Abstract—In this paper, “beam-nulling” is proposed to send signals over a generated subspace orthogonal to the weakest spatial subchannel fed back from the receiver. Signal-to-interference-plus-noise ratio (SINR) of MMSE receiver is derived for beam-nulling. Then the associated average bit-error rate (BER) of beam-nulling is given numerically and verified by simulation results. Simulation results are provided to compare beam-nulling with beamforming at the same data rate. To improve performance further, beam-nulling is concatenated with linear dispersion code. Simulation results are also provided to compare the concatenated scheme with beamforming.

I. INTRODUCTION

Recently, one significant advance in wireless communications is the so-called multiple-input-multiple-output (MIMO) technology [1] [2]. It has been recognized that adaptive techniques proposed for single-input-single-output (SISO) channels [3] [4], can also be applied to improve capacity and performance of MIMO systems.

The ideal scenario is that the transmitter has full knowledge of channel state information (CSI) fed back from the receiver and the CSI keeps constant before the transmitter sends information to the receiver. With such a perfect CSI feedback, the original MIMO channel can be converted to multiple uncoupled SISO channels via singular value decomposition (SVD) at the transmitter and the receiver [1]. These uncoupled channels with various propagation gains will be referred to as “spatial subchannels”.

To achieve better performance, various schemes can be implemented depending on the availability of CSI at the transmitter [5] [6]. If the transmitter has full knowledge about channel matrix, i.e., full CSI, the so-called “water-filling” (WF) principle is performed on each spatial subchannel to maximize the channel capacity [1]. This scheme is optimal in this case. Various WF-based schemes have been proposed, such as [7] [8]. For the WF-based scheme, the feedback bandwidth for the full CSI grows with respect to the number of transmit and receive antennas and the performance is often very sensitive to channel estimation errors.

To mitigate these disadvantages, various beamforming (BF) techniques for MIMO channels have also been investigated intensively. In an adaptive beamforming scheme, complex weights of the transmit antennas are fed back from the receiver. If only one eigenvector can be fed back, eigen-beamforming [5] is optimal in this case. The eigen-beamforming scheme only applies to the strongest spatial subchannel but can achieve full diversity and high signal-to-noise ratio (SNR) [5]. Also, in practice, the eigen-beamforming scheme has to cooperate with the other adaptive parameters to improve performance and/or data rate, such as constellation and coding rate. There are also other beamforming schemes based on various criteria, such as [5]- [11].

Note that the conventional beamforming is optimal in terms of maximizing the SNR at the receiver. However, it is sub-optimal from a MIMO capacity point of view, since only one data stream, instead of parallel streams, is transmitted through the MIMO channel [9]. Inspired by existing beamforming schemes, we propose a new beamforming-like technique called minimum eigenvector “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming. That is, only one eigenvector is fed back to the transmitter. The beam-nulling transmitter is informed with the weakest subchannel and then sends signals over the spatial subspace orthogonal to the weakest spatial subchannel. It is worthy of noting that both transmitter and receiver have to know how to generate the spatial subspace. Hence, the loss of channel capacity as compared to the optimal water-filling scheme can be reduced.

In this paper, performance of beam-nulling are discussed. To improve performance further and maintain reasonable complexity, beam-nulling can be concatenated with linear dispersion code (LDC) [12]- [14]. Although the transmitted symbols are “precoded” according to the feedback, the beam-nulling scheme is different from the other existing precoding schemes with limited feedback channel, which are independent of the instantaneous channel but the optimal precode depends on the instantaneous channel [15] [16]. Numerical and simulation results are provided to compare the new schemes with other schemes.

II. CHANNEL MODEL

In this study, the channel is assumed to be a Rayleigh flat fading channel with \(N_t\) transmit and \(N_r\) (\(N_r \geq N_t\)) receive antennas. Let’s denote the complex gain from transmit antenna \(n\) to receiver antenna \(m\) by \(h_{mn}\) and collect them to form an \(N_r \times N_t\) channel matrix \(\mathbf{H} = [h_{mn}]\). The channel is known perfectly at the receiver. The entries in \(\mathbf{H}\) are assumed to be independently identically distributed (i.i.d.) symmetrical complex Gaussian random variables with zero mean and unit variance.
The singular-value decomposition of $H$ can be written as

$$H = UAV^H$$  \hspace{1cm} (1)$$

where $U$ is an $N_r \times N_r$ unitary matrix, $A$ is an $N_r \times N_t$ matrix with singular values $\{\lambda_i\}$ on the diagonal and zeros off the diagonal, and $V$ is an $N_t \times N_t$ unitary matrix. For convenience, we assume $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N_t}$, $U = [u_1, u_2, \ldots, u_{N_t}]$ and $V = [v_1, v_2, \ldots, v_{N_t}]$. $\{u_i\}$ and $\{v_i\}$ are column vectors. We assume that the rank of $H$ is $\alpha$ ($\alpha \leq N_t$). That is, the number of non-zero singular values is $\alpha$.

From equation (1), the original channel can be considered as consisting of uncoupled parallel subchannels. Each subchannel corresponds to a singular value of $H$. In the following context, the subchannel is also referred to as “spatial subchannel”. For instance, one spatial subchannel corresponds to $\lambda_i$, $u_i$ and $\{v_i\}$.

III. BEAM-NULLING

We assume that the total transmitted power is constrained to $P$. Given the power constraint, different power allocation among spatial subchannels can affect the channel capacity tremendously, such as equal power (EQ), water-filling, and eigen-beamforming. The eigen-beamforming scheme can save feedback bandwidth and is optimized in terms of SNR [10]. However, since only one spatial subchannel is considered, this scheme suffers from loss of channel capacity [9], especially when the number of antennas grows. Inspired by the eigen-beamforming scheme, we will propose a new beamforming-like scheme called “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming. That is, only one eigenvector is fed back to the transmitter. Unlike the eigenbeamforming scheme in which only the best spatial subchannel is considered, in the beam-nulling scheme, only the worst spatial subchannel is discarded. Hence, the loss of channel capacity as compared to the optimal water-filling scheme can be reduced.

In this scheme, the eigenvector associated with the minimum singular value from the transmitter side, i.e., $v_{N_t}$, is feedback to the transmitter. By a certain rule, a subspace orthogonal to the weakest spatial channel is constructed. That is, the following condition should be satisfied.

$$\Phi^Hv_{N_t} = 0$$  \hspace{1cm} (2)$$

The $N_t \times (N_t - 1)$ matrix $\Phi = [g_1, g_2, \ldots, g_{N_t-1}]$ spans the subspace. Note that the rule to construct the subspace $\Phi$ should also be known to the receiver.

An example to construct the orthogonal subspace is presented as follows. We construct an $N_t \times N_t$ matrix

$$A = [v_{N_t}, \Gamma]$$  \hspace{1cm} (3)$$

where $\Gamma = [I_{(N_t-1) \times (N_t-1)}, 0_{(N_t-1) \times 1}]^T$. Applying QR decomposition to $A$, we have

$$A = [v_{N_t}, \Phi] \cdot \Gamma$$  \hspace{1cm} (4)$$

where $\Gamma$ is an upper triangular matrix with the (1,1)-th entry equal to 1. $\Phi$ is the subspace orthogonal to $v_{N_t}$.

At the transmitter, $N_t - 1$ symbols denoted as $x'$ are transmitted only over the orthogonal subspace $\Phi$. The received signals at the receiver can be written as

$$y' = \sqrt{\frac{P}{N_t-1}}H\Phi x' + z' = Hx' + z'$$  \hspace{1cm} (5)$$

where $z'$ is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$ and $H = \sqrt{\frac{P}{N_t-1}}H\Phi$.

Substituting (1) into (5) and multiplying $y'$ by $U^H$, we have

$$\tilde{y} = \sqrt{\frac{P}{N_t-1}}A \begin{pmatrix} B \\ 0^T \end{pmatrix} x' + \tilde{z}$$  \hspace{1cm} (6)$$

where $\tilde{z}$ is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$. With the condition in (2),

$$V^H\Phi = \begin{pmatrix} B \\ 0^T \end{pmatrix}$$  \hspace{1cm} (7)$$

where

$$B = \begin{pmatrix} v_1^H g_1 & v_2^H g_2 & \cdots & v_{N_t-1}^H g_{N_t-1} \\ v_1^H g_2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ v_1^H g_{N_t-1} & \cdots & \cdots & v_{N_t-1}^H g_{N_t-1} \end{pmatrix}$$  \hspace{1cm} (8)$$

$B$ is an $(N_t - 1) \times (N_t - 1)$ unitary matrix. From (6), the available spatial channels are $N_t - 1$. Since the weakest spatial subchannel is “nulled” in this scheme, power can be allocated equally among the left better $N_t - 1$ subchannels. From (6), the associated ergodic channel capacity can be found as

$$C_{bn} = E \left[ \sum_{i=1}^{N_t-1} \log \left( 1 + \frac{P}{(N_t-1)\sigma_z^2} \lambda_i \right) \right]$$  \hspace{1cm} (9)$$

As can be seen, the beam-nulling scheme only needs one eigenvector to be fed back. However, since only the worst spatial subchannel is discarded, this scheme can increase channel capacity significantly as compared to the conventional beamforming scheme.

IV. PERFORMANCE OF BEAM-NULLING

A. MMSE Detector

The close-form error probability for the optimal ML receiver is difficult to find. Other suboptimal receiver can also be implemented. Especially, the MMSE detector is popular due to its low complexity and good performance. In the following context, BER of the MMSE detector is analyzed for beam-nulling scheme.

Equation (5) can also be written as

$$y' = \hat{h}_i x_i + \sum_{j \neq i} \hat{h}_j x_j + z'$$  \hspace{1cm} (10)$$

where $x_i$ is the $i$-th element of $x'$, $\hat{h}_i$ is the $i$-th column of $H$. 

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Without loss of generality, we consider the detection of one symbol, say \( x_i \). Collect the rest of the symbols into a column vector \( x_t \) and denote \( h_i = [h_{i1}, ..., h_{iN_t-1}] \) as the matrix obtained by removing the \( i \)-th column from \( H \).

A linear MMSE detector [17] [18] is applied and the corresponding output is given by

\[
\hat{x}_i = w_i^H y = x_i + \hat{z}_i,
\]

where \( \hat{z}_i \) is the noise term of zero mean. \( \hat{z}_i \) can be approximated to be Gaussian [17]. The corresponding \( w_i \) can be found as

\[
w_i = \left( \tilde{h}_i h_i^H + \mathbf{R}_i \right)^{-1} \tilde{h}_i
\]

where \( \mathbf{R}_i = \tilde{H}_i \tilde{H}_i^H + \sigma_i^2 \mathbf{I} \). Note that the scaling factor \( \frac{1}{\tilde{h}_i h_i^H} \) in the coefficient vector of the MMSE detector \( w_i \) is added to ensure an unbiased detection as indicated by (11). The variance of the noise term \( \hat{z}_i \) can be found from (11) and (12) as

\[
\sigma_i^2 = w_i^H \mathbf{R}_i w_i
\]

Substituting the coefficient vector for the MMSE detector in (12) into (13), the variance can be written as

\[
\sigma_i^2 = \frac{1}{h_i^H \mathbf{R}_i^{-1} h_i}
\]

Then, the SINR of MMSE associated with \( x_i \) is \( 1/\sigma_i^2 \).

\[
\gamma_i = \frac{1}{\sigma_i^2} = \frac{1}{h_i^H \mathbf{R}_i^{-1} h_i}
\]

Closed-form BER for a channel model such as (11) can be found in [19]. The average BER over MIMO fading channel for a given constellation can be found for beam-nulling as follows.

\[
BER_{av} = E_{\gamma_i} \left[ \frac{1}{N_t - 1} \sum_i BER(\gamma_i) \right]
\]

The closed-form formula for the average BER in (16) depends on the distribution of \( \gamma_i \), which is difficult to find. Here, the above average BER is calculated numerically. For example, the average BER for \( 2^{r} \)-PSK is

\[
BER_{av} = E_{\gamma_i} \left[ \frac{1}{N_t - 1} \sum_i Q \left( \sqrt{2 \gamma_i \sin \left( \frac{\pi}{2r} \right)} \right) \right]
\]

and the average BER for rectangular \( 2^{r} \)-QAM is

\[
BER_{av} = E_{\gamma_i} \left[ \frac{1}{N_t - 1} \sum_i Q \left( \sqrt{\frac{3r \gamma_i}{2r - 1}} \right) \right]
\]

where \( Q(\cdot) \) denotes the Gaussian-Q function.

In Fig. 1, numerical and simulation results are compared for QPSK over \( 4 \times 4 \) fading channel. As can be seen, the numerical and simulation results match very well.

**B. Performance Comparison**

In Fig. 2, simulation results are compared for various data rates \( R \) over \( 4 \times 4 \) fading channels. As can be seen, if data rate is low, i.e., constellation size is low, beamforming outperforms beam-nulling. If data rate is high, i.e., constellation size is high, beam-nulling outperforms beamforming at low and medium SNR but at high SNR beamforming outperforms. Also as can be seen, at high data rate, even the beam-nulling scheme with suboptimal MMSE receiver outperforms the beamforming scheme.

**C. Concatenation of Beam-nulling and LDC**

To further improve the performance of beam-nulling with tractable complexity, we propose to concatenate beam-nulling with a linear dispersion code. Note that to meet error-rate requirement, multiple levels of error protection can be implemented. In this study, we focus on space-time coding domain. In this system, the information bits are first mapped into the weakest spatial channel. The generation of the orthogonal symbol stream is denoted by \( x = [x_1, x_2, ..., x_L]^T \) with \( x_i \in \Omega \equiv \{\Omega_m|m = 0, 1, \ldots, 2^r - 1, r \geq 1\} \), i.e., a complex constellation of size \( 2^r \), such as \( 2^r \)-QAM. The average symbol energy is assumed to be 1, i.e., \( \frac{1}{2^r} \sum_{m=0}^{2^r-1} |\Omega_m|^2 = 1 \). Each symbol in a block will be mapped to a dispersion matrix of size \( N_t \times T \) (i.e., \( M_i \)) and then combined linearly to form \( (N_t - 1) \) data streams over \( T \) channel uses. The output \( (N_t - 1) \) data streams are transmitted only over the subspace \( \Phi \) orthogonal to the weakest spatial channel. The generation of the orthogonal subspace \( \Phi \) is described in section III. The received signals at the receiver can be written as

\[
y = \sqrt{\frac{P}{N_t - 1}} \mathbf{H} \Phi \sum_{i=1}^{L} M_i x_i + z
\]

where \( z \) is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance \( \sigma_z^2 \). 

!![](image)
We will compare the concatenated scheme with the original schemes at the same data rate. In Fig. 2, “BF” denotes beam-forming, “BN” denotes beam-nulling, and “BL” denotes beam-nulling with LDC. As can be seen, beam-nulling with LDC outperforms beam-nulling without LDC using the same receiver. The performance of beam-nulling with LDC using MMSE receiver is close to that of beam-nulling without LDC using the optimal ML receiver.

V. CONCLUSIONS

Using the same feedback bandwidth as beamforming, beam-nulling exploits all spatial subchannels except the weakest one and thus can improve channel capacity. Performance of beam-nulling with MMSE receiver is analyzed and verified by numerical and simulation results. Simulation results show that if data rate is low, beamforming outperforms beam-nulling. If data rate is high, beam-nulling outperforms at low and medium SNR but beamforming outperforms at high SNR. To achieve better performance, beam-nulling is concatenated with linear dispersion code. Simulation results show that if data rate is low, beam-nulling with linear dispersion code can approach beamforming at high SNR. If data rate is high, beam-nulling outperforms beamforming even with a suboptimal MMSE receiver.

REFERENCES


