Impact of Successive Interference Cancellation on the Capacity of Wireless Networks: Joint Optimal Link Scheduling and Power Control

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Abstract—In this paper, we study the performance of multi-hop wireless networks when both proper interference control and effective spatial reuse are jointly considered. For the former, we assume nodes endowed with Successive Interference Cancellation (SIC) capabilities at the physical layer and for the latter, we advocate joint link scheduling and power allocation. We study the achievable performance of such networks through a cross-layer design framework. We formulate the joint problem of routing, link scheduling and power allocation as a mixed integer non-linear program (MINLP) and we use column generation (CG) to decompose and linearize it, and hence, solve it. Our numerical results show that when SIC and power control are combined, around 75% performance improvement in dense networks may be achieved in comparison with a network that does not incorporate any advanced interference management techniques.

I. INTRODUCTION

Modern wireless networks are expected to offer competitive and high throughput services in order to satisfy the growing number of users, the ever-increasing demand for higher data rates, and better quality of service. However, current wireless networks are challenged by several technical hurdles, chief among them is interference. Today, interference is considered as a fundamental deterrent for broadband wireless access, preventing the efficient utilization of their limited spectrum resources. Therefore, for wireless networks to provide truly broadband access, proper interference management must be implemented.

One effective approach for interference management is by avoiding it, either through randomizing the transmissions of neighboring senders (e.g., CSMA/CA) or through a more deterministic approach which is scheduling adjacent senders in a way not to interfere one with another (e.g., TDMA). Recently, Successive Interference Cancellation (SIC) [1]–[4] emerged as a new technique to further mitigate the detrimental effect of interference by exploiting, rather than avoiding, it. SIC refers to the node’s capability of decoding more than one packet from an aggregate signal of simultaneous transmissions on the air. While this technique increases the per-node throughput, it also yields significant improvement in network throughput, by allowing more concurrent transmissions. SIC is indeed a physical layer method, as opposed to the MAC layer methods used for resolving collisions and congestions [5], which is implemented through sophisticated signal processing techniques.

In addition to controlling the interference, one must also maintain a high level of transmission concurrency to achieve better network throughput. Promoting transmission concurrency in the network implies an effective spatial reuse of the limited spectrum resources and this may be achieved by controlling the transmission power at the senders [6]. For example, by reducing the transmission power on one link, a neighboring transmission (link) may be scheduled concurrently. Therefore, the total number of concurrent transmissions, which can be accommodated by the network, is increased. Nonetheless, when reducing the transmission power on a link, the perceived signal to interference plus noise ratio (SINR) at the receiver of this link will be smaller as a result of the weaker received signal. Therefore, link scheduling and transmission power adaptation must be done jointly to achieve the best spectrum reuse.

This current work focuses on studying the performance of multi-hop wireless networks. We argue that
for these networks to achieve a better performance, one needs, in addition to controlling the interference, to also promote concurrent transmissions in the network to improve the spectrum spatial reuse. For the former, we use nodes endowed with SIC capabilities and for the latter, we advocate joint link scheduling and power allocation. We study the achievable performance of these networks through a cross-layer design framework. We formulate the joint problem of routing, link scheduling and power allocation as a mixed integer non-linear program (MINLP) and we use column generation (CG) to decompose and linearize it, and hence, solve it. We present the numerical results for several network instances and we show the advantages of our joint design method for dense and heavily loaded networks.

The rest of the paper is organized as follows. In section II, we overview the SIC technique and describe the formulation of the joint problem in section III. Section IV introduces the problem decomposition and numerical results are discussed in section V. We conclude the paper in section VI.

II. BACKGROUND

A. The SINR Interference Model

The Signal to Interference plus Noise Ratio (SINR) model is also known as additive interference model or physical model in the literature [7]. In this model, a receiving node treats the sum of all interference, which are undesired received signals from other ongoing transmissions, as noise. Thus, the SINR at the receiver of link $i$ is given by:

$$SINR_i = \frac{G_{t(i),r(i)}P_{t(i)}}{\eta + \sum_{\forall n \in N_a - \{t(i)\}}^{} G_{n,r(i)}P_n}$$  (1)

where $t(i)$ and $r(i)$ are the transmitter and receiver of link $i$, respectively; $\eta$ is the background noise power in the frequency band of operation, $N_a$ is the set of all active nodes in the network, $P_n$ is the transmitting power at node $n$, and $G_{n,n'}$ is the signal attenuation from node $n$ to node $n'$, which is modeled as $G_{n,n'} = \left(\frac{d_{n,n'}}{d_0}\right)^{-\beta}$. Here, $\beta$ is the path loss exponent and $d_0$ is a close-in distance to the transmitter, where the received power is measured as $P_{t(i)}$.

A link is feasible if the packet from its transmitter to its receiver is decodable. According to the SINR model, link $i$ is feasible if the signal to noise ratio at $r(i)$ is above the SINR threshold of the receiver radio. The SINR threshold is the minimum required signal to noise ratio at the receiver which guarantees the tolerable Bit Error Rate (BER) of the link. Denote that higher SINR reduces the BER of the link. In this case, the following inequality has to be satisfied for link $i$ to be feasible:

$$\frac{G_{t(i),r(i)}P_{t(i)}}{\eta + \sum_{\forall n \in N_a - \{t(i)\}}^{} G_{n,r(i)}P_n} \geq \Gamma$$  (2)

B. Successive Interference Cancellation

Interference cancellation is fundamentally the idea of removing some parts of the interference from the aggregate received signal to improve the SINR. In other words, with interference cancellation, some terms in the summation of denominator in equation (1) are cancelled out and the value of SINR is increased. The interference can conveniently be detected and removed due to its data-like structure. Different methods for interference cancellation have been employed in the literature; namely, parallel [8], successive [9], combination of successive and parallel [10] and iterative [11]. In this paper, we are focusing on Successive Interference Cancellation (SIC) which is shown to be the best scheme for the case of unequal power reception [12].

In this method, the packet with the highest power is decoded in the first step of SIC process. As illustrated in Figure 1, suppose concurrent transmitting nodes are indexed from 1 to $k_n$, based on their signal strength at node $n$. Denote $k_n$ as the maximum number of decodable packets at this node. Thus, the first packet is decodable if equation (2) holds as

$$\frac{G_{t_1,n,t_1}P_{t_1}}{\eta + \sum_{\forall n' \in N_a - \{1\}}^{} G_{n',n}P_{n'}} \geq \Gamma$$

where $P_{t_1}$ is the transmitting power of node 1. To extract the second strongest packet, however, we need to determine and subtract the interference caused by the first packet from the aggregate signal. Therefore, we reconstruct a part of the received signal which is related to the
first packet. This reconstruction is done by using the decoded packet and an estimation of the path loss of the corresponding link. In this work, we assume that packet decoding and reconstruction of the received signals are error free. After removing the interference caused by the first packet, the receiver checks if the threshold requirement for the second packet is met, i.e. \( \eta + \sum_{\forall n' \in N_n \setminus \{1, 2\}} G_{n', n} P_{n'} \geq \Gamma \).

The receiver continues decoding up to \( k_n \) packets; however, it stops whenever the SINR threshold is no longer satisfied. From Figure 1, we observe two main advantages for SIC. The first one is the capability of receiving multiple desired packets from concurrent intended transmitters: 2, 3, ... \( k_n \). The second one is the capability of rejecting the interference from unintended transmitters: 1, 4, ... . For instance, receiver \( n \) is strongly interfered by transmitter 1. However, by employing SIC this interference can be cancelled and \( n \) is still able to receive its desired packets, even with lower signal power. While the first benefit allows for transfusing more data to a node, the second one increases the spatial reuse for other transmissions. These two advantages of SIC indeed improve the throughput significantly. Although decoding more packets and cancelling more interference provides higher capacity to the system, we should note that it causes more latency due to the successive nature of the process [13].

III. Joint Routing, Scheduling and Power Control

A. System Model

In this work, a multi-hop wireless network is modeled using a directed graph \( G = (N, L) \), where \( N \) is the set of all nodes and \( L \) is the set of all transmission links in the network. Considering the same modulation and coding scheme at all nodes, the capacity of all the links, named \( c \), is the same which is normalized to unity. We assume a set of \( M \) end-to-end (unicast) traffic sessions and the \( m^{th} \) session is denoted by \( S_m = \{(s_m, d_m, R_m) : s_m \in N, d_m \in N, R_m > 0, m = 1, \ldots, M\} \). Therefore, in the \( m^{th} \) session, the source node \( s_m \) sends the commodity \( R_m \) to the destination node \( d_m \).

B. Scheduling the Links

We assume a Time Division Multiple Access (TDMA) system. In a TDMA-based MAC layer, time is divided into equal duration slots. At each time slot, a set of links can be active together without violating the requirement for successful communication, i.e. collision-free packet reception at nodes. The set of links that can be active concurrently in the same time slot is defined as a configuration, denoted by \( p \).

A binary variable \( w_n \) equals to 1, whenever node \( n \) is transmitting in \( p \), and otherwise it is zero. Similarly, we denote the activity of a link \( i \) by using a binary variable \( v_i \) which equals 1 when \( i \) is active in \( p \).

1) Interference Constraints: We assume here that nodes can adjust their transmission power; therefore, in the case of employing SIC, the interference constraint for a feasible link \( i \) is given as:

\[
\eta + \sum_{\forall n \in N - \{t(i), r(i)\}} P_{n}^{p} G_{n, r(i)} \geq \Gamma
\]

where variables \( P_{n}^{p} \) and \( P_{n}^{r} \) denote the transmission power of nodes \( t(i) \) and \( r(i) \), respectively. The maximum transmission power of a node is denoted by \( P_{n}^{max} \) which is the required power to connect it to the furthest node; and we assume that the power allocated to an inactive node \( n \) must be zero. Therefore, we can write:

\[
P_{n}^{p} \leq w_n^{p} P_{n}^{max}
\]

We also consider a \( P_{n}^{min} \) for each node through which it can connect to the closest node in the absence of any interference. Therefore:

\[
P_{n}^{p} \geq w_n^{p} P_{n}^{min}
\]

However, the constraints in (3) cannot be represented in an LP format due to the condition \( P_{n}^{p} G_{n, r(i)} \geq P_{n}^{p} G_{n, r(i)} \) which appears in the summation of interference, because this condition must not includes any variable, i.e. \( P_{n}^{p} \) and \( P_{n}^{r} \). However, we overcome this obstacle as will be explained next.

Linear Interference Constraints: Let us define a new binary variable \( e_{n,i}^{p} \) which shows the following relation between each pair of a link \( i \) in \( L \) and a node \( n \in N - \{t(i), r(i)\} \):

\[
e_{n,i}^{p} = \begin{cases} 
1 & \text{if } P_{t(i)}^{p} G_{t(i), r(i)} \geq P_{n}^{p} G_{n, r(i)} \\
0 & \text{if } P_{t(i)}^{p} G_{t(i), r(i)} < P_{n}^{p} G_{n, r(i)} 
\end{cases}
\]

However, \( e_{n,i}^{p} \) can be implemented in an LP format through the following constraints:

\[
P_{t(i)}^{p} G_{t(i), r(i)} - P_{n}^{p} G_{n, r(i)} + (1 - e_{n,i}^{p}) M_{n,i}^{p} \geq 0 \tag{6}
\]

\[
P_{t(i)}^{p} G_{t(i), r(i)} - P_{n}^{p} G_{n, r(i)} - e_{n,i}^{p} M_{n,i}^{p} < 0 \tag{7}
\]
where $M_{n,i}^P$ is an integer parameter, and $M_{n,i}^P \geq |P_{t(i)}^n G_{t(i),r(i)} - P_{n}^P G_{n,r(i)}|$. Therefore, the interference constraints (3) modifies to:

$$\eta + \sum_{n \in N - \{t(i), r(i)\}} e_{n,i}^P P_{n}^P G_{n,r(i)} \geq \Gamma \quad (8)$$

which is still non-linear because of the term $e_{n,i}^P P_{n}^P$. Let us resolve this non-linearity by defining a new variable $h_{n,i}^P = e_{n,i}^P P_{n}^P$. However, this equality is implicitly achievable through the following LP constraints:

$$h_{n,i}^P \leq P_{n}^P + (1 - e_{n,i}^P) P_{n}^{max} \quad (9)$$

$$h_{n,i}^P \geq P_{n}^P - (1 - e_{n,i}^P) P_{n}^{max} \quad (10)$$

$$h_{n,i}^P \leq e_{n,i}^P P_{n}^{max} \quad (11)$$

where if $e_{n,i}^P = 0$, then $h_{n,i}^P$ becomes 0; and whenever $e_{n,i}^P = 1$, then $h_{n,i}^P = P_{n}^P$. Using $h_{n,i}^P$ in (8), and using some mathematical manipulations, the linear interference constraints are given by:

$$P_{t(i)}^n G_{t(i),r(i)} + M_{i}^P (1 - v_i^P) \geq \Gamma (\eta + \sum_{n \neq (i), r(i)} h_{n,i}^P G_{n,r(i)}) \quad (12)$$

where $M_{i}^P$ is a constant parameter, satisfying:

$$M_{i}^P \geq \Gamma (\eta + \sum_{n \in N - \{t(i), r(i)\}} P_{n}^P G_{n,r(i)})$$

2) Radio Constraints: To achieve a network-wide link scheduling, free of conflicts, we have to consider radio constraints as well. Let $L_n$ be a subset of $L$ that includes all the links connected to node $n$. $L_n^+ = \{i \in L : t(i) = n\}$ is the set of all links whose transmitter is node $n$ and $L_n^- = \{i \in L : r(i) = n\}$ is the set of all links whose receiver is node $n$. Therefore, $L_n = \{L_n^+ \cup L_n^-\}$. The radio constraints include half-duplex properties which are given as:

$$v_i^P + v_j^P \leq 1 \quad \forall n \in N : i \in L_n^{-}, j \in L_n^{+} \quad (13)$$

We also note that a node can only transmit to a single receiver in a general scheme, (i.e., no multi-packet transmission capability) which is taken care by:

$$\sum_{i \in L_n^+} v_i^P \leq u_n^P \quad \forall n \in N \quad (14)$$

C. Multihop Routing

The multihop routing constraints are built based on two basic principles as follows.  

1) Flow-balance Constraints: The flows of each session should be balanced at each node. Therefore, for each session $m$, whether a node is the source, destination or just one intermediate hop, we preserve the flows by (15). Hence, for all nodes $n \in N$ and all commodities $m \in M$,

$$\sum_{i \in L_n^+} f_i^m - \sum_{j \in L_n^-} f_j^m = \begin{cases} 0 & \text{if } n \neq s_m, d_m \\ R_m & \text{if } n = s_m \\ -R_m & \text{if } n = d_m \end{cases} \quad (15)$$

where $f_i^m$ denotes the amount of flow from session $m$ on link $i$, and other parameters were defined earlier.

2) Bandwidth Constraints: The flow of traffic on each link should not exceed the link capacity. Thus, one needs to guarantee that a link is active during enough time slots to be able to carry the amount of traffic flowing through it. Similar to [14], this can be mathematically presented as:

$$\sum_{m \in M} f_i^m \leq c_i \sum_{p \in P} \lambda_p v_i^P \quad \forall i \in L \quad (16)$$

where $c_i$ is the time-invariant capacity of link $i$, $\lambda_p$ is the number of time slots in which configuration $p$ is active, and $P$ is the set of all configurations.

IV. Problem Formulation

Our objective is to maximize the throughput of the network by minimizing the overall scheduling time, which is represented as:

$$\min_{\text{P}} \sum_{p=1}^{P} \lambda_p \quad (17)$$

where $P$ is the total number of configurations. This minimization is done through a joint routing and scheduling model, considering all the radio, interference, flow conservation and bandwidth constraints given in the previous sections. In this problem, all the configurations in $P$ should be generated and the routing problem should be solved over all of them, to yield the optimal solution. Moreover, constraints (16) are non-linear due to the term $\lambda_p v_i^P$ in it. However, by using Column Generation (CG) decomposition approach, 1) the optimal solution is obtained without enumerating all configurations, and 2) constraints (16) become linear. In CG approach, the problem is decomposed into smaller subproblems which are called the Restricted Master (RM) problem and the Pricing problem. More details in solving the problem by CG technique are explained in the following paragraphs.
Column Generation Decomposition:

The routing of demands is determined in the RM problem. The objective of the RM is the same as the objective of the original problem and the constraints are flow conservations (15) and bandwidth constraints (16). However, \( v^p_i \) is only a parameter in the RM (and not a variable) and hence, constraints (15) become linear. In the first iteration, a set of initial feasible configurations, \( P_0 \subseteq P \) is available to solve the first instance of the RM. The solution of the RM is the best routing over all possible configurations \( (P_0) \), which is a local optimum. When the RM obtains the local optimal solution, it generates the dual values of the bandwidth constraints, as \( \{\bar{y}_i, \ i \in L\} \) and sends them to the Pricing to find the best link scheduling. We note that \( v^p_i \) is a variable in the Pricing. Then the Pricing sub-problem applies these dual values to construct the Pricing objective as:

\[
\min (1 - \sum_{i \in L} \bar{y}_i v^p_i) \tag{18}
\]

where \( (1 - \sum_{i \in L} \bar{y}_i v^p_i) \) is called the Reduced Cost (RC).

The constraints of the Pricing are the scheduling constraints (4)-(7) and (9)-(14).

From now on, in each iteration, a feasible configuration (known as a column) is generated in the Pricing sub-problem. The value of the Pricing objective is always checked to determine the optimality of the solution. If (18) is a non-negative value, the obtained solution in the RM is the optimal solution to the main problem. In this case, generating more columns in the Pricing would not provide further improvement, because the optimal solution has already been found. A proof of optimality of the solution is given in [15]. On the other hand, if (18) has a negative value, the Pricing generates another configuration named \( p \) and sends it to the RM problem. Then, the RM adds \( p \) into the set of previous configurations, \( P_0 \cup \{p\} \rightarrow P_0 \), and resolves the routing problem. Whenever a solution is produced by the Restricted Master model, the corresponding dual values are passed to the Pricing and the procedure continues until finding the optimal solution.

Table I shows the decomposed model of joint routing and scheduling when SIC is used with variable transmission power at nodes.

V. Numerical Results

In this section, we provide the numerical results using 3 network topologies consisting of 4, 8 and 16 nodes which are uniformly distributed over an area of 100m \( \times \) 100m. The networks are located in an urban environment and the path loss exponent \( \beta \) is equal to 4. We have employed power control technique in which the transmission power of each active node is adjusted to an optimal value varying between \( P_{\text{max}} \) and \( P_{\min} \), explained in Section III.

In all of the networks, the transmitted power is measured at \( d_0 = 1 \text{m} \) from the transmitter; the SINR threshold \( \Gamma \) is set to 1 and the background noise power is assumed to be equal to \( 10^{-3} \text{mW} \). We have evaluated and compared the performance of four models, namely PC, SIC, SIC+PC and BM, which employ power control, successive interference cancellation, both power control and SIC, and neither power control nor SIC techniques, respectively.
VI. Conclusion

We developed a new interference management strategy which includes the benefits of both successive interference cancellation and power control. We showed that using this design improves the performance of dense networks by 75% comparing to the basic model where neither SIC nor power control is used. This remarkable improvement is obtained due to exploiting the interference by SIC and increasing the spatial reuse through power control. Our results light a new direction to enhance the throughput of networks providing popular broadband services.

REFERENCES