

Active Fault Tolerant Control for Quad-Rotor UAV

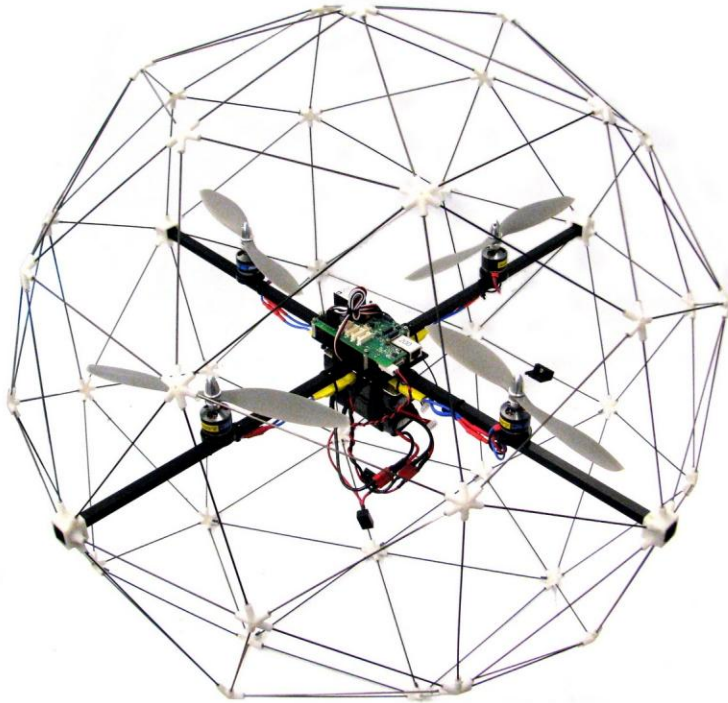
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Outline

- **Quad-Rotor System Modeling**
- **Active Fault Tolerant Control Strategy**
 - ❖ **Adaptive Lyapunov-based Method**
 - ❖ **Adaptive λ – Tracking Method**
 - ❖ **Simulation Results**
- **Linear System Modeling**
- **Active Fault Tolerant Control Strategy**
 - ❖ **LQR and EA Approaches Combined with CGT Method**
 - ❖ **Fault Detection and Diagnosis Integrated with Reconfiguration Mechanism**
 - ❖ **Simulation Results**
- **Conclusion**

Quad – Rotor System Modeling

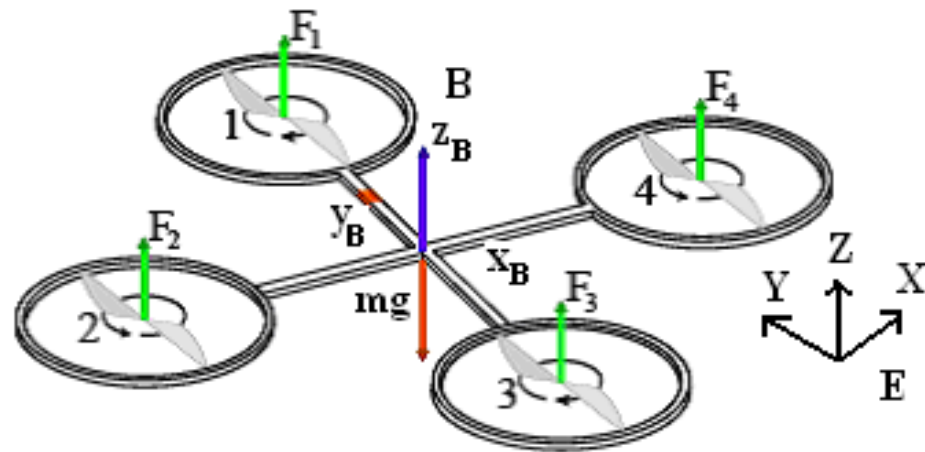


Navigation Equations:

$$\begin{aligned}\ddot{x} &= \frac{F_1 + F_2 + F_3 + F_4}{m} (\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi) - \frac{k_{d1} \dot{x}}{m} \\ \ddot{y} &= \frac{F_1 + F_2 + F_3 + F_4}{m} (\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi) - \frac{k_{d2} \dot{y}}{m} \\ \ddot{z} &= \frac{F_1 + F_2 + F_3 + F_4}{m} \cos \theta \cos \phi - \frac{k_{d3} \dot{z}}{m} - g\end{aligned}$$

Moment Equations:

$$\begin{aligned}\ddot{\phi} &= \frac{1}{J_x} [(F_2 - F_4)l - k_{d4} \dot{\phi} - \dot{\theta} \dot{\psi} (J_z - J_y)] \\ \ddot{\theta} &= \frac{1}{J_y} [(F_1 - F_3)l - k_{d5} \dot{\theta} - \dot{\phi} \dot{\psi} (J_x - J_z)] \\ \ddot{\psi} &= \frac{1}{J_z} [(F_1 - F_2 + F_3 - F_4)l - k_{d6} \dot{\psi} - \dot{\theta} \dot{\phi} (J_y - J_x)]\end{aligned}$$

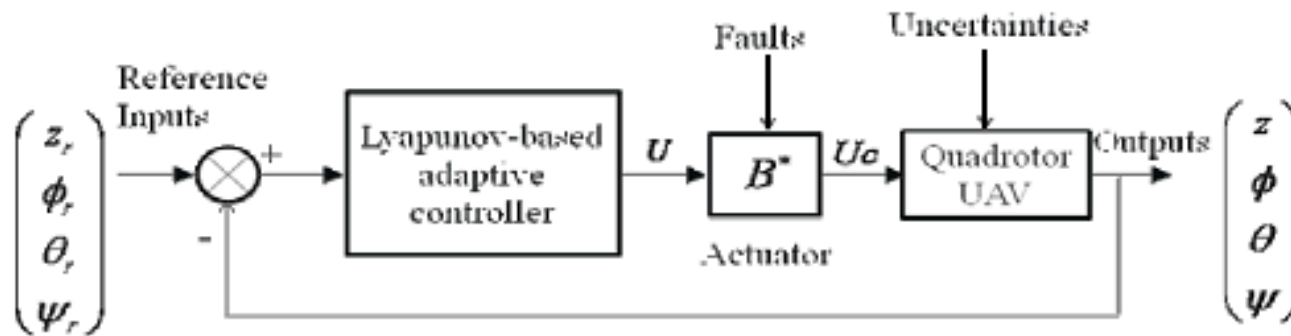


$$F_i = b_i \omega_i^2$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} F_1 + F_2 + F_3 + F_4 \\ F_2 - F_4 \\ F_1 - F_3 \\ F_1 + F_3 - F_2 - F_4 \end{pmatrix} = \begin{pmatrix} b_1 & b_2 & b_3 & b_4 \\ 0 & b_2 & 0 & -b_4 \\ b_1 & 0 & -b_3 & 0 \\ b_1 & -b_2 & b_3 & -b_4 \end{pmatrix} \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix}$$

Active Fault Tolerant Control Strategy

- Adaptive Lyapunov-based Method



$$\begin{pmatrix} \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \begin{pmatrix} -g \\ \frac{-\dot{\theta}\dot{\psi}(J_z - J_y)}{J_x} \\ \frac{-\dot{\phi}\dot{\psi}(J_x - J_z)}{J_y} \\ \frac{-\dot{\theta}\dot{\phi}(J_y - J_x)}{J_z} \end{pmatrix} + \begin{pmatrix} b_1^* \frac{\cos\theta \cos\phi}{m} & 0 & 0 & 0 \\ 0 & b_2^* \frac{l}{J_x} & 0 & 0 \\ 0 & 0 & b_3^* \frac{l}{J_y} & 0 \\ 0 & 0 & 0 & b_4^* \frac{l}{J_z} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

$$B^* = \begin{pmatrix} b_1^* & 0 & 0 & 0 \\ 0 & b_2^* & 0 & 0 \\ 0 & 0 & b_3^* & 0 \\ 0 & 0 & 0 & b_4^* \end{pmatrix}$$

Lyapunov Adaptive Control Algorithm Equations

Quad – Rotor Altitude Equations

$$\ddot{z} = b_1^* \frac{u_1}{m} \cos \theta \cos \phi - g$$

$$u_1 = \hat{\alpha}_1 u_{c1}$$

$$u_{c1} = \frac{m}{\cos \theta \cos \phi} (-c_{12} y_{12} - y_{11} + g + \ddot{z}_r + \dot{\beta}_1)$$

$$\dot{\hat{\alpha}}_1 = -\gamma \frac{u_{c1}}{m} \cos \theta \cos \phi y_{12}$$

$$y_{11}(t) = z(t) - z_r(t)$$

$$y_{12}(t) = \dot{z}(t) - \dot{z}_r(t) - \beta_1$$

$$\beta_1(t) = -c_{11} y_{11}(t)$$

$$\alpha_1 \stackrel{\Delta}{=} \frac{1}{b_1^*}$$

Cont.

Quad – Rotor Angles Equations

$$u_{c2} = \frac{J_x}{I} [-c_{22}y_{22} - y_{21} + \dot{\beta}_2 + \ddot{\phi}_r + \frac{1}{J_x} \dot{\theta}\dot{\psi}(J_z - J_y)]$$

$$u_2 = \hat{\alpha}_2 u_{c2}$$

$$u_{c3} = \frac{J_x}{I} [-c_{32}y_{32} - y_{31} + \dot{\beta}_3 + \ddot{\theta}_r + \frac{1}{J_y} \dot{\phi}\dot{\psi}(J_x - J_z)]$$

$$u_3 = \hat{\alpha}_3 u_{c3}$$

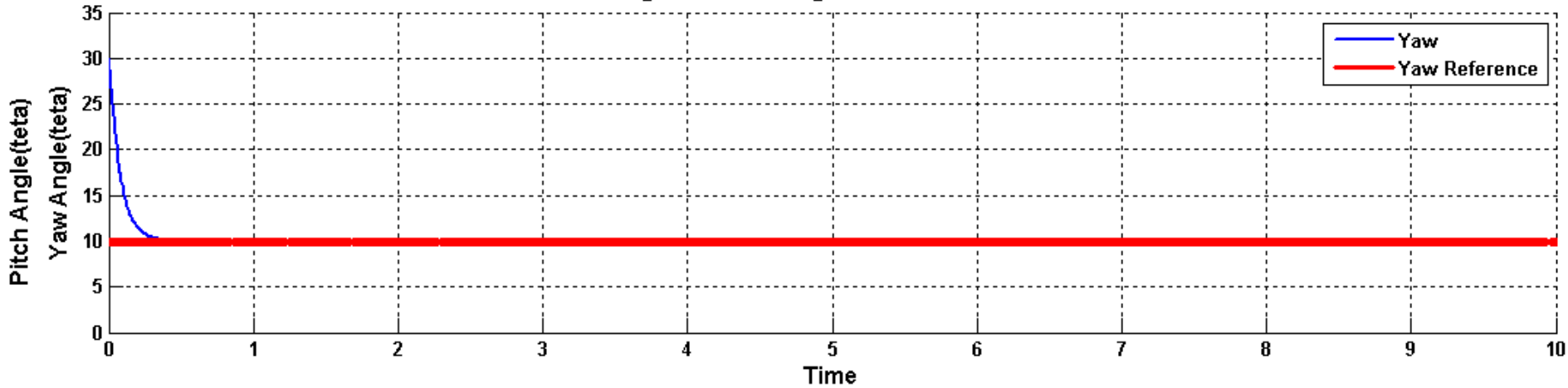
$$u_{c4} = \frac{J_x}{I} [-c_{42}y_{22} - y_{41} + \dot{\beta}_4 + \ddot{\psi}_r + \frac{1}{J_z} \dot{\theta}\dot{\phi}(J_y - J_x)]$$

$$u_4 = \hat{\alpha}_4 u_{c4}$$

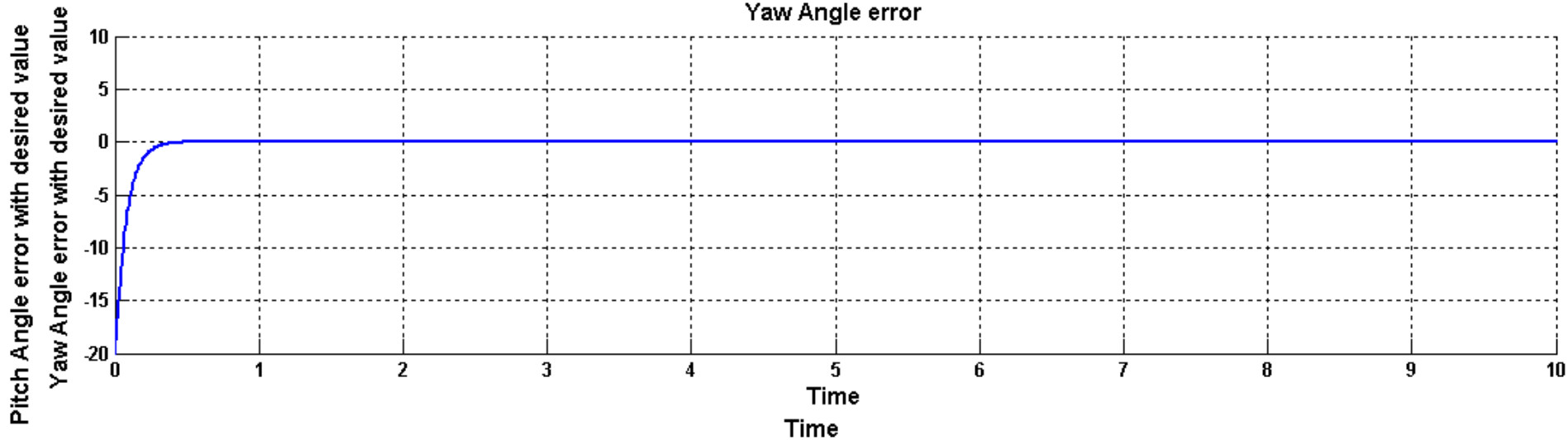
Simulation Results for Lyapunov-Based Adaptive Control

● Normal Case (Without Fault)

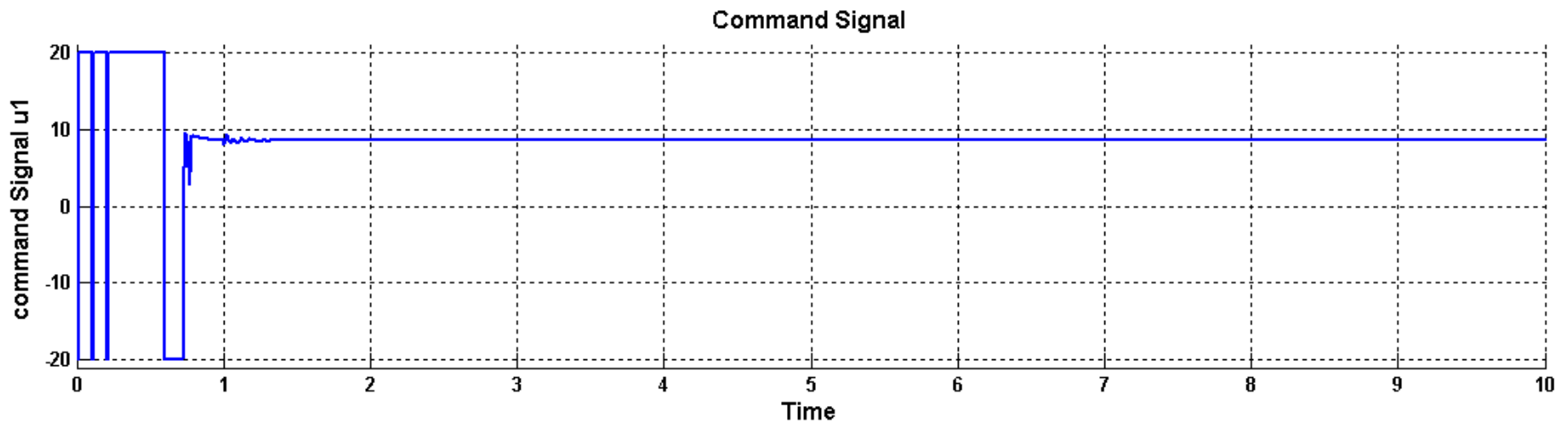
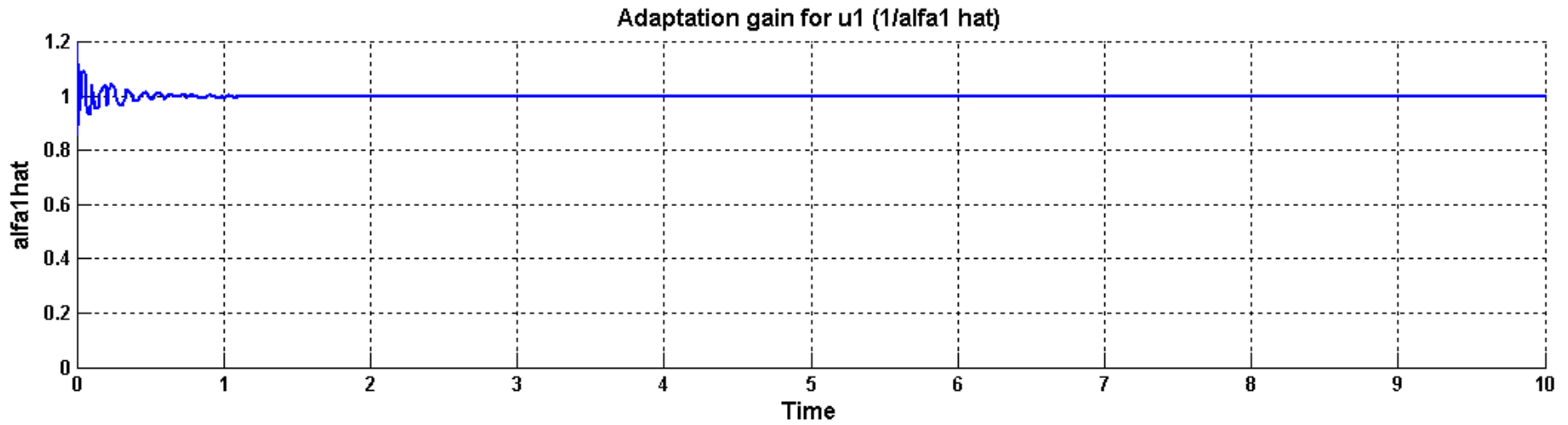
Yaw Angle and Yaw Angle error with normal case



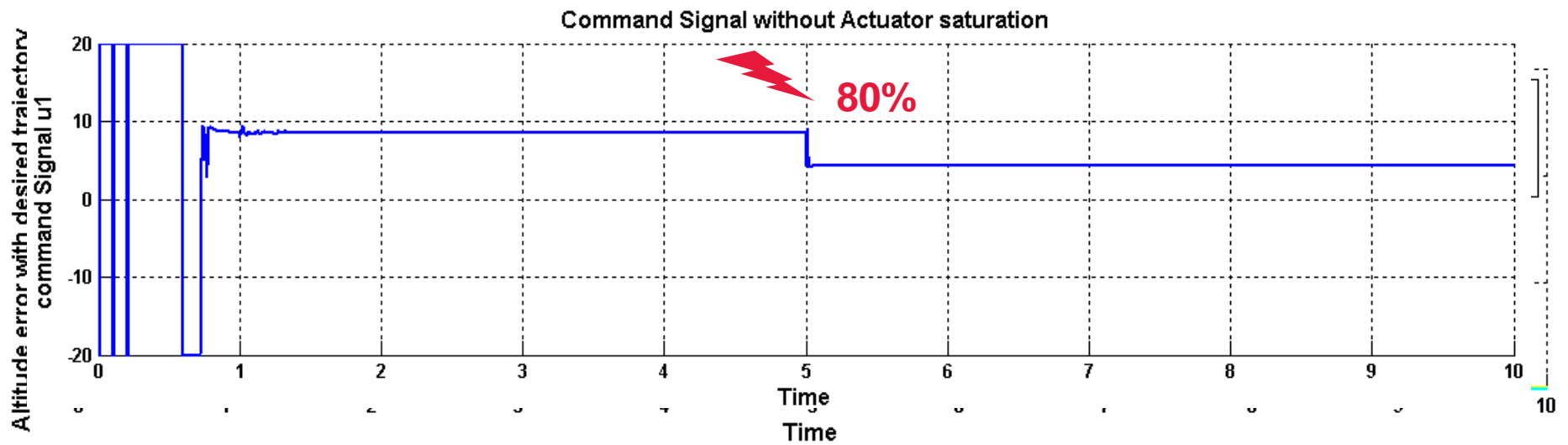
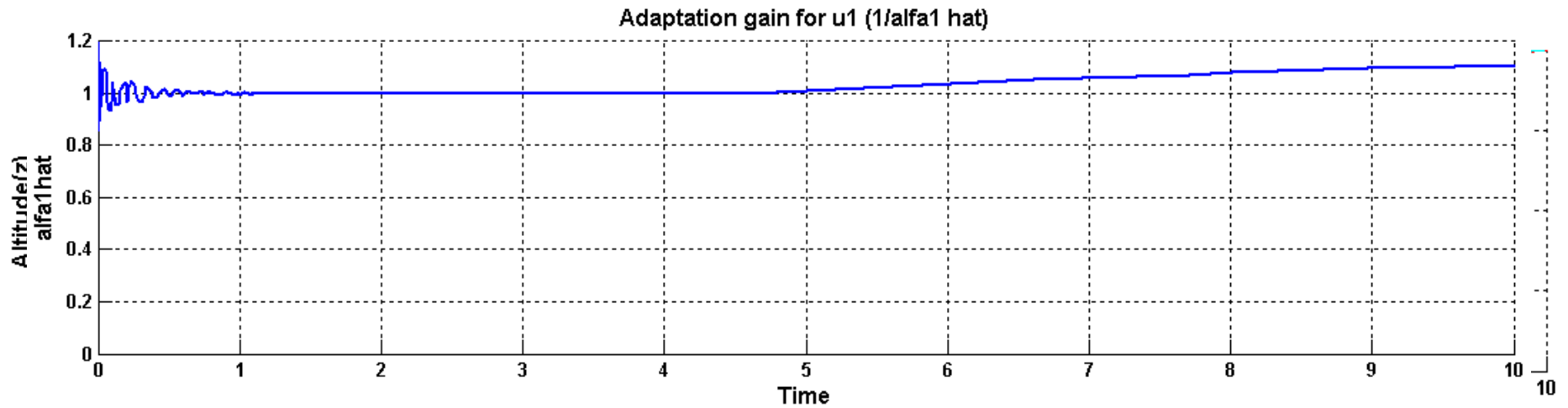
Yaw Angle error



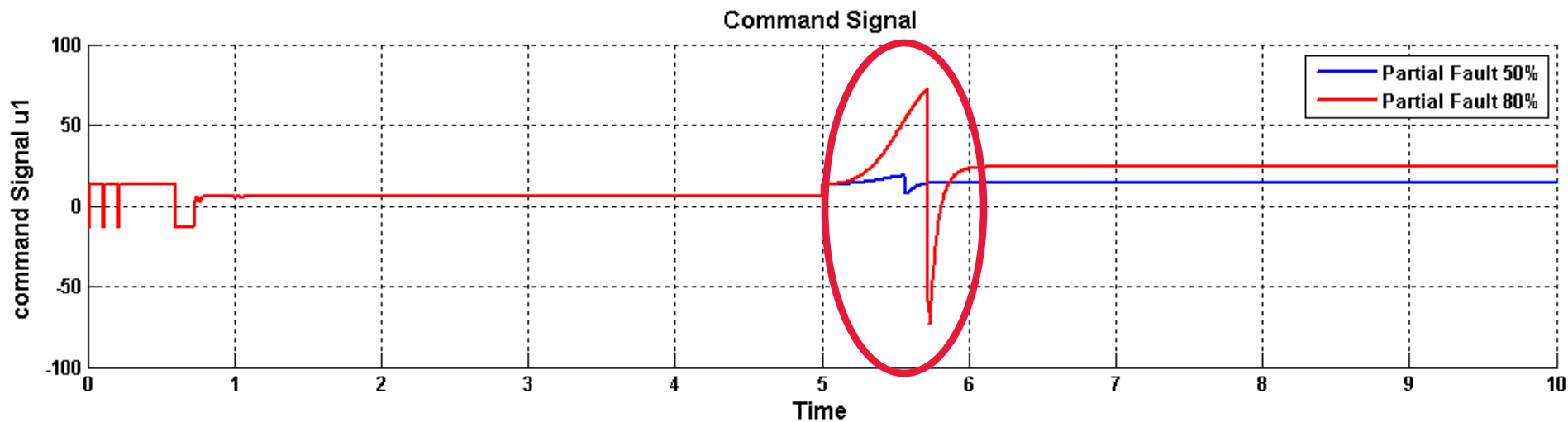
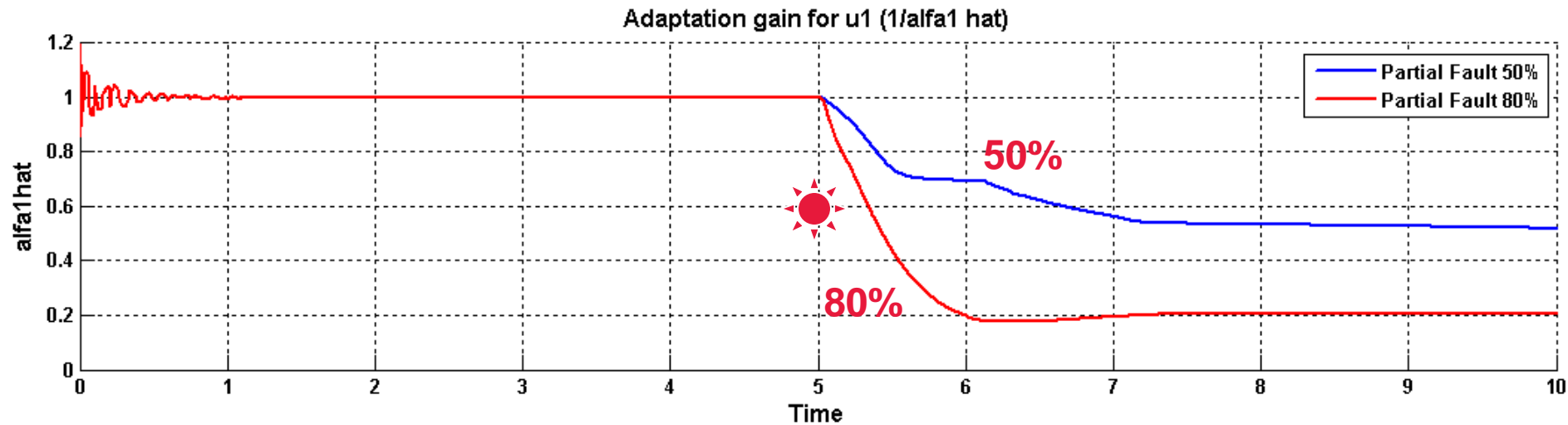
- Normal Case (With Saturation on Altitude Actuator)



- With Uncertainty in System Parameters at $t = 5$ sec

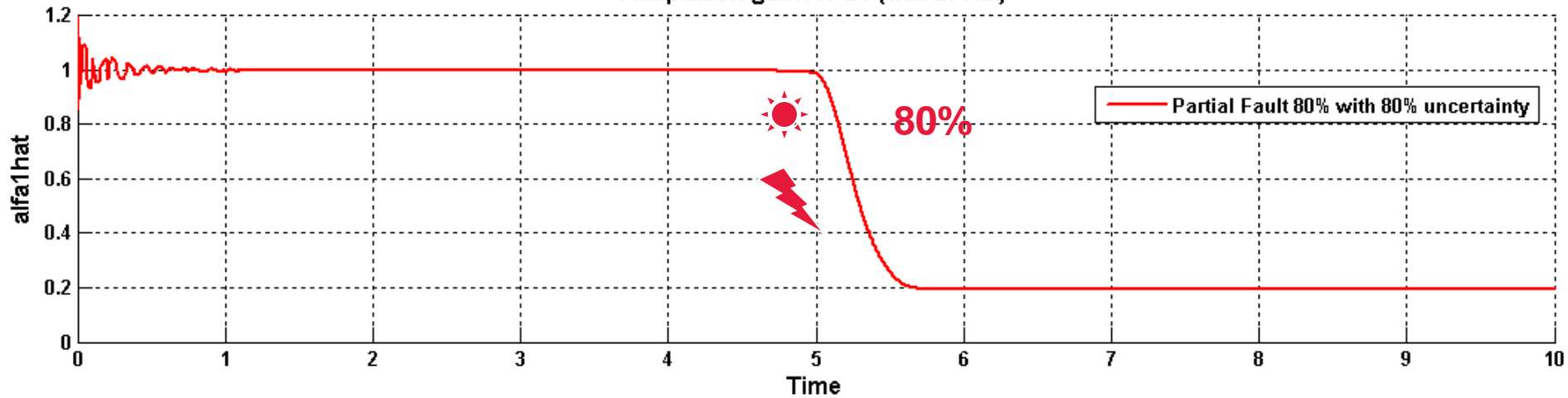


- Partial Fault in Altitude Control Actuator at $t = 5$ sec

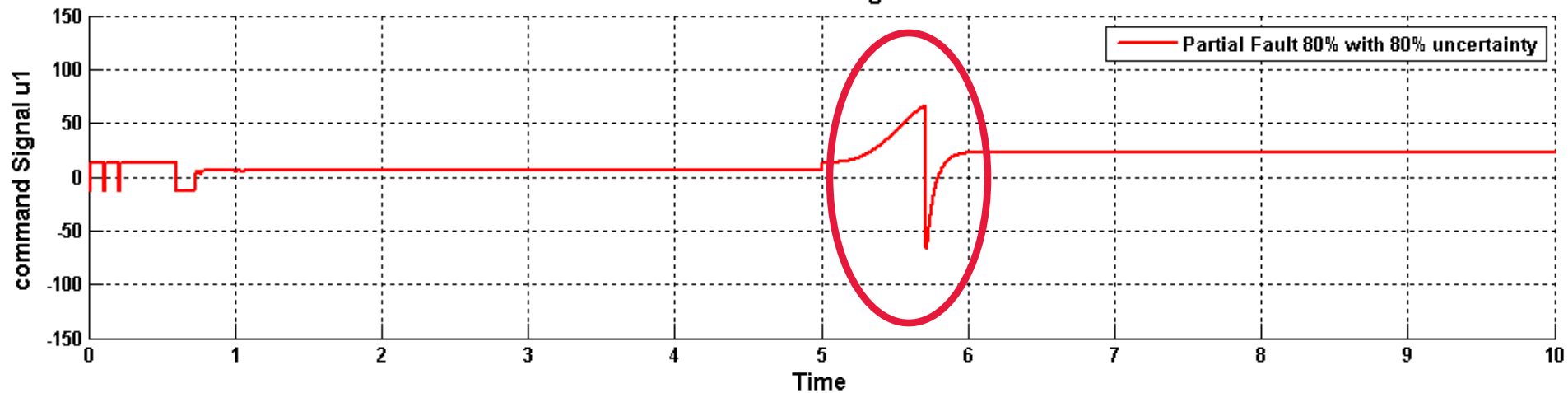


- Partial Fault and Uncertainty at $t = 5$ sec

Altitude and altitude error with normal case
Adaptation gain for u_1 ($1/\hat{\alpha}_1$)



Command Signal



Adaptive λ – Tracking Method

- Introduction to λ – Tracking Method

The output is controlled to a λ neighborhood of the set point.

$$|y(t) - y_{ref}(t)| \leq \lambda$$

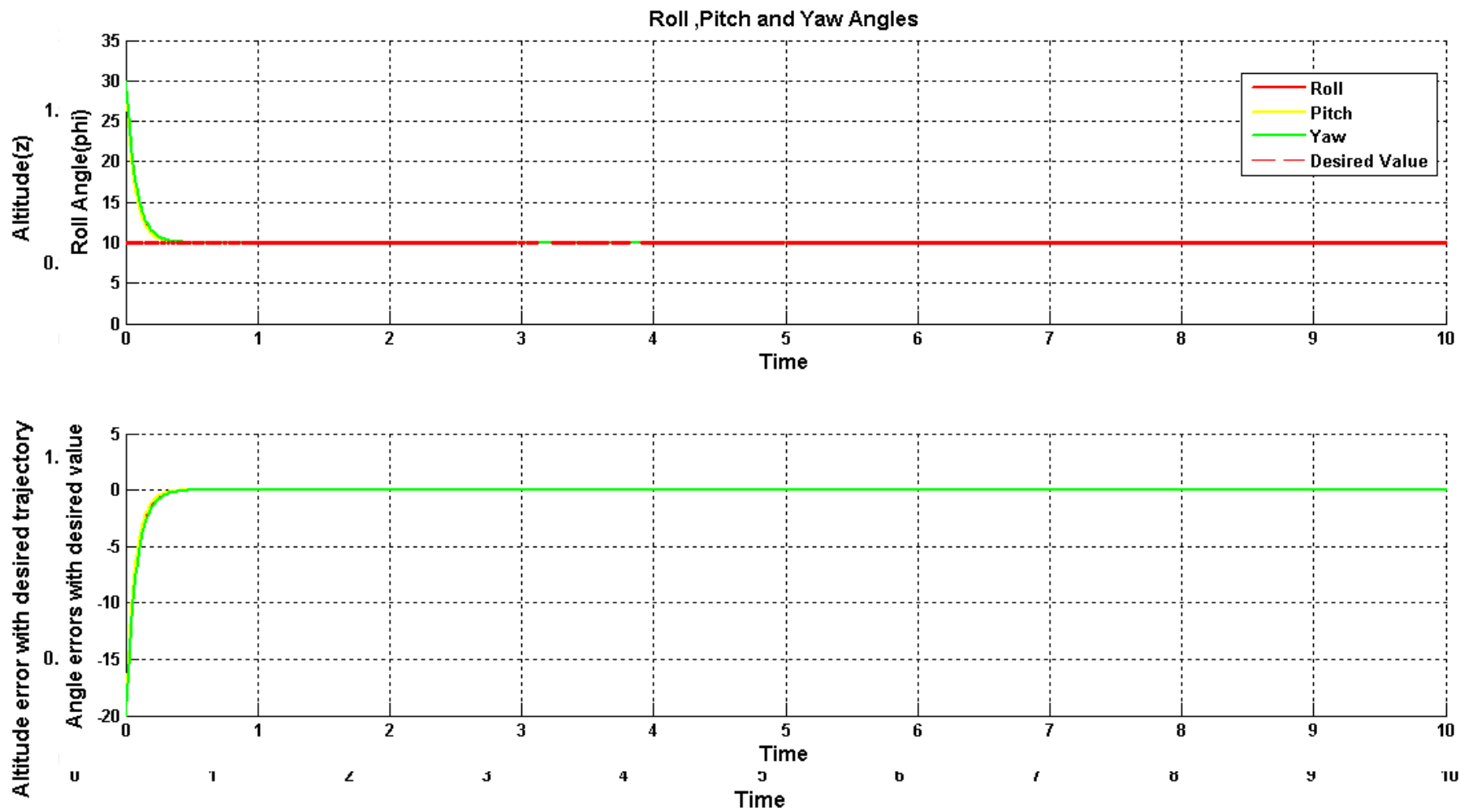
- Structure of the Controller

$$u(t) = -\beta k(t)[y(t) - y_{ref}(t)] + \delta$$

Adaptation Law

$$\dot{k}(t) = \begin{cases} \gamma(y(t) - y_{ref}(t))^p, & \text{if } |y(t) - y_{ref}(t)| \geq \lambda ; (p \geq 1) \\ 0 & , \text{if } |y(t) - y_{ref}(t)| \leq \lambda \end{cases}$$

Simulation Results for λ – Tracking Method



Conclusion of Adaptive Control of Nonlinear System

- Advantages
 - Robustness against parameter uncertainties.
 - Rapid reconfiguration mechanism during fault occurrence.
- Disadvantages
 - Improper transient response
 - Inapplicable without any performance degradation mechanism

Linear System Modeling

- Jacobian matrix around the equilibrium point

$$z = 0 \quad \psi = \frac{\pi}{18} \quad \theta = \frac{\pi}{18} \quad \phi = \frac{\pi}{18}$$

$$\begin{bmatrix} \dot{z} \\ z \\ \dot{\phi} \\ \phi \\ \dot{\theta} \\ \theta \\ \dot{\psi} \\ \psi \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ z \\ \phi \\ \phi \\ \theta \\ \theta \\ \psi \\ \psi \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ g_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & g_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$$y = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} z \\ z \\ \phi \\ \phi \\ \theta \\ \theta \\ \psi \\ \psi \end{bmatrix}$$

$$g_1 = \frac{-2}{m} * b_1 \cos\left(\frac{\pi}{18}\right) * \sin\left(\frac{\pi}{18}\right)$$

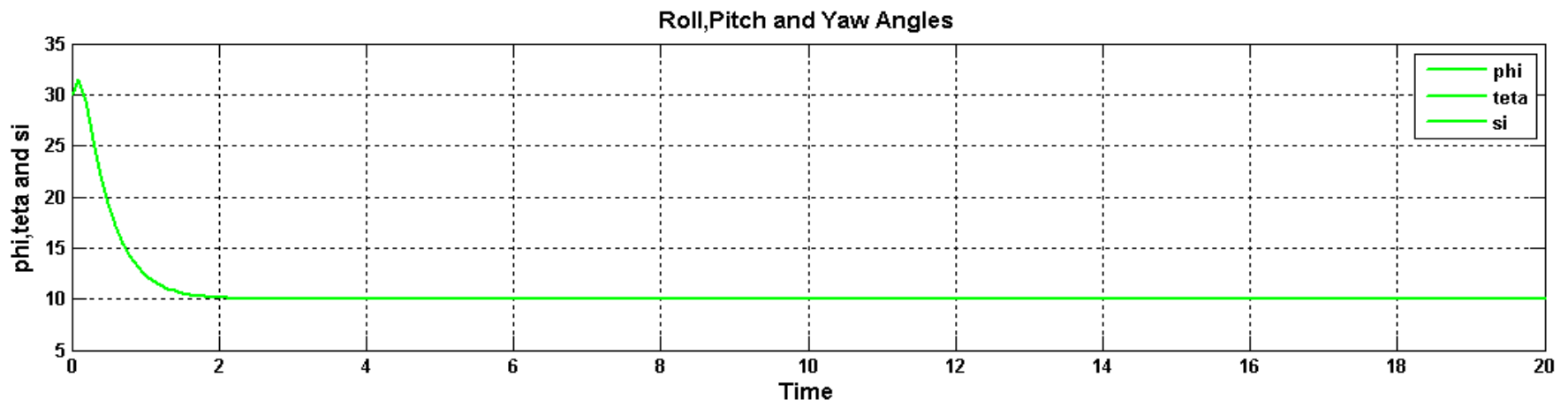
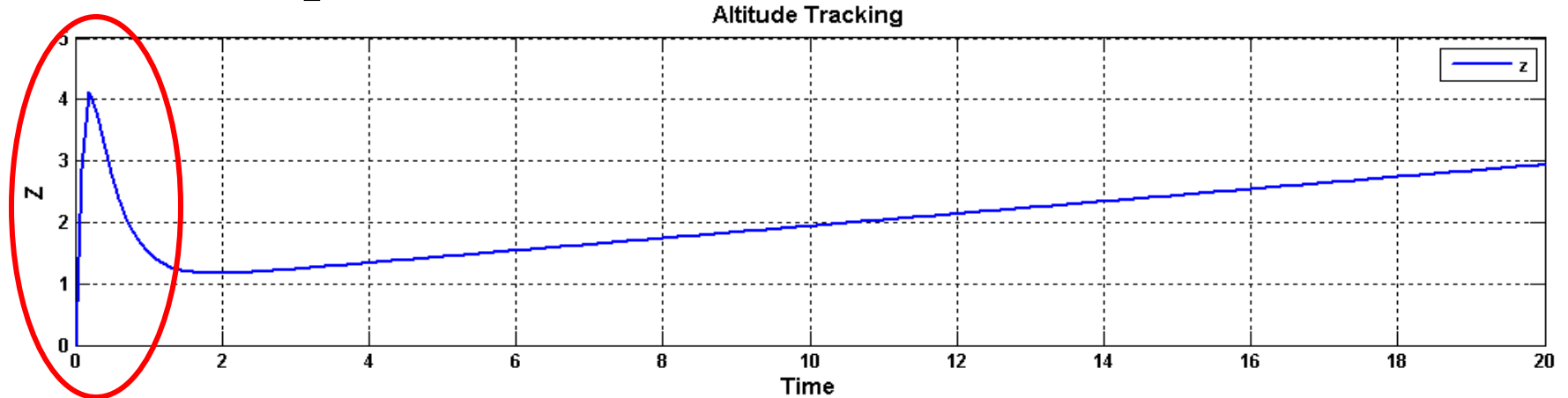
$$g_2 = \frac{l}{j_x} * b_2^*$$

$$g_3 = \frac{b_3^* l}{J_y}$$

$$g_4 = \frac{b_4^* l}{J_z}$$

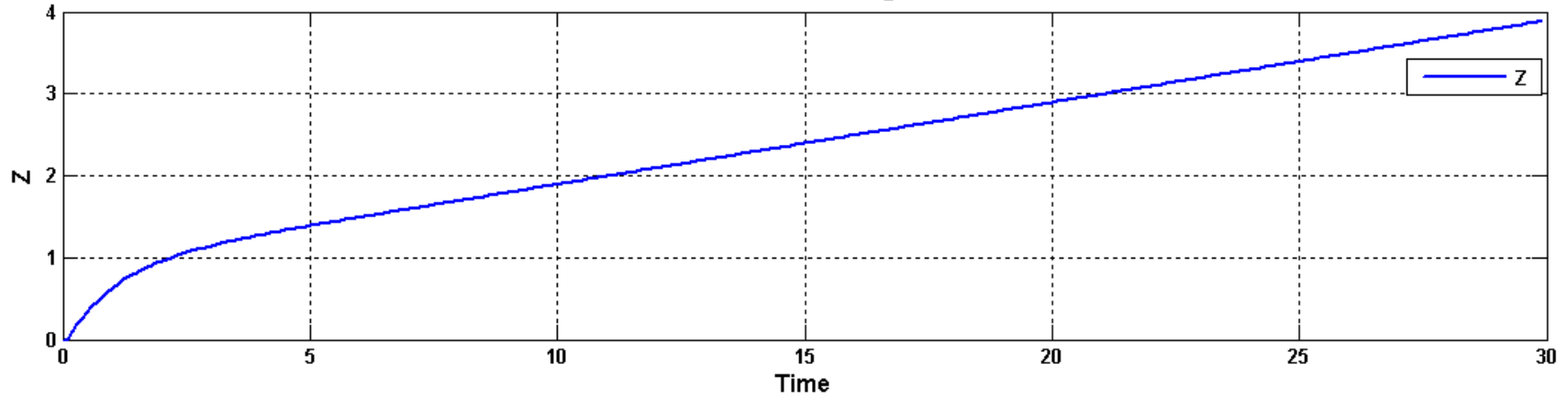
Controlling Mechanisms

- **Linear Quadratic Regulator with Command Governor Technique**

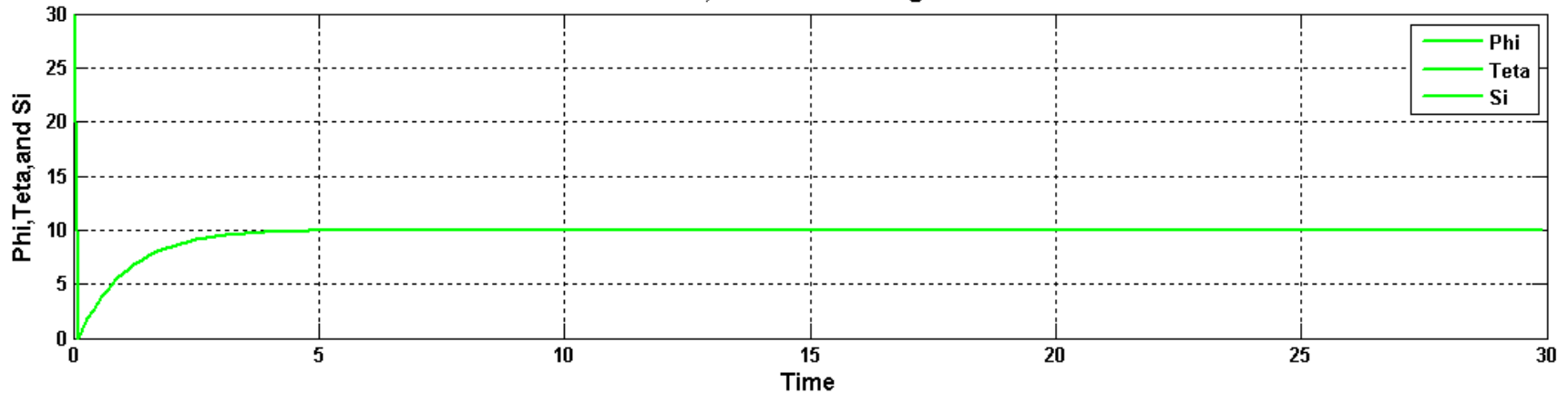


• Eigen Structure Assignment with Command Governor Technique

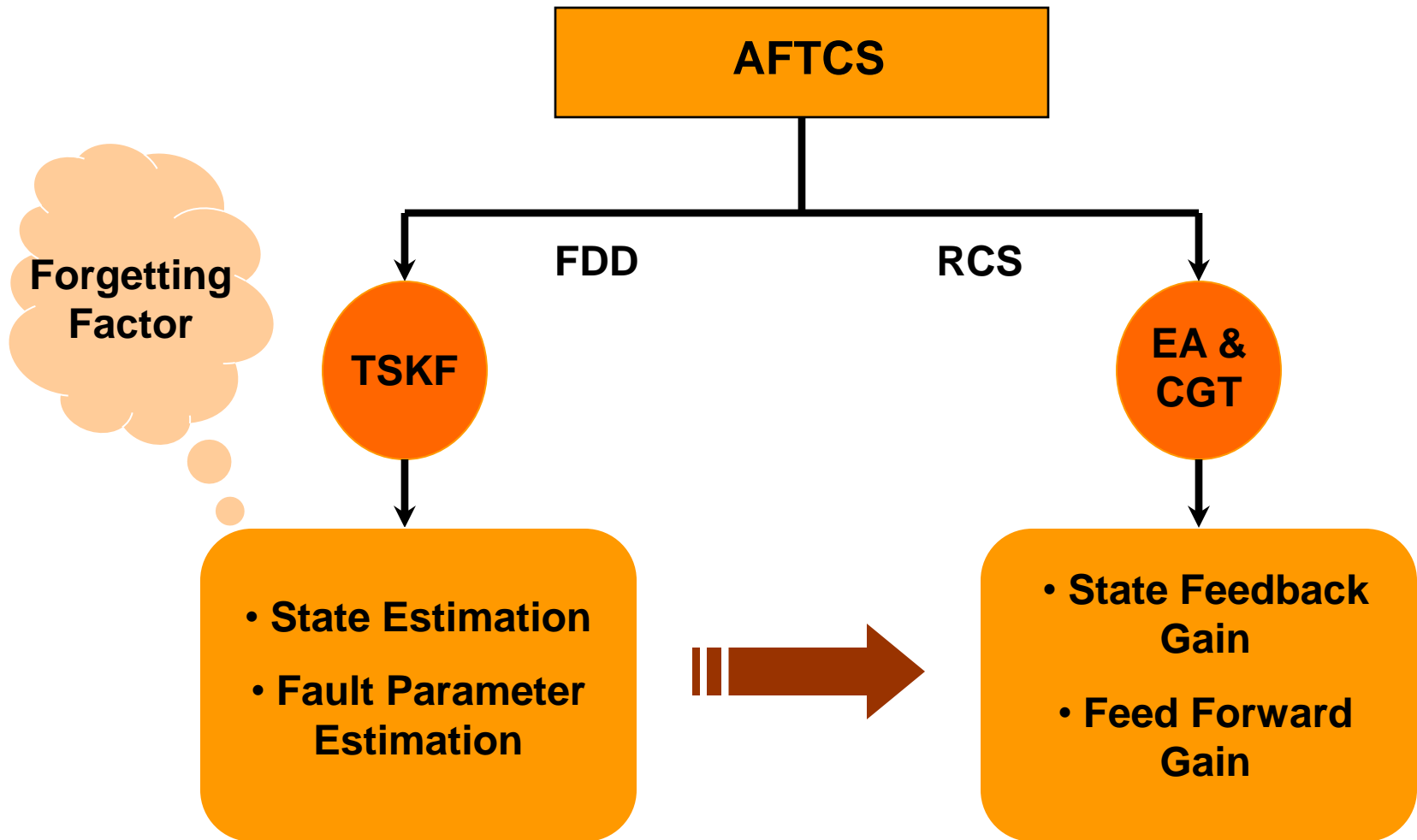
Altitude Tracking



Roll, Pitch and Yaw Angles

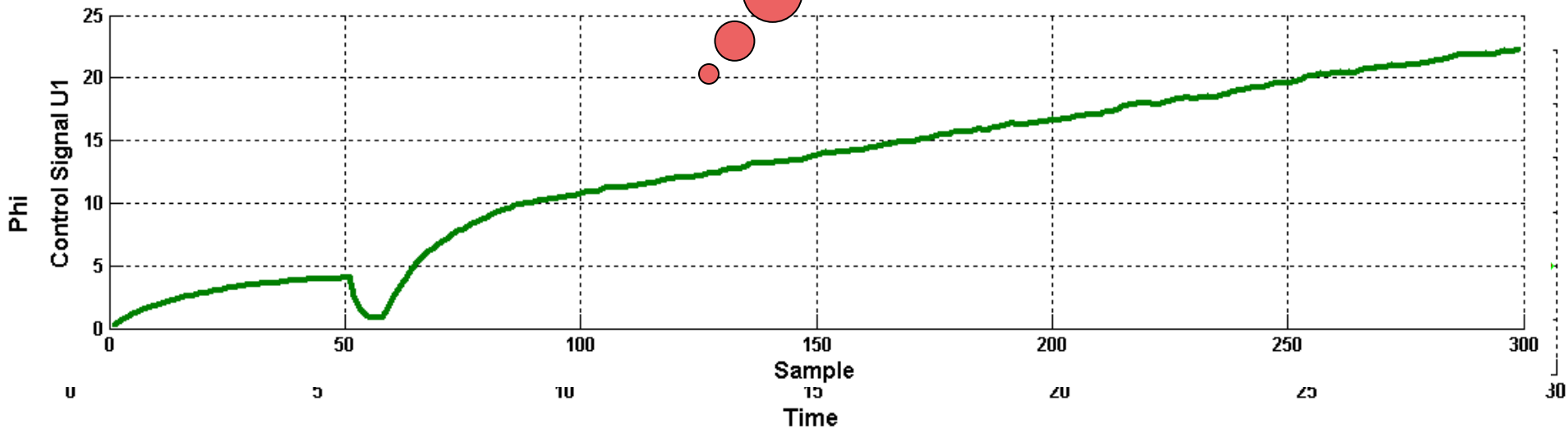
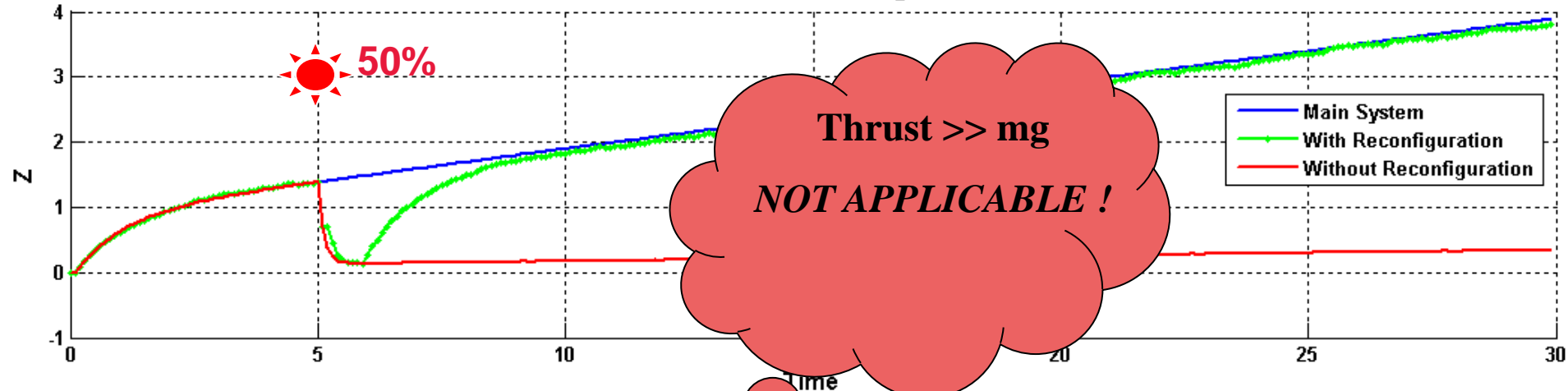


Active Fault Tolerant Control Approach for Linear System

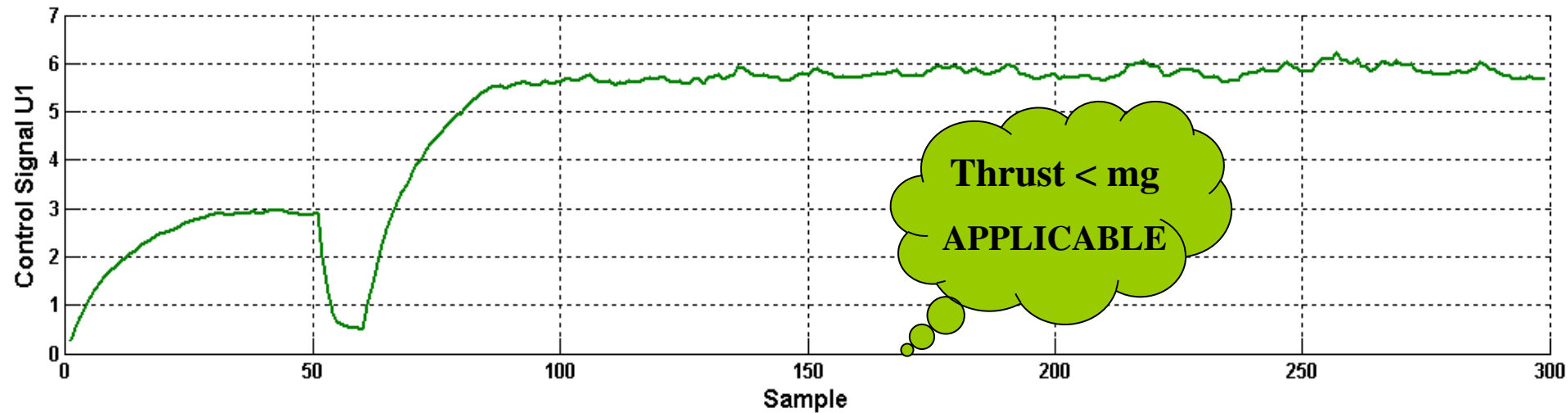
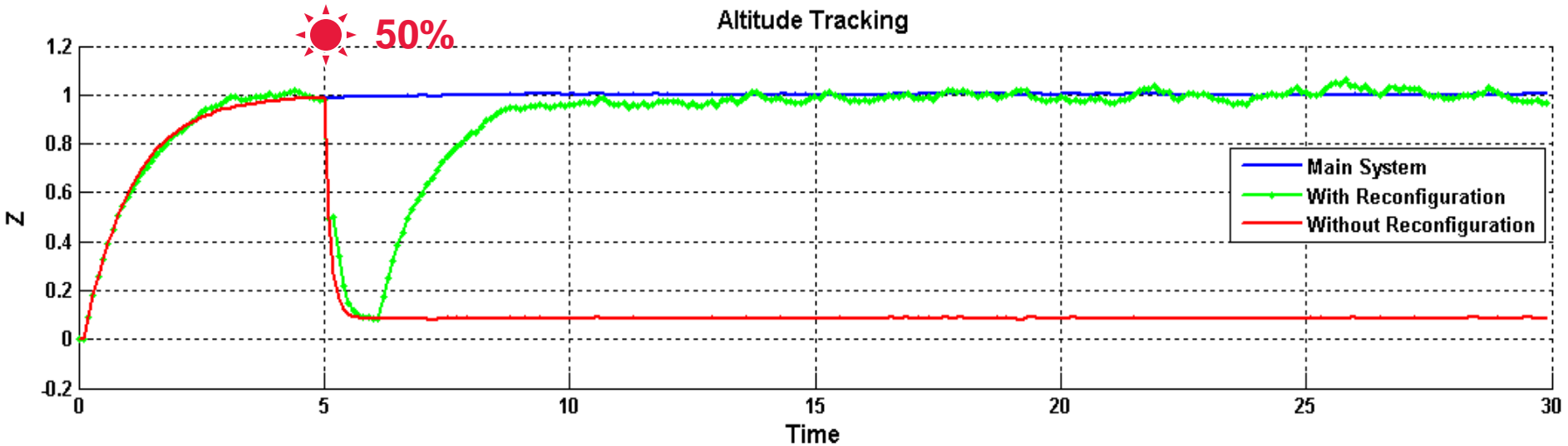


Variable Reference Input & 50% Actuator Fault

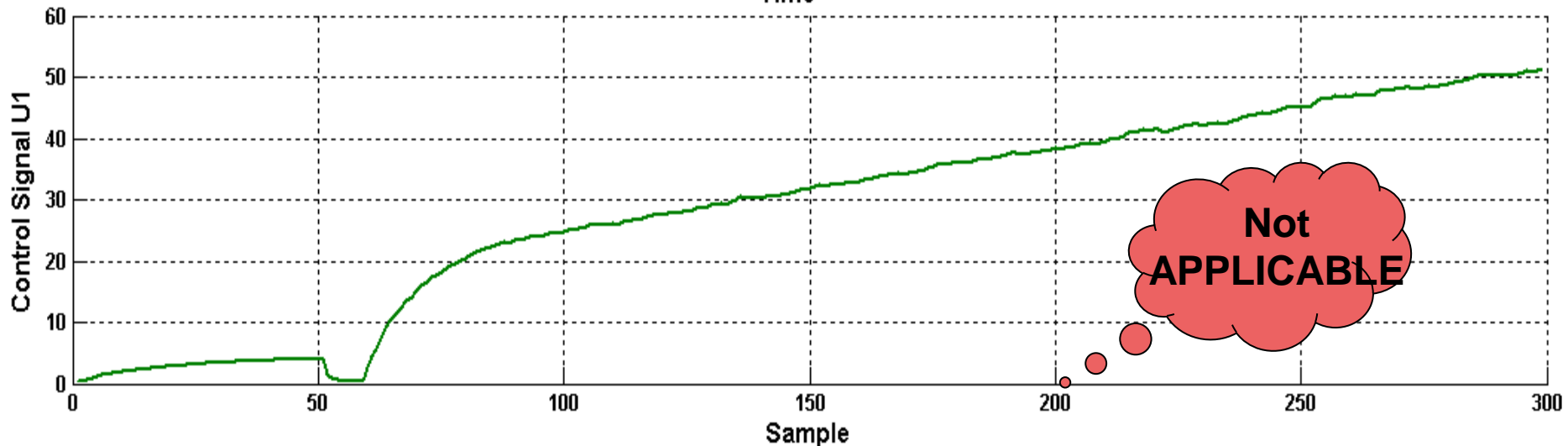
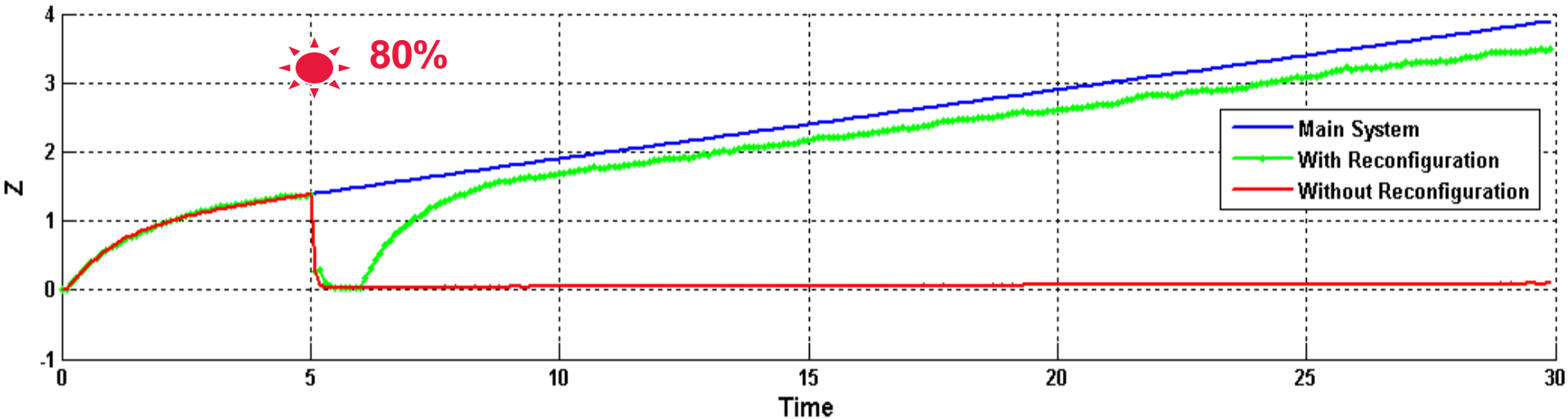
Altitude Tracking



Constant Reference Input & 50% Actuator Fault

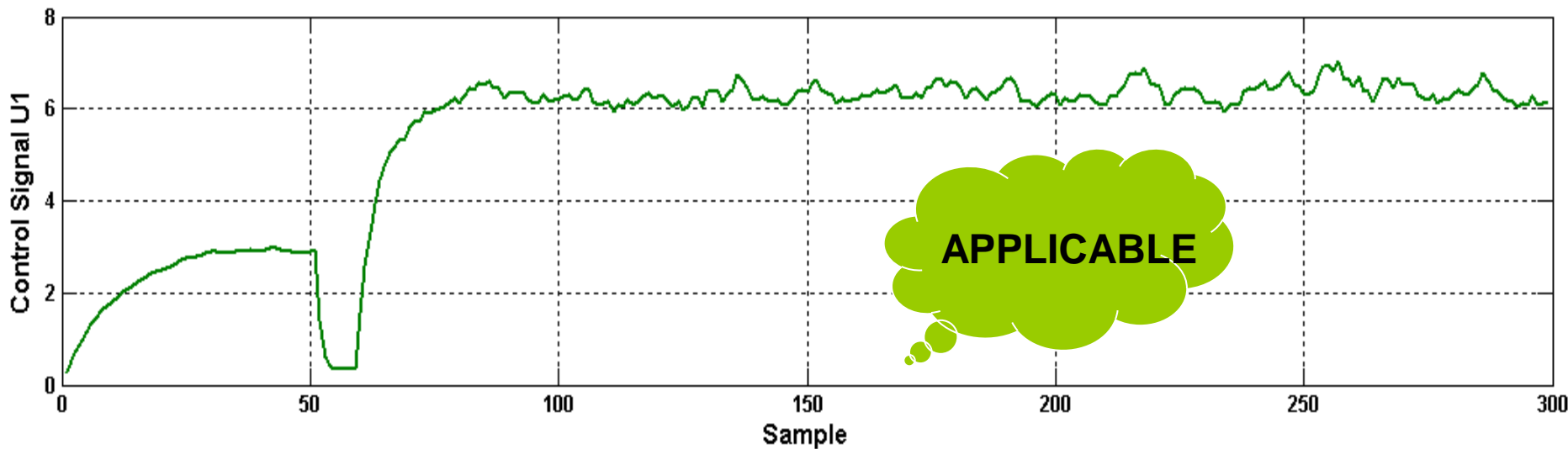
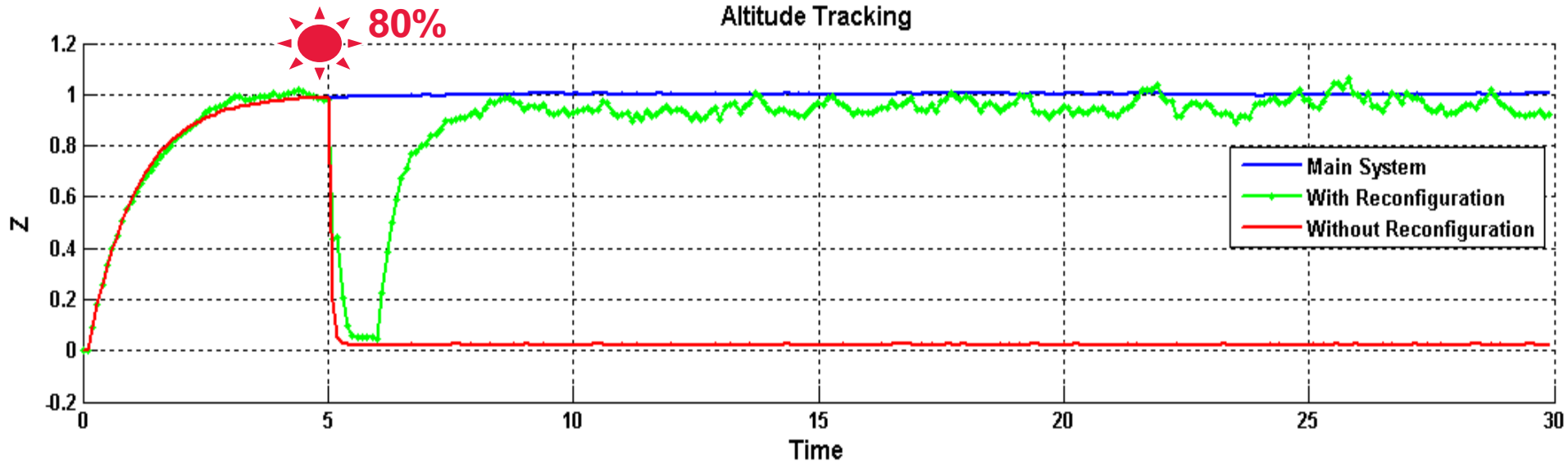


Variable Reference Input & 80% Actuator Fault



Constant Reference Input & 80% Actuator Fault

Altitude Tracking



Conclusion

- Advantages
 - Acceptable transient response.
 - Applicable due to limited command control signals for constant reference input.
- Disadvantages
 - Slower reconfiguration mechanism
 - inapplicable for time varying reference input due to large command controlling signals.
- Suggestion for future work
 - Apply performance degradation mechanism for both linear and nonlinear model.