

Engineering Notes

Fault-Tolerant Attitude Control for Spacecraft Under Loss of Actuator Effectiveness

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I. Introduction

AT PRESENT, a large number of engineers have assorted to fault-tolerant control (FTC) to enhance the system reliability and to guarantee the control performance (see, e.g., [1–4] and references therein), and several investigations on the application of FTC to spacecraft attitude maneuvers have also been considered. For instance, based on the dynamics inversion and time-delay control, a passive fault-tolerant controller was developed in [5] to achieve attitude tracking of a rigid satellite. Cai et al. [6] proposed an indirect adaptive fault scheme for spacecraft attitude tracking problem under thruster faults. In [7], a control augmentation method, similar to adaptive fault-tolerant control, was adopted for the flexible spacecraft attitude tracking. However, all the preceding FTC results can only tolerate limited predetermined faults and have great conservativeness due to its implementing a fixed controller only.

In this work, the attitude stabilization of rigid spacecraft in the presence of partial loss of actuator effectiveness is investigated. Based on adaptive backstepping technique, a nominal attitude controller is derived first for the normal spacecraft system in the face of external disturbances. Then the case of the partial loss of reaction-wheel-effectiveness fault is considered, and the faulty attitude system with time-varying gain is ultimately decoupled into three auxiliary systems by appropriate transformation. Moreover, for each auxiliary system, an implicit estimation filter is proposed to estimate the actuator fault correspondingly, and in the meantime, a new adaptive fault-tolerant controller is synthesized according to the estimated fault to guarantee that outputs of the auxiliary system can follow the normal attitude control command signals. By viewing the tracking error as another disturbance entering the system dynamics, with the robustness of the nominal controller to external disturbances, the online fault tolerance can be achieved. In contrast to preceding FTC results, the main contributions of this study include the following:

1) The proposed strategy can react to the fault online and in real time, and thus the conservativeness of the controller can be reduced largely.

2) Although three implicit estimation filters are developed in the fault-tolerant-controller design, it does not require the precise reconstruction of the faults. Thus, large computation power can be saved and also the response time can be reduced effectively.

This Note is organized as follows: Sec. II briefly presents the spacecraft attitude model and control problems. In Secs. III and IV, adaptive backstepping attitude controllers are derived, respectively, with and without partial loss of actuator effectiveness fault. Simulation results of a rigid spacecraft are given in Sec. V, followed by conclusions in Sec. VI.

II. Dynamical Model and Problem Formulation

In this Note, the unit quaternion to designate the attitude of the rigid spacecraft is adopted, and then the kinematic differential equations can be summarized as [6]

$$\dot{\mathbf{q}} = \frac{1}{2}(\mathbf{q}^\times + q_0 \mathbf{I}_3)\boldsymbol{\omega} \quad (1)$$

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T \boldsymbol{\omega} \quad (2)$$

where $\boldsymbol{\omega}$ is the angular velocity of the spacecraft with respect to an inertial frame \mathcal{I} and expressed in body frame \mathcal{B} , $\mathbf{Q} = (q_0, \mathbf{q}^T)$ denotes the unit quaternion representing the attitude orientation of the spacecraft in \mathcal{B} with respect to the inertial frame \mathcal{I} ; $\mathbf{I}_3 \in \mathbb{R}^{3 \times 3}$ is the identity matrix, and the notation \mathbf{q}^\times denotes the cross-product operator.

As far as the attitude dynamics concerned, the rotational motion of a rigid spacecraft is described by the well-known Euler's equation [5]:

$$\mathbf{J} \dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{E}(t)\mathbf{u} + \mathbf{d} \quad (3)$$

This equation includes provisions for the partial loss-of-effectiveness fault of actuators modeled by a multiplicative factor $\mathbf{E}(t)$, which is given by

$$\mathbf{E}(t) = \text{diag}(e_1(t), e_2(t), e_3(t)) \quad (4)$$

where $e_i(t) = 1$ means that the i th actuator works normally and for $0 < e_i(t) < 1$ corresponding to the case in which the i th actuator loses its effectiveness partially, but still works all the time. Moreover, \mathbf{J} is the spacecraft inertia matrix; \mathbf{u} and \mathbf{d} denote the control torque generated by reaction wheels and the external disturbances, respectively.

Assumption 1. For the effectiveness factor $e_i(t)$ of the i th actuator, its first derivative exists, and there exist an unknown bounded positive constant λ such that $|\dot{e}_i(t)| \leq \lambda$. Further, λ is inside a compact interval $[\lambda_{\min}, \lambda_{\max}]$.

Assumption 2. Examining the environmental disturbances due to solar radiation, magnetic forces and aerodynamic drags, it is reasonable to assume that external disturbances \mathbf{d} considered in Eq. (3) is bounded by a constant T_d such that

$$\|\mathbf{d}\| \leq T_d \quad (5)$$

To this end, the control objective is to develop a fault-tolerant control law, for the faulty attitude control system given in Eqs. (1) and (3) in the presence of external disturbances, to stabilize the attitude equilibrium point ($q_0 = \pm 1, \mathbf{q}^T = 0, \boldsymbol{\omega} = 0$) or guarantee the attitude and angular velocity converge to an arbitrary small set

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containing the equilibrium point; that is, $\|\mathbf{q}(t)\| \leq \varepsilon_1$ and $\|\boldsymbol{\omega}(t)\| \leq \varepsilon_2$.

Remark 1. Note that there exist two equilibrium points: namely, $\mathbf{Q} = [\pm 1 \quad \mathbf{0}^T]^T$ in the closed-loop attitude system. Hence, it is impossible to claim any global property for the equilibrium stability. However, since both equilibrium points correspond to the same physical orientation [8], it is important to make a choice of the equilibrium point to be stabilized. In this Note, aiming for the minimizing the path length for the desired rotation, the equilibrium point is chosen depending on the sign of $q_0(0)$; that is, we chose $\mathbf{Q} = [1 \quad \mathbf{0}^T]^T$ if $q_0(0) > 0$, and $\mathbf{Q} = [-1 \quad \mathbf{0}^T]^T$ otherwise.

III. Normal Attitude Controller Design

For the normal attitude control system equation (3) without actuator faults, introduce the following state variable transformation:

$$\mathbf{z}_1 = \mathbf{q} \quad (6a)$$

$$\mathbf{z}_2 = \boldsymbol{\omega} - \boldsymbol{\alpha}_1 \quad (6b)$$

where $\boldsymbol{\alpha}_1 = -c_1 \mathbf{z}_1$ is a virtual control law with $c_1 > 0$. Then in the following theorem, we summarize our control scheme to the normal attitude stabilization problem.

Theorem 1. Consider the normal attitude control system given in Eqs. (1) and (3) without any actuator fault. If the control law is designed as

$$\mathbf{u}(t) = (\mathbf{z}_2 + \boldsymbol{\alpha}_1)^\times \mathbf{J}(\mathbf{z}_2 + \boldsymbol{\alpha}_1) + \mathbf{J}\dot{\boldsymbol{\alpha}}_1 - \frac{1 + c_2}{2} \mathbf{z}_2 - \frac{\mathbf{z}_2 T_d^2}{\|\mathbf{z}_2\| T_d + \varepsilon e^{-\beta t}} \quad (7)$$

where c_2 , ε and β are positive scalar control gains. Then the closed-loop system is practically stable in the sense that the attitude \mathbf{q} and angular velocity $\boldsymbol{\omega}$ will ultimately converge to an arbitrary set.

Proof. With the second error dynamic Eq. (6b), one obtains

$$\mathbf{J}\dot{\mathbf{z}}_2 = \mathbf{J}\dot{\boldsymbol{\omega}} - \mathbf{J}\dot{\boldsymbol{\alpha}}_1 = -(\mathbf{z}_2 + \boldsymbol{\alpha}_1)^\times \mathbf{J}(\mathbf{z}_2 + \boldsymbol{\alpha}_1) - \mathbf{J}\dot{\boldsymbol{\alpha}}_1 + \mathbf{u} + \mathbf{d} \quad (8)$$

and $\dot{\boldsymbol{\alpha}}_1 = -c_1 \dot{\mathbf{z}}_1 = -0.5c_1(\mathbf{z}_1^\times + q_0 \mathbf{I}_3)(\mathbf{z}_2 + \boldsymbol{\alpha}_1)$ is calculated.

Now, choose a Lyapunov function candidate as $V = 0.5\mathbf{z}_1^T \mathbf{z}_1 + (1 - q_0)^2 + 0.5\mathbf{z}_2^T \mathbf{J}\mathbf{z}_2$. With Eq. (8) and the proposed control-law equation (7), the time derivative of V yields

$$\dot{V} \leq -\frac{c_1}{2} \|\mathbf{z}_1\|^2 - \frac{c_2}{2} \|\mathbf{z}_2\|^2 + \varepsilon e^{-\beta t} \leq -c_0(\|\mathbf{z}_1\|^2 + \|\mathbf{z}_2\|^2) + \varepsilon \quad (9)$$

where $c_0 = \min\{0.5c_1, 0.5c_2\}$. Clearly, if $\|\mathbf{z}_1(t)\| > \sqrt{\varepsilon/c_0}$ and $\|\mathbf{z}_2(t)\| > \sqrt{\varepsilon/c_0}$, then $\dot{V} < 0$, which implies that $V(t)$ decreases monotonically. Therefore, the state signals are bounded ultimately as

$$\lim_{t \rightarrow \infty} [\mathbf{z}_1(t) \mathbf{z}_2(t)] \in \{ \|\mathbf{z}_1\| \leq \sqrt{\varepsilon/c_0}, \|\mathbf{z}_2\| \leq \sqrt{\varepsilon/c_0} \} \quad (10)$$

which is a small set containing the origin $[\mathbf{q}^T \quad \boldsymbol{\omega}] = 0$; Eq. (10) further implies that larger c_0 and smaller ε lead to better control accuracy. Consequently, according to the definition of practical stability in [9], the conclusion stated in Theorem 1 can be summarized. This completes the proof. \square

Remark 2. From the designed controller equation (7), it can be seen that if T_d is appropriately selected, then the inequality (9) can always be guaranteed, and hence great robustness to the external disturbance can be achieved with Eq. (7).

IV. Adaptive Fault-Tolerant Controller Design

In this section, partial loss of actuator effectiveness is considered. To treat with such fault, for the i th actuator, the following auxiliary system is added:

$$\dot{u}_{\text{act}}^i = v_i \quad (11a)$$

$$\dot{v}_i = e_i(t) u_{\text{com}}^i \quad (11b)$$

$$y_i = u_{\text{act}}^i \quad (11c)$$

where $\mathbf{x}_i = [u_{\text{act}}^i \quad v_i]^T \in \mathbb{R}^2$, $u_{\text{com}}^i \in \mathbb{R}$, and $y_i \in \mathbb{R}$ are the auxiliary system state, input, and output. Here, y_i is viewed as the actual output of the i th actuator and the normal controller equation (7) is denoted as u_{nor} , respectively. Note that our control objective can be restated as: designing control input u_{com}^i for auxiliary system equation (11) such that the output y_i can follow the specified desired signal u_{nor}^i , which represents the i th element of u_{nor} .

Definition [10]. A function $N(\chi)$ is called a Nussbaum-type function if the following two properties are satisfied:

$$\limsup_{s \rightarrow +\infty} \frac{1}{s} \int_0^s N(\chi) d\chi = +\infty \quad (12a)$$

$$\liminf_{s \rightarrow -\infty} \frac{1}{s} \int_0^s N(\chi) d\chi = -\infty \quad (12b)$$

For clarity, the even Nussbaum function $N(\chi) = \exp(\chi^2) \cos(\frac{\pi}{2} \chi)$ is used throughout this Note.

Lemma [10]. Let $V(t)$ and $\chi(t)$ be smooth functions defined on $[0, t_f)$ with $V(t) \geq 0$ for $\forall t \in [0, t_f)$ and $N(\chi)$ be an even smooth Nussbaum-type function. If the following inequality holds,

$$V(t) \leq f_0 + e^{-f_1 t} \int_0^t g(\tau) N(\chi) \dot{\chi} e^{-f_1 \tau} d\tau + e^{-f_1 t} \int_0^t \dot{\chi} e^{-f_1 \tau} d\tau \quad (13)$$

where f_0 and f_1 are some positive constants, and $g(\tau)$ is a time-varying parameter, which takes value in the unknown closed intervals $I_1 = [g^-, g^+]$ with $0 \notin I_1$, then $V(t)$, $\chi(t)$, and $\int_0^t g(\tau) N(\chi) \dot{\chi} d\tau$ will be bounded on $[0, t_f)$.

A. State Estimation Filters Design

To derive the fault-tolerant controller, the following two filters are employed [11]:

$$\dot{\boldsymbol{\xi}}_i = \mathbf{A}_i \boldsymbol{\xi}_i + \mathbf{k}_i y_i \quad (14a)$$

$$\dot{\boldsymbol{\Omega}}_i = \mathbf{A}_i \boldsymbol{\Omega}_i + \begin{bmatrix} 0 \\ u_{\text{com}}^i \end{bmatrix} \quad (14b)$$

where $\boldsymbol{\xi}_i = [\xi_{i,1} \quad \xi_{i,2}]^T$, $\boldsymbol{\Omega}_i = [\Omega_{i,1} \quad \Omega_{i,2}]^T$, $\mathbf{k}_i = [k_{i,1} \quad k_{i,2}]^T$, and the matrix

$$\mathbf{A}_i = \begin{bmatrix} -k_{i,1} & 1 \\ -k_{i,2} & 0 \end{bmatrix}$$

is strictly stable.

Define an estimation error $\tilde{\mathbf{x}}_i$ as $\tilde{\mathbf{x}}_i = \mathbf{x}_i - \hat{\mathbf{x}}_i$, where $\hat{\mathbf{x}}_i$ is given by

$$\dot{\hat{\mathbf{x}}}_i = \boldsymbol{\xi}_i + e_i(t) \boldsymbol{\Omega}_i \quad (15)$$

From Eqs. (11b) and (15), it follows that

$$v_i = x_{i,2} = \hat{x}_{i,2} + \tilde{x}_{i,2} = \xi_{i,2} + e_i(t) \Omega_{i,2} + \tilde{x}_{i,2} \quad (16)$$

where $x_{i,2}$, $\hat{x}_{i,2}$, and $\tilde{x}_{i,2}$ are the second elements of \mathbf{x}_i , $\hat{\mathbf{x}}_i$, and $\tilde{\mathbf{x}}_i$. Then the auxiliary subsystem equation (11) can be rewritten as

$$\dot{y}_i = \dot{u}_{\text{act}}^i = v_i = \xi_{i,2} + e_i(t) \Omega_{i,2} + \tilde{x}_{i,2} \quad (17a)$$

$$\dot{\tilde{\Omega}}_{i,2} = -k_{i,2} \tilde{\Omega}_{i,1} + u_{\text{com}}^i \quad (17b)$$

With Eqs. (11) and (15), the estimation error $\tilde{\mathbf{x}}_i$ satisfies

$$\dot{\tilde{\mathbf{x}}}_i = \dot{\mathbf{x}}_i - \dot{\hat{\mathbf{x}}}_i = \mathbf{A}_i \tilde{\mathbf{x}}_i - \dot{e}_i(t) \boldsymbol{\Omega}_i \quad (18)$$

Now we divide the error $\tilde{\mathbf{x}}_i$ into two parts, subject to, $\tilde{\mathbf{x}}_i = \tilde{\mathbf{x}}_i^a + \tilde{\mathbf{x}}_i^b$ satisfying $\dot{\tilde{\mathbf{x}}}_i^a = \mathbf{A}_i \tilde{\mathbf{x}}_i^a$, $\tilde{\mathbf{x}}_i^b = \int_0^t e^{\mathbf{A}_i(t-\tau)} (-\dot{e}_i(t) \boldsymbol{\Omega}_i) d\tau$ with $\tilde{\mathbf{x}}_i^a(0) = \tilde{\mathbf{x}}_i(0)$; then it follows that

$$\|\tilde{\mathbf{x}}_i^b\| \leq \int_0^t \|e^{\mathbf{A}_i(t-\tau)}\| \|\dot{e}_i(t)\| \|\boldsymbol{\Omega}_i\| d\tau \leq \lambda \int_0^t \pi_2 e^{-\pi_1(t-\tau)} \|\boldsymbol{\Omega}_i\| d\tau \quad (19)$$

where π_1 and π_2 are chosen positive constants such that

$$\|e^{\mathbf{A}_i(t-\tau)}\| \leq \pi_2 e^{-\pi_1(t-\tau)} \quad (20)$$

Based on Eq. (19), we have

$$\int_0^t \pi_2 e^{-\pi_1(t-\tau)} \|\boldsymbol{\Omega}_i\| d\tau \leq \pi_2 e^{-\pi_1 t} \int_0^t \pi_2 e^{\pi_1 \tau} \left(\|\boldsymbol{\Omega}_i\|^2 + \frac{1}{4} \right) d\tau \quad (21)$$

Thus, it generates

$$\|\tilde{\mathbf{x}}_i^b\| \leq \lambda \zeta(t) \quad (22)$$

where $\zeta(t)$ is the solution of the following equation:

$$\dot{\zeta}(t) = -\pi_1 \zeta(t) + \pi_2 (\|\boldsymbol{\Omega}_i\|^2 + \frac{1}{4}) \quad (23)$$

B. Fault-Tolerant Controller Design

It can be seen clearly that Eq. (11) is written into a class of triangular nonlinear form Eq. (17). Hence, the standard backstepping procedure can be employed to design the control law. Take the state transformation as

$$z_{i,1} = y_i - u_{\text{nor}}^i \quad (24a)$$

$$z_{i,2} = \boldsymbol{\Omega}_{i,2} - \beta_i \quad (24b)$$

where $\beta_i = N(\chi) \bar{\beta}_i e^{-\delta_i t}$ is a virtual control law [11], and δ_i is a positive constant, χ satisfies $\dot{\chi} = z_{i,1} \bar{\beta}_i$, and $\bar{\beta}_i$ is designed as

$$\bar{\beta}_i = [m_{i,1} + l_{i,1} + \hat{e}_i(t)] z_{i,1} + \xi_{i,2} + z_{i,1} \zeta^2(t) \hat{\lambda} - \dot{u}_{\text{nor}}^i \quad (25)$$

where $m_{i,1}$ and $l_{i,1}$ are two positive constants, $\hat{e}_i(t)$ and $\hat{\lambda}$ are the estimation of $e_i(t)$ and λ , respectively.

Theorem 2. Consider the auxiliary system equation (11) under Assumption 1, with the application of the following controller,

$$u_{\text{com}}^i = -\left(\frac{1}{4} + m_{i,2}\right) z_{i,2} + k_{i,2} \boldsymbol{\Omega}_{i,2} + \dot{\beta}_i \quad (26)$$

and the update laws

$$\dot{\hat{e}}_i = \text{proj}_{[0,1]}(l_{i,3} z_{i,1} z_{i,2}) \quad (27)$$

$$\dot{\hat{\lambda}} = \text{proj}_{[\lambda_{\min}, \lambda_{\max}]}(l_{i,2} z_{i,1}^2 \zeta^2(t)) \quad (28)$$

where $l_{i,2}$ and $l_{i,3}$ are positive design parameters, the projection operator [13] $\text{proj}(\cdot)$ is used to keep the estimate \hat{e}_i of the fault to the i th actuator within $[0,1]$, and $\hat{\lambda}$ restricted in $[\lambda_{\min}, \lambda_{\max}]$, then the system equation (11) is bounded input and bounded output stable, and the output y_i can follow the specified signal u_{nor}^i as close as possible by appropriately choosing the design parameters.

Proof: The following standard backstepping procedures are involved:

In step 1, with Eqs. (24a) and (17a), it follows that

$$\dot{z}_{i,1} = \dot{y}_i - \dot{u}_{\text{nor}}^i = \xi_{i,2} + e_i(t) \boldsymbol{\Omega}_{i,2} + \tilde{x}_{i,2} - \dot{u}_{\text{nor}}^i \quad (29)$$

Choose a Lyapunov function as

$$V_{i,1} = \frac{1}{2} z_{i,1}^2 + \frac{1}{4 l_{i,1}} (\tilde{x}_i^a)^T \mathbf{P}_i \tilde{x}_i^a + \frac{1}{2 l_{i,2}} \tilde{\lambda}^2 + \frac{1}{2 l_{i,3}} \tilde{e}_i^2 \quad (30)$$

where $\tilde{e}_i(t) = e_i - \hat{e}_i(t)$, $\hat{\lambda} = \lambda - \tilde{\lambda}$, and $\mathbf{P}_i \in \mathbb{R}^{2 \times 2}$ is a positive definite matrix such that $\mathbf{P}_i \mathbf{A}_i + \mathbf{A}_i^T \mathbf{P}_i \leq -2 \mathbf{I}_2$.

With Young's inequality $\hat{e}_i(t) z_{i,1} z_{i,2} \leq \tilde{e}_i^2(t) z_{i,1}^2 + \frac{1}{4} z_{i,2}^2$ and

$$z_{i,1} \tilde{x}_i^a \leq l_{i,1} z_{i,1}^2 + \frac{1}{4 l_{i,1}} |\tilde{x}_i^a|^2 \leq l_{i,1} z_{i,1}^2 + \frac{1}{4 l_{i,1}} \|\tilde{\mathbf{x}}_i^a\|^2$$

then the time derivative of $V_{i,1}$ can be calculated as

$$\begin{aligned} \dot{V}_{i,1} &\leq -m_{i,1} z_{i,1}^2 + \frac{1}{l_{i,3}} \tilde{e}_i [l_{i,3} z_{i,1} z_{i,2} - \dot{e}_i(t) + \dot{e}_i] - \frac{1}{4 l_{i,1}} \|\tilde{\mathbf{x}}_i^a\|^2 \\ &\quad + z_{i,1} \tilde{x}_{i,2}^b + [e_i N(\chi) \exp(-\delta_i t) + 1] \dot{\chi} \\ &\quad + \frac{1}{4} z_{i,2}^2 - \frac{1}{l_{i,2}} \tilde{\lambda} \hat{\lambda} - z_{i,1}^2 \zeta^2(t) \hat{\lambda} \end{aligned} \quad (31)$$

On the other hand, from Eq. (22), one has

$$\begin{aligned} z_{i,1} \tilde{x}_{i,2}^b - z_{i,1}^2 \zeta^2(t) \hat{\lambda} &\leq \lambda \left(\zeta^2(t) z_{i,1}^2 + \frac{1}{4} \right) - z_{i,1}^2 \zeta^2(t) \hat{\lambda} \\ &= z_{i,1}^2 \zeta^2(t) \tilde{\lambda} + \frac{\lambda}{4} \end{aligned} \quad (32)$$

With the projection operator property [13]

$$-\tilde{\lambda} \text{proj}(l_{i,2} z_{i,1}^2 \zeta^2(t)) \leq -\tilde{\lambda} l_{i,2} z_{i,1}^2 \zeta^2(t)$$

and

$$-\tilde{e}_i(t) \text{proj}(l_{i,3} z_{i,1} z_{i,2}) \leq -l_{i,3} z_{i,1} z_{i,2} \tilde{e}_i(t)$$

then we have

$$\begin{aligned} \dot{V}_{i,1} &\leq -m_{i,1} z_{i,1}^2 + \frac{1}{l_{i,3}} \tilde{e}_i \dot{e}_i + [e_i N(\chi) \exp(-\delta_i t) + 1] \dot{\chi} \\ &\quad + \frac{\lambda}{4} - \frac{1}{4 l_{i,1}} \|\tilde{\mathbf{x}}_i^a\|^2 + \frac{1}{4} z_{i,2}^2 \end{aligned} \quad (33)$$

In step 2, differentiating Eq. (24b) along with Eq. (17b), one has

$$\dot{z}_{i,2} = \dot{\boldsymbol{\Omega}}_{i,2} - \dot{\beta}_i = -k_2 \boldsymbol{\Omega}_{i,1} - \dot{\beta}_i + u_{\text{com}}^i \quad (34)$$

Define another Lyapunov function as $V_{i,2} = V_{i,1} + 0.5 z_{i,2}^2$, with the updating-law equations (26–28), we have

$$\begin{aligned} \dot{V}_{i,2} &\leq -m_{i,1} z_{i,1}^2 - m_{i,2} z_{i,2}^2 + \frac{1}{l_{i,3}} \tilde{e}_i \dot{e}_i + \frac{\lambda}{4} - \frac{1}{4 l_{i,1}} \|\tilde{\mathbf{x}}_i^a\|^2 \\ &\quad + [e_i N(\chi) \exp(-\delta_i t) + 1] \dot{\chi} \end{aligned} \quad (35)$$

Moreover, it can be observed that

$$V_{i,2} \leq \frac{1}{2} z_{i,1}^2 + \frac{1}{2} z_{i,2}^2 + \frac{1}{4 l_{i,1}} \|\mathbf{P}_i\| \|\tilde{\mathbf{x}}_i^a\|^2 + \frac{1}{2 l_{i,2}} \tilde{\lambda}^2 + \frac{1}{2 l_{i,3}} \tilde{e}_i^2 \quad (36)$$

Then integrating both sides of Eq. (36) from zero to t , we have

$$\begin{aligned} \int_0^t \dot{V}_{i,2} e^{\delta_i \tau} d\tau &\leq - \int_0^t (m_{i,1} z_{i,1}^2 + m_{i,2} z_{i,2}^2) e^{\delta_i \tau} d\tau \\ &+ \int_0^t \dot{\chi} e^{\delta_i \tau} d\tau + \int_0^t e_i N(\chi) \dot{\chi} d\tau + \int_0^t \left(\frac{1}{l_{i,3}} \tilde{e}_i \dot{e}_i + \frac{\lambda}{4} \right) e^{\delta_i \tau} d\tau \\ &- \frac{1}{4l_{i,1}} \int_0^t \|\tilde{\mathbf{x}}_1^a\|^2 e^{\delta_i \tau} d\tau \end{aligned} \quad (37)$$

To this end, it yields

$$\begin{aligned} 0 \leq V_{i,2}(t) &\leq V_{i,2}(0) \\ &+ \underbrace{e^{\delta_i t} \int_0^t e_i N(\chi) \dot{\chi} d\tau + \int_0^t \dot{\chi} e^{-\delta_i(t-\tau)} d\tau}_{\sigma_1} \\ &+ \underbrace{\frac{\delta_i}{2} \int_0^t (\tilde{e}_i^2 + \tilde{\lambda}^2) e^{-\delta_i(t-\tau)} d\tau + \int_0^t \left(\frac{1}{l_{i,3}} \tilde{e}_i \dot{e}_i + \frac{\lambda}{4} \right) e^{-\delta_i(t-\tau)} d\tau}_{\sigma_2} \end{aligned} \quad (38)$$

where $\delta_i = \min\{2m_{i,1}, 2m_{i,2}, \frac{1}{\|\mathbf{P}_i\|^2}\}$. Because of the utilization of projection operator for $\hat{e}_i(t)$ and $\hat{\lambda}$, the boundedness of $\tilde{e}_i(t)$ and $\tilde{\lambda}$ can be obtained, and then σ_2 is also bounded. Thus, according to the Lemma, we know that $V_{i,2}(t)$, $\dot{\chi}$, and $\int_0^t e_i N(\chi) \dot{\chi} d\tau$ are all bounded on $[0, t_f)$. Therefore, $z_{i,1}$, $z_{i,2}$, e_i , λ , and $\tilde{\mathbf{x}}_1^a$ are bounded on $[0, t_f)$ for all $t_f > 0$; in view of Eqs. (11) and (24), it can be known that all the signals in the closed-loop system are bounded on $[0, t_f)$ for all $t_f > 0$. Thus, according to [10], all the signals in the closed-loop system are bounded for $t_f = +\infty$. Moreover, according to Eq. (38), one has

$$|z_{i,1}| = |y_i - u_{\text{nor}}^i| \leq \mu_i \quad (39)$$

where $\mu_i = V_{i,2}(0) + \sigma_1 + \sigma_2$. To this end, the bound input and bound stability of system equation (11) can be concluded. \square

Remark 3. Note that μ_i is dependent on $V_{i,2}(0)$. Thus, by setting $z_{i,j}(0) = 0$ ($j = 1, 2$), the following can be obtained based on the definition $V_{i,2}$,

$$\mu_i = \frac{1}{4l_{i,1}} (\tilde{\mathbf{x}}_1^a(0))^T \mathbf{P}_i \tilde{\mathbf{x}}_1^a(0) + \frac{1}{2l_{i,2}} \tilde{\lambda}^2(0) + \frac{1}{2l_{i,3}} \tilde{e}_i^2(0) + \sigma_1 + \sigma_2 \quad (40)$$

implying that we can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains $l_{i,j}$ ($j = 1, 2, 3$). Further, the closer the initial estimates errors $\tilde{\lambda}(0)$, $\tilde{e}_i(0)$, and $\tilde{\mathbf{x}}_1^a(0)$ are to the true values λ , e_i , and \mathbf{x}_1^a , the better the transient performance.

C. Fault-Tolerance Analysis

Equation (11) yields

$$u_{\text{act}}^i(t) = \int_0^t \int_0^s e_i(\tau) u_{\text{com}}^i d\tau ds \quad (41)$$

According to Theorem 2 and Eq. (39), it is reasonable to suppose that

$$u_{\text{act}}^i - u_{\text{nor}}^i = \Delta u_i \quad (42)$$

where Δu_i satisfies $|\Delta u_i| = |y_i - u_{\text{nor}}^i| \leq \mu_i$. Then under the effect of actual output u_{act}^i of the i th actuator, the faulty Eq. (3) can be rewritten as

$$\mathbf{J} \dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times \mathbf{J} \boldsymbol{\omega} + \mathbf{u}_{\text{act}} + \mathbf{d} = -\boldsymbol{\omega} \times \mathbf{J} \boldsymbol{\omega} + \mathbf{u}_{\text{nor}} + \Delta \mathbf{u} + \mathbf{d} \quad (43)$$

where $\mathbf{u}_{\text{nor}} = [u_{\text{nor}}^1 \quad u_{\text{nor}}^2 \quad u_{\text{nor}}^3]^T$ and $\Delta \mathbf{u} = [\Delta u_1 \quad \Delta u_2 \quad \Delta u_3]^T$ and satisfies

$$\|\Delta \mathbf{u}\| \leq \sqrt{\sum_{i=1}^3 \mu_i^2}$$

In Eq. (43), the item $\Delta \mathbf{u} + \mathbf{d}$ can be viewed as the lumped disturbances. Therefore, if the constant T_d is chosen large enough such that

$$\|\Delta \mathbf{u}\| + \|\mathbf{d}\| \leq \sqrt{\sum_{i=1}^3 \mu_i^2} + \|\mathbf{d}\| \leq T_d \quad (44)$$

Then with the designed normal controller equations (7) and (26), Eq. (9) can be guaranteed. Hence, based on Theorem 1, the following theorem can be obtained.

Theorem 3. Consider the control-law equation (26), if the design parameter T_d is appropriately chosen to satisfy Eq. (44). Then the system equations (1–3) in the closed loop with partial loss of actuator effectiveness fault are practically stable, and the attitude \mathbf{q} and the angular velocity $\boldsymbol{\omega}$ will ultimately converge to an arbitrary set equilibrium point; that is, the objectives as stated in Sec. II can be achieved.

V. Simulation and Comparison Results

To study and verify the effectiveness and performance of our proposed control scheme, the detailed response is numerically simulated for an orbital microsatellite. For this, the model parameters for the rigid spacecraft are chosen [6] as

$$\mathbf{J} = \begin{bmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0.9 & 0 & 15 \end{bmatrix} \text{ kg m}^2$$

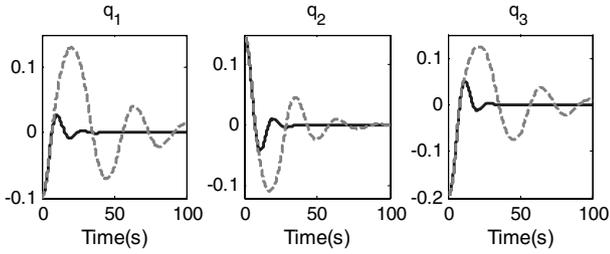
and the external disturbances such as gravity gradient, solar radiation, magnetic field, and aerodynamic disturbances [12] are also incorporated. Moreover, this spacecraft is actuated by three reaction wheels equipped in each axis with the saturation control torque of 0.2 N · m.

In the reaction-wheel fault scenario during attitude stabilization maneuver, each actuator undergoes the following partial loss-of-effectiveness faults:

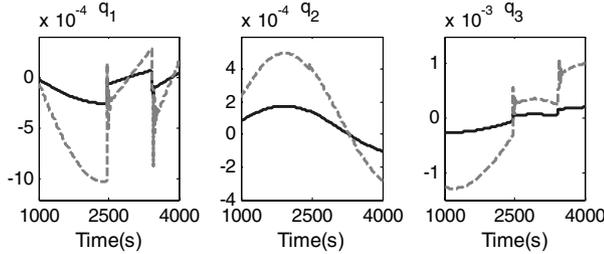
$$\begin{cases} e_1(t) = 0.25, t > 5 \text{ s} \\ e_2(t) = 0.35, t > 8 \text{ s} \\ e_3(t) = 0.20, t > 10 \text{ s} \end{cases} \quad (45)$$

To validate the superior performance of the proposed fault-tolerant-controller equation (26), the following simulations are achieved with fault-scenario equation (45) by comparing with the conventional proportional-derivative (PD) control. To implement the controller, the control parameters for the normal controller equation (7) are chosen as $c_1 = 5$, $c_2 = 20$, $T_d = 1$, $\varepsilon = 0.5$, and $\beta = 2$, and the following choices are made for the various design parameters in the fault-tolerant-controller equation (26): $m_{i,1} = m_{i,2} = 5$, $k_{i,1} = 4$, $k_{i,2} = 2.5$, $l_{i,1} = 1$, $l_{i,2} = 4$, and $l_{i,3} = 2$. Furthermore, in the context of simulation, the initial attitude orientation is set to be $\mathbf{q}(0) = [-0.1 \quad 0.15 \quad -0.2]^T$ with angular velocity $\boldsymbol{\omega}(0) = [0 \quad 0 \quad 0]^T$ rad/s.

Figure 1 shows the simulation results when the PD controller was applied during the attitude stabilization maneuver. We can see clearly that although the PD control scheme can perform the attitude stabilization maneuver with fault-free or partial loss of reaction-wheel-effectiveness fault equation (45), it takes much longer than 100 s to stabilize the attitude and the low control accuracy of $1.5e - 3$ is obtained when fault equation (45) occurs, as we can see in Fig. 1b. Indeed, due to the fault equation (45), the control powers that needed to stabilize the attitude were largely lost; therefore, more time is taken to stabilize the attitude.

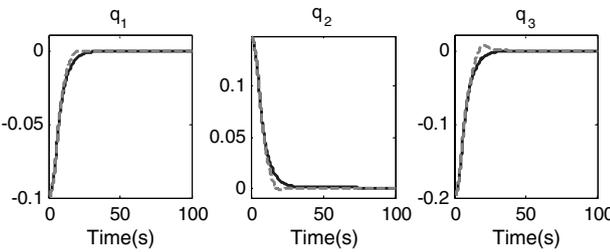


a) Initial response

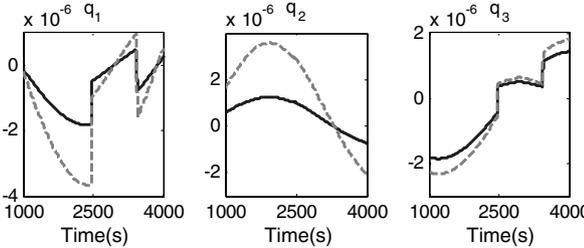


b) Steady-state behavior

Fig. 1 Time response of q^T with PD control: without fault (solid line) and with fault (dashed line).



a) Initial response



b) Steady-state behavior

Fig. 2 Time response of q^T with proposed controller: without fault (solid line) and with fault (dashed line).

We now report the results due to the proposed fault-tolerant-controller equation (26). As expected, we see clearly in Fig. 2 that the control law can achieve the attitude stabilization even under the severe reaction-wheel fault equation (45). Actually, with the designed-estimation-filter equation (14), the outputs of reaction wheel can always follow the signals of nominal control-law equation (7), and hence the attitude stabilization maneuver can be finished almost in the same time, as shown in Fig. 1a. Also, the practical stability and great attitude control accuracy of $2e - 6$ are observed in Fig. 2b. Moreover, after the occurrence of the fault, the designed-filter equation (14) and the update-law equation (27) can accurately estimate the fault value $E(t)$, as shown in Fig. 3.

Summarizing all the cases (normal and fault cases), it is noted that the proposed controllers design method can significantly decrease the attitude stabilization maneuver time and improve the attitude control accuracy over the PD method in both theory and simulations. Also, in the fault case, the proposed methods have better results than those of conventional cases.

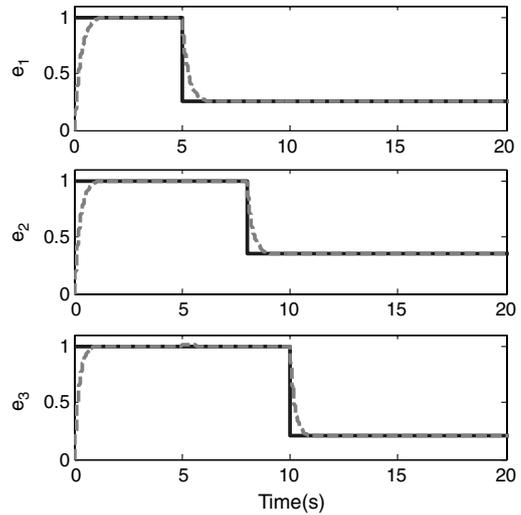


Fig. 3 Time response of fault $E(t)$ with proposed controller: actual fault e_i (solid line) and its estimation \hat{e}_i (dashed line).

VI. Conclusions

An adaptive backstepping control scheme was developed for the spacecraft attitude stabilization system, in which the external disturbances and partial loss of actuator effectiveness fault are considered. More specifically, a normal control law is first proposed to the normal attitude system, and then three auxiliary systems are added. Based on these manipulations, three implicit filters are proposed to estimate the actuator faults, and three new adaptive controllers are designed based on the estimation of fault to let the output of auxiliary system follow the normal control signals. Hence, fault tolerance, robustness against the actuator fault, and high accuracy of attitude stabilization can be achieved. The control designs are evaluated using numerical simulation comparisons between the developed approach and PD control scheme for the normal and faulty actuator scenarios.

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