Decode-Compress-and-Forward with Selective-Cooperation for Relay Networks

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***Abstract*—We propose a new signal-processing scheme, referred to as Decode-Compress-and-Forward with Selective-Cooperation (DCF-SC). In DCF-SC, the relay dedicates a certain amount of time to listen to the message broadcasted by the source and then performs Soft-Input Soft-Output (SISO) decoding. The relay then quantizes the Log-Likelihood Ratio (LLR) values received from the SISO decoder, encodes them and then transmits to the destination. The Selective-Cooperation condition determines whether the destination will accept or reject relay's collaboration. We consider half-duplex relaying with orthogonal channels at the destination and apply turbo coding at both source and relay nodes. We define a trade-off parameter that determines how much of the relay's time should be dedicated to listening and to transmission. We show by simulations that this trade-off factor has an optimal value for which the Block-Error Rate (BLER) is minimized. We compare the error rate performance of the proposed DCF-SC scheme with that of the Decode-Amplify-Forward (DAF) scheme presented in the literature.**

***Index Terms*—Decode-compress-and-forward, iterative decoding, relay channels, turbo codes.**

I. INTRODUCTION

COMMUNICATION over relay channels was first introduced in [1] and several transmission strategies were considered in the fundamental work of [2]. Recently, cooperative transmission techniques have received increasing attention due to their potential gains in increasing transmission rates and providing more reliable communications. Several signal processing techniques at the relay are considered, including Decode-and-Forward (DF) [4] and Amplify-and-Forward (AF) [3]. In DF, the source sends a message that is encoded by a Forward Error-Correcting (FEC) code to the destination node. If the relay decodes the received message with no error, it re-encodes the message and retransmits to the destination. On the contrary, in AF, the relay only amplifies the received signal before retransmission to the destination. The destination then combines the analog signals received from both the source and relay to re-construct the source message. Two other important relaying schemes introduced in the literature are Decode-Amplify-and-Forward (DAF) [5] and Compress-and-Forward (CF) [2]. In DAF, the relay performs Soft-Input Soft-Output (SISO) decoding on the signal received from the source. The soft outputs are in fact the Log-Likelihood Ratios (LLRs). These LLRs (analog values) are then amplified and transmitted to the destination. It has been shown that DAF always outperforms AF and also performs better than DF in some cases. In CF, the relay quantizes the signal received from the source, re-encodes the quantized bits and then transmits to the destination. CF takes advantage of the correlation between the two signals received at the relay and the destination, to decode the message. As CF is similar to distributed compression schemes, decoding techniques used for distributed compression can be applied to develop efficient CF schemes. For example, one of the recent works on CF [6] considers quantization of the source signal received at the relay, followed by encoding using a concatenated code, modulation and transmission to the destination. The destination then performs joint decoding to decode the source signal.

In this paper, we propose a new scheme that we call Decode-Compress-and-Forward with Selective-Cooperation (DCF-SC). In DCF-SC, the relay first performs SISO decoding on the signal received from the source. Then quantizes the LLRs, re-encodes and retransmits them to the destination. The destination takes advantage of the correlation between the signals transmitted by the source and the ones by the relay to decode the message. The Selective-Cooperation condition determines whether the destination will accept or reject relay's collaboration. We define a trade-off factor that determines how much of the relay's time is dedicated to listening to the source and how much time spent in transmitting to the destination. Such trade-off factor is introduced and studied in details for DF schemes in [8], and also studied for DF schemes using rateless codes in [9]. We show by simulations that there is an optimal value for this trade-off factor, for which the Block-Error Rate (BLER) is minimized. We compare the BLER of our DCF-SC scheme with the BLER of the DAF scheme, for half-duplex relaying and assuming orthogonal channels at the destination.

II. SYSTEM MODEL AND DCF-SC SCHEME

We consider a single-relay scenario, consisting of three nodes; source (S), relay (R) and destination (D). Assume that the source intends to transmit a message consisting of K binary unbiased i.i.d. bits to the destination, with possible collaboration from the relay. For this, the source encodes the message by a turbo code, punctures a defined number of parity bits to achieve a target code rate, and starts broadcasting the generated codeword. We assume here that the turbo code consists of two parallel concatenated convolutional codes, separated by a code interleaver. Other variations of turbo codes (e.g. serial concatenation) can be used to further improve the performance, but here we consider parallel concatenated codes and focus on the presentation of the proposed DCF scheme. The codeword broadcasted by the source is received

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by the relay and the destination. We assume that the channels between the S, R, and D can be modeled as independent block-fading channels.

Let $x_s(i)$ represent the ith bit of the generated codeword at the source, $y_{sd}(i)$ represent the ith sample received on the S-D link, h_{sd} represent the complex fading coefficient, g_{sd} represent the channel gain for the S-D link, and $z_{sd}(i)$ be the additive white Gaussian noise. Also, let us use the notation $x_s(i:j)$ to represent the stream (or vector) $x_s(i), x_s(i+1), ..., x_s(j)$. We assume that each codeword is of length N bits, and the code is systematic such that the source starts broadcasting by transmitting the information bits first. In other words, the vector $x_s(1:K)$ represents the information bits, whereas $x_s(K+1:N)$ represents the parity bits. In our scheme, we assume that the destination listens to the whole codeword, before attempting to decode the message. Thus, the decoder at the destination has access to the received vector,

$$y_{sd}(1:N) = h_{sd}\sqrt{g_{sd}}x_s(1:N) + z_{sd}(1:N). \tag{1}$$

Since the source is broadcasting the codeword, the relay is also able to listen and decode the information. Let the relay listen to $N^{'}$ bits before attempting to decode the message, for some $K \leq N^{'} \leq N$. By stating the condition $K \leq N^{'}$, we emphasis that the relay listens to all information bits before attempting to decode. Therefore, we can express $N^{'}$ as

$$N^{'} = K + (N-K)f, \quad 0 \leq f \leq 1 \tag{2}$$

for $0 \leq f \leq 1$. As we will see later, the parameter f represents the trade-off between the time the relay dedicates to listening to the source and the time it spends in transmission to the destination. By listening to the source for $N^{'}$ samples, the relay receives the vector

$$y_{sr}(1:N^{'}) = h_{sr}\sqrt{g_{sr}}x_s(1:N^{'}) + z_{sr}(1:N^{'}). \tag{3}$$

Now the relay applies turbo decoding, considering the parity bits $x_s(N^{'}+1:N)$ as punctured bits. At the end of the decoding process, the relay quantizes the LLR values and re-encodes them by another turbo code. We assume that the quantizer at the relay will follow the modulation format set at the source. Specifically here we assume that the source applies BPSK modulation, so that the quantizer at the relay is a single-threshold hard-decision device that maps positive LLR values to $+1$ and negative LLR values to -1. The output of this quantizer is a (possibly noisy) copy of $x_s(1:K)$. The relay interleaves this copy of the message using an interleaver. Let us denote this interleaved copy of the message by $x_r(1:K)$. The relay then encodes the vector $x_r(1:K)$ by a turbo code that can be different from the one used at the source. Then puncturing is applied to all information bits plus a definite number of parity bits, yielding a codeword $x_r(N^{'}+1:N)$. Note that, we interleave the parity bits before puncturing so that the codeword $x_r(N^{'}+1:N)$ does not represent consequent bits on the code trellis, but it represents the bits that are randomly spread over the trellis. Finally, the relay transmits the generated codeword to the destination during time $N^{'}+1:N$. We assume that the relay and the source transmit to the destination via orthogonal channels. The received signal at the destination is then given by,

$$y_{rd}(N^{'}+1:N) = h_{rd}\sqrt{g_{rd}}x_r(N^{'}+1:N) + z_{rd}(N^{'}+1:N). \tag{4}$$

As indicated above, $x_r(1:K)$ is not necessarily a reliable copy of $x_s(1:K)$, and there can be some positive distortion. However, by estimating this distortion, one can have a measure of correlation between $x_r(1:K)$ and $x_s(1:K)$ which can be exploited by the decoder at the destination. For our system, we consider the normalized Hamming distortion

$$d = \frac{1}{K}\sum_{i=1}^{K}(x_s(i) \oplus x_r(i)) \tag{5}$$

where $\oplus$ represents modula-2 addition. The expected value of this distortion, $E[d]$, represents the average bit-error rate (BER) on the S-R link. We note that, $E[d]$ can be estimated from the instantaneous signal-to-noise ratio (SNR) on the S-R link (given the turbo encoder-decoder and the channel SNR, the BER can be found from SNR-BER graph or look-up table). Therefore, the relay encodes the value of the SNR on S-R link and transmits this information to the destination. The destination can then recover this SNR and evaluate $E[d]$. The destination node uses the received vectors, $y_{sd}(1:N)$, $y_{rd}(N^{'}+1:N)$, and the estimated distortion, $E[d]$, to decode the message $x_s(1:K)$ as explained below.

To perform decoding, we first note that the system model is closely related to the model of distributed source coding schemes. Similar to distributed source coding, source and relay signals are correlated, and both are transmitted to the same destination. Similarity between relay communication and distributed source coding is indicated in several references including the recent work in [6], where a compress-and-forward scheme was proposed to exploit this similarity. We note that due to the similarity between our scheme and distributed source coding schemes, one can apply their decoding algorithms to our proposed DCF scheme. In [7], the authors considered a turbo-coded distributed source coding scenario and proposed a joint decoding algorithm that was shown to be simple and efficient. The key idea of the algorithm in [7] is to perform one turbo decoding iteration for each turbo code, then modify the extrinsic information by considering the effect of correlation, and pass this extrinsic information to the second turbo decoder. To formulate the decoding method, we assume that the decoder at the destination consists of two turbo decoders. *Turbo Decoder* 1, corresponding to the S-D link, and *Turbo Decoder* 2, corresponding to the R-D link. Let the destination begin by allowing one turbo decoding iteration for $y_{rd}(N^{'}+1:N)$ (i.e. for *Turbo Decoder* 2). Now add the extrinsic LLRs from the two SISO decoders corresponding to *Turbo Decoder* 2, to give $L_{E,r}(1:K)$. Similar to [7], we modify $L_{E,r}(1:K)$ as follows:

Step 1 - Define:

$$p^0_{E,r}(1:K) = \exp\frac{L_{E,r}(1:K)}{1+\exp L_{E,r}(1:K)}, \tag{6}$$

$$p^1_{E,r}(1:K) = 1 - p^0_{E,r}(1:K). \tag{7}$$

Step 2 - Define:

$$q^0_{E,r}(1:K) = (1-E[d])p^0_{E,r}(1:K) + E[d]\,p^1_{E,r}(1:K), \tag{8}$$

$$q^{1}_{E,r}(1:K) = E\left[d\right] p^{0}_{E,r}(1:K) + (1 - E\left[d\right])p^{1}_{E,r}(1:K). \tag{9}$$

Step 3 - Modify $L_{E,r}(1:K)$ as:

$$L^{'}_{E,r}(1:K) = \log \frac{q^{0}_{E,r}(1:K)}{q^{1}_{E,r}(1:K)}. \tag{10}$$

$L^{'}_{E,r}(1:K)$ is then passed to the first SISO decoder of *Turbo Decoder* 1. Note that the index r in $L^{'}_{E,r}(1:K)$ denote the modified extrinsic information from the turbo decoder corresponding to relay communication. By the aid of this additional extrinsic information (and its own available extrinsic information), *Turbo Decoder 1* performs one turbo decoding iteration, calculates a modified extrinsic information, $L^{'}_{E,s}(1:K)$, following steps 1-3 stated above, and passes $L^{'}_{E,s}(1:K)$ to the first SISO decoder of *Turbo Decoder* 2. This continues for a defined number of iterations after which a hard decision is made on the LLRs of *Turbo Decoder* 1.

Through simulations, we observed that the joint decoding approach explained above performs well at low SNRs but reaches error floor at higher SNRs. To combat the error floor effect, we equip the decoder with an additional criterion to decide whether to accept or reject relay's collaboration. We implement this criterion as follows: the destination compares the instantaneous SNR on the S-R, SNR_{sr}, with the instantaneous SNR on S-D link, SNR_{sd}. Recall that SNR_{sr} has been sent to the destination by the relay, as a side information. Note that due to the block-fading assumption, these SNRs remain constant for the duration of one codeword and independently change from one codeword to another. Given this, the destination accepts relay's collaboration *only if* $SNR_{sr} \geq SNR_{sd}$, i.e. if the destination realizes that the received signal at the relay is more reliable than the one received at the destination. We refer to this scheme as Decode-Compress-and-Forward with Selective-Cooperation (DCF-SC). In the next section, we present simulation results to assess the performance of our DCF-SC scheme. We also consider the role of the trade-off parameter presented in (2), and show that there is an optimal trade-off for the proposed DCF method.

III. SIMULATION RESULTS AND DISCUSSIONS

For simulations, we assume that both the source and relay use turbo codes with identical, rate $1/2$ recursive convolutional component codes, but employ different interleavers for their codes. For these results, the source performs no puncturing on parity bits. Therefore, the code rate of the source is $1/3$. We also consider a message block length of $K = 1000$ bits. Hence, a codeword of $N = 3000$ bits is broadcasted by the source. We consider 4 iterations for both the joint decoder at the destination and the turbo decoder at the relay. We identify the type of the DCF-SC method by the trade-off factor, f, defined in (2). As explained in section II by selecting a small value of f (close to zero), the relay dedicates a small amount of time to listening and a larger amount to transmit to the destination. Particularly, when $f = 0$, the relay only listens to the information bits; quantizes and encodes them, and transmits the parity bits to the destination. In our simulations, the SNR is defined as the received E_S/N_0 , where E_S is the received signal energy and $N_0/2$ is the variance of the Gaussian noise.

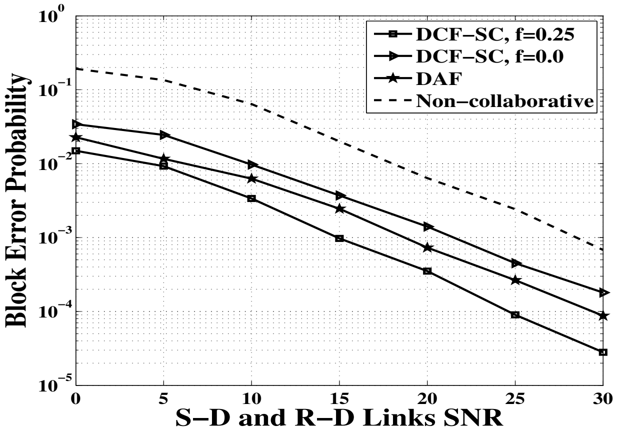

Fig. 1. Error rate for SNR$_{sr}$ $=10dB$ with rate $1/3$ turbo code at the source.

The SNR on the S-D link, $SNR_{SD} = \frac{g_{sd}h^2_{sd}}{2\sigma^2_{sd}}$. We also consider normalized SNRs (normalized by the corresponding channel gain). For instance, we define the normalized SNR on the S-D link as,

$$SNR_{sd} = \frac{SNR_{SD}}{g_{sd}} = \frac{h^2_{sd}}{2\sigma^2_{sd}}. \tag{11}$$

The average SNR on the S-D link is defined as, $\overline{SNR_{sd}} = \frac{E\left[h^2_{sd}\right]}{2\sigma^2_{sd}}$. For Selective-Cooperation at the destination, the instantaneous SNRs (as in (11)) are compared, and given $SNR_{sr} \geq SNR_{sd}$, the relay cooperation is accepted, otherwise the decoder at the destination is switched to non-cooperative mode.

Fig. 1 shows the block error rate (BLER) of the proposed DCF-SC scheme for $f = 0$ and $f = 0.25$ in comparison with the DAF scheme. The average normalized SNR on the S-R is assumed to be 10dB, and channel gains on S-D, S-R and R-D links are set to 0dB, 5dB, and 10dB, respectively. Component convolutional codes have generator polynomials $G(D) = \frac{(1+D^2)}{(1+D+D^2)}$. For the DAF scheme, the a priori LLRs are calculated [[5], eq. (19)] and then turbo decoding is applied to decode the source signal. Note that calculation of a priori LLRs for the DAF scheme requires knowledge of the effective SNR of the block-fading channel (i.e., γ_l in [5], eq. (19)). This effective SNR will be measured by the relay and transmitted to the destination as a side information. Similarly in DCF-SC, the SNR_{sr} is measured and transmitted as a side information to the destination. In both cases, transmission of such side information requires extra resources, however, we assume that for large block lengths the payload introduced by such side information is negligible. From Fig. 1, we observe that DCF-SC with $f = 0.25$ offers approximately $4dB$ gain compared to DAF at a BLER of 10^{-4}. This SNR gain is due to the joint decoding algorithm of the DCF-SC (discussed in Section II) which enhances the error correction capability of the system. Also from Fig. 1 we observe that the DAF scheme outperforms the DCF-SC when $f = 0$, indicating an optimum trade-off parameter for the proposed DCF-SC scheme. In fact, through simulations (see Fig 2), we noted that $f = 0.25$ is an optimal value for which the BLER of DCF-SC scheme is minimized.

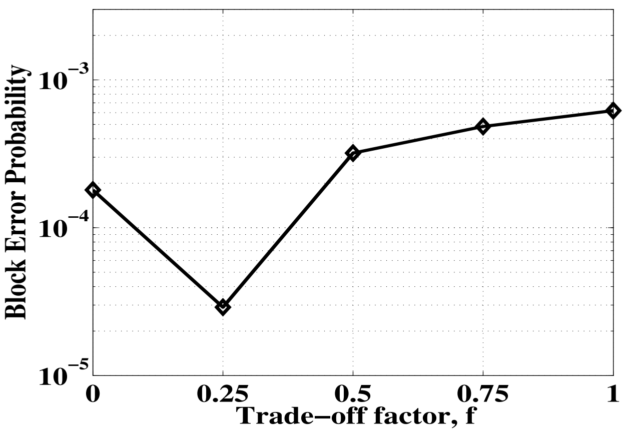

Fig. 2. Trade-off factor f, SNR$_{sr}$= $10dB$ and SNR$_{sd}$=SNR$_{rd}$=30dB.

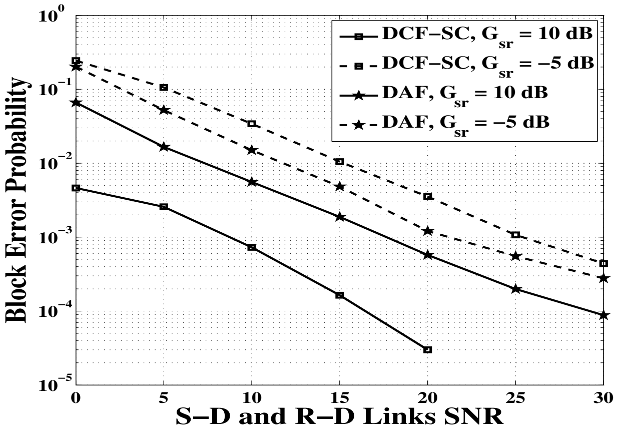

Fig. 3. Block error probability with $g_{sd} = 0dB$, and $g_{rd} = 10dB$. The gain on source-relay channel is either $-5dB$ (dashed) or $10dB$ (solid).

The optimal trade-off value can be intuitively explained as follows: assume that the channel capacity of the S-R channel is fixed. Now, if the capacity of the R-D channel is low, it is more effective for the relay to start transmission as soon as it has received a minimum amount of information from the source (i.e. signifying the case f close to zero). This is due to the large amount of time needed for the relay to reliably transmit the information over the low-capacity R-D link. In this case, when using an FEC code, one selects a low-rate code. On the other hand, when the channel capacity of the R-D link is large, the relay can continue listening to the source for a longer period of time to improve its estimate (i.e. increase the amount of information it has about the message). Since the capacity of the R-D channel is high, the achievable rate for error-free communication over the R-D channel is also high, and the relay can reliably communicate the information at hand to the destination in a short period of time.

Fig. 2 shows the block error rate for the same setup as in Fig. 1, with different values of f when the average normalized SNR on the S-D, and the R-D links is set to $30.0dB$. Note that, setting $f = 1$ has no practical meaning, however, as f approaches one, the scheme behaves similar to a non-cooperative scheme (since the relay dedicates little time to transmission). Therefore, we let $f = 1$ represent a non-cooperative scheme. From Fig. 2, one can see that when f=0.25, the block error rate is minimized.

In Fig. 3, we examine the performance of the DCF-SC scheme when utilizing 3GPP turbo codes with 8-state convolutional codes and $G(D) = \frac{(1+D+D^3)}{(1+D^2+D^3)}$. We compare DAF and DCF-SC under different channel gains, where we consider different channel gains for the S-R link (-5dB and 10dB). The channel gains of the remaining links are the same as in Fig. 1. From Fig. 3 we observe that when the S-R channel gain is 10dB, the DCF-SC outperforms DAF by a considerable margin (about 8dB gain at BLER = 10^{-3}). However, when the S-R channel gain drops to -5dB, DAF outperforms DCF-SC by about 3dB at a BLER = 10^{-3}. This is expected as DAF is able to take advantage of the decoded signal at the relay, with no error propagation at low S-R SNRs. On the other hand, when the S-R channel gain is high, the probability of erroneous decoding reduces and hence DCF-SC outperforms DAF.

IV. CONCLUSION

We proposed a Decode-Compress-and-Forward with Selective-Cooperation (DCF-SC) scheme where the relay performs SISO decoding on the signal received from the source, followed by quantization of LLRs, encoding, and retransmission to the destination. We characterized the DCF-SC scheme by a trade-off factor that determines the trade-off between the listening time and the transmission time of the relay. We showed through simulations that this trade-off factor has an optimal value for which the block error rate is minimized. We provided simulation results for half-duplex relaying with orthogonal channels at the destination and compared the error performance of our DCF-SC scheme with that of the DAF scheme. Our results show that when the channel gain of the S-R link is low, DAF performs better than DCF-SC; however, for higher S-R channel gains, DCF-SC outperforms DAF.

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