Relay Selection for Coded Cooperative Networks with Outdated CSI over Nakagami-*m* Fading Channels

Jules Merlin Moualeu, *Member, IEEE,* Walaa Hamouda, *Senior Member, IEEE,* and Fambirai Takawira, *Member, IEEE*

***Abstract*—In this paper, we consider relay selection for turbo coded cooperative networks subject to Nakagami-*m* fading when the channel state information (CSI) is known at the receiver but is not necessarily ideal. This non-ideality may be due to a feedback delay caused by the difference between the instantaneous CSI during the transmission and the CSI at the time of relay selection, resulting in outdated CSI phenomena. The impact of the outdated CSI on the proposed scheme is well investigated. A closed-form expression for the exact outage probability is derived as well as its asymptotic expression in the high signal-to-noise (SNR) regime. Moreover, upper bounds on the bit-error rate (BER) are presented and a study of the diversity order reveals that for ideal CSI, full diversity in the number of relays and fading parameters *m* is achieved as opposed to outdated CSI where the achievable diversity is equivalent to the diversity of a coded cooperative network with a single relay.**

***Index Terms*—Coded cooperation, diversity order, error rate, selection relaying, turbo codes.**

I. Introduction

SELECTION relaying is a solution to the inefficient utilization of channel resources since it only requires two orthogonal channels regardless of the number of relays. Relay selection schemes in single-hop wireless relaying networks have extensively been investigated (See [1] – [5] and the references therein). Among these selection techniques, opportunistic relay selection (ORS) [1] is a viable strategy from the implementation point of view, due to its low complexity. In time-varying channels, implementing relay selection may cause frequent relay switchings which can be detrimental to the overall system performance. In [6], the authors used an alternative to ORS to study the rate at which the switching of a selected relay occurs in practice. Moreover, frequent relay switchings may cause synchronization issues due to the repeated initializations of the system each time a relay is selected. Hence, this leads to an increase in implementation complexity and poor system performance. So far, the relay selection schemes presented, have dealt with uncoded cooperative relaying networks. In coded cooperation [7], [8], relay selection is seldom studied in the literature. For example in [9], [10], Elfituri *et al.* studied an antenna/relay selection in coded cooperation using convolutional codes. In general, CSI is used to accomplish relay selection and is assumed to be known at the receiver.

The aforementioned studies consider an ideal CSI for relay selection which is impractical in real scenarios. From a practical point of view, the CSI at the time of transmission may be outdated due to a delayed feedback since the relay selection and data transmission instants can differ due to channel conditions. In recent works, the effect of outdated CSI on relay selection has been investigated (for e.g., [11 – [17]). In [11], the authors analyzed the behavior when CSI is subject to delay in the feedback channel for a decode-and-forward (DF)-based protocol over Rayleigh fading channels. In [12], the authors investigated the impact of outdated CSI for relay selection in AF cooperative relaying under both partial and opportunistic relay selection schemes. Chen *et al.* [13] proposed a novel multiple relay selection (MRS) scheme to combat the severe diversity loss. Both amplify-and-forward (AF) and DF are considered in which the N best relays are opportunistically selected under outdated CSI. In [14], partial relay selection (PRS) and best relay selection with delayed CSI are studied for AF wireless relaying. The work in [15] investigated the impact of channel estimation errors and feedback delay in DF with relay selection and Suraweera *et al.* [16] analyzed the effect of outdated CSI on the performance of AF with the k^{th} worst partial relay selection. In [18], the authors considered an AF cooperative system with direct transmission and analyzed the Shannon capacity of the proposed scheme under outdated CSI. The performance analysis of the fixed-gain AF relay systems with interference and thermal noise at the relay and destination was studied in [19]. However, in the above-mentioned works the impact of outdated CSI on AF/DF cooperative networks has been investigated for uncoded systems. So far, no work in the literature considers the effects of outdated CSI on coded cooperative relaying networks. Moreover, the impact of outdated CSI for cooperative relaying systems have been investigated in fewer works under more general fading scenarios such as the Nakagami-*m* fading (See [20] – [22]). However, these studies were undertaken for uncoded systems. In [20], the authors considered an uncoded DF cooperative system and derived the exact closed-form expression of the outage probability as a function of the correlation between the estimated

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and actual channel values. The work in [21] evaluated the transmission quality of a channel under Nakagami-*m* fading with both fading statistics and outdated CSI. Ferdinand *et al.*[22] investigated the uncoded AF with PRS over Nakagami-*m* fading channels with a feedback delay for both variable-gain and fixed-gain relay. However, [20] and [21] only studied the outage probability and no BER analysis was investigated. In [23], Wang *et al.* considered a multi-hop system in order to address the issue of multi-user interference and design efficient transmission. In their work, they studied the sum degrees of freedom (DoF) of a system in which a K-antenna source intends to communicate with K destination nodes through multiple layers each consisting of K full-duplex single-antenna relays with delayed channel state information at transmitters (CSIT). It was shown that by treating the multi-hop multiple-input multiple-output (MIMO) broadcast network as an entity, better maximum multiplexing gain could be achieved over the cascade approach with individual single-hop. However, outdated CSIT can be detrimental to the DoF gain. In [24], it was shown that by properly designing an interference alignment (IA) scheme, the maximum multiplexing gain achieving the sum DoF of a K-user MIMO broadcast channel could be attained with outdated CSI. In [25], the authors proposed a novel IA scheme at the relays that combines perfect delayed CSIT and imperfect instantaneous CSIT to achieve higher sum DoF for a two-hop multiple-input single output (MISO) system. In [26], the authors investigated the design of efficient network codes for multi-user multi-relay wireless networks with slow fading channels. A non-binary network code construction based on maximum distance separable (MDS) codes was proposed in order to achieve maximum diversity order for an arbitrary number of sources and relays. Xiao and Skoglund [27] proposed a multi-user cooperative wireless networking system based on linear network codes. The use of diversity network codes (DNCs) was also proposed in order to exploit in an efficient manner the diversity available by time-varying fading and cooperation. Eid *et al.* [28] studied a multi-relay coded cooperation for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems over slow fading channels. They showed that by suppressing multi-user interference at the cooperative end, the full benefits of coded cooperation can be achieved.

To the best of our knowledge, the study of relay selection in coded cooperation taking into account outdated CSI over Nakagami-*m* fading channels is not presently available in the literature. To fill this void, we investigate the impact of outdated CSI for relay selection in turbo-coded cooperation over Nakagami-*m* fading channels. We consider a turbo coded system with a transmission scheme different from the traditional turbo coded system. We derive a closed-form solution of the outage probability for the scheme under study over Nakagami-*m* fading channels with any fading figure. Furthermore, a high-SNR approximation of the exact outage probability is derived to evaluate the diversity order and is shown to be dependent on the correlation factor ρ. The system performance of the proposed system is also investigated through the pairwise error probability (PEP) analysis. A closed-form expression of the PEP for all values of the fading parameter *m* (integer and non-integer) is provided and its asymptotic expression at high SNR is examined to assess the diversity order of the scheme for ideal and delayed CSI. We then provide a BER performance analysis obtained by using the transfer function bounds method and the limit-before-average-technique.

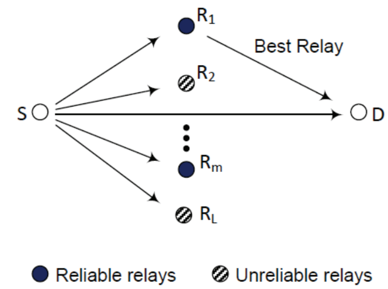

Fig. 1. Cooperative system model.

The remainder of the paper is organized as follows. In Section II, we describe the system model. In Section III, we analyze the outage probability of the proposed scheme followed by the diversity analysis in Section IV. In Section V, the BER analysis is presented. This includes the derivation of the closed-form expression of the PEP and its bahaviour at high SNR. In Section VI, the diversity-multiplexing tradeoff (DMT) of the underlying scheme is discussed. Numerical results are presented in Section VII. Finally conclusions are drawn in Section VIII.

II. System Model

We consider the cooperative relaying system illustrated in Fig. 1, which consists of a single source node s communicating with a single destination node d with the help of L relay nodes denoted by r_i, $i = 1, \cdots, L$. The relays operate in half-duplex mode, i.e., they cannot receive and transmit simultaneously. All nodes are equipped with a single transmit and/or receive antenna. It is also assumed that the fading coefficients in the source-to-relay (s–r) and relay-to-destination (r–d) links are independent and identically distributed (i.i.d.), hence the subscript i denoting the relay number can be left out in the subsequent analysis.

The cooperative transmission takes place in two phases. During the first phase, the source encodes a *K*-bit message by a turbo code of rate R_c and broadcasts the generated codeword of length N to the relays and destination. We assume that the destination listens to the entire codeword whereas the relays only listen to a fraction of the entire codeword [1].

We consider that the relays only listen to a noisy version of the systematic bits before attempting to decode. The received signals at the destination and the i^{th} relay are given respectively by

$$y_{sd}(1:N) = \sqrt{P_s} h_{sd} x(1:N) + n_{sd}(1:N), \quad (1)$$

$$y_{sr_i}(1:K) = \sqrt{P_s} h_{sr_i} x(1:K) + n_{sr_i}(1:K), \quad (2)$$

where r_i represents the i^{th} relay, the vector $x(1:N)$={$x(1)$, $\cdots$, $x(K)$, $x(K+1)$, $\cdots$, $x(N)$} is binary phase shift keying

[1]In [29], an approach in which pre-defined phase durations for broadcast and cooperation commonly presented in the literature is proposed. In this approach, the time allocated for the relay to listen could vary depending on the relay received channel. Hence, in practical scenarios, it is possible that relays only listen to a fraction of the codeword whereas the destination listens to the entire codeword.

(BPSK) modulated, with $x(1:K)$ denoting the systematic bits and $x(K+1:N)$ representing the parity bits, h_{sd} and h_{sr_i} are the Nakagami-*m* fading coefficients for the source-destination and source-relays links respectively, with unit variance, P_s is the transmit power at the source for the $s-d$ and $s-r_i$ links, $n_{sd}(1:N)$ and $n_{sr_i}(1:K)$ represent the i.i.d. additive white Gaussian noise (AWGN) modeled as $\mathcal{CN}(0, N_0/2)$.

All the relays employ a turbo iterative decoder to estimate the information sent by the source and a cyclic redundancy check (CRC) code for error detection. In the second phase, the relay with the highest relay-destination instantaneous SNR belonging to the decoding set (the set of relays that have successfully decoded the source message), is selected to forward the parity bits to the destination. The received signal at the destination is given by

$$y_{r_kd}(K+1:N) = \sqrt{P_r}h_{r_kd}\hat{x}(K+1:N)+n_{r_kd}(K+1:N), \tag{3}$$

where the vector $\hat{x}(K+1:N)$ denotes the estimated parity bits, h_{r_kd} is the fading coefficient for the best relay-destination link and is subjected to a Nakagami-*m* distribution with unit variance, P_r is the transmit power at the best relay node, $n_{r_kd}(K+1:N) \sim \mathcal{CN}(0, N_0/2)$ and

$$k = \arg\max_{j\in\mathcal{D}(s)} \{\hat{\gamma}_{r_jd}\}, \tag{4}$$

where $\mathcal{D}(s)$ represents the decoding set and $\hat{\gamma}_{r_jd}$ is the instantaneous SNR at the selection instant, which can be different from the actual SNR γ_{r_jd} (used for retransmission) due to the time delay in the feedback channel. By definition, $\hat{\gamma}_{r_jd} = |\hat{h}_{r_jd}|^2\bar{\gamma}$, where $\hat{h}_{r_jd}$ is an outdated version of h_{rd}. Both $\hat{h}_{r_jd}$ and h_{r_jd} are jointly Gaussian RV, and according to [11], h_{r_jd} conditioned on $\hat{h}_{r_jd}$ follows a Gaussian distribution given by

$$h_{r_jd}|\hat{h}_{r_jd} \sim \mathcal{CN}\left(\rho\hat{h}_{r_jd}, \sqrt{1-\rho^2}\right), \tag{5}$$

where ρ is the correlation factor between $\hat{h}_{r_jd}$ and h_{r_jd}, modeled according to Jakes' autocorrelation given by [30]

$$\rho = J_0\left(2\pi f_{d,r_jd}T_d\right), \tag{6}$$

with J_0 denoting the zeroth order Bessel function of the first kind, f_{d,r_jd} is the maximum Doppler frequency on the r_j-d link, and T_d is the time difference between the actual channel value and its estimate.

III. Outage Probability Analysis

Outage probability represents an important performance metric in wireless communications and is defined as the probability that an instantaneous capacity C falls below a target rate R_c. It can be mathematically formulated as

$$\begin{aligned} P_{out} &= \Pr\{C(\gamma) < R_c\} = \Pr\{\log_2(1+\gamma) < R_c\} \\ &= \int_0^{2^{R_c}-1} p(\gamma)\mathrm{d}\gamma, \end{aligned} \tag{7}$$

where γ and $\Pr\{x\}$ denote the instantaneous SNR and the probability of x respectively and $p(x)$ denotes the probability density function (PDF) of x.

In the proposed scheme, the destination listens to the entire codeword transmitted by the source with a code R_c. The relays listen only to the systematic bits sent by the source with a code rate $R_1 = \frac{R_c}{\alpha}$ and only the best relay forward to the destination with a code rate $R_2 = \frac{R_c}{1-\alpha}$, where $\alpha = K/N$ represents the cooperation ratio. The end-to-end outage probability can be represented by

$$\begin{aligned} P_{out} =& \Pr\left\{\gamma_{sd} < 2^{R_c}-1\right\}\left(\Pr\left\{\gamma_{sr} < 2^{R_c/\alpha}-1\right\}\right)^L \\ &+ \sum_{\vartheta=1}^{L}\binom{L}{\vartheta}\left(\Pr\left\{\gamma_{sr} < 2^{R_c/\alpha}-1\right\}\right)^{L-\vartheta} \\ &\times\left(\Pr\left\{\gamma_{sr} > 2^{R_c/\alpha}-1\right\}\right)^{\vartheta} \\ &\times \Pr\left\{\left(1+\gamma_{sd}\right)\left(1+\gamma_{rd}\right)^{1-\alpha} < 2^R\right\}, \end{aligned} \tag{8}$$

where $\vartheta = |\Theta|$ with Θ representing the set of indices of cooperating relays given by $\Theta = \{j_1, j_2, \cdots, j_\vartheta\} \subset \{1, 2, \cdots, L\}$ and $|x|$ denotes the the cardinality of x. The first part of (8) represents $\mathcal{D}(s) = \emptyset$ (with $\emptyset$ denoting the empty set), i.e., all the relays in network are unreliable, whereas the second expression of (8) represents the case where at least one relay is reliable.

Following (7), the expression in (8) can be rewritten as

$$\begin{aligned} P_{out} =& \left(\int_0^{2^{R_c}-1} p(\gamma_{sd})\mathrm{d}\gamma_{sd}\right)\left(\int_0^{2^{R_c/\alpha}-1} p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^L \\ &+ \sum_{\vartheta=1}^{L}\binom{L}{\vartheta}\left(\int_0^{2^{R_c/\alpha}-1} p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^{L-\vartheta} \\ &\times\left\{\left(1-\int_0^{2^{R_c/\alpha}-1} p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^{\vartheta}\right. \\ &\times\left.\underbrace{\int_0^a\int_0^b}_{\mathcal{A}} p(\gamma_{sd})p(\gamma_{rd})\mathrm{d}\gamma_{rd}\mathrm{d}\gamma_{sd}\right\}, \end{aligned} \tag{9}$$

where $p(\gamma_{sd})$ and $p(\gamma_{sr})$ denote the PDF of the RVs γ_{sd} and γ_{sr}, respectively. For Nakagami-*m* fading channels, $p(\gamma_{ij}) = \frac{m_{ij}^{m_{ij}}\gamma^{m_{ij}-1}}{\Gamma(m_{ij})\bar{\gamma}^{m_{ij}}}\exp\left(-\frac{m_{ij}\gamma_{ij}}{\bar{\gamma}_{ij}}\right)$ where $\{i,j\} \in \{s,(r,d)\}$, and $\mathcal{A}$ corresponds to the region of integration given by

$$\mathcal{A}\left\{(\gamma_{sd},\gamma_{rd})|\gamma_{sd}\geq 0, \gamma_{rd}\geq 0, (1+\gamma_{sd})\left(1+\gamma_{rd}\right)^{1-\alpha} < R_c\right\}. \tag{10}$$

Considering (10), we can rewrite the constraint $\mathcal{A}$ as $\gamma_{r_kd} = f\left(\gamma_{sd}\right)$ given by

$$\gamma_{rd} < \frac{2^{R_c/(1-\alpha)}}{\left(1+\gamma_{sd}\right)^{1/(1-\alpha)}} - 1 \triangleq b. \tag{11}$$

Since $\gamma_{rd} > 0$, we can easily obtain

$$\gamma_{sd} < 2^{R_c} - 1 \triangleq a. \tag{12}$$

However, the PDF of γ_{rd}, i.e., $p_{\gamma_{r_kd}}(\gamma_{rd})$ is not straighforward since it involves relay selection based on outdated CSI and can be obtained using the cumulative density function (CDF) derived in [20] by using the fact $p(x) = \partial\mathrm{F}(x)/\partial x$

$$p_{\gamma_{r_k d}}(\gamma_{rd}) = \vartheta \sum_{n=0}^{\infty} \frac{\rho^n m_{rd}^{m_{rd}+n} \gamma_{rd}^{m_{rd}+n-1} \bar{\gamma}^{-(m_{rd}+n)}}{n!(1-\rho)^{m_{rd}+2n} \Gamma(m_{rd}) \Gamma(m_{rd}+n)} \times \exp\left(-\frac{m_{rd}\gamma_{rd}}{(1-\rho)\bar{\gamma}}\right) \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \times \chi(m_{rd}+n, \rho, j), \tag{13}$$

where

$$\chi(m_{rd}+n,\rho,j) = \frac{\Gamma(j+1)}{\left(\frac{1}{1-\rho}+j\right)^{m_{rd}+n}} \times \sum_{\substack{\tau_0=\tau_1=\cdots=\tau_{m_{rd}-1}=0 \\ \tau_0+\tau_1+\cdots+\tau_{m_{rd}-1}=j}} \prod_{i=0}^{m_{rd}-1} \frac{\left(\frac{1}{i!\left(\frac{1}{1-\rho}+j\right)^i}\right)^{\tau_i}}{\Gamma(\tau_i+1)} \times \left(m_{rd}+n-1+\sum_{i=0}^{m_{rd}-1} i\tau_i\right)! \tag{14}$$

Substituting (13) in (9), the resulting outage probability expression P_{out} also contains a double integral as in (9), that is not easy to evaluate. In what follows, we evaluate the double integral expression (contained in the new outage probability expression after substituting (13) in (9)) denoted by I_d which can be written as

$$I_d = \vartheta \left(\int_0^a \int_0^b \frac{m_{sd}^{m_{sd}} \gamma^{m_{sd}-1}}{\Gamma(m_{sd}) \bar{\gamma}^{m_{sd}}} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}}\right) \times \sum_{n=0}^{\infty} \frac{\rho^n m_{rd}^{m_{rd}+n} \gamma_{rd}^{m_{rd}+n-1}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}} \times \gamma_{rd}^{m_{rd}+n-1} \exp\left(-\frac{m_{rd}\gamma_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right) \mathrm{d}\gamma_{rd} \mathrm{d}\gamma_{sd} \right) \times \sum_{j=0}^{\vartheta-1} (-1)^j \chi(m_{rd}+n,\rho,j). \tag{15}$$

After some manipulations and evaluating the inner integral using [31, Eq. 3.351.1], we obtain the following expression shown at the top of the next page.

In order to evaluate the integral in (16), the alternative form of the lower incomplete Gamma function given in [31, Eq. 8.352.4] by $\gamma(a,x) = (a-1)!\left(1-\exp(-x)\sum_{m=0}^{n-1}\frac{x^m}{m!}\right)$ can be used and the expression in equation (16) can be rewritten as shown in equation (17) on the next page.

The expression in (17) can be expanded using the binomial expansion $(1-y)^m = \sum_{k=0}^{m} \binom{m}{k} (-1)^k y^k$, to give (in (18) as shown on the next page) where we define $\nu = \frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}$ and $\beta = \frac{1}{1-\alpha}$. In (18), the inner integral can be evaluated using [31, Eq. 3.351.1], whereas the outer integral can be rearranged as shown in (19).

Examining the exponential function in (19), one can rewrite it as shown in (20).

After substituting (20) in (19) and performing some manipulations, (19) can further be expressed as in (21).

Finally, using [31, Eq. 3.194.1] in (21) and substituting in (18), the outage probaility is given by (22), where ${}_2F_1(w,x;y;z)$ is the Gauss Hypergeometric function defined in [31, Eq. 9.111]. The derived outage probability in (22) includes a series form (infinite series) that is not practical for numerical computation. Through some numerical simulations, it is noted that the derived expression converges after a finite number of terms.

IV. Diversity Analysis

In order to find useful insights into the diversity order d_o of our scheme, an easy to analyze expression of the outage probability provided in (22) is needed. In what follows, an expression of the asymptotic approximation of the outage probability is derived. We consider that $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma} \to \infty$ and without loss of generality, we assume that $m_{sr_j} = m_{sr}$, $m_{r_j d} = m_{rd}$ since the fading in all the respective links is i.i.d.

At high SNR, the outage probability in (8) can be reduced to

$$P_{out} \approx \sum_{\vartheta=1}^{L} \Pr\left\{ \left(1+\gamma_{sd}\right)\left(1+\gamma_{rd}\right)^{1-\alpha} < 2^R \right\}. \tag{23}$$

Corollary 1: The diversity gain of the coded cooperative system with outdated CSI over Nakagami-*m* fading is given by

$$d_o = \begin{cases} m_{sd}+m_{rd}, & \text{if } \rho < 1 \\ m_{sd}+Lm_{rd}, & \text{if } \rho = 1 \end{cases} \tag{24}$$

Proof: The proof is provided in the Appendix. ■

V. Upper Bounds on the Bit Error Probability

In this section, we derive upper bounds on the BER for the proposed scheme under outdated CSI. To this end, we first evaluate an exact expression for the unconditional PEP used to derive the BER expression of a coded system. It is worth noting that it is possible to get some insight of the system through the PEP analysis. In addition, we obtain asymptotic an expression of the derived PEP at high SNR to determine the diversity order of the scheme under consideration.

A. Pairwise Error Probability

Assuming slow fading channel and perfect knowledge of the CSI at the receivers with the effects of delayed feedback, the end-to-end conditional PEP is given by

$$P(d_H|\gamma_{sd},\gamma_{sr},\gamma_{r_k d}) = Q\left(\sqrt{2d_H\gamma_{sd}}\right)\left(Q\left(\sqrt{2d_1\gamma_{sr}}\right)\right)^L + \sum_{\vartheta=1}^{L}\binom{L}{\vartheta}\left(Q\left(\sqrt{2d_1\gamma_{sr}}\right)\right)^{L-\vartheta}\left(1-Q\left(\sqrt{2d_1\gamma_{sr}}\right)\right)^{\vartheta} \times Q\left(\sqrt{2d_H\gamma_{sd}+2d_2\gamma_{rd}}\right), \tag{25}$$

where $Q(\bullet)$ denotes the Gaussian *Q*-function, $d_H = d_1 + d_2$ is the Hamming distance between the transmitted codeword $\mathbf{c}$ and the erroneous codeword $\tilde{\mathbf{c}}$, d_1 is the Hamming distance corresponding to the frame in the $s-r$ links and d_2 is the Hamming distance corresponding to the frame in the $r_k - d$ link. Moreover, similar to (8), the first term in (25) represents the case where all relays are unreliable, whereas the second term indicates that at least one relay is reliable.

$$
\begin{aligned}
I_d = &\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}\Gamma(m_{sd})\bar{\gamma}_{sd}} \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^{-m_{rd}-n} \\
&\times \left(\int_0^{2^{R_c}-1} \gamma\left(m_{rd}+n, \frac{2^{R_c/(1-\alpha)} m_{rd}}{(1-\rho)(1+\gamma_{sd})^{1/(1-\alpha)}\bar{\gamma}_{rd}} - \frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right) \gamma_{sd}^{m_{sd}-1} \right. \\
&\left. \times \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}}\right) \mathrm{d}\gamma_{sd} \right) \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \chi(m_{rd}+n,\rho,j).
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
I_d = &\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}\Gamma(m_{sd})\bar{\gamma}_{sd}} \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^{-m_{rd}-n} \\
&\times \left(\int_0^{2^{R_c-1}} (m_{rd}+n-1)! \left[1 - \exp\left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}} \left(1 - \frac{2^{R_c/(1-\alpha)}}{(1+\gamma_{sd})^{1/(1-\alpha)}}\right)\right) \sum_{m=0}^{m_{rd}+n-1} \frac{1}{m!} \right.\right. \\
&\left.\left. \times \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^m \left(\frac{2^{R_c/(1-\alpha)}}{(1+\gamma_{sd})^{1/(1-\alpha)}}\right)^m \right] \mathrm{d}\gamma_{sd} \right) \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \chi(m_{rd}+n,\rho,j).
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
I_d = &\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}\Gamma(m_{sd})\bar{\gamma}_{sd}} \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^{-m_{rd}-n} \left[\int_0^{2^{R_c-1}} \Gamma(m_{rd}+n) \right. \\
&\times \gamma_{sd}^{m_{sd}-1} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}}\right) \mathrm{d}\gamma_{sd} - \int_0^{2^{R_c}-1} \Gamma(m_{rd}+n) \gamma_{sd}^{m_{sd}-1} \exp\left(\nu\left(1 - \frac{2^{R_c\beta}}{(1+\gamma_{sd})^{\beta}}\right)\right) \\
&\left. \times \sum_{k=0}^{m_{rd}+n-1} \frac{\nu^k}{k!} \sum_{l=0}^{k} \binom{k}{l} (-1)^{l-k} \frac{2^{R_c l\beta}}{(1+\gamma_{sd})^{l\beta}} \mathrm{d}\gamma_{sd} \right] \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \chi(m_{rd}+n,\rho,j),
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
I_d = &\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}\Gamma(m_{sd})\bar{\gamma}_{sd}} \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^{-m_{rd}-n} \left[\Gamma(m_{rd}+n) \right. \\
&\times \left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{-m_{sd}} \gamma\left(m_{sd}, \frac{m_{sd}}{\bar{\gamma}_{sd}}\left(2^{R_c}-1\right)\right) - \Gamma(m_{rd}+n)\exp(\nu) \sum_{k=0}^{m_{rd}+n-1} \sum_{l=0}^{k} \frac{\nu^k}{k!} \binom{k}{l} (-1)^{l-k} 2^{R_c l\beta} \\
&\left. \times \int_0^{2^{R_c}-1} \frac{\gamma_{sd}^{m_{sd}-1}}{(1+\gamma_{sd})^{l\beta}} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}} - \frac{2^{R_c\beta}}{(1+\gamma_{sd})^{\beta}}\right) \mathrm{d}\gamma_{sd} \right] \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \chi(m_{rd}+n,\rho,j),
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}} - \frac{2^{R_c\beta}}{(1+\gamma_{sd})^{\beta}}\right) &= \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \left(\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}_{sd}} + \frac{2^{R_c\beta}}{(1+\gamma_{sd})^{\beta}}\right)^r \\
&= \sum_{r=0}^{\infty} \sum_{s=0}^{r} \frac{2^{R_c\beta s}(-1)^r}{r!} \binom{r}{s} \left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{r-s} \frac{\gamma_{sd}^{r-s}}{(1+\gamma_{sd})^{\beta s}}.
\end{aligned} \tag{20}
$$

Using the alternative representation of the Gaussian Q-function [32] in (25) and averaging over the fading distribution, and after some manipulations with the aid of the moment generating function (MGF) defined in [33], yields a closed-form expression of the end-to-end unconditional PEP for integer values of the fading figure m. To derive an exact expression for the average PEP for both integer and non-integer values of fading parameter m, we use an accurate and simple approximate expression given by the sum of two exponential functions, in which $\mathrm{erfc}(x) \approx \frac{1}{6}\exp\left(-x^2\right) + \frac{1}{2}\exp\left(-\frac{4}{3}x^2\right)$ [34]. Therefore, the resulting Gaussian Q-function is given by

$$
Q(x) = \frac{1}{12}\exp\left(-\frac{x^2}{2}\right) + \frac{1}{4}\exp\left(-\frac{2}{3}x^2\right). \tag{26}
$$

$$I_d = \sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{rd}+n)\bar{\gamma}^{m_{rd}+n}\Gamma(m_{sd})\bar{\gamma}_{sd}} \left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^{-m_{rd}-n} \Bigg[\Gamma(m_{rd}+n) \times \left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{-m_{sd}} \gamma\left(m_{sd}, \frac{m_{sd}}{\bar{\gamma}_{sd}}\left(2^{R_c}-1\right)\right) - \Gamma(m_{rd}+n)\exp(\nu) \sum_{k=0}^{m_{rd}+n-1} \sum_{l=0}^{k} \frac{\nu^k}{k!}\binom{k}{l}(-1)^{l-k} 2^{R_c l \beta} \times \sum_{r=0}^{\infty}\sum_{s=0}^{r} \frac{2^{R_c \beta s}(-1)^r}{r!}\binom{r}{s}\left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{r-s} \int_0^{2^{R_c}-1} \frac{\gamma_{sd}^{msd+r-s-1}}{(1+\gamma_{sd})^{\beta(l+s)}} \mathrm{d}\gamma_{sd}\Bigg] \sum_{j=0}^{\vartheta-1}\binom{\vartheta-1}{j}(-1)^j \times \chi(m_{rd}+n,\rho,j). \tag{21}$$

$$P_{out} = \frac{1}{\Gamma(m_{sd})\Gamma^L(m_{sr})}\left[\gamma\left(m_{sd}, \frac{m_{sd}}{\bar{\gamma}_{sd}}(2^{R_c}-1)\right)\right]\left[\gamma\left(m_{sr}, \frac{m_{sr}}{\bar{\gamma}_{sr}}(2^{R_c/\alpha}-1)\right)\right]^L + \sum_{\vartheta=1}^{L}\binom{L}{\vartheta}\frac{1}{\Gamma^L(m_{sr})}\left[\gamma\left(m_{sr}, \frac{m_{sr}}{\bar{\gamma}_{sr}}(2^{R_c/\alpha}-1)\right)\right]^{L-\vartheta}\left[\gamma\left(m_{sr_j}, \frac{m_{sr_j}}{\bar{\gamma}_{sr_j}}(2^{R_c/\alpha}-1)\right)\right]^{\vartheta} \times \sum_{n=0}^{\infty}\frac{\vartheta\rho^n m_{rd}^{m_{rd}+n} m_{sd}}{n!(1-\rho)^{m_{rd}+2n}\Gamma(m_{rd})\Gamma(m_{sd})\bar{\gamma}_{rd}\bar{\gamma}_{sd}}\left(\frac{m_{rd}}{(1-\rho)}\bar{\gamma}_{rd}\right)^{-m_{rd}-n} \times \left[\left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{-m_{sd}}\gamma\left(m_{sd}, \frac{m_{sd}}{\bar{\gamma}_{sd}}(2^{R_c}-1)\right) - \exp\left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right) \times \sum_{i=0}^{m_{rd}+n-1}\sum_{l=0}^{i}\sum_{r=0}^{\infty}\sum_{s=0}^{r}\sum_{j=0}^{M-1}\frac{1}{i!}\left(\frac{m_{rd}}{(1-\rho)\bar{\gamma}_{rd}}\right)^i\binom{i}{l}(-1)^l 2^{R_c/(1-\alpha)}\frac{(-1)^r}{r!}\binom{r}{s}\left(\frac{m_{sd}}{\bar{\gamma}_{sd}}\right)^{r-s} \times \frac{2^{\frac{R_c s}{1-\alpha}}\left(2^{R_c}-1\right)^{m_{sd}-r-s}}{m_{sd}-r-s}\,{}_2F_1\left(\frac{l+s}{1-\alpha}, m_{sd}-r-s; m_{sd}-r-s+1; 1-2^{R_c}\right)\right] \times \binom{\vartheta-1}{j}(-1)^j\chi(m_{rd}+n,\rho,j), \tag{22}$$

$$P(d_H) = \left(\int_0^\infty \left[\frac{1}{12}\exp\left(-d_H\gamma_{sd}\right) + \frac{1}{4}\exp\left(-\frac{4}{3}d_H\gamma_{sd}\right)\right]p(\gamma_{sd})\mathrm{d}\gamma_{sd}\right)\left(\int_0^\infty \left[\frac{1}{12}\exp\left(-d_1\gamma_{sr}\right) + \frac{1}{4}\exp\left(-\frac{4}{3}d_1\gamma_{sr}\right)\right]p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^L + \sum_{\vartheta=1}^{L}\binom{L}{\vartheta}\left(\int_0^\infty\left[\frac{1}{12}\exp\left(-d_1\gamma_{sr}\right) + \frac{1}{4}\exp\left(-\frac{4}{3}d_1\gamma_{sr}\right)\right]p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^{L-\vartheta}\left(1-\int_0^\infty\left[\frac{1}{12}\exp\left(-d_1\gamma_{sr}\right) + \frac{1}{4}\exp\left(-\frac{4}{3}d_1\gamma_{sr}\right)\right] \times p(\gamma_{sr})\mathrm{d}\gamma_{sr}\right)^{\vartheta}\int_0^\infty\int_0^\infty\left[\frac{1}{12}\exp\left(-d_H\gamma_{sd}-d_2\gamma_{rd}\right) + \frac{1}{4}\exp\left(-\frac{4}{3}(d_H\gamma_{sd}+d_2\gamma_{rd})\right)\right]p(\gamma_{sd}) \times p(\gamma_{rd})\mathrm{d}\gamma_{sd}\mathrm{d}\gamma_{rd}. \tag{27}$$

Using (26) in (25) and averaging over the fading distribution, the average PEP is given by (27).

Substituting (13), $p(\gamma_{sd})$ and $p(\gamma_{sr})$ given in Section III, in (27), and after performing some manipulations, the exact expression of the average PEP is given by (28) where the derived closed-form expression in (28) holds for all values of m_{sd}, m_{sr} and m_{rd}. Furthermore, the average PEP derived in (28) contains some infinite series that are not suited for numerical computation for practical SNRs. The convergence of power series is noted through numerical simulations after a finite number of terms.

In what follows, we derive the asymptotic expression of the average PEP in the high SNR regime. For this analysis, it is worth rewriting the average PEP expression using the aforementioned Craig's formula as (29).

Using the following assumption $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma} \rightarrow$

$$P(d_H) = \frac{m_{sd}^{m_{sd}} m_{sr}^{Lm_{sr}}}{\gamma_{sd}^{m_{sd}} \gamma_{sr}^{Lm_{sr}}} \left(\frac{1}{12} \left(d_H + \frac{m_{sd}}{\bar{\gamma}_{sd}} \right)^{-m_{sd}} + \frac{1}{4} \left(\frac{4d_H}{3} + \frac{m_{sd}}{\bar{\gamma}_{sd}} \right)^{-m_{sd}} \right) \left(\frac{1}{12} \left(d_1 + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} \right.$$
$$\left. + \frac{1}{4} \left(\frac{4d_1}{3} + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} \right)^L + \sum_{\vartheta=1}^{L} \binom{L}{\vartheta} \left(\frac{1}{12} \left(d_1 + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} + \frac{1}{4} \left(\frac{4d_1}{3} + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} \right)^{L-\vartheta}$$
$$\times \left(\frac{1}{12} \left(d_1 + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} + \frac{1}{4} \left(\frac{4d_1}{3} + \frac{m_{sr}}{\bar{\gamma}_{sr}} \right)^{-m_{sr}} \right)^{\vartheta} \sum_{n=0}^{\infty} \frac{\rho^n m_{rd}^{m_{rd}+n}}{n!\,(1-\rho)^{m_{rd}+2n}\, \bar{\gamma}_{rd}^{m_{rd}+n}} \tag{28}$$
$$\times \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} (-1)^j \chi(m_{rd}+n, \rho, j) \frac{m_{sd}^{m_{sd}}}{\bar{\gamma}_{sd}^{m_{sd}}} \left(\frac{1}{12} \left(d_H + \frac{m_{sd}}{\bar{\gamma}_{sd}} \right)^{-m_{sd}} \left(d_2 + \frac{m_{rd}}{(1-\rho)\,\bar{\gamma}_{rd}} \right)^{-m_{rd}-n} \right.$$
$$\left. + \frac{1}{4} \left(\frac{4d_H}{3} + \frac{m_{sd}}{\bar{\gamma}_{sd}} \right)^{-m_{sr}} \left(\frac{4d_2}{3} + \frac{m_{rd}}{(1-\rho)\,\bar{\gamma}_{rd}} \right)^{-m_{rd}-n} \right)$$

$$P(d_H) = \left(\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{d_H \gamma_{sd}}{\sin^2\theta} \right) p(\gamma_{sd}) \mathrm{d}\gamma_{sd} \mathrm{d}\theta \right) \left(\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{d_1 \gamma_{sr}}{\sin^2\theta} \right) p(\gamma_{sr}) \mathrm{d}\gamma_{sr} \mathrm{d}\theta \right)^L$$
$$+ \sum_{\vartheta=1}^{L} \binom{L}{\vartheta} \left(\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{d_1 \gamma_{sr}}{\sin^2\theta} \right) p(\gamma_{sr}) \mathrm{d}\gamma_{sr} \mathrm{d}\theta \right)^{L-\vartheta} \left(1 - \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \exp\left(-\frac{d_1 \gamma_{sr}}{\sin^2\theta} \right) \right. \tag{29}$$
$$\left. \times\, p(\gamma_{sr}) \mathrm{d}\gamma_{sr} \mathrm{d}\theta \right)^{\vartheta} \left(\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \int_0^{\infty} \exp\left(-\frac{d_H \gamma_{sd} + d_2 \gamma_{rd}}{\sin^2\theta} \right) p(\gamma_{sd}) p(\gamma_{rd}) \mathrm{d}\gamma_{sd} \mathrm{d}\gamma_{rd} \mathrm{d}\theta \right).$$

∞, the expression in (29) can be reduced to

$$P(d_H) \overset{\bar{\gamma}\to\infty}{\approx} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \int_0^{\infty} \int_0^{\infty} \exp\left(-\frac{d_H \gamma_{sd} + d_2 \gamma_{rd}}{\sin^2\theta} \right) \times p(\gamma_{sd}) p(\gamma_{rd}) \mathrm{d}\gamma_{sd} \mathrm{d}\gamma_{rd} \mathrm{d}\theta. \tag{30}$$

After some algebraic manipulations and using the approximations $\left(1 + \frac{d_H \bar{\gamma}_{ij}}{m_{ij}}\right)^{-m_{ij}} \approx \left(\frac{d_H \bar{\gamma}_{ij}}{m_{ij}}\right)^{-m_{ij}}$, it is easy to obtain

$$P(d_H) \overset{\bar{\gamma}\to\infty}{\approx} \frac{2m_{rd}}{(1-\rho)^{m_{rd}}\, \Gamma(m_{rd}) \bar{\gamma}^{m_{sd}+m_{rd}}} \left(\frac{d_H}{m_{sd}} \right)^{-m_{sd}} \times \left(d_2 + \frac{m_{rd}}{(1-\rho)\,\bar{\gamma}} \right)^{-m_{rd}} \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} \times (-1)^j \chi(m_{rd}+n, \rho, j). \tag{31}$$

Case 1: $0 \le \rho < 1$

In (31), the following approximation can be made $\left(d_2 + \frac{m_{rd}}{(1-\rho)\bar{\gamma}}\right)^{-m_{rd}} \approx d_2^{-m_{rd}}$. In what follows, the resulting expression corresponds to the high-SNR PEP for $\rho < 1$ and is given by

$$P(d_H) \overset{\bar{\gamma}\to\infty}{\approx} G_{c_1} \bar{\gamma}^{-(m_{sd}+m_{rd})}, \tag{32}$$

where G_{c_1} is given by

$$G_{c_1} = \frac{2m_{rd} m_{sd}^{-m_{sd}}}{(1-\rho)^{rd} d_H^{m_{sd}} d_2^{m_{rd}} \Gamma(m_{rd})} \sum_{j=0}^{\vartheta-1} \binom{\vartheta-1}{j} \times (-1)^j \chi(m_{rd}+n, \rho, j) \tag{33}$$

Case 2: $\rho = 1$

Substituting $\rho = 1$ in (31) cannot hold since $(1-\rho)\,\bar{\gamma}$ yields an indeterminate value. Using the alternative expression for $p(\gamma_{rd})$ as expressed in (47) and after performing some integrations, the expression in (30) can further be expressed as

$$P(d_H) \overset{\bar{\gamma}\to\infty}{\approx} \frac{L}{2} \left(\frac{d_H}{m_{sd}} \right)^{-m_{sd}} \bar{\gamma}^{-m_{sd}} \int_0^{\infty} \frac{m_{rd}^{2m_{sd}}}{\xi^{m_{rd}} \Gamma^2(m_{rd}) \bar{\gamma}^{m_{rd}}} \times \gamma_{rd}^{m_{rd}-1} \exp\left(-d_2 \gamma_{rd} - \frac{m_{rd}\gamma_{rd}}{\xi} \right) \sum_{j=1}^{L-1} \binom{L-1}{j} \times \left(\frac{-1}{\Gamma(m_{rd})} \right)^j \int_0^{\infty} x^{m_{rd}} \exp\left(-\frac{m_{rd} x}{\xi} \right) \times \Gamma^j \left(m_{rd}, \frac{m_{rd} x}{\bar{\gamma}} \right) \mathrm{d}x \mathrm{d}\gamma_{rd}, \tag{34}$$

where $\xi = (1-\rho)\,\bar{\gamma}$. Following the steps similar to those in the Appendix for the case $\rho = 1$, and with some additional manipulations the asymptotic PEP is given by

$$P(d_H) \overset{\bar{\gamma}\to\infty}{\approx} \underbrace{\frac{m_{sd}^{m_{sd}} m_{rd}^{m_{rd}+L-1} L}{2 d_H^{m_{sd}} \Gamma(m_{sd})}}_{G_{c_2}} \bar{\gamma}^{-(m_{sd}+Lm_{rd})}, \tag{35}$$

where G_{c_2} is also a constant.

B. Bit Error Probability

In the sequel, we evaluate the performance of the proposed scheme in terms of BER using the limit-before-average method and the transfer bounds technique. The upper bound on the average BER is given by [36]

$$\bar{P}_b \leq \frac{1}{K} \sum_{d_H=d_f}^{\infty} a(d_H) P(d_H), \tag{36}$$

where d_f denotes the free Hamming distance and $a(d_H)$ is the sum of bit errors for error event of distance d_H and can be obtained using the transfer function bounds method as in [37]

$$a\left(d_H\right) = \underbrace{\sum_{i=1}^{K} \sum_{d_1=1}^{K} \sum_{d_2=1}^{K}}_{d_H=i+d_1+d_2} \frac{i}{K} \binom{i}{K} p\left(d_1|i\right) p\left(d_2|i\right), \tag{37}$$

where $p\left(d_{1/2}|i\right) = t(K, i, d_{1/2})/\binom{K}{i}$, with $t(K, i, d_{1/2})$ obtained from the transfer function of the given turbo code $T\left(J, I, D\right) = \sum_{j\geq 0} \sum_{i \geq 0} \sum_{d\geq 0} J^j I^i D^d t\left(j, i, d\right)$ by a recursive method using MATLAB, $J^j I^i D^d$ is a monomial with j equal to 1, i and d are input and output dependent and take on the value 0 or 1. However, using (36) yields loose bounds on the BER. A remedy is to use the limit-before-average technique [38] that yields much tighter bounds. Hence, the upper bounds on the BER can be expressed as

$$\bar{P}_b \leq \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} \min \left(\frac{1}{2}, \frac{1}{K} \sum_{d=d_f}^{\infty} a(d_H) P(d_H|\gamma_{sd}, \gamma_{sr}, \gamma_{rd}) \times p(\gamma_{sd}) p(\gamma_{sr}) p(\gamma_{rd}) \mathrm{d}\gamma_{sd} \mathrm{d}\gamma_{sr} \mathrm{d}\gamma_{rd} \right). \tag{38}$$

A closed-form expression of (38) is difficult to obtain, since the summation and integration are not interchangeable due to the minimization expression. Therefore, we resort to numerical methods to find the solution to (38).

VI. Diversity-Multiplexing Tradeoff

The diversity-multiplexing tradeoff (DMT) is a fundamental tradeoff in the design of a diversity-achieving wireless communication system. The diversity and multiplexing gains are respectively defined as [35]

$$d_o \doteq \lim_{\bar{\gamma}\to\infty} -\frac{\log\left(P_{out}(\bar{\gamma})\right)}{\log\left(\bar{\gamma}\right)}, \tag{39}$$

$$r \doteq \lim_{\bar{\gamma}\to\infty} \frac{C(\bar{\gamma})}{\log\left(\bar{\gamma}\right)}, \tag{40}$$

where $C(\bar{\gamma})$ is the data rate a given scheme can support given by $C(\bar{\gamma}) = 1 - P_{out}(\bar{\gamma})$ and $\bar{\gamma}$ is the average SNR.

In order to derive the DMT of the underlying scheme at high SNR, it is essential to determine the behavior of the multiplexing gain $r = C(\bar{\gamma}) \log \bar{\gamma}$ at high SNR. It can be noted that at high SNR, $P_{out}(\bar{\gamma})$ tends to zero, hence $r \to 1$. Using *corollary 1* and (40), it is easy to obtain the DMT.

Theorem 2: The diversity-multiplexing gain of the proposed scheme is

$$d_o(r) = \begin{cases} (m_{sd} + m_{rd})(1-r), & \text{if } \rho < 1 \\ (m_{sd} + Lm_{rd})(1-r), & \text{if } \rho = 1 \end{cases} \tag{41}$$

Fig. 2 depicts the DMT tradeoff of the proposed scheme for perfect and outdated CSI. It can be seen that the DMT is invariant regardless of the number of relays for outdated CSI, whereas in the case of perfect CSI, the DMT is a function of the number of relays. Furthermore, in both cases, the DMT is a function of the fading parameters m_{sd} and m_{rd}.

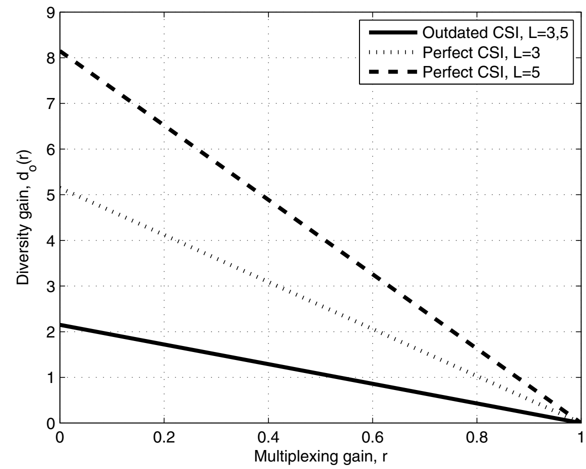

Fig. 2. Diversity-multiplexing tradeoff of the turbo coded system scheme for outdated and perfect CSI. $m_{sd} = 0.65, m_{r_i d} = 1.5, i = 1, \cdots, L$.

VII. Numerical Results

In this section, we present the numerical results for the outage probability of the underlying transmission scheme. In all simulations, the message length is $K = 128$ bits. We consider a turbo code with code rate $R_c = 1/3$, generator polynomial (1, 17/13) in octal form and without loss of generality assume that all the average SNRs are equal and $m_{sr_j} = m_{sr}$, $m_{r_j d} = m_{rd}$.

Fig. 3 shows a performance comparison of the simulated and exact expression of the outage probability for $\rho < 1$ ($\rho = \{0.1, 0.5, 0.8, 0.99\}$). It is clear that the exact outage probability derived is in good agreement with the simulated one. As noted, the achievable diversity order for outdated CSI is the same and independent of ρ and the number of available relays as can be confirmed by the same slope. Moreover, this diversity order is equal to the one of an adaptive turbo-coded DF with a single relay. This can be intuitively explained by noting that even for the most outdated CSI, the selected relay is reliable (source-to-relay link is good) which translates to no error propagation at the relay.

In Fig. 4, the performance of the outage probability for $L = 4$ and $\rho = 0.5$ is presented for different fading figures m. A comparison of the simulated and exact outage probability can help verify the accuracy of the latter.

In Fig. 5, we show the outage probability versus the correlation factor ρ for both $\bar{\gamma} = 10$–15dB. For $L = 1$, full diversity is always achieved and the outage probability is not a function of the correlation factor as the former does not vary with the correlation factor ρ. However, for $L > 1$, it can be seen that the outage probability is a function of ρ and slightly varies for $\rho < 1$.

In Fig. 6, similar to Fig. 5 we present the outage probability performance as a function of the correlation factor ρ. It can be noted that for $\rho \neq 1$, the performance improves slowly as ρ increases. For $\rho = 1$, there is a sudden change in slope which

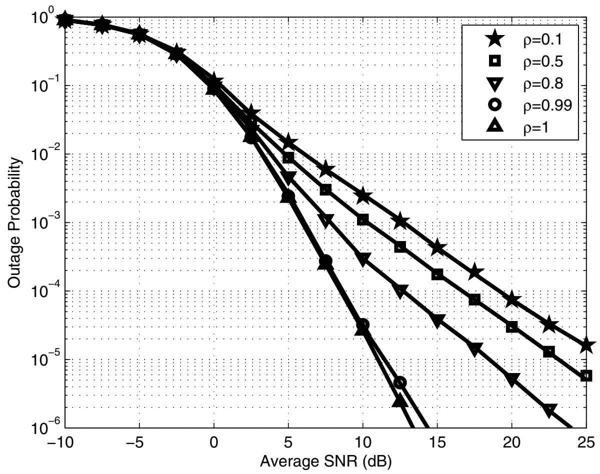

Fig. 3. Simulated (symbols) versus exact (solid lines) outage probability for $\rho = 0.1, 0.5, 0.8, 0.99, 1$ and $L = 4$. $m_{sd} = 0.65$ and $m_{rd} = 0.85$.

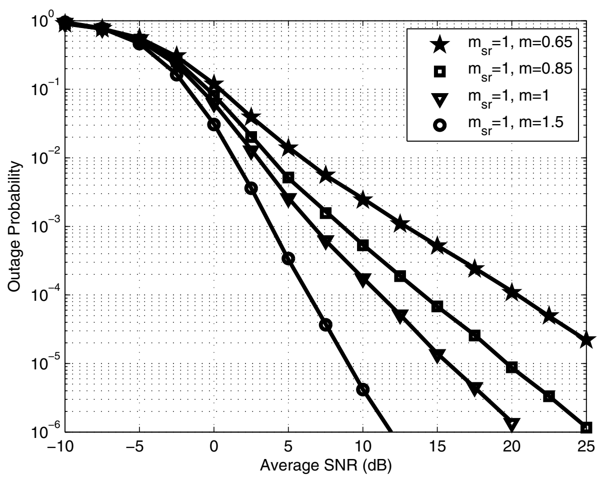

Fig. 4. Simulated (symbols) versus exact (solid lines) outage probability for $\rho = 0.5$ and $L = 4$. $m_{sd} = m_{rd} = m$.

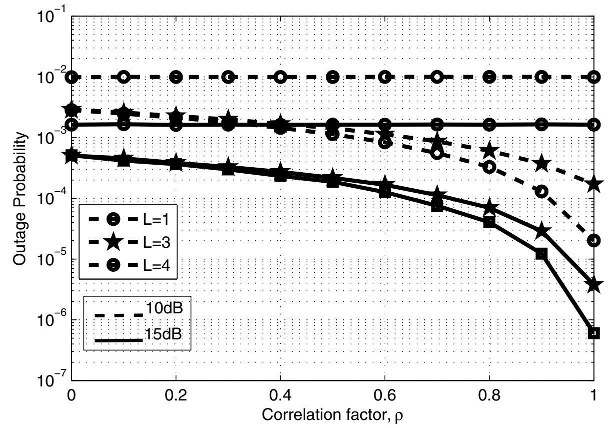

Fig. 5. Outage probability versus correlation factor ρ for various SNRs and number of relays. $m_{sd_i} = m_{sr_i} = m_{r_i d} = 1$, $i = 1 \cdots L$.

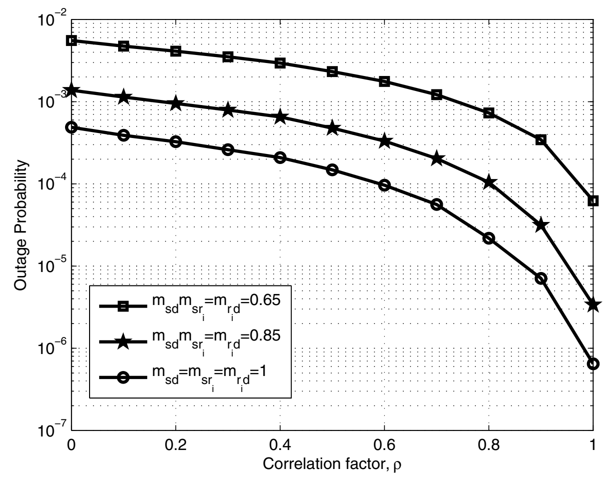

Fig. 6. Outage probability versus correlation factor ρ for different m values and $L = 5$. $m_{sd_i} = m_{sr_i} = m_{r_i d}$, $i = 1 \cdots L$.

can be attributed to the fact that full diversity is achieved at that instance.

We also consider the effects of pathloss which represents a practical scenario. For simplicity, a line topology is considered, i.e., the relay nodes are situated on the same line between the source and the destination, with the distance between s and d normalized to 1. Furthermore, the distance between s and r_i, and r_i and d is denoted as d_{sr} and d_{rd} respectively, where $d_{sr} = d$ and $d_{rd} = 1 - d$. In this scenario, the variances $\mathbb{E}\langle h_{sd}^2 \rangle = 1$, $\mathbb{E}\langle h_{sr}^2 \rangle = 1/d^\eta$ and $\mathbb{E}\langle h_{rd}^2 \rangle = 1/(1-d)^\eta$ are assumed where η is the pathloss coefficient. For our simulations, without loss of generality, we consider $d = 0.3$ and $\eta = 3$ in Fig. 7. For both cases, $\{m_{sd} = 0.65, m_{sr} = 1, m_{rd} = 0.85\}$ and $\{m_{sd} = 0.85, m_{sr} = 1, m_{rd} = 1.5\}$, it can be seen that the diveristy order $d_o = m_{sd} + m_{rd}$ is achieved as predicted from our analysis for outdated CSI.

In Figs. 8 and 9, various levels of imperfect CSI are considered where one can note that full diversity in the number of available relays L and fading figure m can be achieved with ideal CSI. However, for non-ideal CSI, the achievable diversity order is identical to a single relay scenario. Moreover, both Figs. 8 and 9 show a comparison between the union bounds on the BER and the simulated BER. As seen, the bounds on the BER are in good agreement with the simulations for different values of ρ. This validates the accuracy of our analytical framework presented in this work.

VIII. Conclusion

The effect of outdated CSI on turbo-coded cooperation with relay selection subject to Nakagami-*m* fading was investigated. A closed-form expression for the outage probability and its asymptotic expression at high SNR are derived. It was noted that the outage probability performance is dependent on the level of CSI imperfection, with full diversity achieved for ideal CSI only. In addition, a closed-form expression of the PEP is derived and the diversity analysis of the PEP corroborates with the conclusions extracted from the outage probability study.

Appendix

We derive the high-SNR expression of the outage probability for turbo-coded cooperation with outdated CSI. Two cases arise: $\rho < 1$ and $\rho = 1$.

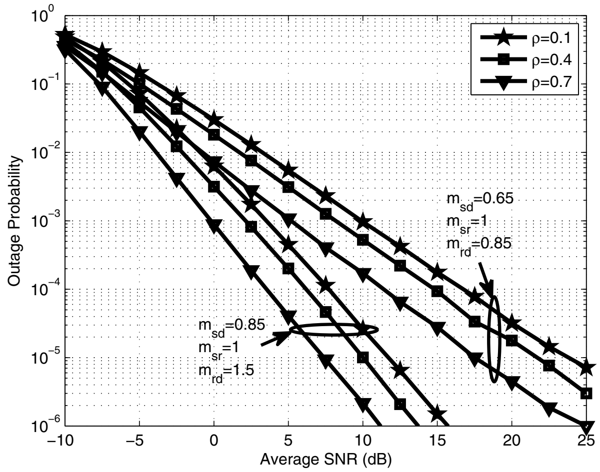

Fig. 7. Outage probability performance for $\rho = 0.1, 0.4, 0.7$. $\mathbb{E}\langle h_{sd}^2\rangle = 1$, $\mathbb{E}\langle h_{sr}^2\rangle = 1/0.3^3$ and $\mathbb{E}\langle h_{rd}^2\rangle = 1/0.7^3$.

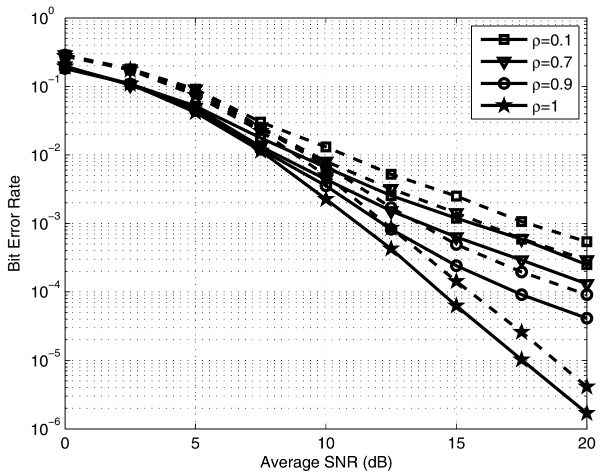

Fig. 8. BER of the proposed scheme versus average SNR for various correlation values ρ. $L = 3$ and $m_{sd} = 0.65$, $m_{sr} = 1$, $m_{rd} = 0.85$. Solid lines: simulations, dashed lines: bounds.

Case 1: $0 \le \rho < 1$

It is easy to rewrite (23) as in (15) and assuming that $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma} \to \infty$,

$$
\begin{aligned}
P_{out} &\overset{\bar{\gamma}\to\infty}{\approx} \sum_{\vartheta=1}^{L}\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n}\bar{\gamma}^{-(m_{rd}+n)}}{n!\,(1-\rho)^{m_{rd}+2n}\,\Gamma(m_{rd})\Gamma(m_{rd}+n)} \\
&\times \left(\int_0^a \frac{m_{sd}^{m_{sd}}\gamma^{m_{sd}-1}}{\Gamma(m_{sd})\bar{\gamma}^{m_{sd}}} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right)\right. \\
&\times \left.\int_0^b \gamma^{m_{rd}+n-1} \exp\left(-\frac{m_{rd}\gamma_{rd}}{(1-\rho)\,\bar{\gamma}}\right) \mathrm{d}\gamma_{rd} d\gamma_{sd}\right) \\
&\times \sum_{j=0}^{\vartheta-1}\binom{\vartheta-1}{j}(-1)^j \chi\left(m_{rd}+n,\rho,j\right).
\end{aligned} \tag{42}
$$

To solve the inner integral, we can use [31, Eq. 3.351.1] and after some manipulations,

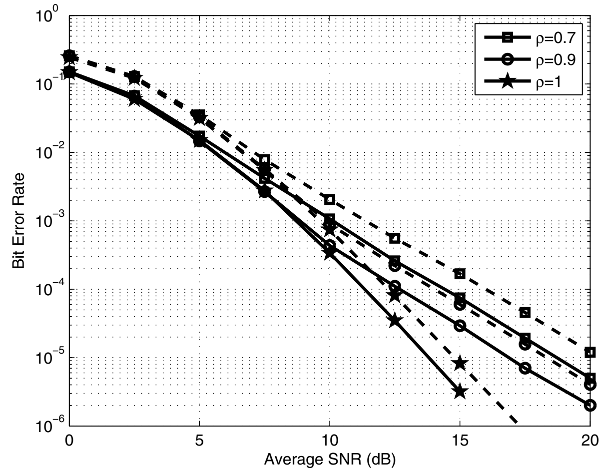

Fig. 9. BER of the proposed scheme versus average SNR for various correlation values ρ. $L = 3$ and $m_{sd} = 1.5$, $m_{sr} = 1$, $m_{rd} = 0.85$. Solid lines: simulations, dashed lines: bounds.

$$
\begin{aligned}
P_{out} &\overset{\bar{\gamma}\to\infty}{\approx} \sum_{\vartheta=1}^{L}\sum_{n=0}^{\infty} \frac{\vartheta \rho^n m_{rd}^{m_{rd}+n} m_{sd}^{m_{sd}} \bar{\gamma}^{-(m_{sd}+m_{rd}+n)}}{n!\,(1-\rho)^{m_{rd}+2n}\,\Gamma(m_{sd})\Gamma(m_{rd})\Gamma(m_{rd}+n)} \\
&\times \left(\int_0^a \gamma_{sd}^{m_{sd}-1} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right)\left(\frac{m_{rd}}{(1-\rho)\,\bar{\gamma}}\right)^{-m_{rd}-n}\right. \\
&\times \left.\gamma\left(m_{rd}+n, \frac{m_{rd}b}{(1-\rho)\,\bar{\gamma}}\right)\mathrm{d}\gamma_{sd}\right)\sum_{j=0}^{\vartheta-1}\binom{\vartheta-1}{j} \\
&\times (-1)^j \chi\left(m_{rd}+n,\rho,j\right).
\end{aligned} \tag{43}
$$

It can easily be noticed that at high SNR: (a) the maximum of the outage probability in (43) occurs when $\vartheta = L$. (b) The dominant term in (43) is $n = 0$ of the infinite series. Following these observations and using the identity $\gamma(a,x) = x^a{}_1F_1\left(a, 1+a; -x\right)$ in [31, Eq. 8.352.1],

$$
\begin{aligned}
P_{out} &\overset{\bar{\gamma}\to\infty}{\approx} \frac{L m_{sd}^{m_{sd}} m_{rd}^{m_{rd}}}{(1-\rho)^{m_{rd}}\,\Gamma(m_{sd})\Gamma^2(m_{rd})\bar{\gamma}^{m_{sd}+m_{rd}}} \\
&\times \left(\int_0^a \gamma_{sd}^{m_{sd}-1}\exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right)\right. \\
&\times \left. {}_1F_1\left(m_{rd}, 1+m_{rd}; -\frac{m_{rd}b}{(1-\rho)\,\bar{\gamma}}\right)\mathrm{d}\gamma_{sd}\right) \\
&\times \sum_{j=0}^{L-1}\binom{L-1}{j}(-1)^j \chi\left(m_{rd},\rho,j\right),
\end{aligned} \tag{44}
$$

where ${}_1F_1\left(x,y;z\right)$ denotes the confluent Hypergeometric function defined in [31, Eq. 9.210.1]. As $\bar{\gamma} \to \infty$, the hypergeometric function in (44) reduces to 1. Hence,

$$
P_{out} \overset{\bar{\gamma}\to\infty}{\approx} G_{c_1}^{'}\bar{\gamma}^{-(m_{sd}+m_{rd})}, \tag{45}
$$

where $G^{'}_{c_1}$ is a constant value and is given by

$$G^{'}_{c_1} = \sum_{j=0}^{L-1} \binom{L-1}{j} (-1)^j \chi(m_{rd}, \rho, j) \times \int_0^{2^{R_c}-1} \gamma_{sd}^{m_{sd}-1} \left(\frac{2^{R_c/(1-\alpha)}}{(1+\gamma_{sd})^{1/(1-\alpha)}} - 1 \right)^{m_{rd}} \mathrm{d}\gamma_{sd}, \tag{46}$$

where the integral part can be computed numerically yielding a constant value.

Case 2: $\rho = 1$

In this case, the observations (a) and (b) made for $\rho < 1$ are still valid. Furthermore, a less simplified expression of the CDF in [20] and $p(x) = \partial \mathrm{F}(x)/\partial x$ are used, and the asymptotic outage probability (i.e., $\bar{\gamma}_{sd} = \bar{\gamma}_{sr} = \bar{\gamma}_{rd} = \bar{\gamma} \to \infty$ is assumed) can be given by

$$P_{out} \overset{\bar{\gamma}\to\infty}{\approx} \frac{m_{sd}^{m_{sd}} m_{rd}^{2m_{rd}}}{(1-\rho)^{m_{rd}} \Gamma(m_{sd}) \Gamma^2(m_{rd}) \bar{\gamma}^{m_{sd}+2m_{rd}}} \times \left(\int_0^a \gamma_{sd}^{m_{sd}-1} \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right) \int_0^b \gamma_{rd}^{m_{rd}-1} \times \exp\left(-\frac{m_{rd}\gamma_{rd}}{(1-\rho)\bar{\gamma}}\right) \mathrm{d}\gamma_{rd} \mathrm{d}\gamma_{sd} \right) \sum_{j=0}^{L-1} \binom{L-1}{j} \times \frac{(-1)^j}{\Gamma^j(m_{rd})} \int_0^\infty x^{m_{rd}-1} \exp\left(-\frac{m_{rd}x}{(1-\rho)\bar{\gamma}}\right) \times \Gamma^j\left(m_{rd}, \frac{m_{rd}x}{\bar{\gamma}}\right) \mathrm{d}x. \tag{47}$$

The expression in (47) can further be simplified using [31, Eq. 3.351.1]. Substituting $\Gamma^j\left(m, \frac{mx}{\bar{\gamma}}\right) \approx \Gamma^j(m) \exp\left(-\frac{mjx}{\gamma}\right)$ [31, Eq. 3.352.7] for high values of $\bar{\gamma}$, and after some algebraic manipulations, the expression in (47) can be rewritten as

$$P_{out} \overset{\bar{\gamma}\to\infty}{\approx} \frac{m_{sd}^{m_{sd}} m_{rd}^{m_{rd}}}{\Gamma(m_{sd}) \Gamma^2(m_{rd}) \bar{\gamma}^{m_{sd}+2m_{rd}}} \left(\int_0^a \gamma_{sd}^{m_{sd}-1} \times \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right) \gamma\left(m_{rd}, \frac{m_{rd}b}{(1-\rho)\bar{\gamma}}\right) \mathrm{d}\gamma_{sd} \right) \times \int_0^\infty x^{m_{rd}-1} \exp\left(-\frac{m_{rd}x}{(1-\rho)\bar{\gamma}}\right) \sum_{j=0}^{L-1} \binom{L-1}{j} \times (-1)^j \exp\left(-\frac{m_{rd}jx}{\bar{\gamma}}\right) \mathrm{d}x. \tag{48}$$

The Binomial expansion $(1-y)^m = \sum_{j=0}^m \binom{m}{j} (-1)^j y^j$ can be identified in (48). Moreover, at high values of y, $\exp\left(-\frac{1}{y}\right) \approx 1 - \frac{1}{y^m}$. Hence (48) can be expressed as

$$P_{out} \overset{\bar{\gamma}\to\infty}{\approx} \frac{m_{sd}^{m_{sd}} m_{rd}^{L}}{\Gamma(m_{sd}) \Gamma^2(m_{rd}) \gamma^{m_{sd}+Lm_{rd}}} \left(\int_0^a \gamma_{sd}^{m_{sd}-1} \times \gamma\left(m_{rd}, \frac{m_{rd}b}{\bar{\gamma}}\right) \exp\left(-\frac{m_{sd}\gamma_{sd}}{\bar{\gamma}}\right) \mathrm{d}\gamma_{sd} \right) \times \int_0^\infty x^{m_{rd}+L-2} \exp\left(-\frac{m_{rd}x}{(1-\rho)\bar{\gamma}}\right) \mathrm{d}x. \tag{49}$$

From (49), at high $\bar{\gamma}$, $\delta \approx 0$ and hence $\gamma(m, \frac{1}{\delta}) \approx \Gamma(m)$ and $\exp\left(-\frac{1}{\delta}\right) \approx 0$. After some manipulations, the asymptotic outage probability can be expressed as

$$P_{out} \overset{\bar{\gamma}\to\infty}{\approx} G^{'}_{c_2} \bar{\gamma}^{m_{sd}+Lm_{rd}}, \tag{50}$$

$$G^{'}_{c_2} = \frac{K m_{sd}^{m_{sd}} m_{rd}^{L}}{\Gamma(m_{sd})\Gamma(m_{rd})} \left(2^{R_c} - 1\right)^{m_{sd}}, \tag{51}$$

with $K = \int_0^\infty \exp\left(-\frac{m_{rd}x}{\delta}\right) \mathrm{d}x$ being a constant.

By adopting the diversity order definition in [35], i.e., $d \doteq \lim_{\bar{\gamma}\to\infty} -\log(P_{out}) / \log(\bar{\gamma})$, the diversity order for both cases ($\rho < 1$ and $\rho = 1$) can be obtained.

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References

[1] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative communications with outage-optimal opportunistic relaying," *IEEE Trans. Wireless Commun.*, vol. 6, no. 9, pp. 3450-3460, Sept. 2007.

[2] A. Bletsas, H. Shin, and M. Z. Win, "Outage optimality of opportunistic amplify-and-forward relaying," *IEEE Commun. Lett.*, vol. 11, no. 3, pp. 261-263, Mar. 2007.

[3] I. Krikidis, J. S. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 3, pp. 235-237, Apr. 2008.

[4] M. Seyfi, S. Muhaidat, J. Liang, and M. Uysal, "Relay selection in dual-hop vehicular networks," *IEEE Signal Process. Lett.*, vol. 18, no. 2, pp. 134-137, Feb. 2011.

[5] H. A. Saleh and W. Hamouda, "Cross-layer based transmit antenna selection for decision-feedback detection in correlated Ricean MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1677-1682, Apr. 2009.

[6] D. S. Michalopoulos, A. S. Lioumpas, G. K. Karagiannidis, and R. Schober, "Selective cooperative relaying over time-varying channels," *IEEE Trans. Commun.*, vol. 58, no. 8, pp. 2402-2412, July 2010.

[7] T. Hunter and A. Nosratinia, "Diversity through coded cooperation," *IEEE Trans. Wireless Commun.*, vol. 5, no. 2, pp. 283-289, Feb. 2006.

[8] M. Janani, A. Hedayat, T. E.Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: space-time transmission and iterative decoding," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 362-371, Feb. 2004.

[9] M. Elfituri, A. Ghrayeb, and W. Hamouda, "Antenna/Relay selection for coded cooperative networks with AF relaying," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2580-2584, Sept. 2009.

[10] M. Elfituri, A. Ghrayeb, and W. Hamouda, "Antenna/Relay selection for coded wireless cooperative networks," in *Proc. 2008 IEEE International Conference on Communications*.

[11] J. L. Vicario, A. Bel, J. A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2872-2876, June 2009.

[12] M. Soysa, H. Suraweera, C. Tellambura, and H. K. Garg, "Partial and opportunistic relay selection with outdated channel estimates," *IEEE Trans. Commun.*, vol. 60, no. 3, pp. 840-850, Mar. 2012.

[13] M. Chen, T. C.-K. Liu, and X. Dong, "Opportunistic multiple relay selection with outdated channel state information," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1333-1345, Mar. 2012.

[14] D. Michalopoulos, H. A. Suraweera, G. Karagiannidis, and R. Schober, "Amplify-and-forward relay selection with outdated channel estimated," *IEEE Trans. Commun.*, vol. 60, no. 5, pp.1278-1290, May 2012.

[15] M. Seyfi, S. Muhaidat and J. Liang, "Performance analysis of relay selection with feedback delay and channel estimation errors," *IEEE Signal Process. Lett.*, vol. 18, no. 1, pp. 67-70, Jan. 2011.

[16] H. A. Suraweera, M. Soysa, C. Tellambura, and H. K. Garg, "Performance analysis of partial relay selection with feedback delay," *IEEE Signal Process. Lett.*, vol. 17, pp. 531-534, June 2010.

[17] R. Mo, Y. H. Chew and C. Yuen, "Information rate and relay precoder design for amplify-and-forward MIMO relay networks with imperfect channel state information," *IEEE Trans. Veh. Technol.*, vol. 61, no. 9, pp. 3958-3968, Nov. 2012.

[18] M. Torabi and D. Haccoun, "Capacity analysis of opportunistic relaying in cooperative systems with outdated channel information," *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1137-1139, Dec. 2012.
[19] H. A. Suwaweera, D. S. Michalopoulos, and C. Yuen, "Performance analysis of fixed-gain AF relay system with interference in Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1457-1463, Mar. 2012.
[20] D. S. Michalopoulos, N. D. Chatzidiamantis, R. Schober, and G. K. Karagiannidis, "Relay selection with outdated channel estimates in Nakagami-*m* fading," in *Proc. 2011 IEEE International Conference on Communications*.
[21] Y. Li, Q. Yin, L. Sun, H. Chen, and H. Wang, "A channel quality metric in opportunistic selection with outdated CSI over Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 3, pp. 1427-1432, Mar. 2012.
[22] N. S. Ferdinand, N. Rajatheva, and M. Latva-aho, "Effects of feedback delay in partial relay selection over Nakagami-*m* fading channels," *IEEE Trans. Veh. Technol.*, vol. 61, no. 4, pp. 1620-1634, May 2012.
[23] Z. Wang, M. Xiao, C. Wang, and M. Skoglund, "Degrees of freedom of multi-hop MIMO broadcast networks with delayed CSIT," *IEEE Wireless Commun. Lett.*, vol. 2, no. 2 , pp. 207-210, Apr. 2013.
[24] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Trans. Inf. Theory*, vol. 58, no. 7, p. 4418-4431, July 2012.
[25] Z. Wang, M. Xiao, C. Wang, and M. Skoglund, "On the degrees of freedom of two-hop MISO broadcast networks with mixed CSIT," in *Proc. 2013 IEEE Global Communications Conference*.
[26] M. Xiao, J. Kliewer, and M. Skoglund, "Design of network codes for multiple-user multiple-relay wireless networks," *IEEE Trans. Commun.*, vol. 60, no. 12, pp.. 3755-3766, Dec. 2012.
[27] M. Xiao and M. Skoglund, "Multiple-user cooperative communications based on linear network coding," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3345-3351, Dec. 2010.
[28] A. Eid, W. Hamouda, and I. Dayoub, "Performance of multi-relay coded cooperative diversity in asynchronous code-division multiple-access over fading channels", *IET Commun.*, vol. 5, no. 5, pp. 683-692, 2011.
[29] P. Mitran, H. Ochiai, and V. Tarokh, "Space-time diversity enhancements using collaborative communications," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2041-2057, June 2005.
[30] W. C. Jakes, *Microwave Mobile Communication*. Wiley & Sons, 1974.
[31] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 3rd ed. New York: Academic, 2007.
[32] J. W. Craig, "A new, simple and exact result for calculating the probability of error for two-dimensional signal constellations," in *Proc. 1991 IEEE Military Communications Conference*, pp. 571-575.
[33] M. K. Simon and M.–S. Alouini, *Digital Communications over Fading Channels: A Unified Approach to Performance Analysis.* New York: John Wiley and Sons, 2000.
[34] M. Chiani, D. Dardari, and M. K. Simon, "New exponential bounds and approximations for the computation of error probability in fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 840-845, July 2003.
[35] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073-1096, May 2003.
[36] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
[37] D. Divsalar, S. Dolinar, and F. Pollara, "Transfer function bounds on the performance of turbo codes," *TDA Progress Report 42-122*, pp. 44-55, Aug. 1995.
[38] E. Malkamaki and H. Leib, "Evaluating the performance of convolutional codes over block fadings" *IEEE Trans. Inf. Therory*, vol. 45, no. 5, pp. 1643-1646, July 1999.

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