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Assignment 2

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Assignment answers should be submitted on Moodle in PDF format.

1 Preference of a group of decision makers

Suppose that there are two decision makers, with preference relations that admit a vNM representation using utility functions u_1 and u_2. They communicate their utility functions to a third person, who computes a function $u = \frac{1}{2}u_1 + \frac{1}{2}u_2$. Under what conditions is u a utility function?

Suppose that the third person derives a preference relation $>_u$ from u according to the vNM representation. Give an example where $>_u$ coincides with the preference relation of the first decision maker in some cases, and with the preference relation of the second decision maker in other cases.

2 From preference to representation U

Suppose that you know that a decision maker has a preference relation over the set of real numbers $[-100, 100]$, with a representation of the form $U(z) = b - (z - a)^2$, where a, b are constants that are unknown. Suppose that the decision maker tells you that $U(1) < U(4)$, and $U(4) > U(5)$ and $U(5) > U(8)$. Can you conclude that $U(4.5) > U(2)$? Can you conclude that $U(4.5) > U(7)$?

Suppose that you can ask the decision maker questions of the form "Do you prefer x or y?" What is a sequence of such questions that you can ask in order to approximately find the number z^* that the decision maker prefers the most?

Bonus: How can you minimize the number of such questions asked?

3 Managing risk

As in lecture 4, consider a mix $\lambda \in [0, 1]$ of a random payoff X and a fixed amount c. The payoff of a mix λ is

$$X_\lambda = (1 - \lambda)X + \lambda c. \tag{1}$$

Let f_λ denote the probability density function of X_λ.

1. (10 points) Consider a decision maker with $u(z) = 2\sqrt{z}$, $c = 0.42$, X uniform on $[0, 1]$. What is the optimal λ?

2. (10 points) Consider a decision maker with $u(z) = 2\log(1+z)$, $c = 0.42$, X uniform on $[0,1]$. What is the optimal λ?

3. (10 points) Consider a decision maker with $u(z) = \frac{2z}{1+z}$, $c = 0.42$, X uniform on $[0,1]$. What is the optimal λ?

4. (10 points) Consider a decision maker with $u(z) = 2\sqrt{z}1_{[z\in[0,\infty)]}$, $c = 0.42$, X normal with mean 0.5 and variance 0.5. What is the optimal λ?

5. (10 points) Consider a decision maker with $u(z) = 2\log(1+z)1_{[z\in[0,\infty)]}$, $c = 0.42$, X normal with mean 0.5 and variance 0.5. What is the optimal λ?

6. (10 points) Consider a decision maker with $u(z) = \frac{2z}{1+z}1_{[z\in[0,\infty)]}$, $c = 0.42$, X normal with mean 0.5 and variance 0.5. What is the optimal λ?

7. (10 points) What is the take-away lesson from this exercise?