Optimized Signal Mappings for BICM-ID Systems
Over Fast Fading MIMO Channels

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Abstract— As shown in the literature, the performance of bit-interleaved coded modulation systems (BICM-ID) depends on the binary sequences mapping to constellation symbols. In this paper, we present a new multidimensional mapping for multi-input multi-output communication systems employing BICM-ID over fast fading channel. To introduce the optimized mapping, cost function is derived based on upper bound on the pairwise error probability over fast fading. Analytical and simulation results show that the optimized mapping outperforms the conventional mapping, and other proposed mappings.

I. INTRODUCTION

Until the introduction of BICM by Zehavi [1], Coded modulation (TCM) technique proposed by Ungerboeck [2], where coding and modulation are optimized together is believed to provide better performance in digital communications especially in AWGN channels. In BICM, coding and modulation are optimized separately. Through the use of random bit-interleaver, the diversity of code is increased at the cost of decreasing free Euclidean distance. Therefore, the system performance improves over fading channels and degrades over AWGN channels. To improve the system over AWGN and further over fading channels, BICM with iteration between demapper and decoder was developed [3]. In [3], it was shown that a good design of mapper can increase the harmonic mean of the squared Euclidean distance after ideal feedback ($d_h^2$). Higher harmonic mean of the squared Euclidean distance after ideal feedback gives a better asymptotic performance in BICM-ID systems. Further, in [4] based on $d_h^2$ and using binary switching algorithm (BSA), optimum mappings were found for BICM-ID.

In [5], it was shown that MIMO provides a significant increase in data rate, bandwidth efficiency and system capacity. The increasing demand for wireless multimedia, which requires higher data rate and better power efficiency compared to current wireless communication systems, motivates the use of BICM-ID with multiple antennas at the transmitter and receiver side. In [6] the bit error rate performance of BICM-ID with MIMO was studied. Recently, the use of multidimensional mapping to further improve MIMO with BICM-ID has been investigated in [7]. In [7] locally optimized multidimensional mappings were found for MIMO-BICM-ID systems.

In this paper, we propose an optimized multidimensional mapping for MIMO-BICM-ID over fast Rayleigh fading channel. Our mappings are optimized based on $d_h^2$ which yields in the asymptotic optimum performance of the system. Analysis and simulation results show the advantage of the optimized mapping over conventional mapping, and locally optimized mapping in [7].

This paper is organized as follows: In section II, the system model is introduced. In section III, an upper bound on the pairwise error probability over Rayleigh fast fading is derived and cost function is proposed. In section IV, proposed mapping is introduced. In section V, simulation results are provided. The conclusion is given in section VI.

II. SYSTEM MODEL

Figure (1) shows the structure of transmitter and receiver of the system considered in this paper. The communication system consists of convolutional encoder concatenated in parallel with pseudorandom bit-interleaver and multidimensional mapper. Since the design of encoder is independent from the mapper and the focus of this paper is on the optimization of mapper, we use rate $\frac{1}{2}$, 8-state simple channel encoder ($g_1 = 15, g_2 = 17$, octal). Bit-interleaver was designed according to the approach given in [3]. Let’s define $M = 2^m$, where $m$ is the number of bits in the conventional constellation symbol, and $M$ is the number of symbols in conventional constellation. The multidimensional mapper takes simultaneously $mn$ bits and map them to one $2n$-dimensional signal $x$, where $n$ is the number of conventional constellation points $s$ in one multidimensional signal $x$. This creates a bigger constellation $\psi$ consisting of $M^l$ multidimensional signals. The bandwidth efficiency does not change as a result of the introduction of this multidimensional mapping. Because of its high data rate, V-Blast is used as space-time transmission technique and the multidimensional signal $x$, is sent through the multiple transmit antennas.

Let’s consider a system with $M$ transmit antennas, and $N_r$ receive antennas. The received signal at the $j$-th receive antenna, and time $l$ is given by:
In the above equation, \( h_{i,j,l} \) is fading coefficient representing the path from transmit antenna \( i \) to receive antenna \( j \) at time \( l \). \( h_{i,j,l} \) is a complex Gaussian random variable with unit variance and zero mean. \( n_j(l) \) is a complex white Gaussian noise at receive antenna \( j \) and time \( l \). \( n_j(l) \) has zero mean with power spectral density of \( N_0/2 \) per dimension.

As shown in figure (1), the receiver includes a posteriori probability (APP) soft-output demapper and a soft-input soft-output (SISO) decoder. The SISO decoder used in this paper is explained in [10]. The output of demapper is the extrinsic probability of interleaved coded bits \( P(V, O) \). The output of SISO decoder is the extrinsic probability of coded bits \( P(C, O) \). The demapper and decoder exchange the extrinsic information \( P(V, O) \) and \( P(C, O) \) in an iterative manner. After de-interleaver \( P(V, O) \) becomes priori probability \( P(V, I) \). After interleaver \( P(C, O) \) becomes priori probability \( P(C, I) \). At the last iteration, hard decision is made based on a posteriori probability of information bits.

III. UPPER BOUND ON THE PEP AND MAPPING CRITERIA

Let \( X \) and \( \hat{X} \) denote two multidimensional symbol sequences which differ in \( d \) consecutive symbols. The probability that the decoder makes mistake by choosing the sequence \( \hat{X} \) instead of the transmitted sequence \( X \) is called the pairwise error probability (PEP). The conditional pairwise error probability of the considered system is given as,

\[
r_j(l) = \sum_{i=1}^{M} h_{i,j,l} s_i(l) + n_j(l) \quad j = 1, \ldots, N_j
\]

Following the procedure described in [5], the squared Euclidean distance between \( X \) and \( \hat{X} \) can be expressed as,

\[
d^2_{j,i}(X, \hat{X}) = \sum_{j=1}^{N_j} \sum_{i=1}^{M_i} h_{i,j,i}(s_{i,j} - \hat{s}_{i,j})^2
\]

where \( h_{i,j,i} \) are Rayleigh distributed random variables.

Using Chernoff bound

\[
Q(x) \leq \exp \left( - \frac{x^2}{2} \right)
\]

and averaging the conditional pairwise error probability over Rayleigh random variables \( h_{i,j,i} \), the unconditional PEP becomes:

\[
P(X, \hat{X}) \leq \prod_{j=1}^{N_j} \prod_{i=1}^{M_i} \left( 1 + \frac{1}{\sqrt{4 N_0}} \left| x_j - \hat{x}_j \right|^2 \right)^{-1}
\]
The average pairwise error probability \( f(d, \psi, \mu) \) can be obtained by averaging over all possible multi-dimensional signals.

\[
f(d, \psi, \mu) \leq \frac{1}{mn} 2^{-m} \sum_{x, x'} \sum_{q=1}^m \left( \frac{1}{1 + \frac{1}{4N^m} (\gamma - \bar{e}(q))^2} \right)^{N-1} d^{\mu}
\]  

(6)

For reasonably high SNR, the cost function that shows the impact of multidimensional mapping on the asymptotic performance of BICM-ID systems over fast fading MIMO channels is defined as,

\[
\delta(\psi, \mu) = \frac{1}{mn} 2^{-m} \sum_{x, x'} \sum_{q=1}^m (\gamma - \bar{e}(q))^2
\]

(7)

where \( \bar{e}(q) \) has the same bits as \( x \) except at position \( q \) where it has the complement bit. The inverse of (7) is the harmonic mean of the squared Euclidean distance among the three mappings, and the highest harmonic mean Euclidean distance among the three mappings. This implies that our mapping outperforms both Gray mapping and the proposed mapping in [7] when it is used with BICM-ID systems. On the other hand, the Gray mapping has the highest value of cost function, and the lowest harmonic mean Euclidean distance among the three mappings. This suggests that BER performance of Gray mapping is poorer than the BER performance of the other two mappings when BICM-ID system is considered. Since the cost function and the harmonic mean Euclidean distance in table (I) are for BICM systems with ideal iterative decoding (i.e. BICM-ID system), they do not give us an insight into the performance of the three mappings when only one iteration is used (i.e. BICM system). The simulation results in the next section confirm the above discussion and illustrate the performance of the mappings after one iteration.

IV. OPTIMIZED MAPPING

The cost function in equation (7) is used to find the optimum mapping for any multidimensional \( M \)-ary constellation over fast Rayleigh fading channel. In this paper, the case of four dimensional 8 QAM is considered. The optimum mapping is the one that minimizes the cost function as given by (7). Binary switching algorithm is used to find such mapping for 4 dimensional 8 QAM case. To obtain globally optimum mapping, we implement BSA several times, and each time we start with different initial mapping. Table (I) lists the cost function values and the harmonic mean of the squared Euclidean distance after ideal feedback for 8QAM Gray mapping, locally optimized four dimensional 8QAM mapping in [7], and our optimized four dimensional 8QAM mapping over fast fading channel. The average energy of constellation symbols is normalized to three. Implementing BSA gives the results in Table (II) for our optimized four dimensional 8 QAM mapping. Figure (2) depicts the conventional 8QAM constellation.

Our optimized mapping has the lowest value of cost function among the three mappings, and the highest harmonic mean Euclidean distance among the three mappings. This implies that our mapping outperforms both Gray mapping and the proposed mapping in [7] when it is used with BICM-ID systems. On the other hand, the Gray mapping has the highest value of cost function, and the lowest harmonic mean Euclidean distance among the three mappings. This suggests that BER performance of Gray mapping is poorer than the BER performance of the other two mappings when BICM-ID system is considered. Since the cost function and the harmonic mean Euclidean distance in table (I) are for BICM systems with ideal iterative decoding (i.e. BICM-ID system), they do not give us an insight into the performance of the three mappings when only one iteration is used (i.e. BICM system). The simulation results in the next section confirm the above discussion and illustrate the performance of the mappings after one iteration.

TABLE I

The \( \delta(\psi, \mu) \) and \( d_f^2 \) for DIFFERENT 8QAM MAPPINGS

<table>
<thead>
<tr>
<th>Mapping</th>
<th>( \delta(\psi, \mu) )</th>
<th>( d_f^2 )</th>
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<tr>
<td>8QAM Gray mapping</td>
<td>0.3667</td>
<td>2.727</td>
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<tr>
<td>8QAM Proposed in [7]</td>
<td>0.0622</td>
<td>16.072</td>
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<td>Our optimized mapping</td>
<td>0.0527</td>
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TABLE II

The INDEX ASSIGNMENTS of the OPTIMIZED 4 DIMENSIONAL 8QAM MAPPING

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<th>(S5,S1)</th>
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Fig. 2. The conventional 8 QAM constellation
V. SIMULATION RESULTS

Simulation is carried out to verify the advantages of optimized mapping over fast Rayleigh fading channel. For fair comparison between our optimized mapping and the other mappings, we use the same system structure for all considered mappings. In our simulation, rate ½, 8 state convolutional code is used. The pseudo random bit-interleaver is designed according to the rules in [3]. The length of interleaver is 10000 coded bits. V-Blast is used as space-time transmission technique. MIMO system with two transmit antennas and two receive antennas is employed. We assume that channel state information (CSI) is available at the receiver side. The channel is frequency non-selective fast Rayleigh fading which means the fading coefficients change independently in each channel realization.

Figures 3, 4, and 5 present the bit error rate performance of MIMO-BICM-ID system employing our optimized mapping along with proposed mapping in [7] and Gray mapping after 1, 2, and 8 iterations respectively. Figures 3, 4, and 5 clearly show that our optimized mapping outperforms the proposed mapping in [7] in the whole range of SNR, BER levels, and number of iterations. This coding gain is achieved with neither complexity increase nor bandwidth expansion. On the other hand, the optimized mapping outperforms the Gray mapping if 2 iterations or more are used (i.e. BICM-ID system). More specifically, at the first iteration (i.e. BICM system), the optimized mapping performs worse than Gray mapping and that is due to the fact that our mapping is not optimized for BICM systems, but rather for BICM-ID systems. At the 2nd iteration (i.e. BICM-ID), our optimized mapping outperforms Gray mapping by 1.4 dB at the BER level of $10^{-5}$. At the 8th iteration, the optimized mapping significantly outperforms the gray mapping by 4 dB at the BER of $10^{-5}$.

To get insight into the convergence behavior of the optimized mapping, BER performance of the latter with iterations from 1 to 8 is illustrated in figure 6. The bit error rate performance improves with iterations and that is because of the increased reliability of the soft feedback information.
VI. CONCLUSION

An upper bound on the pairwise error probability for MIMO-BICM-ID systems employing multidimensional signal mapping over fast Rayleigh fading channel was derived. Cost function for designing the optimum mappings over fast Rayleigh fading was proposed. Based on the cost function and using binary switching algorithm, optimum four dimensional 8 QAM mapping was proposed. Analytical and simulation results confirm the advantage of the proposed mapping over other optimized mappings.

REFERENCES
