

# SPACE-TIME SPREADING AND DIVERSITY IN ASYNCHRONOUS CDMA SYSTEMS OVER FREQUENCY-SELECTIVE FADING CHANNELS

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## ABSTRACT

In this paper, the performance of space-time spreading (STS) introduced in the literature is analyzed for asynchronous direct sequence code division multiple access (DS-CDMA) systems in frequency-selective slowly-fading environment. The transmit diversity scheme is based on two transmit and one receive antenna. The bit-error-rate performance of the underlying system is investigated for an uplink transmission where a decorrelator detector is used at the base station receiver. The proposed receiver has the advantage of mitigating the multipath fading effects and suppressing the multiuser interference (MUI). Both simulation and analytical results show significant performance gains over the conventional STS receiver. Different from the decorrelator detector, the achieved diversity order of the conventional detector is shown to deteriorate as a function of the number of users.

## 1. INTRODUCTION

Multi-input multi-output systems (MIMO) systems allow the receiver to see independent versions of the information which yields to spatial diversity and/or coding gain compared to single antenna systems. One approach that uses multiple transmit antennas and, if possible, multiple receive antennas to provide reliable and high data rate communication is space-time coding (STC). It has been shown that STCs can offer these gains by introducing both temporal and spatial correlation into the transmitted signals from different antennas without increasing the total transmitted power or transmission bandwidth [1]. Depending on the structure of the STC used, one can achieve a coding gain and/or diversity gain [1],[2].

MIMO systems have recently been recognized as an efficient method for improving the overall system capacity and signal reliability in wireless communication system [1]. One of the most widely used techniques is the space time coding. There are two major schemes: space time trellis codes (STTC) [1] and space time block codes (STBC) [2],[3]. It is known that STBC has the advantage of less complexity than STTC while achieving the same diversity gain.

In [4], a space time transmit diversity scheme suitable for DS-CDMA systems was proposed. This scheme was examined in flat fading environment in [4]. Recently significant research efforts have aimed at the integration of these codes with DS-CDMA systems (e.g.,[5]-[9]).

In this paper, we study the performance of the scheme proposed in [4] for asynchronous DS-CDMA over frequency selective fading channel. The proposed receiver is a Rake-type receiver. The Rake receiver exploits the path diversity inherent to multipath propagation. Then, a decorrelator detector is used to mitigate the multiple access interference (MAI) and the known near-far problem [10].

The remainder of this paper is organized as follows: The following section describes the asynchronous DS-CDMA system over frequency selective fading channels. In section 3, the structure of the multiuser receiver is discussed. Section 4 introduces simulation results and the discussions. Finally, the conclusion is given in section 5.

## 2. SYSTEM MODEL

Consider an uplink transmission for asynchronous DS-CDMA system with  $K$  users. The system employs two transmit antennas at the transmitter side and one receive antenna at the receiver side. We consider the space-time spreading system proposed in [4]. The proposed scheme can be summarized as follows. Assuming  $x_1, x_2$  are data symbols assigned to each user in two consecutive symbol intervals, the space-time block coded signals transmitted during the first transmission period from antenna 1 and 2 are  $x_1^* s_1 + x_2^* s_2$  and  $x_1 s_2 - x_2 s_1$  respectively, where  $s_1$  and  $s_2$  are the spreading codes. These space-time coded signals are switched with respect to the antenna order during the second transmission period.

Consider a multipath channel with  $P$  paths for each transmit antenna during each transmission period, the low pass equivalent of the received signal at the base station can be expressed as

$$r(t) = \sum_{k=1}^K \sum_{p=1}^P \sqrt{E} (h_{1p}^k (x_1^{k*} s_1^k(t - \tau_k - \tilde{\tau}_p) + x_2^{k*} s_2^k(t - \tau_k - \tilde{\tau}_p)) + h_{2p}^k (x_1^k s_2^k(t - \tau_k - \tilde{\tau}_p) - x_2^k s_1^k(t - \tau_k - \tilde{\tau}_p)) + h_{1p}^k (x_1^k s_2^k(t + T_b - \tau_k - \tilde{\tau}_p) - x_2^k s_1^k(t + T_b - \tau_k - \tilde{\tau}_p)) + h_{2p}^k (x_1^{k*} s_1^k(t + T_b - \tau_k - \tilde{\tau}_p) + x_2^{k*} s_2^k(t + T_b - \tau_k - \tilde{\tau}_p))) + n(t). \quad (1)$$

In (1),  $E$  is the received signal energy for the single user,  $x_1^k$  and  $x_2^k$  are the even and odd  $k^{th}$  user data symbols,  $s_1^k(t)$

and  $s_2^k(t)$  are the two spreading codes assigned to the  $k^{\text{th}}$  user with processing gain  $(T_b/T_c)$  where  $T_b$  is the bit period and  $T_c$  is the chip period, and  $\tau_k$  is the random transmit delay of the  $k^{\text{th}}$  user which is assumed uniformly distributed along the symbol period. We assume that the delay of each user,  $\tau_k$ , to be fixed during two consecutive symbol periods. The parameter  $\tilde{\tau}_p$  represents the delay of the  $p^{\text{th}}$  path of each transmit antenna during one transmission period. This delay taken to be an integer number of chips and small compared to the symbol period to neglect the effect of intersymbol interference (ISI). We also assume that

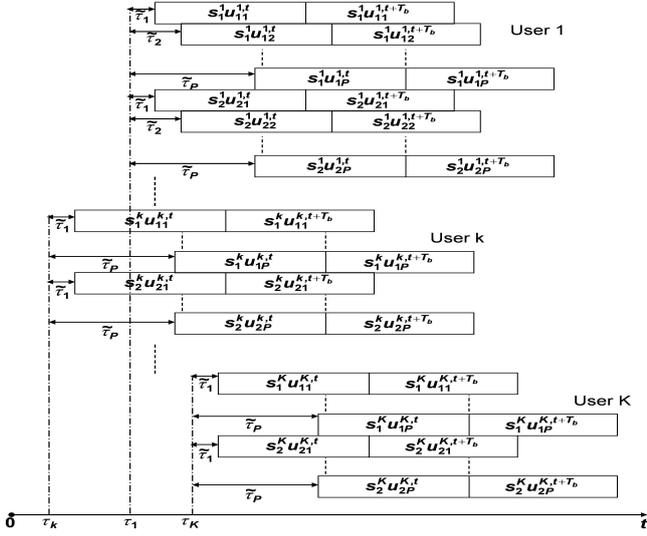


Fig. 1. Received signal for K-user system.

each pair of paths from the two transmitter antennas of any user arrives with the same set of delays at the receiver antenna. The channel coefficient  $h_{mp}^k$  ( $m = 1, 2$ ) models the fading channel corresponding to the  $k^{\text{th}}$  user,  $p^{\text{th}}$  path from the  $m^{\text{th}}$  transmit antenna to the base station. The fading coefficients are assumed to be constant during two symbol periods and modeled as independent Gaussian random variables with zero mean and unit variance. The noise  $n(t)$  is Gaussian with zero mean and variance  $\sigma_n = N_o/2$ . From (1), the received signal can be represented in a more compact form as

$$r(t) = \sum_{k=1}^K \sum_{p=1}^P s_1^k(t - \tau_k - \tilde{\tau}_p) u_{1p}^{k,t} + s_2^k(t - \tau_k - \tilde{\tau}_p) u_{2p}^{k,t} + s_1^k(t + T_b - \tau_k - \tilde{\tau}_p) u_{1p}^{k,t+T_b} + s_2^k(t + T_b - \tau_k - \tilde{\tau}_p) u_{2p}^{k,t+T_b} + n(t) \quad (2)$$

where

$$\begin{aligned} u_{1p}^{k,t} &= \sqrt{E}(h_{1p}^k x_1^{k*} - h_{2p}^k x_2^k), \\ u_{2p}^{k,t} &= \sqrt{E}(h_{1p}^k x_2^{k*} + h_{2p}^k x_1^k), \\ u_{1p}^{k,t+T_b} &= \sqrt{E}(-h_{1p}^k x_2^k + h_{2p}^k x_1^{k*}), \\ u_{2p}^{k,t+T_b} &= \sqrt{E}(h_{1p}^k x_1^k + h_{2p}^k x_2^{k*}). \end{aligned}$$

The received signal structure for  $K$ -user system is shown in Fig. 1.

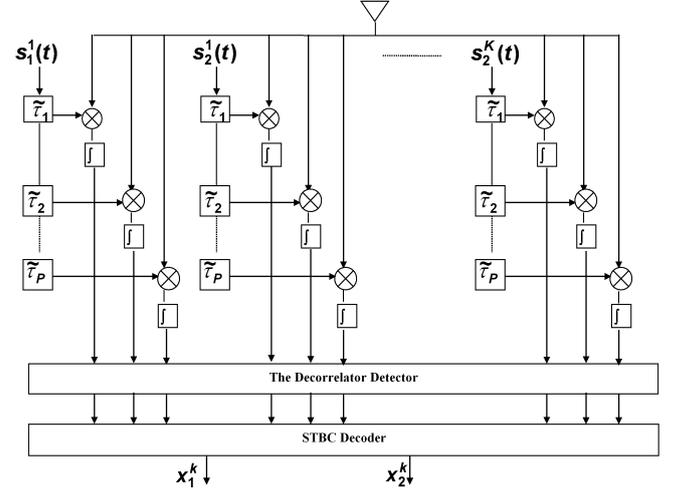


Fig. 2. Multiuser receiver structure.

### 3. THE MULTIUSER RECEIVER

The multiuser detector consists of  $2PK$  filters matched to the delayed versions of the normalized signature waveforms of each user as shown in Fig. 2. The output of this filter bank sampled at the chip rate during one ST-block interval, which is equivalent to two symbol intervals ( $L=2$  where  $L$  is the length of one block interval), is given in a vector form by

$$Y = RU + N. \quad (3)$$

The  $(2LPK \times 1)$  vector  $Y$  consists of the bank of matched filters output at time  $t$  and  $t + T_b$ , given by

$$Y = [y_{1,1,1}^t y_{1,2,1}^t \dots y_{1,2,P}^t y_{1,1,1}^{t+T_b} \dots y_{K,1,1}^t \dots y_{K,2,P}^{t+T_b}]^T \quad (4)$$

where the superscript  $T$  denotes vector transpose and  $y_{k,l,p}^t, y_{k,l,p}^{t+T_b}$  ( $l = 1, 2$ ) represent the outputs of the filter matched to the  $p^{\text{th}}$  path of the  $l^{\text{th}}$  sequence of the  $k^{\text{th}}$  user at time  $t$  and  $t + T_b$  respectively. The vector  $U$  represents the faded data for the  $K$ -user system, and is given by

$$U = [U_1^T U_2^T \dots U_k^T \dots U_K^T]^T. \quad (5)$$

In (5), the  $(2LP \times 1)$  vector  $U_k$  represents the faded data transmitted by the  $k^{\text{th}}$  user over two successive symbols, defined as

$$U_k = [u_{11}^{k,t} u_{21}^{k,t} u_{12}^{k,t} \dots u_{2P}^{k,t} u_{11}^{k,t+T_b} u_{21}^{k,t+T_b} \dots u_{2P}^{k,t+T_b}]^T. \quad (6)$$

The  $(2LPK \times 2LPK)$  cross correlation matrix  $R$  is given by

$$R = \begin{bmatrix} R_{11} & R_{12} & \dots & R_{1K} \\ \vdots & \dots & \dots & \vdots \\ R_{K1} & \dots & \dots & R_{KK} \end{bmatrix} \quad (7)$$

where  $R_{kw}$  ( $w = 1, \dots, K$ ) is a  $(2LP \times 2LP)$  sub-matrix defined according to

$$R_{kw} = \int_{\tau_k}^{\tau_k + LT} S_k(t) S_w^H(t) dt, \quad (8)$$

with  $H$  denotes Hermitian transpose. In (8),  $S_k(t)$  represents all the delayed versions of the two codes assigned to the  $k^{th}$  user during the two symbol periods, and is given by

$$S_k(t) = \begin{bmatrix} s_1^k(t - \tau_k - \tilde{\tau}_1) \\ s_2^k(t - \tau_k - \tilde{\tau}_1) \\ \vdots \\ s_1^k(t - \tau_k - \tilde{\tau}_P) \\ \vdots \\ s_1^k(t + T_b - \tau_k - \tilde{\tau}_1) \\ \vdots \\ s_2^k(t + T_b - \tau_k - \tilde{\tau}_P) \end{bmatrix}.$$

The  $(2LPK \times 1)$  noise vector  $N$  is given by

$$N = [N_1^T N_2^T \dots N_k^T \dots N_K^T]^T \quad (9)$$

with

$$N_k = [n_{11}^{k,t} n_{21}^{k,t} n_{12}^{k,t} \dots n_{2P}^{k,t} n_{11}^{k,t+T_b} n_{21}^{k,t+T_b} \dots n_{2P}^{k,t+T_b}]^T \quad (10)$$

and each of the elements  $n_{lp}^{k,t}, n_{lp}^{k,t+T_b}$  ( $l = 1, 2$  and  $p = 1, \dots, P$ ) is modeled as complex Gaussian random variable, with variance  $N_o/2$  per dimension. Note that the output of the matched filter bank suffers from MAI which can be eliminated using the decorrelator detector [11]. In this case, the output of the matched filter bank  $Y$  is applied to a linear mapper  $Z = R^{-1}Y$ , where  $R^{-1}$  is the inverse of the cross correlation. The  $(2LPK \times 1)$  vector  $Z$  represents the output of the decorrelator during two successive symbol periods. It includes the  $P$  replicas of the transmitted signals from the two antennas for each user during one space-time block interval.

Now, the combining scheme in [4] (designed for fast-fading channels) needs to be modified to compensate for the multipath fading effects. Assuming, for simplicity, that  $P = 2$  and considering user 1 as a desired user, the output of the decorrelator can be expressed as

$$Z = [Z_1^T Z_2^T \dots Z_k^T \dots Z_K^T]^T \quad (11)$$

where  $Z_k$  is defined by

$$Z_k = [z_{11}^{k,t} z_{21}^{k,t} z_{12}^{k,t} z_{22}^{k,t} z_{11}^{k,t+T_b} z_{21}^{k,t+T_b} z_{12}^{k,t+T_b} z_{22}^{k,t+T_b}]^T \quad (12)$$

with the entries  $z_{lp}^{k,t}, z_{lp}^{k,t+T_b}$  represent the output of the decorrelator corresponding to the  $p^{th}$  path of the  $l^{th}$  sequence for the  $k^{th}$  user at time  $t$  and  $t + T_b$ , respectively. Now, we can extract the two transmitted symbols of the  $k^{th}$  user according to

$$\hat{x}_1^k = h_{11}^1 Z_{11}^{k,t*} + h_{21}^{1*} Z_{21}^{k,t} + h_{21}^1 Z_{11}^{k,t+T_b*} + h_{11}^{1*} Z_{21}^{k,t+T_b} \\ + h_{12}^1 Z_{12}^{k,t*} + h_{22}^{1*} Z_{22}^{k,t} + h_{22}^1 Z_{12}^{k,t+T_b*} + h_{12}^{1*} Z_{22}^{k,t+T_b} \quad (13)$$

$$\hat{x}_2^1 = h_{11}^1 Z_{21}^{k,t*} - h_{21}^{1*} Z_{11}^{k,t} - h_{11}^{1*} Z_{21}^{k,t+T_b} + h_{21}^1 Z_{21}^{k,t+T_b*} \\ + h_{12}^1 Z_{22}^{k,t*} - h_{22}^{1*} Z_{12}^{k,t} - h_{12}^{1*} Z_{12}^{k,t+T_b} + h_{22}^1 Z_{22}^{k,t+T_b*}. \quad (14)$$

From (13), the estimated data symbol for the first user,

$$\hat{x}_1^1 = 2\sqrt{E}(|h_{11}^1|^2 + |h_{21}^1|^2 + |h_{12}^1|^2 + |h_{22}^1|^2)x_1^1 \\ + h_{11}^1(R^{-1}N)_{11}^* + h_{21}^{1*}(R^{-1}N)_{21} + h_{21}^1(R^{-1}N)_{51}^* \\ + h_{11}^{1*}(R^{-1}N)_{61} + h_{12}^1(R^{-1}N)_{31}^* + h_{22}^{1*}(R^{-1}N)_{41} \\ + h_{22}^1(R^{-1}N)_{71}^* + h_{12}^{1*}(R^{-1}N)_{81} \quad (15)$$

where  $(R^{-1}N)_{i1}$ ,  $i = 1, \dots, 8$  represents the  $i^{th}$  element of the  $(8K \times 1)$  vector  $R^{-1}N$ . The probability of error in  $\hat{x}_1^1$  conditioned on the channel coefficients can be expressed as

$$P_b(\hat{x}_1^1 = 1 | h_{11}^1, h_{12}^1, h_{21}^1, h_{22}^1) =$$

$$Q\left(\frac{2\sqrt{E}(|h_{11}^1|^2 + |h_{12}^1|^2 + |h_{21}^1|^2 + |h_{22}^1|^2)}{\sqrt{\sigma_x^2}}\right) \quad (16)$$

where

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\nu^2}{2}\right) d\nu \quad (17)$$

and  $\sigma_x^2$  is the variance of the noise term in (16). It is easy to show that

$$\sigma_x^2 = \sigma_n^2(|h_{11}^1|^2(R_{11}^{-1} + R_{66}^{-1}) + |h_{21}^1|^2(R_{22}^{-1} + R_{55}^{-1}) \\ + |h_{12}^1|^2(R_{33}^{-1} + R_{88}^{-1}) + |h_{22}^1|^2(R_{44}^{-1} + R_{77}^{-1})) \quad (18)$$

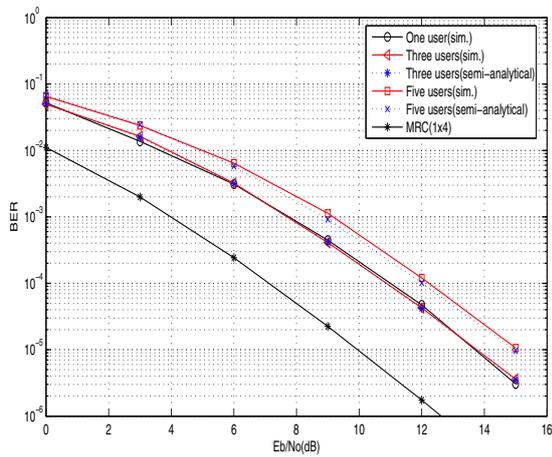
where  $R_{ii}^{-1}$  is the  $i^{th}$  element of the inverse of the cross correlation matrix in (7).

#### 4. SIMULATION RESULTS

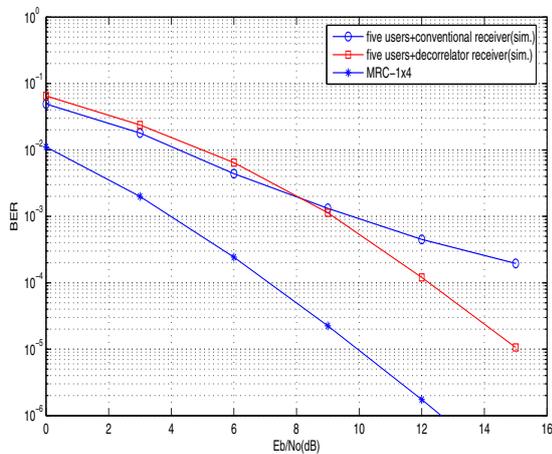
We consider the space-time spreading scheme in [4] with two transmit and one receive antennas. An asynchronous DS-CDMA system with Binary-Phase Shift Keying (BPSK) transmission where every user data is spread using Walsh codes of length 64 chips is considered. For the asynchronous channel, the delay between users,  $\tau_k$ , is uniformly distributed along the symbol period. To neglect the effect of ISI, the delay of each path,  $\tilde{\tau}_p$ , is taken as a multiple of chip periods of length less than 10% of the symbol period. Furthermore, we assume perfect knowledge of the channel coefficients at the receiver. To simplify the simulations, we consider a multipath channel with  $P = 2$  per transmit antenna per transmission interval.

Fig. 3 shows both simulations and analytical results as a function of the number of users. The semi-analytical results are obtained by averaging the conditional probability of error in (16) over  $10^6$  channel realizations. The results show that the proposed system is able to deliver the same diversity order as the maximal-ratio combiner (MRC) with four diversity branches. Note that the diversity order of four is due to the  $M=2$  transmit antennas and  $P=2$  paths.

Fig. 4 shows the performance of both the conventional space-time receiver (without MAI removal) and the decorrelator detector for a 5-user system. Unlike the decorrelator, the conventional receiver fails to maintain the full system diversity where the bit error rate is shown to degrade as the signal-to-noise ratio (SNR) gets higher.



**Fig. 3.** BER performance for asynchronous DS-CDMA systems over frequency-selective slow fading channels.



**Fig. 4.** BER performance of the conventional and decorrelator detectors in a 5-user asynchronous DS-CDMA system.

## 5. CONCLUSION

We examined the performance of transmit diversity using space-time spreading in asynchronous DS-CDMA systems over frequency-selective slow fading channels. For the multipath fading channel with  $P$  paths, we formulated a space-time combining technique to extract the full system diversity. The performance of the decorrelator detector was examined where we derived a semi-analytical expression for the probability of bit errors as a function of both the number of users and paths. Different from the conventional matched filter detector, our results have shown that the decorrelator detector is able to deliver the full system diversity.

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