the above equation for $Y_1(s)$ yields

$$\frac{Y_1(s)}{R_2(s)} = \frac{G_2(s)G_3(s)G_4(s)G_5(s)G_8(s) + G_3(s)G_4(s)G_5(s)G_9(s)}{1 + G_3(s)H_1(s) + G_5(s)H_2(s) + G_3(s)G_5(s)H_1(s)H_2(s)}.$$

E2.14 For loop 1, we have

$$R_1 i_1 + L_1\frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)dt + R_2(i_1 - i_2) = v(t)\,.$$

And for loop 2, we have

$$\frac{1}{C_2}\int i_2 dt + L_2\frac{di_2}{dt} + R_2(i_2 - i_1) + \frac{1}{C_1}\int (i_2 - i_1)dt = 0\,.$$

E2.15 The transfer function from $R(s)$ to $P(s)$ is

$$\frac{P(s)}{R(s)} = \frac{4.2}{s^3 + 2s^2 + 4s + 4.2}\,.$$

The corresponding signal flow graph is shown in Figure E2.15 for

$$P(s)/R(s) = \frac{4.2}{s^3 + 2s^2 + 4s + 4.2}\,.$$

P2.18 The signal flow graph is shown in Figure P2.18.

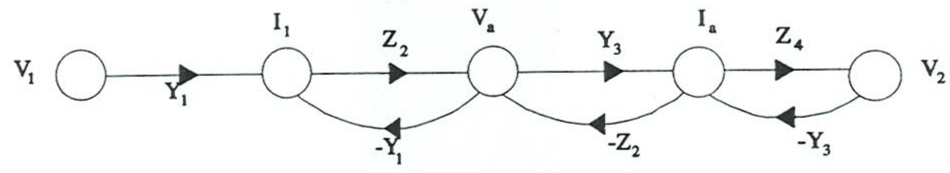

FIGURE P2.18
Signal flow graph.

P2.32 The signal flow graph shows three loops:

$$L_1 = -G_1 G_2 H_1$$
$$L_2 = G_5 G_6 H_2$$
$$L_3 = G_3 G_4 G_6 H_2$$

L_1 and L_2 are nontouching, so

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2) .$$

(a) For Y_1/R_1, L_2 does not touch the path, so

$$\Delta_1 = 1 - L_2 ,$$

and

$$\frac{Y_1}{R_1} = \frac{G_1 G_2 \Delta_1}{\Delta} .$$

(b) For Y_2/R_1, the path touchs all loops, so

$$\frac{Y_2}{R_1} = \frac{G_1 G_4 G_6}{\Delta} .$$

P2.33 The signal flow graph shows three loops:

$$L_1 = -G_1 G_3 G_4 H_2$$
$$L_2 = -G_2 G_5 G_6 H_1$$
$$L_3 = -H_1 G_8 G_6 G_2 G_7 G_4 H_2 G_1 .$$

The transfer function Y_2/R_1 is found to be

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_1 G_8 G_6 \Delta_1 - G_2 G_5 G_6 \Delta_2}{1 - (L_1 + L_2 + L_3) + (L_1 L_2)} ,$$

where for path 1

$$\Delta_1 = 1$$

and for path 2

$$\Delta_2 = 1 - L_1 .$$

Since we want Y_2 to be independent of R_1, we need $Y_2/R_1 = 0$. Therefore, we require

$$G_1 G_8 G_6 - G_2 G_5 G_6 (1 + G_1 G_3 G_4 H_2) = 0 .$$

P2.34 The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G_3(s) G_1(s)(G_2(s) + K_5 K_6)}{1 - G_3(s)(H_1(s) + K_6) + G_3(s) G_1(s)(G_2(s) + K_5 K_6)(H_2(s) + K_4)} .$$