POWER ALLOCATION STRATEGY FOR MIMO SYSTEM BASED ON BEAM-NULLING

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ABSTRACT
In this paper, we propose a scheme called “beam-nulling” using the same feedback bandwidth as beamforming but with higher capacity. In the beam-nulling scheme, the eigenvector of the weakest subchannel is fed back and then signals are sent over a generated subspace orthogonal to the weakest subchannel. Hence, the scheme can achieve high capacity. The capacities of water-filling, equal power, beamforming and beam-nulling are compared through theoretical analysis and numerical results. It is shown that at medium signal-to-noise ratio, beam-nulling approaches the optimal water-filling scheme. Additionally, the existing beamforming and new proposed beam-nulling can be extended if more than one eigenvector is available at the transmitter. The new extended schemes are called multi-dimensional (MD) beamforming and MD beam-nulling. Theoretical analysis and numerical results in terms of capacity are also provided to evaluate the new extended schemes.

1. INTRODUCTION
It has been recognized that adaptive techniques, proposed for single-input-single-output (SISO) channels [3] [4], can also be applied to improve MIMO channel capacity [1] [2]. The ideal scenario in adaptive schemes is that the transmitter has full knowledge of channel state information (CSI) which is fed back from the receiver. With such a perfect CSI feedback, the original MIMO channel can be converted to multiple uncoupled SISO channels via singular value decomposition (SVD) at the transmitter and the receiver [1]. In other words, the original MIMO channel can be decomposed into several orthogonal “spatial subchannels” with various propagation gains. To achieve better performance, various strategies to allocate constrained power to these subchannels can be implemented depending on the availability of CSI at the transmitter [6]-[8].

If the transmitter has full knowledge about channel matrix, i.e., full CSI, the so-called “water-filling” (WF) principle is performed on each spatial subchannel to maximize the channel capacity [1]. This scheme is optimal in this case. Various WF-based schemes have been proposed, such as [11]-[14]. For the WF-based scheme, the feedback bandwidth for the full CSI grows with respect to the number of transmit and receive antennas and the performance is often very sensitive to channel estimation errors.

Various beamforming techniques for MIMO channels have also been investigated intensively, which can mitigate the above disadvantages. In an adaptive beamforming scheme, complex weights of the transmit antennas are fed back from the receiver. If only partial CSI is available at the transmitter such as the eigenvector associated with the strongest spatial subchannel, eigen-beamforming [7] is optimal. The eigen-beamforming scheme only allocates power to the strongest spatial subchannel but can achieve full diversity and high signal-to-noise ratio (SNR). Also, in practice, the eigen-beamforming scheme has to cooperate with the other adaptive parameters to improve performance or/and data rate, such as constellation and coding rate. There are also other beamforming schemes based on various criteria, such as [7]-[9].

Note that the conventional beamforming is optimal in terms of maximizing the SNR at the receiver. However, it is sub-optimal from a MIMO capacity point of view, since only one data stream, instead of parallel streams, is transmitted through the MIMO channel [15]. In this paper, we propose a new technique called “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming. That is, only one eigenvector is fed back to the transmitter. The beam-nulling transmitter is informed with the weakest spatial subchannel and then sends signals over a generated spatial subspace orthogonal to the weakest subchannel. Note that both transmitter and receiver should know how to generate the same spatial subspace. Hence, the loss of channel capacity as compared to the optimal water-filling scheme can be reduced. Although the transmitted symbols are “precoded” according to the feedback, the beam-nulling scheme is different from the other existing precoding schemes with limited feedback channel, which are independent of the instantaneous channel but the optimal precode depends on the instantaneous channel [16] [17]. Additionally, if more than one eigenvector, e.g. $k$ eigenvectors, can be available at the transmitter, the ex-
isting beamforming scheme and the proposed beam-nulling scheme can be further extended, respectively. The extended schemes will exploit or discard $k$ spatial subchannels and they will be referred to as “multi-dimensional (MD)” beamforming and “multi-dimensional” beam-nulling, respectively.

2. CHANNEL MODEL

In this study, the channel is assumed to be a Rayleigh flat fading channel with $N_t$ transmit and $N_r$ ($N_r \geq N_t$) receive antennas. Let’s denote the complex gain from transmit antenna $n$ to receiver antenna $m$ by $h_{mn}$ and collect them to form an $N_r \times N_t$ channel matrix $H = [h_{mn}]$. The channel is known perfectly at the receiver. The entries in $H$ are assumed to be independently identically distributed (i.i.d.) symmetrical complex Gaussian random variables with zero mean and unit variance.

The symbol vector at the $N_t$ transmit antennas is denoted by $x = [x_1, x_2, \ldots, x_{N_t}]^T$. According to information theory [5], the optimal distribution of the transmitted symbols is Gaussian. Thus, the elements $\{x_i\}$ of $x$ are assumed to be i.i.d. Gaussian variables with zero mean and unit variance, i.e., $E(x_i) = 0$ and $E|x_i|^2 = 1$.

The singular-value decomposition of $H$ can be written as

$$H = U \Lambda V^H$$

where $U$ is an $N_r \times N_r$ unitary matrix, $\Lambda$ is an $N_r \times N_t$ matrix with singular values $\{\lambda_i\}$ on the diagonal and zeros off the diagonal, and $V$ is an $N_t \times N_t$ unitary matrix. For convenience, we assume $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N_t}$, $U = [u_1, u_2, \ldots, u_{N_r}]$ and $V = [v_1, v_2, \ldots, v_{N_t}]$, $\{u_i\}$ and $\{v_i\}$ are column vectors.

From equation (1), the original channel can be considered as consisting of uncoupled parallel subchannels. Each subchannel corresponds to a singular value of $H$. In the following context, the subchannel is also referred to as “spatial subchannel”. For instance, one spatial subchannel corresponds to $\lambda_i$, $u_i$ and $\{v_i\}$.

3. POWER ALLOCATION AMONG SPATIAL SUBCHANNELS

We assume that the total transmitted power is constrained to $P$. Given the power constraint, different power allocation among spatial subchannels can affect the channel capacity tremendously. Depending on power allocation strategy among spatial subchannels, four schemes are presented which are equal power, water-filling, eigen-beamforming, and the new power allocation which is beam-nulling.

If the transmitter has no knowledge about the channel, the most judicious strategy is to allocate the power to each transmit antenna equally, i.e., equal power. In this case, the received signals can be written as

$$y = \sqrt{\frac{P}{N_t}} H x + z$$

$z$ is the additive white Gaussian noise (AWGN) vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$. The associated ergodic channel capacity can be written as [1]

$$C_{eq} = E\left[\sum_{i=1}^{N_t} \log\left(1 + \frac{\rho \lambda_i}{\sigma_z^2}\right)\right]$$

where $E[\cdot]$ denotes expectation with respect to $H$ and $\rho = \frac{P}{\sigma_z^2}$ denotes SNR. If the transmitter has full knowledge about the channel, the most judicious strategy is to allocate the power to each spatial subchannel by water-filling principle [1]. In water-filling scheme, the received signals can be written as

$$y_i = \sqrt{P_i \lambda_i} x_i + z_i$$

where $\sum_{i=1}^{N_t} P_i = P$ as a constraint and $z_i$ is the AWGN random variable with zero mean and $\sigma_z^2$ variance. Following the method of Lagrange multipliers, $P_i$ can be found [1] and the total ergodic channel capacity is

$$C_{wf} = E\left[\sum_{i=1}^{N_t} \log\left(1 + \frac{P_i}{\sigma_z^2} \lambda_i\right)\right]$$

To save feedback bandwidth, beamforming can be considered. For the MIMO model, the optimal beamforming is called “eigen-beamforming” [7] [10], or simply beamforming. We assume one symbol, saying $x_1$, is transmitted. At the receiver, the received vector can be written as

$$y_1 = \sqrt{P} H v_1 x_1 + z_1$$

where $z_1$ is the additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$. The associated ergodic channel capacity can be written as

$$C_{bf} = E\left[\log\left(1 + \rho \lambda_1^2\right)\right]$$

The eigen-beamforming scheme can save feedback bandwidth and is optimized in terms of SNR [10]. However, since only one spatial subchannel is considered, this scheme suffers from loss of channel capacity [15], especially when the number of antennas grows.

3.1. Beam-Nulling

Inspired by the eigen-beamforming scheme, we will propose a new beamforming-like scheme called “beam-nulling” (BN). This scheme uses the same feedback bandwidth as beamforming. That is, only one eigenvector is fed back to the transmitter. Unlike the eigen-beamforming scheme in which only the best spatial subchannel is considered, in the beam-nulling scheme, only the worst spatial subchannel is discarded. Hence, the loss of channel capacity can be reduced as compared to the optimal water-filling scheme.
In this scheme, the eigenvector associated with the minimum singular value from the transmitter side, i.e., $v_{N_t}$, is feedback to the transmitter. By a certain rule, a subspace orthogonal to the weakest spatial channel is constructed. That is, the following condition should be satisfied.

$$\Phi^H v_{N_t} = 0$$  \hspace{1cm} (8)

The $N_t \times (N_t - 1)$ matrix $\Phi = [g_1 g_2 \ldots g_{N_t-1}]$ spans the subspace. Note that the rule to construct the subspace $\Phi$ should also be known to the receiver.

An example to construct the orthogonal subspace is presented as follows. We construct an $N_t \times N_t$ matrix

$$A = [v_{N_t} I']$$  \hspace{1cm} (9)

where $I' = [I_{(N_t-1) \times (N_t-1)} 0_{(N_t-1) \times 1}]^T$. Applying QR decomposition to $A$, we have

$$A = [v_{N_t} \Phi] \cdot R$$  \hspace{1cm} (10)

where $R$ is an upper triangular matrix with the (1,1)-th entry equal to 1. $\Phi$ is the subspace orthogonal to $v_{N_t}$.

At the transmitter, $N_t - 1$ symbols denoted as $x'$ are transmitted only over the orthogonal subspace $\Phi$. The received signals at the receiver can be written as

$$y' = \sqrt{\frac{P}{N_t - 1}} H \Phi x' + z'$$  \hspace{1cm} (11)

where $z'$ is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$.

Substituting (1) into (11) and multiplying $y'$ by $U^H$, we have

$$\bar{y} = \sqrt{\frac{P}{N_t - 1}} \Lambda \begin{pmatrix} B \\ 0^T \end{pmatrix} x' + \bar{z}$$  \hspace{1cm} (12)

where $\bar{z}$ is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$. With the condition in (8),

$$v^H \Phi = \begin{pmatrix} B \\ 0^T \end{pmatrix}$$  \hspace{1cm} (13)

where

$$B = \begin{pmatrix} v^H_1 g_1 & v^H_1 g_2 & \ldots & v^H_1 g_{N_t-1} \\ v^H_2 g_1 & \ldots & \ldots & \ldots \\ \vdots & \ldots & \ldots & \ldots \\ v^H_{N_t-1} g_1 & \ldots & v^H_{N_t-1} g_{N_t-1} \end{pmatrix}$$  \hspace{1cm} (14)

$B$ is an $(N_t - 1) \times (N_t - 1)$ unitary matrix. As can be seen from (12), the available spatial channels are $N_t - 1$. Since the weakest spatial subchannel is “nulled” in this scheme, power can be allocated equally among the left better $N_t - 1$ subchannels. Equation (12) can be rewritten as

$$\bar{y}' = \sqrt{\frac{P}{N_t - 1}} A' B x' + \bar{z}'$$  \hspace{1cm} (15)

where $\bar{y}'$ and $\bar{z}'$ are column vectors with the first $(N_t - 1)$ elements of $\bar{y}$ and $\bar{z}$, respectively, and $A' = \text{diag} [\lambda_1, \lambda_2, \ldots, \lambda_{(N_t-1)}]$. From (15), the associated ergodic channel capacity can be found as

$$\bar{C}_{bn} = E \left[ \sum_{i=1}^{N_t-1} \log \left( 1 + \frac{\rho}{N_t - 1} \lambda_i^2 \right) \right]$$  \hspace{1cm} (16)

As can be seen, the beam-nulling scheme only needs one eigenvector to be fed back. However, since only the worst spatial subchannel is discarded, this scheme can increase channel capacity significantly as compared to the conventional beamforming scheme.

4. COMPARISONS AMONG THE FOUR SCHEMES

In this section, we compare the new proposed beam-nulling scheme with the other schemes. Water-filling is the optimal solution among the four schemes for any SNR.

Differentiating the above ergodic capacities with respect to $\rho$ respectively, we have

$$\frac{\partial \bar{C}_{eq}}{\partial \rho} = E \left[ \sum_{i=1}^{N_t} \frac{1}{\rho + \frac{N_t}{\lambda_i^2}} \right]$$  \hspace{1cm} (17)

$$\frac{\partial \bar{C}_{bf}}{\partial \rho} = E \left[ \frac{1}{\rho + \frac{N_t}{\lambda_i^2}} \right]$$  \hspace{1cm} (18)

$$\frac{\partial \bar{C}_{bn}}{\partial \rho} = E \left[ \sum_{i=1}^{N_t-1} \frac{1}{\rho + \frac{N_t - 1}{\lambda_i^2}} \right]$$  \hspace{1cm} (19)

The differential will also be referred to as “slope”. Since the second order differentials are negative, the above ergodic capacities are concave and monotonically increasing with respect to $\rho$.

For the case of $N_t = 2$, beamforming and beam-nulling have the same capacity for any $\rho$ as can be seen from equations of capacity and slope. If $\rho \to 0$, i.e., at low SNR, it can be easily found that

$$\frac{\partial \bar{C}_{bf}}{\partial \rho} \leq \frac{\partial \bar{C}_{bn}}{\partial \rho} \geq \frac{\partial \bar{C}_{eq}}{\partial \rho}, \rho \to 0$$  \hspace{1cm} (20)

If $\rho \to \infty$, i.e., at high SNR, it can be easily found that

$$\frac{\partial \bar{C}_{eq}}{\partial \rho} \geq \frac{\partial \bar{C}_{bn}}{\partial \rho} \geq \frac{\partial \bar{C}_{bf}}{\partial \rho}, \rho \to \infty$$  \hspace{1cm} (21)

Note that $\bar{C}_{bf} = \bar{C}_{bn} = \bar{C}_{eq} = 0$ when $\rho = 0$ or minus infinity in dB. Hence, at medium SNR, $\frac{\partial \bar{C}_{bn}}{\partial \rho}$ has the largest value.
region from nulling is the closest to the optimal water-filling, e.g., the SNR the water-filling scheme can only allocate power to one or two low 3 closest to the optimal water-filling, e.g., the SNR region be-
different SNR regions. At low SNR, the beamforming is the any SNR region. The other schemes perform differently at


\[ \frac{\partial C_{k,bf}}{\partial \rho} = E \left( \sum_{i=1}^{k} \log \left( 1 + \frac{\rho}{k} \lambda_i \right) \right) \] (23)

It is readily checked that the capacity of MD beamforming is also concave and monotonically increasing with respect to \( \rho \). Differentiating the above ergodic capacity with respect to \( \rho \), we have

\[ \frac{\partial C_{k,bf}}{\partial \rho} \bigg|_{\rho = 0} = E \left( \sum_{i=1}^{k} \frac{1}{\rho + \frac{1}{N_t}} \right) \] (24)

If \( \rho \to 0 \), i.e., at low SNR.

\[ \frac{\partial C_{(k-1),bf}}{\partial \rho} \bigg|_{\rho = 0} > \frac{\partial C_{k,bf}}{\partial \rho} \bigg|_{\rho = 0} \] (25)

If \( \rho \to \infty \), i.e., at high SNR.

\[ \frac{\partial C_{k,bf}}{\partial \rho} \bigg|_{\rho = \infty} > \frac{\partial C_{(k-1),bf}}{\partial \rho} \bigg|_{\rho = \infty} \] (26)

That is, at low SNR, the capacity of the \( k \)-D beamforming scheme is worse than the \( (k-1) \)-D beamforming scheme and while at high SNR, the capacity of the \( k \)-D beamforming scheme is better than the \( (k-1) \)-D beamforming scheme at the cost of more feedback bandwidth.

5.2. MD Beam-nulling

For MD beam-nulling, similar to 1D-beam-nulling, by a cer-
tain rule, a subspace orthogonal to the \( k \) weakest spatial chan-
el is constructed. That is, the following condition should be satisfied.

\[ v_n^H \Phi^{(k)} = 0^T, \forall n = N_t - k + 1, \ldots, N_t. \] (27)

The \( N_t \times (N_t - k) \) matrix \( \Phi^{(k)} = [g_1 \ g_2 \ \ldots \ g_{N_t-k}] \) spans the \( (N_t - k) \) dimension subspace.
At the transmitter, $N_t - k$ symbols denoted as $x^{(k)}$ are transmitted only over the orthogonal subspace $\Phi^{(k)}$. The received signals at the receiver can be written as

$$y^{(k)} = \sqrt{P/N_t - k} H\Phi^{(k)} x^{(k)} + z^{(k)}$$  \hspace{1cm} (28)

where $z^{(k)}$ is additive white Gaussian noise vector with i.i.d. symmetrical complex Gaussian elements of zero mean and variance $\sigma_z^2$. From (28), the associated ergodic channel capacity can be found as

$$\bar{C}_{bn}^{(k)} = E \left[ \sum_{i=1}^{N_t-k} \log \left( 1 + \frac{\rho}{N_t - k} \lambda_i^2 \right) \right]$$  \hspace{1cm} (29)

It is readily checked that the capacity of MD beam-nulling is also concave and monotonically increasing with respect to $\rho$. Differentiating the above ergodic capacity with respect to $\rho$, we have

$$\frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho} = E \left( \sum_{i=1}^{N_t-k} \frac{1}{\rho + N_t - k} \lambda_i^2 \right)$$  \hspace{1cm} (30)

If $\rho \to 0$, i.e., at low SNR,

$$\frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho} > \frac{\partial \bar{C}_{bn}^{(k-1)}}{\partial \rho} , \rho \to 0$$  \hspace{1cm} (31)

If $\rho \to \infty$, i.e., at high SNR,

$$\frac{\partial \bar{C}_{bn}^{(k-1)}}{\partial \rho} > \frac{\partial \bar{C}_{bn}^{(k)}}{\partial \rho} , \rho \to \infty$$  \hspace{1cm} (32)

That is, at low SNR, the capacity of the $k$-D beam-nulling scheme is better than the $(k-1)$-D beam-nulling scheme at the cost of more feedback bandwidth and while at high SNR, the capacity of the $k$-D beam-nulling scheme is worse than the $(k-1)$-D beam-nulling scheme.

For example, in Fig. 2, capacities of 1D-beam-nulling and 2D-beam-nulling schemes are compared with WF and equal power scheme over $5 \times 5$ Rayleigh fading channel at different SNR regions. At relatively low SNR, i.e., less than 13dB, the 2D-beam-nulling scheme outperforms the 1D-beam-nulling scheme in terms of capacity at the price of feedback bandwidth. While at relatively high SNR, i.e., more than 13dB, the 1D-beam-nulling scheme outperforms the 2D-beam-nulling scheme as predicted.

5.3. Comparison

Here, over $5 \times 5$ Rayleigh fading channel, the MD schemes are compared with WF and equal power schemes as shown in Fig. 3. It can be readily checked that, at relatively low SNR, MD beamforming schemes are better than MD beam-nulling schemes; while at relatively high SNR, the results are opposite. Specifically, at very low SNR, i.e. less than 0dB, the 1D beamforming scheme outperforms the other MD schemes. At the SNR region between 0dB and 5.5dB, the 2D beamforming scheme outperforms the other MD schemes. At the SNR region between 5.5dB and 12.7dB, the 2D beam-nulling scheme outperforms the other MD schemes. At the SNR region between 12.7dB and 23dB, the 1D beam-nulling scheme outperforms the other MD schemes. Again, when SNR is more than 23dB, the equal power scheme outperforms the other suboptimal schemes.

6. CONCLUSIONS

Based on the concept of spatial subchannels and inspired by the beamforming scheme, we proposed a novel scheme called “beam-nulling”. Using the same feedback bandwidth as beamforming, the new scheme exploits all spatial subchannels except the weakest one and thus achieves significant high capacity near the optimal water-filling scheme at medium signal-to-noise ratio. If more than one eigenvector can be fed back to the transmitter, new extended schemes based on the existing beamforming and the proposed beam-nulling could be proposed. The new schemes are called multi-dimensional beamforming and multi-dimensional beam-nulling. The theoretical analysis and numeric results in terms of capacity are also pro-
vided to evaluate the new proposed schemes. The comparison showed that at low signal-to-noise ratio, beamforming is the closest to the optimal water-filling, at medium signal-to-noise ratio, beam-nulling is the closest to the optimal solution, and at high signal-to-noise ratio, equal power is the closest to the optimal solution.

7. REFERENCES


