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Decentralized Receding Horizon Control for Cooperative Multiple Vehicles Subject to Communication Delay

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I. Introduction

In this paper, a new approach is proposed for the decentralized receding horizon control (DRHC) of multiple cooperative vehicles with the possibility of communication failures leading to large intervehicle communication delay. Such large communication delays can lead to poor performance and even instability. The neighboring vehicles exchange their predicted trajectories at each sample time to maintain the cooperation objectives. It is assumed that the communication failure is partial in nature, which in turn leads to large communication delays of the exchanged trajectories. The proposed fault-tolerant DRHC is based on two extensions of existing work for the case of large communication delays. The first contribution is the development of a new DRHC approach that estimates the trajectory of the neighboring vehicles for the tail of the prediction horizon, which would otherwise not be available due to the communication delay. In this approach, the tail of the cost function is estimated by adding extra decision variables in the cost function. A relatively small amount of existing work has investigated the implementation issues associated with exchange of trajectory information, but so far no work has proposed a tail estimation process to compensate for large delays. For instance, in [1–3], no prediction or estimation for the trajectory of neighboring vehicles is performed, and it is assumed that the neighboring vehicles remain at the last delayed states broadcasted by them. Such assumptions may yield poor performance for large communication delays because the constant state vector is not a good estimation of a trajectory of states in general. Similar issues are also investigated in [4,5].

The second contribution of this paper is an extension of the tube-based model predictive control (MPC) approach [6,7] for the case of the large communication delays in order to guarantee the safety of the fleet against possible collisions during formation control problems. The concept of the tube MPC [or tube receding horizon control (RHC)] in existing work [6,7] is normally used to calculate a robust bound on the states due to system uncertainty, whereas in this paper, the approach is used to calculate bounds that arise from large communication delays of the exchanged neighbor trajectories.

The proposed algorithms in this paper are presented in the context of fault-tolerant control, as the communication delay/break may occur due to any failure and malfunction in the communication devices. Some examples of communication failures for the team of cooperative vehicles can be found in [8–10]. In [8], the wireless communication packet loss/delay is considered; once the packet loss/delay occurs, the previous available trajectory of the faulty unmanned aerial vehicle (UAV) is extrapolated to predict the future reference trajectory. Also, in [9], the communication failure in formation flight of multiple UAVs leads to a break in the communicated messages that forces the fleet to redefine the communication graph.

This paper is organized as follows. Section II deals with a general formulation of the decentralized receding horizon controller, and the corresponding algorithm for a fault-free (delay-free) condition. In Section III, a faulty condition is first defined, and a reconfigurable fault-tolerant controller is developed. A safety guarantee method for the faulty condition is also developed based on the concept of tube RHC. In Section IV, the proposed algorithms are tested through simulation of a leaderless formation controller for a fleet of unmanned vehicles.

II. Decentralized Receding Horizon Control Formulation

Consider a team of vehicles with uncoupled dynamics. Each vehicle in the team is equipped with measurement sensors, a communication channel, and a computation resource. Moreover, each vehicle has a dynamic model of its neighboring vehicles available to predict their trajectory when required. It is also assumed that there are no sensor errors, actuator errors, model uncertainty, or communication noise. These assumptions allow the paper to focus on the main problem of communication delays. However, it is thought that the proposed approach can be extended to the preceding cases by suitably modifying the tube calculation approach to account for these nonideal effects.

The following indirect graph topology [11,12] is used to present the interaction among vehicles:

$$G(t) = \{V, E\} \tag{1}$$

where V is the set of nodes (vehicles) and $E \subseteq VV$ is the set of arcs (i, j), with $i, j \in V$. Also, let N_n^i denote the number of neighbors of vehicle i.

A. Decentralized Receding Horizon Control Notation and Terminology

In RHC, a cost function is optimized over a finite time called the prediction horizon T. The first portion of the computed optimal input is applied to the plant during a period of time called the execution horizon δ or the sampling period. The reader is referred to [13] for a comprehensive review of RHC schemes.

It is assumed that the execution horizon δ is equal to the communication period. The discrete timing is then given by t_k, where $t_{k+1} = t_k + \delta$ (or $t_k = k\delta$) and $t_0 = 0$.

The possible state vectors are introduced as follows:

1) $x^i(t)$ is the actual state vector of the ith vehicle at time t.

2) $x_{t_k}^{j,i}(t)$ is the state vector of the jth vehicle at time t, computed (estimated) by the ith vehicle at time step t_k.

The state of vehicle i calculated by itself at time t_k is represented by $x_{t_k}^{i,i}(t)$ (predicted). Further, the sequence of these states over the prediction horizon is called the state trajectory of vehicle i calculated by itself and is represented by $x^i(t_k\colon t_k + T)$; for example

$$x^i(t_k\colon t_k + T) = \{x_{t_k}^{i,i}(t) | t \in [t_k, t_k + T]\};\quad u^i(t_k\colon t_k + T) = \{u_{t_k}^{i,i}(t) | t \in [t_k, t_k + T]\} \tag{2}$$

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Then let the following represent the concatenated state and input trajectories of the neighbors of the ith vehicle at time t_k:

$$\breve{x}^i(t_k{:}\ t_k+T)=[\ldots,x^j(t_k{:}\ t_k+T),\ \ldots]'; \quad j\in V; \quad (i,j)\in E;$$
$$\breve{u}^i(t_k{:}\ t_k+T)=[\ldots,u^j(t_k{:}\ t_k+T),\ \ldots]'; \quad j\in V; \quad (i,j)\in E \tag{3}$$

B. Fault-Free Decentralized Receding Horizon Control Formulation

In this section, a brief overview of the fault-free DRHC problem and its implementation are described. More details can be obtained from [4,11,14]. For the DRHC scheme presented in this paper, the predicted trajectories are exchanged instead of being estimated, thereby reducing the online computational time. The information set of the ith vehicle for the case of fault-free DRHC is introduced as follows:

$$\Gamma^i(t_k)=\{x^i(t_k),\ \breve{x}^i(t_{k-1}{:}\ t_{k-1}+T)\} \tag{4}$$

where set $\Gamma^i(t_k)$ contains the updated information available to the ith vehicle at time t_k and is referred to the information set in this paper. This collects 1) the instant state vector of the ith vehicle and 2) the concatenated state trajectory of neighbors calculated at the previous time step $\breve{x}^i(t_{k-1}{:}\ t_{k-1}+T)$.

For the particular case of formation control, the fault-free decentralized cost function for the ith vehicle in the team at time t_k is defined as follows:

$$J^i[\Gamma^i(t_k)]=\int_{t_k}^{t_k+T}\left[\|x_{t_k}^{i,i}(t)-x^{T,i}\|_Q^2+\|u_{t_k}^{i,i}(t)\|_R^2\right]\mathrm{d}t + \|x_{t_k}^{i,i}(t_k+T)-x^{T,i}\|_P^2 + \sum_{j|(i,j)\in E}\int_{t_k}^{t_k+T}\|x_{t_k}^{i,i}(t)-x_{t_k}^{j,j}(t)-r^{i,j}\|_S^2\,\mathrm{d}t \tag{5}$$

where $\|x\|_Q^2=x'Qx$ and P, Q, R, and S are positive, definite, and symmetric matrices, $x^{T,i}$ is the state vector of target of vehicle i, and $r^{i,j}$ is the vector of desired relative position between agents i and j.

1. Fault-Free Decentralized Receding Horizon Control Problem

Assume the following equation represents the linear dynamics of the homogeneous vehicles:

$$\dot{x}(t)=Ax(t)+Bu(t); \qquad x(t_0)=x_0 \tag{6}$$

Then, the fault-free DRHC problem $P^i(t_k)$ is defined for the ith vehicle at time t_k as follows:

Problem 1: Fault-free DRHC problem $P^i(t_k)$ $(i\in V)$

$$\min_{\{u^i(t_k{:}\ t_k+T),\,x^i(t_k{:}\ t_k+T)\}} J^i[\Gamma^i(t_k)] \tag{7}$$

subject to:

$$\dot{x}_{t_k}^{i,i}(t)=Ax_{t_k}^{i,i}(t)+Bu_{t_k}^{i,i}(t); \qquad x_{t_k}^{i,i}(t_k)=x^i(t_k); \qquad t\in[t_k,\ t_k+T] \tag{8a}$$

$$x_{t_k}^{i,i}(t)\in X^i; \qquad u_{t_k}^{i,i}(t)\in U^i; \qquad t\in[t_k,t_k+T] \tag{8b}$$

$$x_{t_k}^{i,i}(t_k+T)\in X_f^i \tag{8c}$$

where X^i, U^i, and X_f^i denote the set of admissible states, inputs, and final states (terminal region), respectively, for the ith vehicle.

2. Fault-Free Decentralized Receding Horizon Control Algorithm

The following algorithm is presented for the online implementation of the preceding fault-free DRHC problem. The algorithm is formulated for the ith vehicle as follows:

Algorithm 1: Fault-free DRHC with one-step delay (online)

1) Let $k=0$, measure $x^i(t_k)$ and GOTO step 3.

2) Receive the trajectory $x^j(t_{k-1}{:}\ t_{k-1}+T)$ from neighbors j [where $(i,j)\in E$] if available, measure $x^i(t_k)$ and update the information set $\Gamma^i(t_k)$ from Eq. (4).

3) Solve $P^i(t_k)$ and generate the control action $u^i(t_k{:}\ t_k+T)$ and the state trajectory $x^i(t_k{:}\ t_k+T)$.

4) Send the trajectory $x^i(t_k{:}\ t_k+T)$ to the neighboring vehicles.

5) Execute the control action for the individual vehicle i during $[t_k,\ t_{k+1}]$:

$$u^i(t)=u_{t_k}^{i,i}(t); \qquad t\in[t_k,\ t_{k+1}] \tag{9}$$

6) $k=k+1$, GOTO step 2.

This algorithm is repeated until the assigned targets are reached. The targets are assumed to be known and assigned to each agent a priori.

III. Fault-Tolerant Decentralized Receding Horizon Control

This section develops a new fault-tolerant reconfigurable DRHC approach. The safety guarantee in faulty conditions is also discussed. It is assumed that each vehicle is equipped with a high performance communication channel and a low performance communication channel as a redundant backup. In the fault-free condition, the high performance communication channel is used, which leads to small communication delays, typically smaller than the sampling time. In the faulty condition, the low performance communication channel is used that leads to large communication delays. It is assumed the delay in the faulty condition is applied to both the received and transmitted information from/to a faulty vehicle. Then, the faulty condition is defined as when the high performance communication channel of one vehicle in the team fails.

In this paper, it is assumed that an effective fault detection scheme is available to determine when a fault occurs and which vehicle is faulty in the team [8,9]. The following sections describe the presented approach for handling these type of failures that lead to a large communication delay.

A. Faulty Cost Function

Once the fault is detected and the faulty vehicle is identified in the team, the faulty vehicle switches to the backup low-performance communication channel. This will cause the neighboring vehicles to receive the messages from the faulty vehicle with a large communication delay, and the faulty vehicle receives the messages from all the neighbors with a large communication delay. The decentralized receding horizon controllers of the neighbors of a faulty vehicle and the faulty vehicle then have to be modified (reconfigured) to account for large communication delays (i.e., to use the available delayed information instead of the unavailable delay-free information).

In the faulty condition, all the vehicles involved in the fault (the faulty vehicle and those that have a faulty neighbor) will construct the set of faulty neighbors, which is denoted by V_F^i, the set of faulty neighbors of vehicle i. The vehicles that have a faulty neighbor assign the faulty neighbor to this set, and the faulty vehicle assigns all of its neighbors to this set (even though they are not faulty) because the faulty vehicle receives the information from healthy neighbors with a large delay.

Because, in the faulty condition, the vehicles receive the neighbor's trajectory with a delay, for the tail of the cost function, there is no trajectory to set the formation. It is assumed at time t_k that the vehicle i receives the information from neighbor j with the time delay τ. Then, the trajectory of neighbor j for only the interval $[t_k-\tau,\ t_k+T-\tau]$ is available to vehicle i, although according to the cost function of Eq. (5), vehicle i needs the trajectory of neighbor j for the entire segment $[t_k,\ t_k+T]$. Hence, for the portion $[t_k+T-\tau,\ t_k+T]$, the trajectory of j is not available due to the delay. When the time delay is small, this lack of information is not important, but for large communication delays, the tail of the cost function during $[t_k+T-\tau,\ t_k+T]$ becomes large and (as shown in the example section IV) it can lead to poor performance and even instability (see also [12,15]). One remedy to this problem is proposed here by estimating the tail of the cost function by including extra decision variables in the cost function.

It is assumed that $\tau > \delta$ in the faulty condition. Further, it is assumed that $(d-1)\delta \le \tau \le d\delta$, where $d \in N$; hence, in this paper, d represents the (discrete) communication time delay. This is used instead of τ in most of the cases to provide synchronization between the communication delay and the RHC sampling time.

As mentioned, in the faulty conditions, the vehicles receive the delayed information from the faulty neighbor and nondelayed information from the fault-free neighbors; consequently, the information set is updated as the following general form [compare with Eq. (4)]:

$$\Gamma^i(t_k) = \{x^i(t_k),\ \breve{x}^i(t_{k-d}\colon t_{k-d}+T)\} \tag{10}$$

where $d = 1$ for healthy neighbors and $d > 1$ for faulty neighbors:

$$\begin{cases} d = 1 & (i,j) \in E \quad \text{and} \quad j \notin V_F^i \\ d > 1 & (i,j) \in E \quad \text{and} \quad j \in V_F^i \end{cases} \tag{11}$$

The information set $\Gamma^i(t_k)$ represents updated information available to the ith vehicle at time t_k. It implies that at time t_k, each vehicle i has access to its own delay-free information and delayed information from its neighbors. This includes the delay-free (small delay) information of healthy neighbors and the delayed information (larger delay) of its faulty neighbors.

The cost function for the faulty conditions (large communication delay) can now be presented as follows for the ith vehicle in the team at time t_k:

$$J_F^i[\Gamma^i(t_k)] = J_1^i + J_2^i + J_3^i \tag{12}$$

where

$$J_1^i = \int_{t_k}^{t_k+T}\left[\|x_{t_k}^{i,i}(t) - x^{T,i}\|_Q^2 + \|u_{t_k}^{i,i}(t)\|_R^2\right]\mathrm{d}t + \|x_{t_k}^{i,i}(t_k+T) - x^{T,i}\|_P^2 \tag{13}$$

$$J_2^i = \sum_{j|(i,j)\in E}\left[\int_{t_k}^{t_{k-d}+T}\|x_{t_k}^{i,i}(t) - x_{t_{k-d}}^{j,j}(t) - r^{i,j}(t)\|_S^2\,\mathrm{d}t + \int_{t_{k-d}+T}^{t_k+T}\|x_{t_k}^{i,i}(t) - x_{t_k}^{j,i}(t) - r^{i,j}(t)\|_S^2\,\mathrm{d}t\right] \tag{14}$$

$$J_3^i = \sum_{j|(i,j)\in E}\left\{\int_{t_{k-d}+T}^{t_k+T}\left[\|x_{t_k}^{j,i}(t) - x^{T,j}\|_Q^2 + \|u_{t_k}^{j,i}(t)\|_R^2\right]\mathrm{d}t + \|x_{t_k}^{j,i}(t_k+T) - x^{T,j}\|_P^2\right\} \tag{15}$$

The subscript F stands for the faulty condition. The faulty decentralized cost function of each vehicle i includes two main parts:

1) The first part is associated with the cost of local vehicle i and therefore uses the delay-free information. It is used to compute the trajectory of local vehicle i over the time interval $[t_k,\ t_k+T]$ [see Eq. (13)].

2) The second part [Eqs. (14) and (15)] is associated with the cost of the neighbors and then uses the information subject to delay (only a one-step delay for healthy and a larger delay for faulty vehicles). It is used to compute the coupling cost over the time interval $[t_k,\ t_k+T]$ [see Eq. (14)]. For the tail of the cost function during $[t_{k-d}+T,\ t_k+T]$, an estimation of the trajectory of the neighbor is required; hence, the cost of Eq. (15) is added to incorporate some decision variables for this portion.

B. Safety Guarantee Using Tube Decentralized Receding Horizon Control

The collision avoidance constraint is difficult to include in the optimization problem of DRHC because of its nonconvex nature. To avoid this problem, the desired distance in the formation can be chosen large enough to ensure collision avoidance. However, in the faulty conditions due to the large communication delays, the lack of updated information on the trajectory of the neighbors can make collisions possible if the desired distances do not account for the delays. To address this problem, the tube DRHC approach is employed to avoid collisions by adding an extra distance to the desired relative distance between healthy and faulty vehicles. In this approach, the neighbors of the faulty vehicle consider a tube-shaped trajectory set around the trajectory of the faulty vehicle instead of a line-shaped trajectory. The radius of this tube is added as the extra distance to the desired relative distance. This will put the faulty vehicle in a safe zone (tube) where the neighbors of the faulty vehicles are not allowed.

The radius of this tube is a function of the communication time delay, maneuverability, and time. In such cases, if a constraint is imposed on the maneuverability of the faulty vehicle, then the reachable set (tube) of the faulty vehicle can be computed by neighboring vehicles using the available (albeit delayed) information from the faulty vehicle. The maneuverability of the faulty vehicle is restricted by imposing an input constraint in its optimization problem, such that at any time instant, the computed inputs do not deviate too far from the previous one.

To implement the tube DRHC, the radius of the tube at any time is added to the desired relative position $r^{i,j}(t)$ to avoid nonconvexity, as the tube has a nonconvex nature. In fact, the desired relative position $r^{i,j}$ between vehicle i and the faulty neighbor j is increased as follows to account for the communication delay:

$$r^{i,j}(t) \leftarrow r^{i,j}(t) + \mathrm{sign}[r^{i,j}(t)]\Delta r^{i,j}(t) \tag{16}$$

In fact, the vehicle i adds the margin $\Delta r^{i,j}(t)$ to its desired relative distance to vehicle j to ensure safety; the margin $\Delta r^{i,j}(t)$ is the radius of the tube at any time t. As an example, the following problem represents a method for calculating the radius of the tube for linear systems using a state transition matrix method [Eq. (20)].

Problem 2: Consider the homogenous subsystems with the linear dynamics of Eq. (6). Also assume the change in the control input (maneuverability) of the faulty vehicle j is restricted as follows:

$$-\bar{\mu} \le u_{t_k}^j(t) - u_{t_{k-1}}^j(t) \le \bar{\mu}; \qquad t \in [t_k,\ t_{k-1}+T] \tag{17}$$

where $\bar{\mu}$ is a vector with an appropriate length containing the bound on the change in control inputs. Then if, at time t_k, vehicle i receives the information from faulty neighbor j with a delay of d steps [i.e., $x^j(t_{k-d}\colon t_{k-d}+T)$], then calculate the reachable set of faulty vehicle j at time t_k; this reachable set is denoted by $\hat{X}^j(t)$ and the boundaries of this reachable set determine the radius of the tube.

For the solution, the updated trajectory of faulty vehicle j is approximated from its delayed trajectory by neighbor i as follows:

$$x^j(t) = x^j(t - d\delta) + \Delta x^j(t); \qquad t \in [t_k\colon t_k+T] \tag{18}$$

where $\Delta x^j(t)$ is due to any variation in control input during $[t_{k-d},\ t_k]$ and then is calculated as follows. Using the state transition method, the solution of the differential Eq. (6) for faulty vehicle j is given as

$$x^j(t) = \varphi(t,t_0)x^j(t_0) + \int_{t_0}^{t}\varphi(t,s)Bu^j(s)\,\mathrm{d}s \tag{19}$$

Then, assume a perturbation in the input of faulty vehicle as $u^j \leftarrow u^j + \Delta u^j$; hence,

$$x^j + \Delta x^j = \varphi(t,t_0)x^j(t_0) + \int_{t_0}^{t}\varphi(t,s)B[u^j(s) + \Delta u^j(s)]\,\mathrm{d}s$$
$$\Rightarrow \Delta x^j(t) = \int_{t_0}^{t}\varphi(t,s)B\Delta u^j(s)\,\mathrm{d}s \tag{20}$$

To find $\Delta u^j(s)$ after a delay of the d step, the input constraint of Eq. (17) can be used sequentially as follows (the superscript j is dropped temporarily):

$$
\begin{array}{ll}
-\bar{\mu} \le u_{t_k}(t) - u_{t_{k-1}}(t) \le \bar{\mu} & t \in [t_k,\ t_{k-1} + T] \\
-\bar{\mu} \le u_{t_{k-1}}(t) - u_{t_{k-2}}(t) \le \bar{\mu} & t \in [t_{k-1},\ t_{k-2} + T] \\
\vdots & \vdots \\
-\bar{\mu} \le u_{t_{k-d+1}}(t) - u_{t_{k-d}}(t) \le \bar{\mu} & t \in [t_{k-d+1},\ t_{k-d} + T] \\
d\bar{\mu} \le u_{t_k}(t) - u_{t_{k-d}}(t) \le d\bar{\mu} & t \in [t_k,\ t_{k-d} + T]
\end{array} \tag{21}
$$

Hence,

$$
-\,d\bar{\mu} \le \Delta u^j(t) \le d\bar{\mu}; \qquad t \in [t_k,\ t_{k-d} + T] \tag{22}
$$

By substituting all possible values for Δu^j from Eq. (22) into Eq. (20), all possible Δx^j can be found, and then the set of reachable states $\hat{X}^j(t)$ can be calculated using Eq. (18). The boundaries of this reachable set determine the radius of the tube; in fact, the radius of the tube is equal to $\max[\Delta x^j(t)]$. The procedure presented in this subsection for tube calculation is summarized in the following algorithm:

Algorithm 2: Tube calculator (offline)

Assuming the vehicle i calculates the tube around the trajectory of neighbor j:

1) For all $t \in [0,\ T]$, solve the following optimization problem (discretize $[0,\ T]$ as appropriate) to get a sequence of tube radii $\Delta r^{ij}(t)$:

$$
\Delta r^{i,j}(t) = \max\left[\int_{t_0}^{t} \varphi(t,\ s) B \Delta u^j(s)\, \mathrm{d}s\right] \tag{23}
$$

subject to

$$
-\,\bar{\mu} \le \Delta u^j(t) \le \bar{\mu}
$$

2) Save $\Delta r^{i,j}(t)$ vs t to be used for online tube calculation.

The output of this algorithm will be used in the online Algorithm 2, presented in the next subsections.

Note that Eq. (23) implies that the radius of the tube is time dependent and hence increases with time (over the prediction horizon). It implies also that the radius of the tube is a function of maneuverability Δu^j and, according to Eq. (22), the maneuverability is a function of delay; hence, the tube radius is a function of time delay as well. It should be noted that, because the tube is being used to represent the uncertainty in the faulty vehicle trajectory, the tube size will increase with time and communication delay. Furthermore, a larger maneuverability implies a larger more conservative tube so that there is a tradeoff between the two factors.

Calculation of $\Delta r^{i,j}(t)$ does not impose any online computation time as the set of all $\Delta x^j(t)$ can be computed offline. This is because the only parameter that may not be known before a mission is d, which can be decomposed from the formula of Δu^j in Eq. (22) and multiplied by the computed bound when determined online.

C. Reconfigurable Decentralized Receding Horizon Control Problem Formulation

The reconfigurable DRHC problem $P^i_F(t_k)$ for the faulty conditions is defined next at time t_k for any ith vehicle that involves in the fault (either faults with itself or its neighbors). The outputs of this decentralized optimization problem are 1) the input, 2) the state trajectory of the local vehicle over the prediction horizon, and 3) the trajectory of neighboring vehicles during the tail of the cost function:

Problem 3: Reconfigurable DRHC problem $P^i_F(t_k)$

$$
\min_{\{u^i(t_k:\ t_k+T),\, x^i(t_k:\ t_k+T),\, \check{x}^i(t_{k-d}T:\ t_k+T)\}} J^i_F[\Gamma^i(t_k)] \tag{24}
$$

subject to:

$$
\dot{x}^{i,i}_{t_k}(t) = A x^{i,i}_{t_k}(t) + B u^{i,i}_{t_k}(t); \qquad x^{i,i}_{t_k}(t_k) = x^i(t_k); \qquad t \in [t_k, t_k + T] \tag{25a}
$$

$$
x^{i,i}_{t_k}(t) \in \mathrm{X}^i; \qquad u^{i,i}_{t_k}(t) \in U^i; \qquad t \in [t_k, t_k + T] \tag{25b}
$$

$$
\dot{x}^{j,i}_{t_k}(t) = A x^{j,i}_{t_k}(t) + B u^{j,i}_{t_k}(t); \qquad x^{j,i}_{t_k}(t_{k-d} + T) = x^{j,j}_{t_{k-d}}(t_{k-d} + T); \qquad t \in [t_{k-d} + T,\ t_k + T]; \qquad (i, j) \in E \tag{25c}
$$

$$
x^{j,i}_{t_k}(t) \in \mathrm{X}^j; \qquad u^{j,i}_{t_k}(t) \in U^j; \qquad t \in [t_{k-d} + T, t_k + T]; \qquad (i, j) \in E \tag{25d}
$$

$$
x^{j,i}_{t_k}(t_k + T) \in X^i_f; \qquad x^{j,i}_{t_k}(t_k + T) \in X^j_f; \qquad (i,\ j) \in E \tag{25e}
$$

$$
|u^{i,i}_{t_k}(t) - u^{i,i}_{t_{k-1}}(t)| \le \mu; \qquad t \in [t_k, t_{k-1} + T] \tag{25f}
$$

In Eq. (24), J^i_F is calculated from Eq. (12). Constraints (25a) and (25b) are the same as (8a) and (8b) for the fault-free problem $P^i(t_k)$ and are applied to the trajectory for calculating the cost (13). Constraints (25c) and (25d) are applied to the trajectory of the neighbors and hence correspond to the cost function term (15). Constraint (25e) is the terminal constraint and the same as (8c) for $P^i(t_k)$. Constraint (25f) is imposed for safety guarantee purposes (Problem 2).

Removing Eq. (25f) from problem $P^i_F(t_k)$ and setting $d = 0$, the problem $P^i_F(t_k)$ reduces to a fault-free problem $P^i(t_k)$. In $P^i_F(t_k)$, it is then perfectly valid to choose $i \in V$.

D. Reconfigurable Decentralized Receding Horizon Control Algorithm

The following algorithm is presented for the online implementation of the proposed reconfigurable DRHC problem $P^i_F(t_k)$. The algorithm is formulated for the ith vehicle; in fact, all vehicles run this algorithm during the mission simultaneously:

Algorithm 3: Reconfigurable DRHC

1) Let $k = 0$, measure $x^i(t_k)$, and GOTO step 4.

2) Receive the trajectory

$$
x^j(t_{k-d}:\ t_{k-d} + T); \qquad (i,\ j)E
$$

(with appropriate d for each neighbor) if available, measure the current states $x^i(t_k)$, and update the information set $\Gamma^i(t_k)$ from Eq. (10).

3) Take Δr^{ij} (calculated offline from Algorithm 2), multiply by d, and update r^{ij} in the cost function of Eq. (12).

4) Solve $P^i_F(t_k)$ and generate the control action $u^i(t_k:\ t_k + T)$ and the state trajectory $x^i(t_k:\ t_k + T)$.

5) Send the trajectory $x^i(t_k:\ t_k + T)$ to the neighboring vehicles.

6) Execute the control action for individual vehicle i during $[t_k,\ t_{k+1}]$.

7) $k = k + 1$. GOTO step 2.

This algorithm is a modified version of Algorithm 1 and handles the large communication delays for faulty conditions; it also provides the safety guarantee by executing step 3 using the tube DRHC approach.

IV. Simulation Results

In this section, the proposed approach is tested on the formation problem of a fleet of unmanned vehicles with the following two-DOF dynamics:

$$
\dot{x}_1 = x_2; \qquad \dot{x}_2 = -x_2 + u_1; \qquad \dot{x}_3 = x_4; \qquad \dot{x}_4 = -x_4 + u_2 \tag{26}
$$

where x_1 and x_2 represent the components of the position vector in the x–y coordinate and x_3 and x_4 are their corresponding velocity components. The input vector is given by $u = [u_1,\ u_2]$.

In the first simulation example, the effect of the tail cost added to the cost function is investigated. The simulation is run for two cases:

1) The first case uses the cost function without the tail cost. In this case, the control input is set to $u = 0$ for the tail of cost function (15).

The extra decision variables for tail cost estimation are not included in the optimization problem (24).

2) The second case uses the cost function with the tail cost. In this case, the tail of cost function (15) is estimated using the extra decision variables in the optimization problem (24).

The matrix penalties in the cost function are chosen as follows: $Q = R = I$ (where I is the identity matrix), $P = \mathrm{diag}(0.72,\ 0.5,\ 0.72,\ 0.5)$, and $S = \mathrm{diag}(2,\ 1,\ 2,\ 1)$. The final penalty matrix P is calculated from the Lyapunov equation [16]. The optimization horizon and execution horizon are given by $T = 3.0$ s and $\delta = 0.1$ s, respectively. In all cases, no disturbances, sensor noise, or model uncertainty are considered in the simulations in order to focus on the effect of the communication delay.

A triangular leaderless formation of three vehicles is first considered. To measure the deviation from the desired equilateral triangle formation, the decentralized formation error is calculated as the performance index as follows:

$$E^i(t) = \sum_{j|(i,j)\in E} \| x^i(t) - x^j(t) - r^{ij}(t) \|_S^2 \tag{27}$$

The simulation was repeated for cases with different communication delays and the results are gathered in Fig. 1, which illustrates the average and maximum of the formation error (27), with each point representing a single simulation. It can be seen from Fig. 1 that using the tail of the cost function yields a smaller error, and in some cases it can reduce the error by 150%. It can also enhance the stability of the formation; for this particular example, it is seen that if the communication delay is increased to around $d = 30$ time steps (or $\tau = 3.0$ s), the formation becomes unstable when using the cost function without the tail cost. However, it is still stable with the proposed cost function, including the tail cost. This result is consistent with that of [17,18], in which a final cost is added to the cost function for formation stability, although they did not consider communication delays. The overall trend of the graphs in Fig. 1 shows that the error goes up with delay. The small downward fluctuations are associated with time-delay-related nonlinearities and imperfect numerical optimization.

It is also seen in some simulations that during faulty conditions, although adding the final cost can lead to more precise estimation and a stable formation, the vehicles still may get too close to each other and collide. Hence, in the next simulation, the effect of the proposed tube DRHC is investigated. This case involves the triangular formation control of six vehicles. The communication graph topology is set as follows:

$$\begin{aligned} &V = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}; \\ &E = \{(1,\ 2),\ (1,\ 3),\ (2,\ 3),\ (2,\ 4),\ (3,\ 6),\ (4,\ 5),\ (5,\ 6)\} \end{aligned} \tag{28}$$

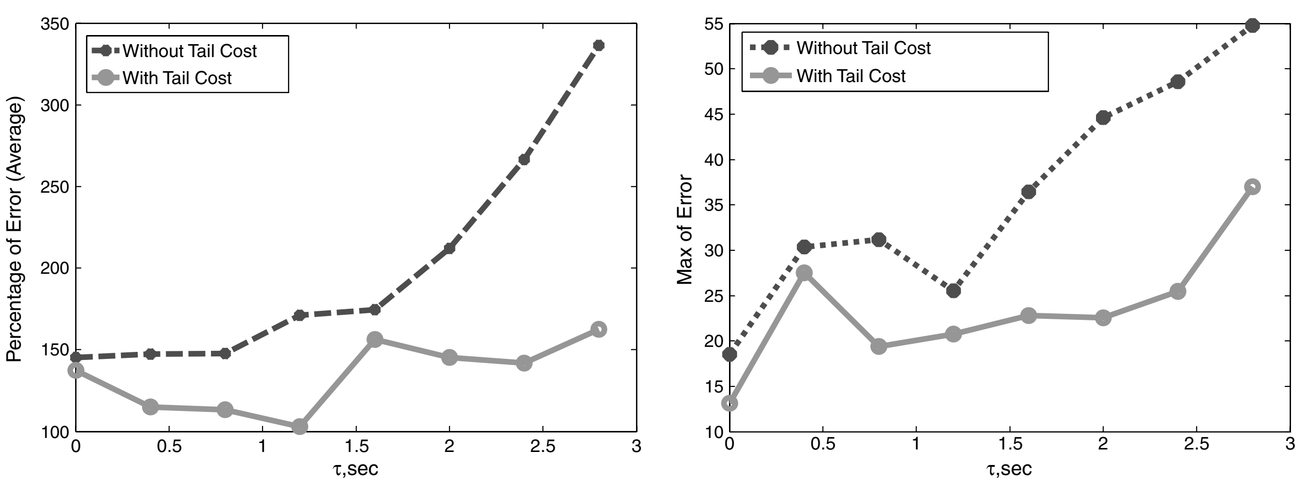

Fig. 1 Percentage of average (left) and maximum (right) error versus communication delay for a triangle configuration of three vehicles.

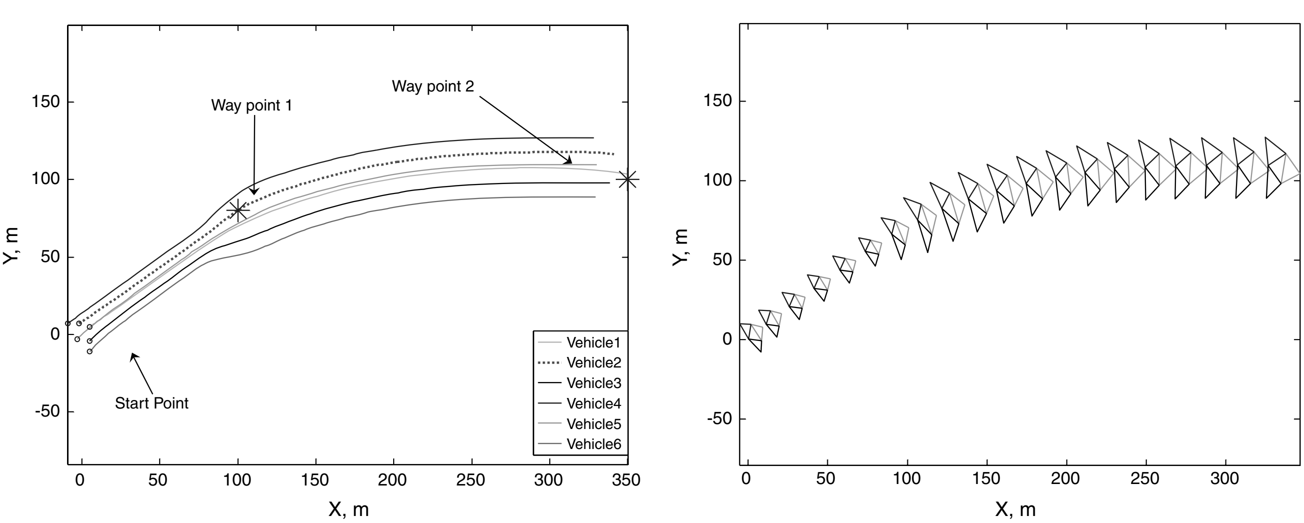

Fig. 2 Trajectory (left) and snapshot (right) of a six-vehicle triangle configuration experiencing a communication failure (delay): the formation expands upon fault occurrence.

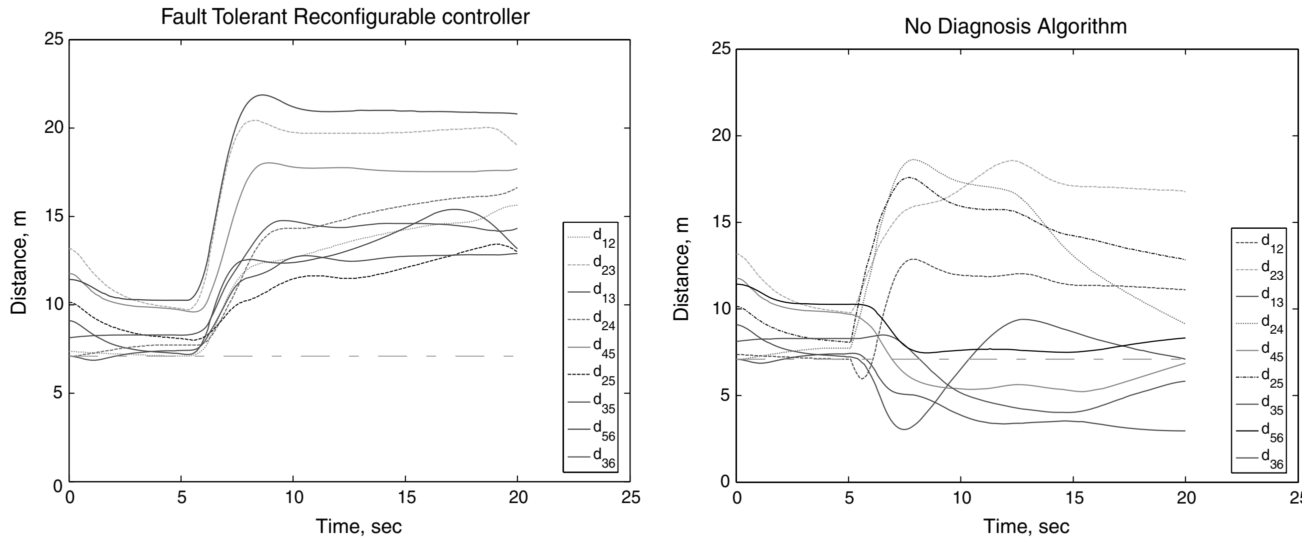

Fig. 3 Distances between each pair of vehicles in a six-vehicle triangle configuration experiencing a communication failure (delay) (at $t = 5$ s): Algorithm 3 (left) and Algorithm 1 (right).

The neighbor assignment was performed manually before the mission by selecting the two or more vehicles that were closest in the desired formation. The results are shown in Figs. 2 and 3. In this case, two sets of way points were considered to be visited by the fleet. At first, the fleet was not faulty, but after 5 s [around point (70,60)], vehicle 2 (for which the trajectory is dotted) became faulty, which lead to a $d = 8$ time-steps delay in the messages communicated to and from vehicle 2. As seen from Fig. 2, the vehicles started to keep a larger distance, and the formation was expanded for safety upon fault occurrence as the result of using the tube DRHC approach.

The distances between each pair of neighboring vehicles are shown in Fig. 3 for two cases: 1) faulty without any fault-tolerant algorithm (Algorithm 1) and 2) faulty with the proposed fault-tolerant Algorithm 3. It is desired that vehicles keep a 7 m distance from neighbors. As seen from Fig. 3 (right) in the case of Algorithm 1, the vehicles get too close to each other and may collide. However, the reconfigurable Algorithm 2 offers a larger distance (Fig. 3, left) and the formation is safe as a consequence of using the tube DRHC approach.

V. Conclusions

A new reconfigurable fault-tolerant DRHC approach is proposed that can address faults leading to large communication delays. The proposed approach provides two key features. The first is that the tail of the cost function is estimated to account for large communication delays. The second aspect is the development of the tube-based RHC to provide guaranteed formation safety from possible collisions. Simulations illustrate that the proposed approach can reconfigure effectively in the presence of communication failures. It is also demonstrated that using a prediction for the tail of the cost function can lead to better overall performance and stability. Together, these results provide a new approach to deal with communication faults in DRHC problems that ensure safe formations and improved performance.

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References

[1] Franco, E., Parisini, T., and Polycarpou, M. M., "Cooperative Control of Discrete-Time Agents with Delayed Information Exchange: a Receding-Horizon Approach," *Proceedings of the 43rd IEEE Conference on Decision and Control*, Vol. 4, Inst. of Electrical and Electronics Engineers, New York, 2004, pp. 4274–4279.

[2] Franco, E., Parisini, T., and Polycarpou, M. M., "Stable Receding-Horizon Cooperative Control of a Class of Distributed Agents," *Proceedings of the American Control Conference*, Vol. 7, Inst. of Electrical and Electronics Engineers, New York, 2005, pp. 4673–4678.

[3] Franco, E., Parisini, T., and Polycarpou, M. M., "Cooperative Control of Distributed Agents with Nonlinear Dynamics and Delayed Information Exchange: a Stabilizing Receding-Horizon Approach," *Proceedings of the 44th IEEE Conference on Decision and Control and the European Control Conference*, Inst. of Electrical and Electronics Engineers, New York, 2005, pp. 2206–2211.

[4] Richards, A. G., and How, J. P., "Implementation of Robust Decentralized Model Predictive Control," AIAA Guidance, Navigation, and Control Conference, AIAA, Paper 2005-6366, Aug. 2005.

[5] Venkat, A. N., Rawlings, J. B., and Wright, S. J., "Stability and Optimality of Distributed Model Predictive Control," *Proceedings of the 44th IEEE Conference on Decision and Control and the European Control Conference*, Inst. of Electrical and Electronics Engineers, New York, 2005, pp. 6680–6685.

[6] Alvarado, I., Limon, D., Alamo, T., and Camacho, E. F., "Output Feedback Robust Tube Based MPC for Tracking of Piece-Wise Constant References," *Proceedings of the 46th IEEE Conference on Decision and Control*, Inst. of Electrical and Electronics Engineers, New York, 2007, pp. 2175–2180.

[7] Trodden, P., and Richards, A., "Robust Distributed Model Predictive Control Using Tubes," *Proceedings of the American Control Conference*, Inst. of Electrical and Electronics Engineers, New York, 2006, pp. 2034–2039.

[8] Zhang, X., Xu, R., Kwan, C., Haynes, L., Yang, Y., and Polycarpou, M. M., "Fault Tolerant Formation Flight Control of UAVs," *International Journal of Vehicle Autonomous Systems*, Vol. 2, Nos. 3–4, 2004, pp. 217–235.
doi:10.1504/IJVAS.2004.006108

[9] Innocenti, M., and Pollini, L., "Management of Communication Failures in Formation Flight," *Journal of Aerospace Computing, Information, and Communication*, Vol. 1, No. 1, 2004, pp. 19–35.
doi:10.2514/1.2608

[10] Cheng, L., Wang, Y., and Zhu, Q., "A Communication-Based Multirobot Rigid Formation Control System: Design and Analysis," *International Journal of Modelling, Identification, and Control*, Vol. 1, No. 1, 2006, pp. 13–22.
doi:10.1504/IJMIC.2006.008643

[11] Keviczky, T., Borrelli, F., and Balas, G. J., "Decentralized Receding Horizon Control for Large Scale Dynamically Decoupled Systems," *Automatica*, Vol. 42, No. 12, 2006, pp. 2105–2115.
doi:10.1016/j.automatica.2006.07.008

[12] Olfati-Saber, R., and Murray, R. M., "Consensus Problems in Networks of Agents with Switching Topology and Time Delays," *The IEEE Transactions on Automatic Control*, Vol. 49, No. 9, 2004, pp. 1520–533.doi:10.1109/TAC.2004.834113

[13] Mayne, D. Q., Rawlings, J. B., Rao, C. V., and Scokaert, P. O., "Constrained Model Predictive Control: Stability and Optimality," *Automatica*, Vol. 36, No. 6, 2000, pp. 789–814.

doi:10.1016/S0005-1098(99)00214-9

[14] Richards, A. G., and How, J. P., "Robust Distributed Model Predictive Control," *International Journal of Control*, Vol. 80, No. 9, 2007, pp. 1517–1531.
doi:10.1080/00207170701491070

[15] Garagic, D., "On Delay-Dependent Stability of a Swarm of Networked Autonomous Vehicles under Communication Constraints," *Proceedings of the 2005 IEEE Swarm Intelligence Symposium*, Inst. of Electrical and Electronics Engineers, New York, 2005, pp. 289–293.

[16] Chen, H., and Allgower, F., "A Quasi Infinitive Horizon Nonlinear Model Predictive Control Scheme with Guaranteed Stability," *Automatica*, Vol. 34, No. 10, 1998, pp. 1205–1217.
doi:10.1016/S0005-1098(98)00073-9

[17] Dunbar, W., and Murray, R., "Model Predictive Control of Coordinated Multi-Vehicle Formations," *Proceedings of the 41st IEEE Conference on Decision and Control*, Vol. 4, Inst. of Electrical and Electronics Engineers, New York, 2002, pp. 4631–4636.

[18] Gu, D., and Yang, E., "A Suboptimal Model Predictive Formation Control," *Proceedings of the 2005 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Inst. of Electrical and Electronics Engineers, New York, 2005, pp. 1295–1300.