Phase-Shift-Based Layered Linear Space-Time Codes

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Abstract

In this paper, a new layered space-time coding scheme for flat fading MIMO channels is presented. Based on linear dispersive coding that preserves the channel capacity, the proposed scheme employs phase shifts among input symbols to maximize both diversity and coding gains without loss of mutual information. In addition, this coding scheme provides simple spatial multiplexing (channel layering) through antenna weighting. As a result, a class of linear block space-time codes can be designed to achieve various tradeoffs between performance and spectral efficiency. Simulation results are provided to show the merits of the proposed scheme compared to conventional schemes.

1. Introduction

Due to limited spectrum resource, there has been a growing demand on the bandwidth efficiency of wireless communications. This has prompted a great deal of research and development interest in MIMO systems that provide significantly increased channel capacity [1, 2]. In addition to higher bandwidth efficiency, the use of multiple antennas provides spatial diversity, leading to improved reliability of communications over fading channels [3, 4]. Aimed at improved performance (reliability), a number of trellis and block space-time codes have been proposed [5, 6, 7]. These codes were designed to maximize the diversity and coding gains under a fixed data rate. As the number of multiple transmit and receive antennas increases, these schemes suffer from high decoding complexity as in the case of trellis codes [5] or loss in performance and rate as in the case of block codes [6, 7]. On the other hand, spatial multiplexing schemes have been proposed [8, 9] with the aim of high bandwidth efficiency. This is done by creating parallel independent data streams between transmit and receive antennas. These approaches often result in loss of diversity gain and, hence, loss of performance.

Recently, a great deal of research attention has been devoted to the design of MIMO transmission schemes that provide tradeoff between performance and bandwidth efficiency [10, 11]. In [10], linear dispersive codes were designed to maximize the mutual information. However, they can’t guarantee diversity and coding gains. In contrast, full-diversity full-rate codes have been proposed [11] via constellation rotation. Although these codes achieve full diversity, a large portion of their codeword pairs have very small determinants which results in small pairwise coding gain. In summary, some of the diversity orders cannot be achieved unless the SNR is extremely high.

In this paper, we introduce a new layered space-time coding scheme for flat fading MIMO channels. The proposed scheme is based on linear dispersive coding proposed in [10] and allows various rate versus performance tradeoffs. When a maximal number of data streams (layers) are created, the channel capacity is preserved. In addition to the above advantages, we propose to apply phase shifts among layers to simultaneously optimize the diversity and coding gains. As a result, significant performance gain can be achieved as compared to the linear dispersive codes in [10] and [11].

The rest of the paper is organized as follows: We describe the system model and the design criteria for linear dispersive codes in section II. In section III, we introduce the proposed layered space-time coding scheme. Simulation results are presented in section IV and conclusions are drawn in section V.

2. Preliminaries

In this section, we present the system model and design criteria for linear dispersive space-time codes.
2.1. System Model

We consider a MIMO communication system consisting of \( N_t \) transmit and \( N_r \) receive antennas over a flat fading channel. The complex gain of the channel between transmit antenna \( i \) and receive antenna \( j \) is denoted by \( h_{ij} \). It is assumed that these gains are samples of independent complex Gaussian random variables with zero mean and variance 1. A block fading channel model is considered in which the channel is constant over the duration of a block and varies independently from block to block. Also, it is assumed that the receiver has perfect knowledge about the channel whereas the transmitter does not. Let us consider a block of symbols to be transmitted during \( T \) symbol periods where symbols are taken from a complex constellation. Every block of symbols is encoded to generate the codeword matrix. Within one block, the received signal can be written as

\[
Y = \sqrt{\frac{E_s}{N_t}} HS + V
\]

where \( H \) is the \( N_r \times N_t \) complex channel matrix whose \((j,i)\)th entry is \( h_{j,i} \), \( S \) is the \( N_t \times T \) codeword matrix with entries \( s_{t,i} \) to be transmitted at antenna \( i \) and time \( t \) with \( t = 0, \ldots, T-1 \), \( V \) is the \( N_r \times T \) additive complex Gaussian noise matrix with i.i.d entries, i.e., \( v_{i,t} \sim CN(0, N_0) \) and \( E_s \) is the symbol energy.

2.2. Linear dispersive space-time coding Design

Linear dispersive codes are constructed as linear combinations of the codeword dispersion matrices as [10, 12]

\[
S = \sum_{k=1}^{K} M_k s_k
\]

where \([s_1, s_2, \ldots, s_K]^T \) is a block of symbols to be encoded and \([M_k]_{k=1}^{K} \) is a set of \( N_t \times T \) codeword dispersion matrices. Note that maximizing the mutual information doesn’t necessarily lead to good performance [12]. Thus, for a code to achieve optimal tradeoff between performance and data rate, it must satisfy the following conditions:

1. The codewords have to be chosen to maximize mutual information that, in the best case, reaches channel capacity.
2. The rank of the difference matrix between any two codewords has to be maximized. When the matrix has full rank, full diversity order is achieved.
3. The coding gain also has to be maximized. To do that the determinant of the difference matrix has to be maximized.

Our goal in this paper is to consider all the three conditions in our design.

3. Proposed Phase-Shift-Based Layered Space-Time Coding Scheme

In this section, we propose a new layered space-time coding scheme based on linear dispersive codes. The proposed scheme allows simple spatial multiplexing (channel layering) that increases the data rate in the system. The creation of multiple layers can be achieved by applying antenna weighting. Columns of Walsh or FFT matrices could be used as spatial weighting vectors since orthogonal weighting preserves channel capacity as can be seen later. In the proposed scheme the encoding is performed block by block. Each encoding block takes \( NK \) symbols as input, \( K \) symbols per layer, and outputs a codeword matrix of size \( N_t \times K \). Specifically, if the transmission of each block is chosen to take \( T \) channel uses, \( K = T \) symbols will be transmitted for each layer during one encoding block. Hence, the transmission rate is \( N \) symbols per channel use if \( N \) layers are used. The dispersion matrix for the \( k \)th symbol of layer \( n \) is given by

\[
M_{k,n} = \frac{e^{j\phi_n} D_n P_k}{\sqrt{N}}
\]

where \( \{\phi_n\}_{n=1}^{N} \) is a set of phase shifts among layers to be optimized, \( D_n \) is an \( N_t \times N_t \) diagonal matrix whose \( i^{th} \) diagonal entry is \( \exp\left(\frac{-j2\pi (i-1)(n-1)}{N} \right) \), and \( P_k \) is an \( N_t \times K \) circulant matrix whose first row has 1 as its \( k^{th} \) entry and zero elsewhere, i.e,

\[
P_k = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 1
\end{bmatrix}
\]
Then, the codeword matrix for one block is constructed as

$$S = \sum_{n=1}^{N} \sum_{k=1}^{K} M_{k,n} s_{k,n}$$  \hspace{1cm} (5)

where \( s_{k,n} \) is the \( k \)th symbol of layer \( n \). As can be seen from (3) and (5), the \( k \)th symbol of the \( n \)th layer will be spatially spread using the spreading vector \( \text{diag}(D_n) \) and phase shifted before it is combined with the \( k \)th symbols of other layers. If we define

$$x_{kl} = \frac{e^{j\phi_k} q_{11}s_{k,1} + e^{j\phi_2} q_{12}s_{k,2} + \cdots + e^{j\phi_N} q_{1N}s_{k,N}}{\sqrt{N}}$$  \hspace{1cm} (6)

\( l = 0, \ldots, N_l - 1 \)

then, \( S \) can be written as

$$S = \begin{bmatrix} x_{1,1} & x_{2,1} & \cdots & x_{K,1} \\ x_{1,2} & x_{2,2} & \cdots & x_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{K-N_l+2,N_l} & x_{K-N_l+3,N_l} & \cdots & x_{K,N_l} \end{bmatrix}$$  \hspace{1cm} (7)

The received signal can be written as

$$Y = \sqrt{\frac{E_s}{N_t}} HS + V.$$  \hspace{1cm} (8)

Denote \( s = [s_{1,1}, \ldots, s_{1,N_l}, \ldots, s_{K,N_l}]^T \) as the vector collecting the \( NK \) symbols to be encoded, \( y \) as the vector of length \( N_tK \) formed by concatenating the \( K \) columns of \( Y \), i.e., \( y = \text{vec}(Y) \), and \( v = \text{vec}(V) \). Then (8) can be rewritten as

$$y = \sqrt{\frac{E_s}{N_t}} \overline{H} MS + v$$  \hspace{1cm} (9)

where \( \overline{H} = I_T \otimes H \) with the operation \( \otimes \) standing for the kronecker product and \( M = [\text{vec}(M_{1,1}) \ldots \text{vec}(M_{1,N}) \ldots \text{vec}(M_{K,N})] \). The capacity under the assumption of Gaussian input symbols with zero mean and covariance matrix \( R_x = I_{NK} \) is given by

$$C = \frac{1}{K} \mathbb{E} \left[ \log_2 \det \left( \frac{I_{N,K} + \frac{E_s}{N_tN_0} \overline{H} MM^H \overline{H}^H}{1} \right) \right]$$  \hspace{1cm} (10)

When \( N = N_l \) layers are used, it can be checked from (3) that \( MM^H = I_{N,K} \) and hence

$$C = \frac{1}{K} \mathbb{E} \left[ \log_2 \det \left( I_{N,K} + \frac{E_s}{N_tN_0} \overline{H} \overline{H}^H \right) \right]$$  \hspace{1cm} (11)

which is exactly the capacity of the original channel \( H \) [1,2]. That is, the proposed scheme preserves the channel capacity when \( N_l \) layers are used.

On the other hand, the pair-wise error probability of codeword \( S \) being transmitted but the decision being erroneously made in favour of \( \hat{S} \) can be found as

$$P(S \rightarrow \hat{S}) \leq \prod_{i=1}^{N} (1 + d_i \frac{E_s}{N_tN_0})^{-N_l}$$  \hspace{1cm} (12)

where \( \frac{E_s}{N_0} \) is signal to noise ratio and \( d_i \) is the \( i \)th eigenvalue of the matrix \( (S - \hat{S})(S - \hat{S})^H \). When \( \frac{E_s}{N_0} >> 1 \), the pair-wise error probability can be approximated as

$$P(S \rightarrow \hat{S}) \leq \prod_{i=1}^{N} d_i^{-N_l} \left( \frac{E_s}{N_tN_0} \right)^{-N_l}$$  \hspace{1cm} (13)

where \( r \) is the rank of matrix \( (S - \hat{S})(S - \hat{S})^H \). It can be seen that a diversity gain of \( rN_t \) and coding gain of \( \left( \prod_{i=1}^{r} d_i^{-1} \right)^{-\frac{1}{r}} \) are achieved.

We now proceed to maximize the coding gain of those vulnerable codeword pairs. For this, we consider two codewords that differ by just 1 symbol, say \( x_{1,1} \). It is worthy of mentioning that this is the worst case that dominates the probability of error. Since \( x_{1,1} \) is a linear combination of \( s_{1,n}, n = 1, \ldots, N \) this implies that the codeword pair will also differ at symbols \( x_{1,2}, x_{1,3}, \ldots, x_{1,N_l} \). Then the determinant of the
difference matrix is the product of $N_t$ distances and it is given by

$$D = \left| \frac{d_1 d_2 \cdots d_{N_t}}{(N_t N)^{N_t/2}} \right|$$  \hspace{1cm} (14)$$

where

$$d_i = e^{i\phi_i} q_i(s_{1,i} - s'_{1,i}) + e^{i\phi_i} q_i(s_{1,i} - s'_{1,i}) + \cdots + e^{i\phi_i} q_i(s_{N,i} - s'_{N,i}),$$

$s_{1,i}$ and $s'_{1,i}$ are different but taken from the same constellation, $n = 1,2,\ldots, N$. From (14), if the phase shifts $\phi_i$ are zeros, $D$ can be zero. For instance, as long as $\sum_{n=1}^{N_t} (s_{1,n} - s'_{1,n}) = 0$, then $d_i = 0$. This shows that a large percentage of codeword pairs will not enjoy full diversity. This motivated us to use the phase shifts among layers to guarantee nonzero $D$. In addition, the phase shifts among layers can be chosen to maximize the minimum determinant $D$ taken over all possible pairs $\{s, s'\}$, i.e.,

$$\{\phi_i\}_{i=1}^{N_t} = \arg \max_{\{s, s'\}} \min D.$$  \hspace{1cm} (15)$$

It can be checked that $r$ is equal to $N_t$ (i.e., full rank), except rare cases when each layer has the same $K$ symbols for both of the two codewords of concern, i.e., $(s_{1,n} = s_{j,n}$ and $s'_{1,n} = s'_{j,n}$, $\forall i,j,n$). It is worthy of noting that the codeword pairs without full diversity have large coding gain which is $K \sum_{n=1}^{N_t} |s_{1,n} - s'_{1,n}|^2$.

This shows that the proposed scheme achieves full diversity gain except for the aforementioned codeword pairs.

Equation (15) can be solved efficiently by using computer search and some results are provided in tables 1, 2 and 3. In these tables, it is assumed that $\phi_i = 0$ as reference.

As shown in tables 1, 2 and 3, for the same transmitted power and data rate the multi-layer codes provide better coding gain than the single-layer codes. For example, the two-layer code with QPSK modulation has much larger coding gain than the one-layer code with 16PSK or 16QAM modulation. Also by using 3 transmit antennas, the three-layer code with BPSK modulation has much larger coding gain than the one-layer code with 8PSK modulation. This shows that the proposed scheme provides good performance and multiplexing gain.

### Table 1. Coding gain for one-layer codes.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$N_t$</th>
<th>$N$</th>
<th>Coding gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>QPSK</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8PSK</td>
<td>2</td>
<td>1</td>
<td>0.2929</td>
</tr>
<tr>
<td>16PSK</td>
<td>2</td>
<td>1</td>
<td>0.0761</td>
</tr>
<tr>
<td>16QAM</td>
<td>2</td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>8PSK</td>
<td>3</td>
<td>1</td>
<td>0.0863</td>
</tr>
</tbody>
</table>

### Table 2. Optimum phase shifts and coding gain for two-layer codes.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$N_t$</th>
<th>$N$</th>
<th>$\phi_2$</th>
<th>Coding gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>2</td>
<td>2</td>
<td>30°, 150°</td>
<td>1</td>
</tr>
<tr>
<td>QPSK</td>
<td>2</td>
<td>2</td>
<td>30°, 60°</td>
<td>0.5</td>
</tr>
<tr>
<td>8PSK</td>
<td>2</td>
<td>2</td>
<td>22.5°</td>
<td>0.1097</td>
</tr>
<tr>
<td>16PSK</td>
<td>2</td>
<td>2</td>
<td>11.25°</td>
<td>0.0145</td>
</tr>
<tr>
<td>16QAM</td>
<td>2</td>
<td>2</td>
<td>30°, 31°, 43°, -47°, 59°, 60°</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### Table 3. Optimum phase shifts and coding gain for three-layer code.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$N_t$</th>
<th>$N$</th>
<th>$\phi_2$</th>
<th>$\phi_3$</th>
<th>Coding gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>3</td>
<td>3</td>
<td>20°</td>
<td>160°</td>
<td>0.2963</td>
</tr>
</tbody>
</table>

### 4. Simulation Results

In this section, we provide simulation results to compare the proposed scheme with the transmission schemes that provide tradeoff between the performance and bandwidth efficiency [10,11]. The channel model described in section II was used in all simulations. Also, maximum likelihood decoding was used for all schemes.

Fig.1 compares the proposed scheme and two other schemes presented in [10] and [11]. All three schemes use 2 layers, BPSK modulation and block of 12 bits. Two transmit and two receive antennas were used for all three schemes. As can be observed, at BLER = 0.002, the performance gain of the proposed scheme is approximately 2 dB better than the other two schemes. That is because the proposed scheme maximizes the diversity and coding gains. The scheme in [10] preserves channel capacity and doesn’t guarantee good performance.
performance gain since the diversity and coding gains were not explicitly optimized. The scheme in [11] maximizes the diversity gain and preserves channel capacity but doesn't necessarily yield high coding gain which affects the performance of the system.

Fig. 2 compares the performance of one-layer and two-layer codes with same transmitted power and same data rate. For one-layer codes, 16PSK or 16QAM modulation was used and block of 12 bits was assumed. In the two-layer codes, QPSK modulation was used and each layer has 6 bits per block. Two transmit and two receive antennas were used for all cases of Fig. 2. It is shown that approximately a 2.5 dB performance gain is achieved for the QPSK two-layer code compared to 16QAM one layer code at BLER = 0.01. Also, a 6.25 dB is achieve for the QPSK two-layer code compared to 16PSK one-layer code at BLER = 0.02.

In Fig. 2, the performance of the scheme presented in [10] is also given. As expected, similar behavior is observed as in the case shown in Fig. 1.

5. Conclusions

In this paper, we have introduced a new layered space-time coding scheme for flat fading channels. Based on linear dispersive coding, the proposed scheme allows various rate-performance tradeoffs and preserves channel capacity when full rate is required. In addition, phase shift among layers are proposed to maximize diversity and coding gains. Simulation results have demonstrated that significant performance gain can be achieved.

Figure 1. BLER performance comparison of FDFR, LDC and the proposed scheme: \( N_t = N_r = 2 \), 2 layers, and BPSK modulation.

Figure 2. BLER performance comparison of LDC and the proposed scheme: \( N_t = N_r = 2 \).

6. Acknowledgement

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7. References


