AN EXTENDED SPACE-TIME TRANSMIT DIVERSITY
SCHEME FOR FREQUENCY SELECTIVE FADING CHANNELS

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Abstract
This paper presents a transmit diversity scheme for frequency selective fading channels. The proposed scheme uses a space-time block code (ST-BC) with three transmit antennas combined with Orthogonal Frequency Division Multiplexing (OFDM). This scheme provides a diversity order as the maximal-ratio receiver combining (MRRC) scheme. It achieves diversity of $3n$ for $n$ receive antennas and has code rate of $3/4$. A maximum-likelihood detector with linear processing is used which results in low computational complexity. Simulation results are provided to show the comparison between the proposed scheme and MRRC.

Keywords: ST-BC; diversity; OFDM; fading.

1. INTRODUCTION

Time-varying multipath fading is fundamental problem in wireless communication systems. To combat this problem, one way would be achieving diversity by using multiple transmit and receive antennas. In some applications, it is expensive or not practical to employ multiple receive antennas (for example on a remote station). Therefore, transmit diversity has taken a lot of attention as a suitable technique to eliminate the effects of fading channels and to maintain high bandwidth efficiency. A number of space-time coded systems have been proposed [1-4] for wireless flat-fading channels. However, the performance of these algorithms is degraded by multipath fading. In recent years, several space-time trellis and block codes in conjunction with OFDM have been proposed [5-8] for high data-rate wireless communications over frequency selective fading channels. The schemes in [5, 6] are based on space-time trellis codes whereas the schemes in [7, 8] are based on space-time block codes with two transmit antennas [4].

The scheme in [2] uses three transmit antennas for flat-fading channels. This scheme has diversity order and computational complexity similar to MRRC. Therefore, in this paper, we extend this scheme to the case of frequency selective fading channel. Simulation results are provided to show the significant improvement in performance achieved by this scheme.

In section 2, we describe the space-time block code with three transmit antennas presented in [2]. Wireless communication system model for frequency selective fading channel and the proposed scheme are presented in section 3. We discuss simulation results in section 4 to show the validity and efficiency of the proposed scheme. Finally, conclusions are presented in section 5.

2. TRANSMIT DIVERSITY SCHEME

Fig. 1 shows the space-time block coded system with three transmit antennas and one receive antenna under the assumption of flat-fading channel [2].

Fig. 1. ST-BC with three transmit antennas and one receive antenna

Let the coefficients $h_i(k)$ be the channel gain from transmit antenna $i$ to the receive antenna at time $k$ where $i=1,2,3$. The channel gains are assumed to be samples of independent complex Gaussian random variables with variance 0.5 per one dimension. Consider the noise $n(k)$ at time $k$ be independent zero-mean complex Gaussian random variable. In [2] it is also assumed that the channel doesn’t change over four symbol periods. Now let us take four symbol periods, every period, symbols $s_p$ are encoded and transmitted according to Table 1, where the subscript $p=1, 2, 3$ is the symbol index.
Table 1. Encoding and transmission scheme for flat fading channels

<table>
<thead>
<tr>
<th>k</th>
<th>Antenna 1</th>
<th>Antenna 2</th>
<th>Antenna 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$s_1$</td>
<td>$s_2$</td>
<td>$s_1/\sqrt{2}$</td>
</tr>
<tr>
<td>2</td>
<td>$-s_2^*$</td>
<td>$s_1^*$</td>
<td>$s_2/\sqrt{2}$</td>
</tr>
<tr>
<td>3</td>
<td>$s_2/\sqrt{2}$</td>
<td>$s_2^*/\sqrt{2}$</td>
<td>$-s_1-s_2+s_2^*/2$</td>
</tr>
<tr>
<td>4</td>
<td>$s_2^*/\sqrt{2}$</td>
<td>$-s_2/\sqrt{2}$</td>
<td>$s_1-s_1^<em>+s_2+s_2^</em>/2$</td>
</tr>
</tbody>
</table>

The received signal can be written as:

$$
\begin{bmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
  y_4
\end{bmatrix} = \begin{bmatrix}
  s_1 \\
  s_2 \\
  -s_2^* \\
  s_1^*
\end{bmatrix} + \begin{bmatrix}
  h_1 \\
  h_2 \\
  h_3 \\
  h_4
\end{bmatrix} + \begin{bmatrix}
  n(1) \\
  n(2) \\
  n(3) \\
  n(4)
\end{bmatrix}
$$

where the superscript * is the complex conjugate. With some manipulations, we can write the received signal as:

$$
\begin{bmatrix}
  y_r(1) \\
  y_m(1) \\
  y_r(2) \\
  y_m(2) \\
  y_r(3) \\
  y_m(3) \\
  y_r(4) \\
  y_m(4)
\end{bmatrix} = \tilde{H}\begin{bmatrix}
  s_{1r} \\
  s_{2r} \\
  s_{1m} \\
  s_{2m}
\end{bmatrix} + \begin{bmatrix}
  n_r(1) \\
  n_m(1) \\
  n_r(2) \\
  n_m(2) \\
  n_r(3) \\
  n_m(3) \\
  n_r(4) \\
  n_m(4)
\end{bmatrix}
$$

where the subscripts $r$ and $m$ denote the real and imaginary parts, respectively. $\tilde{H}$ is defined as:

$$
\tilde{H} = \begin{bmatrix}
  h_{1r} & -h_{1m} & -h_{2m} & h_{2r} \\
  h_{1m} & h_{2r} & h_{2m} & h_{1r} \\
  h_{2r} & h_{2m} & h_{1r} & h_{1m} \\
  h_{2m} & h_{1m} & h_{1r} & h_{2r} \\
  -h_{2r} & 0 & 0 & h_{2m} \\
  0 & h_{1r} & h_{2r} & h_{1m} \\
  -h_{2m} & 0 & 0 & h_{2r} \\
  0 & -h_{1r} & 0 & h_{2m} \\
  0 & -h_{1m} & 0 & h_{2r} \\
  0 & h_{1m} & 0 & h_{2r}
\end{bmatrix}
$$

It is assumed that the receiver has perfect knowledge of the channel. Hence, from (2) the estimates $\hat{s}_i$, $\hat{\delta}_i$ and $\hat{\delta}_2$ can be evaluated as:

$$
\begin{bmatrix}
  \hat{s}_1 \\
  \hat{s}_2 \\
  \hat{\delta}_1 \\
  \hat{\delta}_2 \\
  \hat{s}_3 \\
  \hat{s}_4 \\
  \hat{\delta}_3 \\
  \hat{\delta}_4
\end{bmatrix} = \tilde{H}^H
\begin{bmatrix}
  y_r(1) \\
  y_m(1) \\
  y_r(2) \\
  y_m(2) \\
  y_r(3) \\
  y_m(3) \\
  y_r(4) \\
  y_m(4)
\end{bmatrix}
$$

where the superscript $H$ denotes the complex conjugate. These estimates are sent to the optimum detector where the maximum likelihood decision rule is applied.

3. THE PROPOSED SCHEME

Consider the system shown in Fig. 1 under the assumption of frequency selective fading channel (delay spread in the channel). At a given time $k$, signal $x_i(k)$ is transmitted from antenna $i$ and signal $y(k)$ is received, where $k = 0, 1, \ldots, K-1$ and $K$ is the size of data block. We can model the delay spread channel between antenna $i$ and the receive antenna by

$$
\begin{bmatrix}
  h_i(0) \\
  \vdots \\
  h_i(L)
\end{bmatrix}
$$

where $L$ is the delay spread in the channel. The received vector $y$ can be written as:

$$
y = \begin{bmatrix}
  y(0) \\
  \vdots \\
  y(K+L-1)
\end{bmatrix} = \sum_{i=1}^{3} H_i \begin{bmatrix}
  x_i(0) \\
  \vdots \\
  x_i(K-1)
\end{bmatrix} + n
$$

where $L$ is the delay spread in the channel and vector $n$ is complex white Gaussian noise.

Fig. 2 shows the baseband representation of the proposed scheme which uses space-time block code combined with OFDM technique.
Every block of data is encoded by the space-time block code, then a cyclic prefix (CP) of length $L$ is added to the encoded block $\bar{x}^{(j)} = [x^{(j)}_1(0), \ldots, x^{(j)}_1(K-1)]^T$ which results in $x^{(j)} = [x^{(j)}_1(K-L), \ldots, x^{(j)}_1(K-1), x^{(j)}_1]^T$ where $j=1, 2, 3, 4$ is the data block index. The Inverse Discrete Fourier Transform (IDFT) of this block is transmitted. By employing Discrete Fourier Transform (DFT) in the receiver and canceling the first and the last $L$ entries of the received signal, the vector $y^{(j)}$ can be written as:

$$y^{(j)} = \sum_{i=1}^{3} \tilde{H} \bar{x}^{(j)} + \bar{n}^{(j)} \quad (7)$$

where $\tilde{H}$ is a block circulant matrix and vector $\bar{n}^{(j)}$ is the discrete Fourier transform of the channel noise. Now consider a block of symbols $s_p = [s_p(0), \ldots, s_p(K-1)]$ and choose $\bar{x}^{(j)}$ according to Table 2.

**Table 2. Encoding and transmission scheme for delay spread channels**

<table>
<thead>
<tr>
<th>Antenna 1</th>
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<th>Antenna 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}^{(j)}_1$</td>
<td>$\bar{x}^{(j)}_2$</td>
<td>$\bar{x}^{(j)}_3$</td>
</tr>
<tr>
<td>1</td>
<td>$s_1$</td>
<td>$s_2$</td>
</tr>
<tr>
<td>2</td>
<td>$s'_1$</td>
<td>$s'_2$</td>
</tr>
<tr>
<td>3</td>
<td>$s'_1/\sqrt{2}$</td>
<td>$s'_2/\sqrt{2}$</td>
</tr>
<tr>
<td>4</td>
<td>$s'_1/\sqrt{2}$</td>
<td>$-s'_2/\sqrt{2}$</td>
</tr>
</tbody>
</table>

The vectors $y^{(j)}$ can be written as:

$$\begin{bmatrix}
    y^{(1)}_1 \\
    y^{(1)}_2 \\
    y^{(3)}_1 \\
    y^{(3)}_2 \\
    y^{(4)}_1 \\
    y^{(4)}_2 \\
    y^{(3)}_3 \\
    y^{(3)}_4 \\
    y^{(4)}_3 \\
    y^{(4)}_4
\end{bmatrix} = \Lambda \begin{bmatrix}
    s_1 \\
    s_2 \\
    s'_1 \\
    s'_2 \\
    s'_3 \\
    s'_4 \\
    s'_3 \\
    s'_4 \\
\end{bmatrix} + \begin{bmatrix}
    \bar{n}^{(1)}_1 \\
    \bar{n}^{(1)}_2 \\
    \bar{n}^{(3)}_1 \\
    \bar{n}^{(3)}_2 \\
    \bar{n}^{(4)}_1 \\
    \bar{n}^{(4)}_2 \\
    \bar{n}^{(3)}_3 \\
    \bar{n}^{(3)}_4 \\
    \bar{n}^{(4)}_3 \\
    \bar{n}^{(4)}_4
\end{bmatrix} \quad (8)$$

In (8), $A_i$ is diagonal matrix where its diagonal elements are DFT of the channel impulse response $h(k)$. With some manipulations, we can write the received signal as:

$$A = \begin{bmatrix}
    A_{11} & A_{12} & A_{13} & A_{14} \\
    A_{21} & A_{22} & A_{23} & A_{24} \\
    A_{31} & A_{32} & A_{33} & A_{34} \\
    A_{41} & A_{42} & A_{43} & A_{44}
\end{bmatrix} \begin{bmatrix}
    A_{41} & A_{42} & A_{43} & A_{44}
\end{bmatrix}$$

These estimates are sent to the optimum detector where the maximum likelihood decision rule is applied. In the same manner presented in [2], it is straightforward to extend this scheme to multiple receive antennas. Note that this scheme provides a diversity order as the MRRC scheme. It achieves diversity of $3n$ for $n$ receive antennas. On the other hand, this code exploits $\frac{n}{4}$ of the maximum transmission rate and uses maximum-likelihood decoder with linear processing. Using OFDM technique with cyclic prefix causes a reduction in the bandwidth proportional to $L/(K+L)$. This fraction can be reduced by...
increasing the block size $K$ if it is guaranteed that the channel doesn’t change over four block periods.

4. SIMULATION RESULTS

This section provides simulation results of the proposed scheme and the MRRC scheme combined with OFDM. Let us consider a wireless communication system with 3 transmit antennas and 1 receive antenna under the assumption that the channel is frequency selective fading with delay spread of 4 bits and the size of the transmitted block equals 128. The channel path gains are assumed as samples of independent complex Gaussian random variables with variance 0.5 per one dimension (Rayleigh fading). The noise is also considered to be complex white Gaussian. Further, it is assumed that the total transmitted power from three transmit antennas of the proposed scheme is the same as the transmitted power from the single transmit antenna for MRRC.

Fig. 3 shows the bit error probability with respect to signal to noise ratio of the proposed scheme compared to the MRRC in conjunction with OFDM and BPSK modulation for frequency selective fading channel. It is shown in Fig. 3 that the performance of the proposed scheme is 4 dB worse than the three transmit antennas MRRC because as mentioned above, the total transmit power of our scheme equals to the transmitted power from the single transmit antenna for MRRC. If we make the power of each block of $s_1$, $s_2$ and $s_3$ in our scheme equal to the transmitted power from the single transmit antenna for MRRC, the Bit Error Rate (BER) of the two schemes will be the same and the curves will overlap with each other.

5. CONCLUSIONS

A ST-BC/OFDM transmit diversity scheme for frequency selective fading channels has been presented. It is shown that this scheme is efficient for frequency selective fading channels with low complexity and good performance. This scheme provides a diversity order as the MRRC scheme. It achieves diversity of $3n$ for $n$ receive antennas and has code rate of $3/4$. Based on the scheme presented in [2] the extension of this scheme to multiple receive antennas is straightforward.

References