Fault Tolerant Control of a Quad-rotor UAV Using Flatness Based Control

Fault Detection and Fault Tolerant Control Course

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Outline

- Quad-rotor Equations of Motions
- Introduction to Flatness Based Control
- Trajectory Planning
- Fault Diagnosis and Isolation
- Fault Scenarios
- Results

Quadrotor Equations of Motions

$$\ddot{x} = u_1\left(cos\phi\ sin\theta\ cos\psi + sin\phi\ sin\psi\right) - \frac{k_1}{m}\dot{x}$$

$$\ddot{y} = u_1\left(cos\phi\ sin\theta\ sin\psi - sin\phi\ cos\psi\right) - \frac{k_2}{m}\dot{y}$$

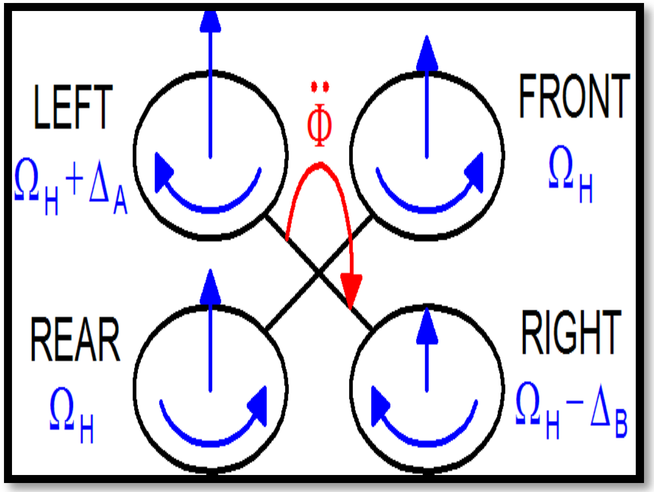
$$\ddot{z} = -g + u_1\left(cos\phi\ cos\theta\right) - \frac{k_3}{m}\dot{z}$$

$$\ddot{\theta} = u_2 - l\frac{k_4}{J_1}\dot{\theta}$$

$$\ddot{\phi} = u_3 - l\frac{k_5}{J_2}\dot{\phi}$$

$$\ddot{\psi} = u_4 - \frac{k_6}{J_3}\dot{\psi}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ \frac{-l}{J_1} & \frac{-l}{J_1} & \frac{l}{J_1} & \frac{l}{J_1} \\ \frac{-l}{J_2} & \frac{l}{J_2} & \frac{l}{J_2} & \frac{-l}{J_2} \\ \frac{C}{J_3} & \frac{-C}{J_3} & \frac{C}{J_3} & \frac{-C}{J_3} \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix}$$



Flatness Based Control

- The flatness property is described as follows.
- A dynamical system: $\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases}$

  is flat if and only if there exist variables $F \in \Re^m$

  called the flat outputs such that:

$$x = \Xi_1(F, \dot{F}, \cdots, F^{(n-1)})$$

$$y = \Xi_2(F, \dot{F}, \cdots, F^{(n-1)})$$

$$u = \Xi_3(F, \dot{F}, \cdots, F^{(n)})$$

Flatness Based Control In Quadrotor

$$F_1 = z \quad F_2 = x \quad F_3 = y \quad F_4 = \varphi$$

$$\theta = atan\left\{\frac{cosF_4\ddot{F}_2 + sinF_4\ddot{F}_3}{\ddot{F}_1 + g}\right\} \; ; \; \phi = atan\left\{\frac{cos\theta\left(sinF_4\ddot{F}_2 - cosF_4\ddot{F}_3\right)}{\ddot{F}_1 + g}\right\}$$

$$u_1 = \frac{\ddot{F}_1 + g}{cos\phi \; cos\theta}; \; u_2 = \ddot{\theta}; \; u_3 = \ddot{\phi}; \; u_4 = \ddot{F}_4$$

$$\ddot{F}_1 = \bar{u}_1; \; F_2^{(4)} = \bar{u}_2; \; F_3^{(4)} = \bar{u}_3; \; \ddot{F}_4 = \bar{u}_4,$$

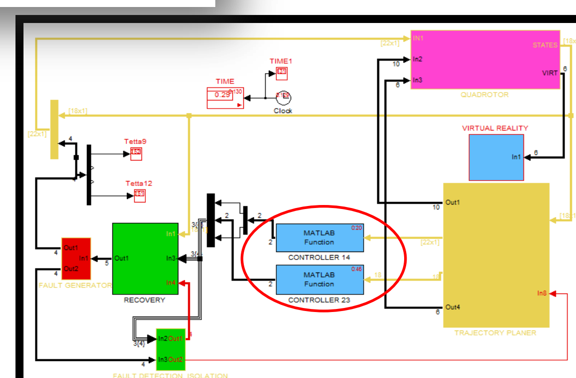
$$\bar{u}_1 = \ddot{F}_1^* + K_{11}(\dot{F}_1^* - \dot{F}_1) + K_{12}\,(F_1^* - F_1)$$

$$\bar{u}_2 = F_2^{(4)*} + K_{21}(F_2^{(3)*} - F_2^{(3)}) + K_{22}(\ddot{F}_2^* - \ddot{F}_2) + K_{23}(\dot{F}_2^* - \dot{F}_2) + K_{24}\,(F_2^* - F_2)$$

$$\bar{u}_3 = F_3^{(4)*} + K_{31}(F_3^{(3)*} - F_3^{(3)}) + K_{32}(\ddot{F}_3^* - \ddot{F}_3) + K_{33}(\dot{F}_3^* - \dot{F}_3) + K_{34}\,(F_3^* - F_3)$$

$$\bar{u}_4 = \ddot{F}_4^* + K_{41}(\dot{F}_4^* - \dot{F}_4) + K_{42}\,(F_4^* - F_4)$$

$$u_1 = \frac{\bar{u}_1 + g}{cos\phi \; cos\theta}; u_2 = \bar{\ddot{\theta}}; u_3 = \bar{\ddot{\phi}}; u_4 = \bar{u}_4$$



Trajectory Planning

$$F_i^* = a_5^i t^5 + a_4^i t^4 + a_3^i t^3 + a_2^i t^2 + a_1^i t + a_0^i \quad ; \quad (i = 1, 4)$$

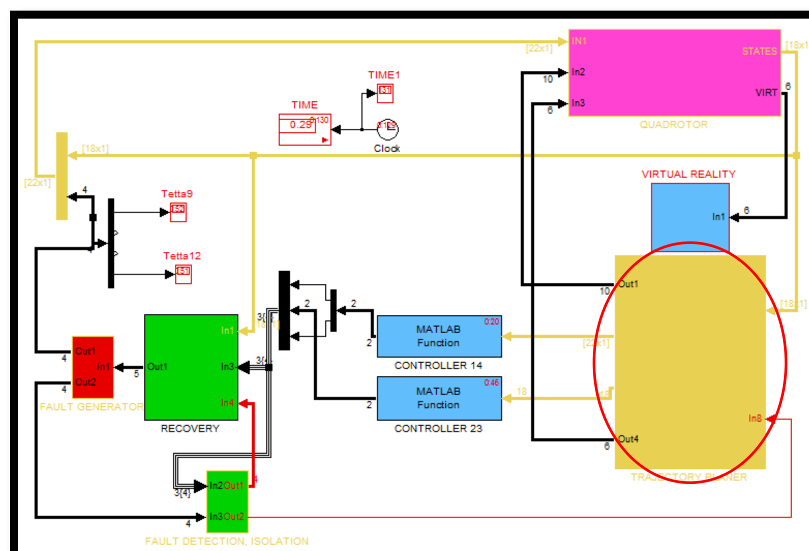
$$F_i^* = a_9^i t^9 + a_8^i t^8 + a_7^i t^7 + a_6^i t^6 + a_5^i t^5 + a_4^i t^4 + a_3^i t^3 + a_2^i t^2 + a_1^i t + a_0^i \quad ; \quad (i = 2, 3)$$

Initial Conditions F_{1i} $\dot{F}_{1i}$ $\ddot{F}_{1i}$

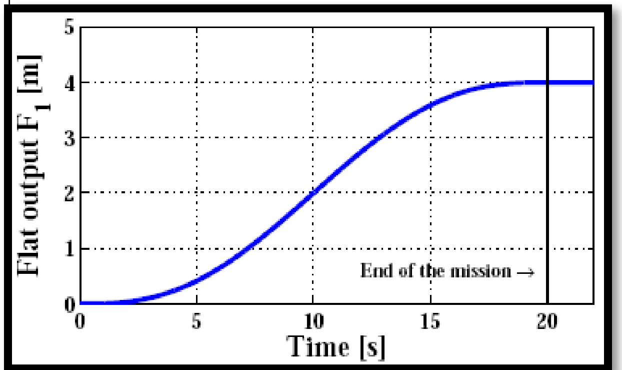
Final Conditions F_{1f} $\dot{F}_{1f}$ $\ddot{F}_{1f}$

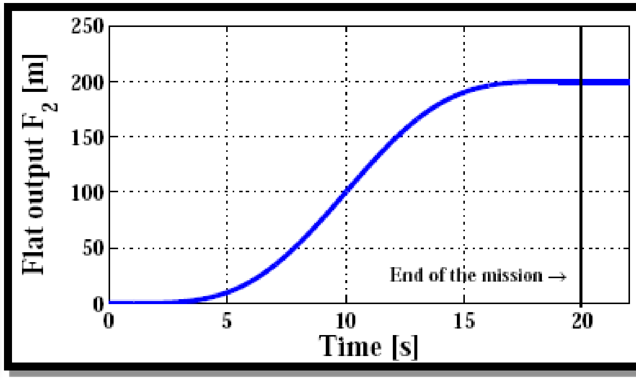
Initial Conditions F_{2i} $\dot{F}_{2i}$ $\ddot{F}_{2i}$ $\dddot{F}_{2i}$ ${F_{2i}}^{(4)}$

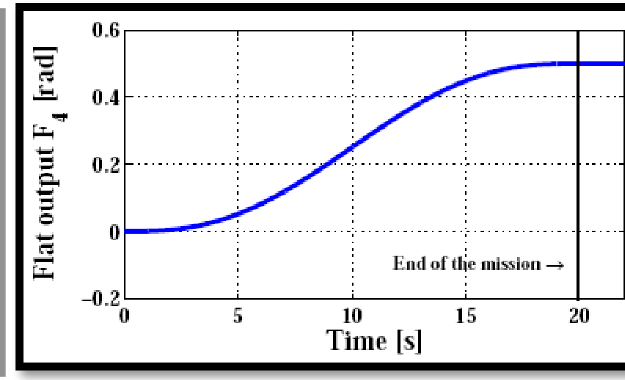
Final Conditions F_{2f} $\dot{F}_{2f}$ $\ddot{F}_{2f}$ $\dddot{F}_{2f}$ ${F_{2f}}^{(4)}$

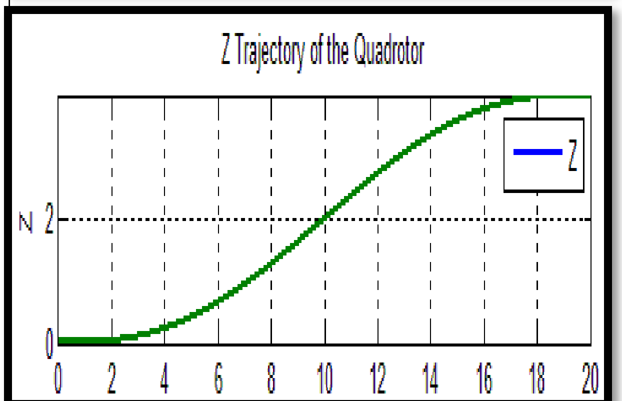


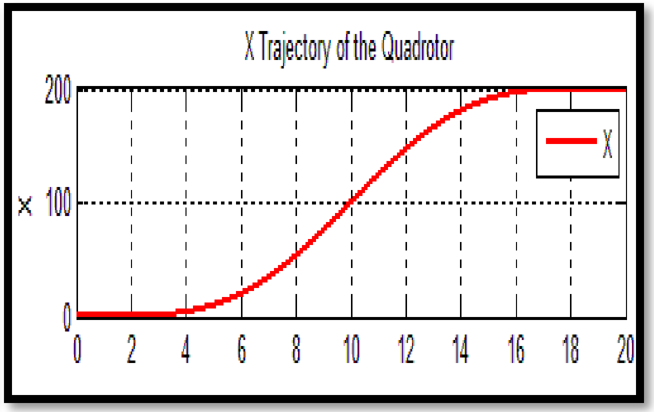
Comparing Results with Paper

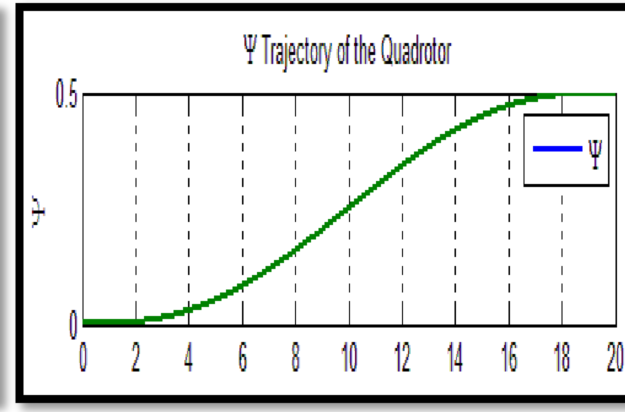


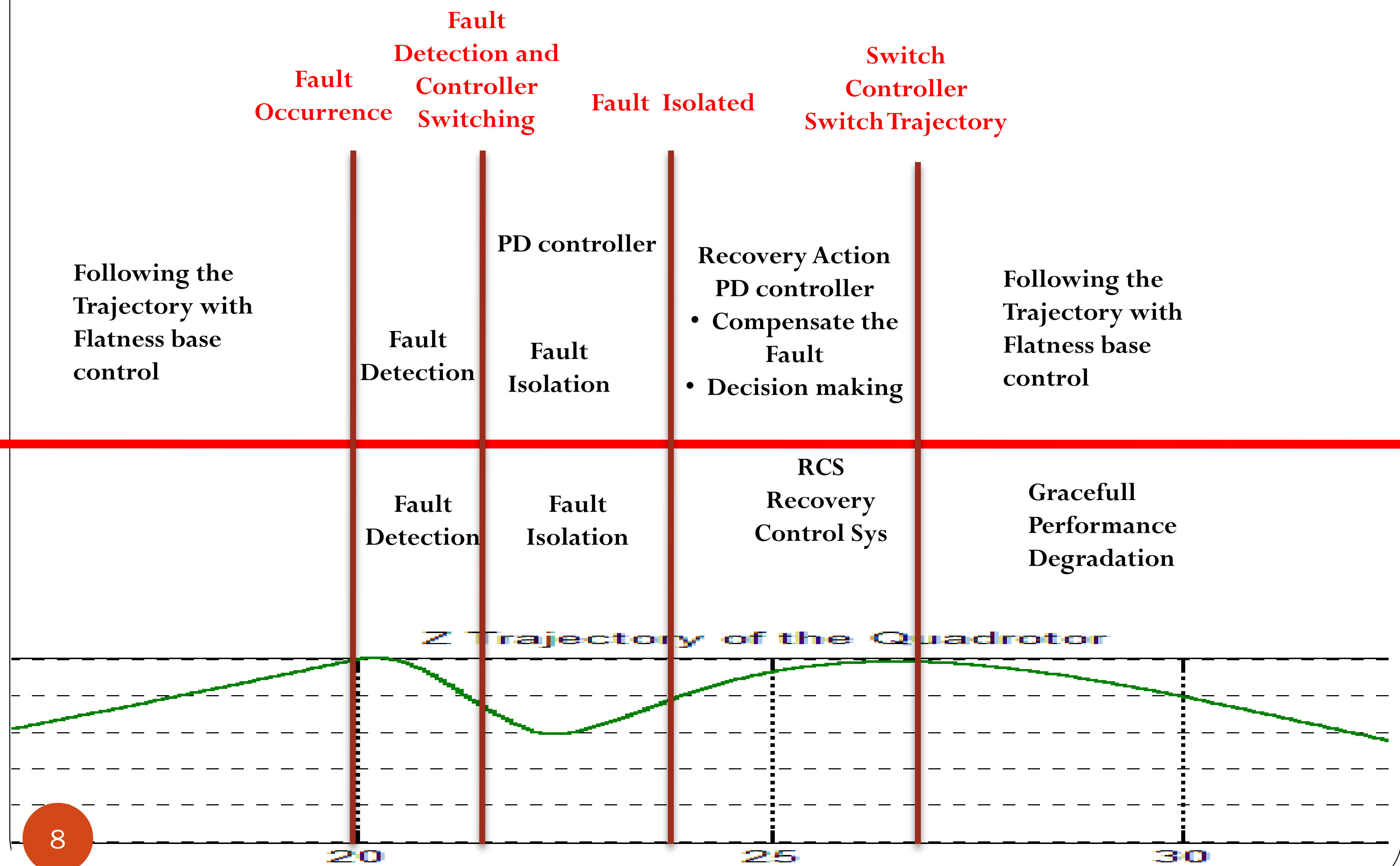




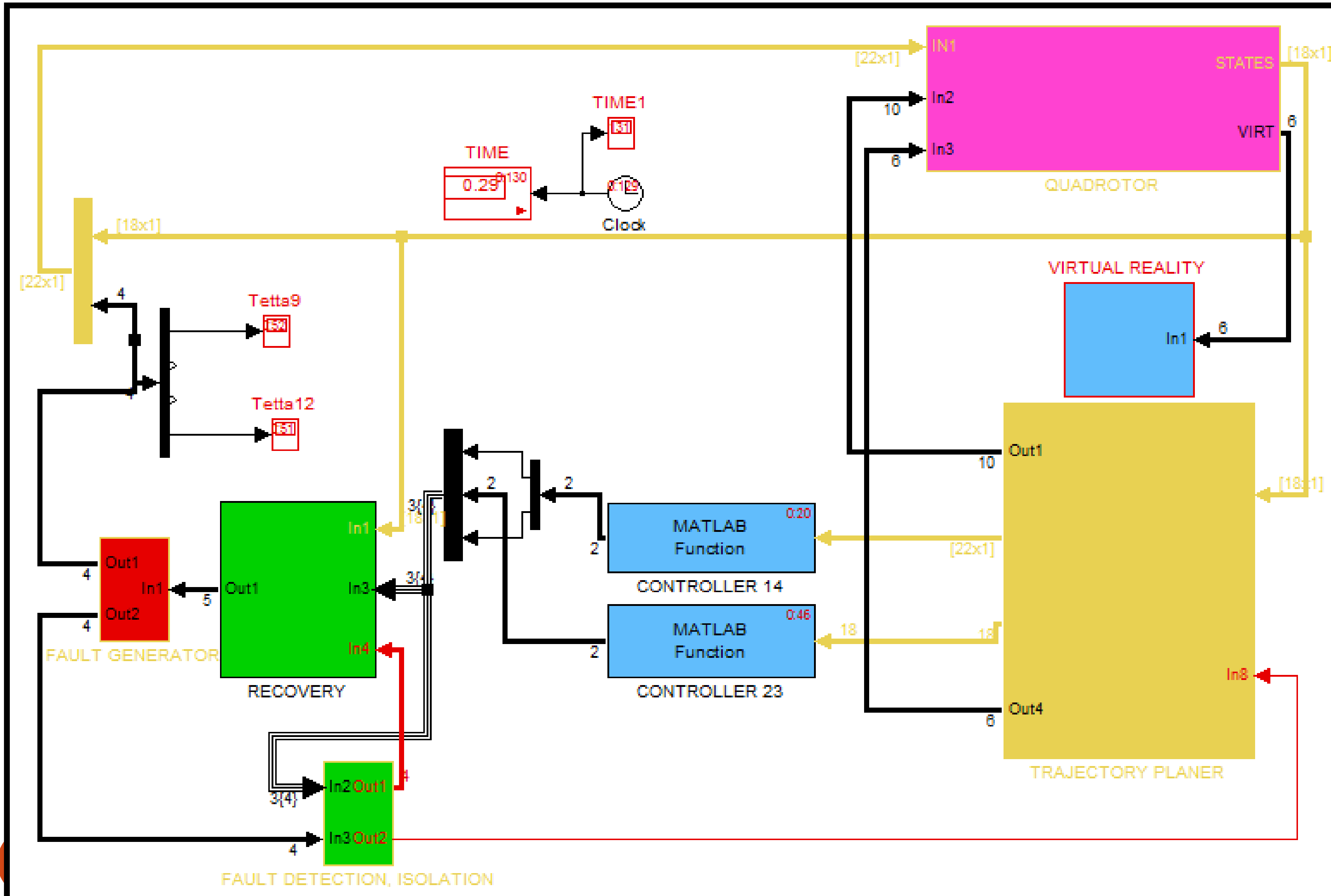







Fault
Occurrence
Fault
Detection and
Controller
Switching
Fault Isolated
Switch
Controller
Switch Trajectory
Following the
Trajectory with
Flatness base
control
Fault
Detection
PD controller
Fault
Isolation
Recovery Action
PD controller
• Compensate the
Fault
• Decision making
Following the
Trajectory with
Flatness base
control
Fault
Detection
Fault
Isolation
RCS
Recovery
Control Sys
Gracefull
Performance
Degradation
Z Trajectory of the Quadrotor
20
25
30

Simulation Model



Fault Detection & Isolation

$$\ddot{x} = u_1 \left(cos\phi \ sin\theta \ cos\psi + sin\phi \ sin\psi \right) \qquad \ddot{\theta} = u_2$$

$$\ddot{y} = u_1 \left(cos\phi \ sin\theta \ sin\psi - sin\phi \ cos\psi \right) \qquad \ddot{\phi} = u_3$$

$$\ddot{z} = -g + u_1 \left(cos\phi \ cos\theta \right) \qquad \ddot{\psi} = u_4$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ \frac{-l}{J_1} & \frac{-l}{J_1} & \frac{l}{J_1} & \frac{l}{J_1} \\ \frac{-l}{J_2} & \frac{l}{J_2} & \frac{l}{J_2} & \frac{-l}{J_2} \\ \frac{C}{J_3} & \frac{-C}{J_3} & \frac{C}{J_3} & \frac{-C}{J_3} \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix}$$

$$T_f = \begin{pmatrix} f_1 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \\ 0 & 0 & 0 & f_4 \end{pmatrix} * T$$

$$\begin{pmatrix} u_{1f} \\ u_{2f} \\ u_{3f} \\ u_{4f} \end{pmatrix} = \begin{pmatrix} +\frac{1}{4} & +\frac{1}{4} & +\frac{1}{4} & +\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} & +\frac{1}{2} & -\frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} & +\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} f1 & 0 & 0 & 0 \\ 0 & f2 & 0 & 0 \\ 0 & 0 & f3 & 0 \\ 0 & 0 & 0 & f4 \end{pmatrix} \begin{pmatrix} 1 & -0.5 & -0.5 & 0.5 \\ 1 & -0.5 & 0.5 & -0.5 \\ 1 & 0.5 & 0.5 & 0.5 \\ 1 & 0.5 & -0.5 & -0.5 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}$$

$$\begin{pmatrix} u_{1f} \\ u_{2f} \\ u_{3f} \\ u_{4f} \end{pmatrix} = \begin{pmatrix} \frac{8u_1-25u_2-25u_3+10u_4}{32} & \frac{8u_1-25u_2+25u_3-10u_4}{32} & \frac{8u_1+25u_2+25u_3+10u_4}{32} & \frac{8u_1+25u_2-25u_3-10u_4}{32} \\ \frac{-8u_1+25u_2+25u_3-10u_4}{100} & \frac{-8u_1+25u_2-25u_3+10u_4}{100} & \frac{8u_1+25u_2+25u_3+10u_4}{100} & \frac{8u_1+25u_2-25u_3-10u_4}{100} \\ \frac{-8u_1+25u_2+25u_3-10u_4}{100} & \frac{8u_1-25u_2+25u_3-10u_4}{100} & \frac{8u_1+25u_2+25u_3+10u_4}{100} & \frac{-8u_1-25u_2+25u_3+10u_4}{100} \\ \frac{+8u_1-25u_2-25u_3+10u_4}{10} & \frac{-8u_1+25u_2-25u_3+10u_4}{10} & \frac{8u_1+25u_2+25u_3+10u_4}{10} & \frac{-8u_1-25u_2+25u_3+10u_4}{10} \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix}$$

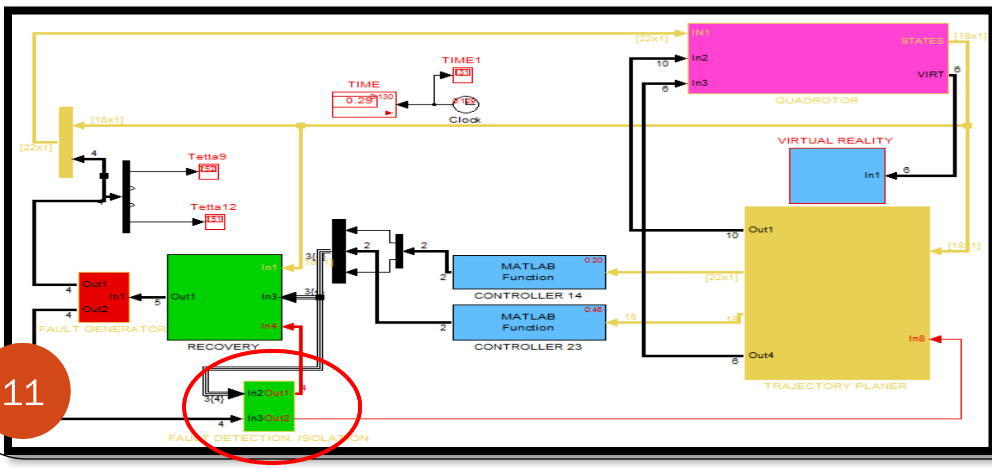
Fault Recovery

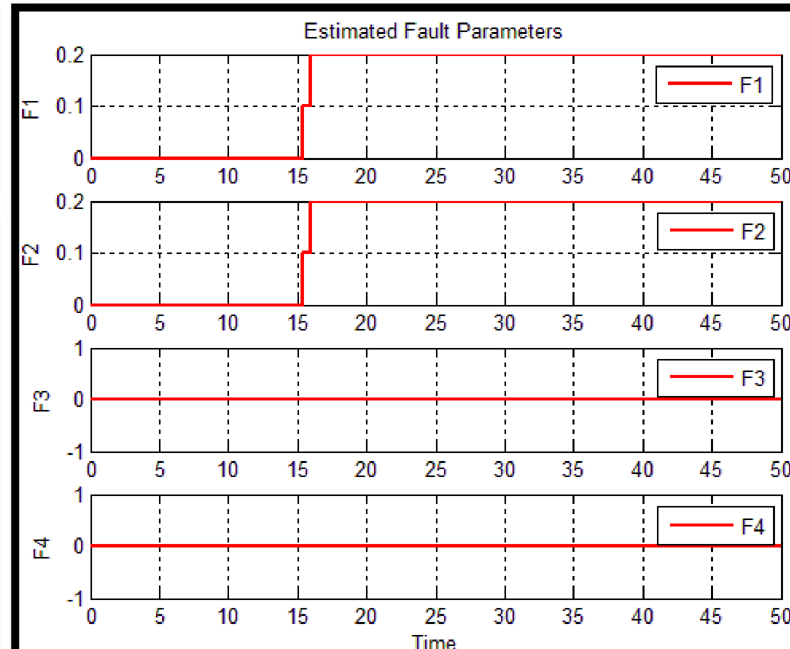
- We want to reconfigure our controller to produce an output command such that the Final Uf would be equal to the original controllers output without reconfiguration from previous section we have

$$U_{f1} = A^{-1} * F * A * U_1$$
$$U_{f2} = A^{-1} * F * A * U_2$$

- Our reconfigured controller will produce U2 instead of U1 but the overall U that take effect in system dynamic is U1

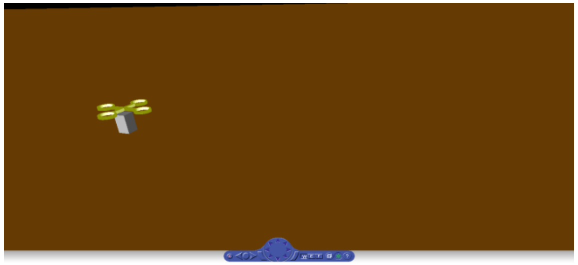
$$U_{f2} = U_1$$
$$U_2 = A^{-1} * F^{-1} * A * U_{f2} = A^{-1} * F^{-1} * A * U_1$$





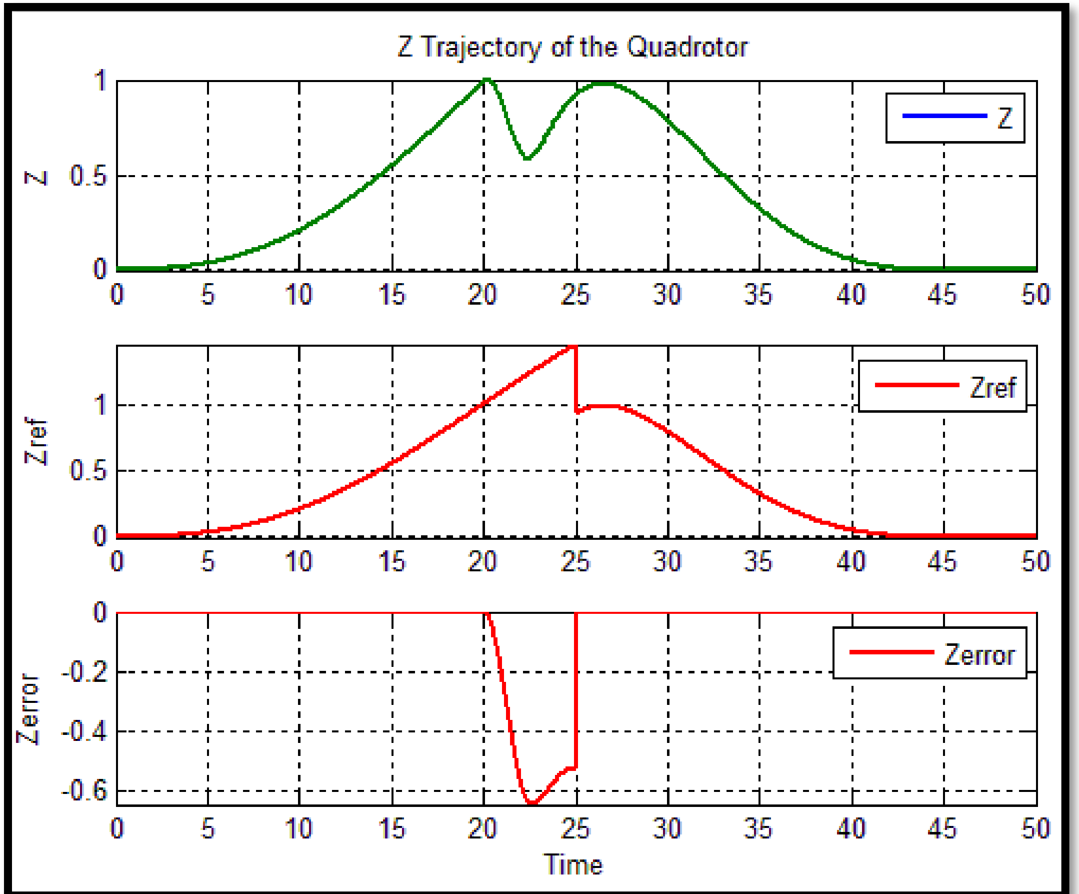
Fault Scenario 1

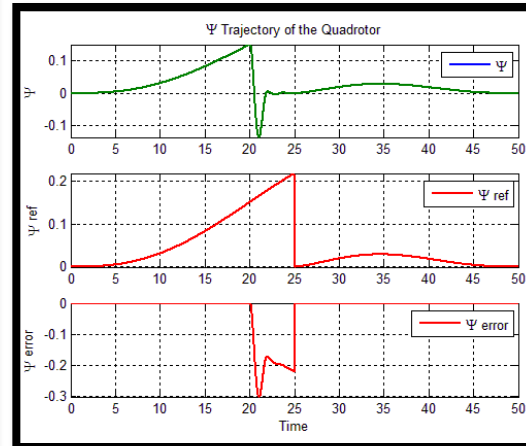
- Fault: 20% in Actuator 1
- Noise: Noise in Accelerations Measurements with Variance 0.00001
- Final time: 40
- Delay: 30 steps
- Time Steps:0.01
- Time of fault occurrence: 20
- Detection time: 20.33
- Isolation Time: 21.06

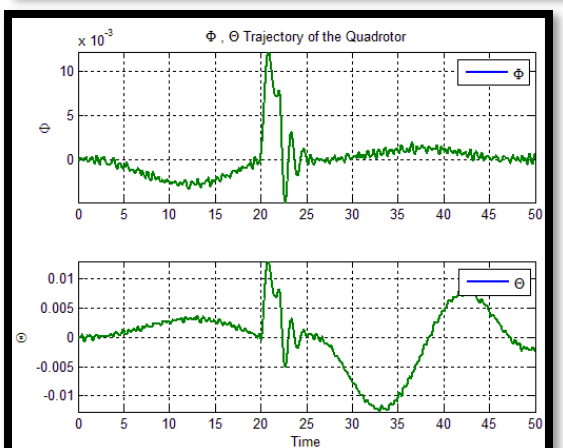


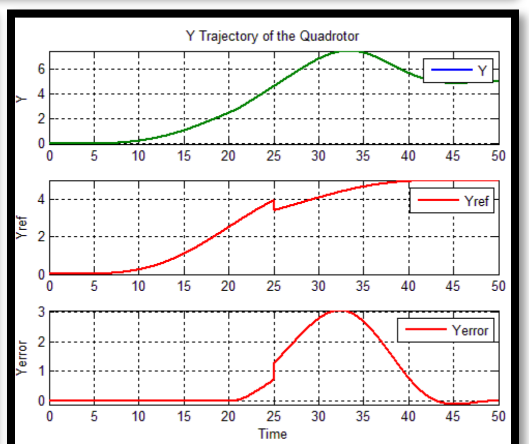
	Initial Con.	Final Con.	Recon Traj.
X	0	5	5
Y	0	5	5
Z	0	2	0
ψ	0	.3	0

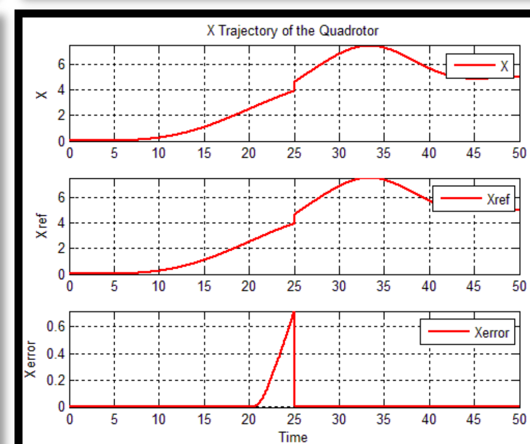
Scenario 1 Results









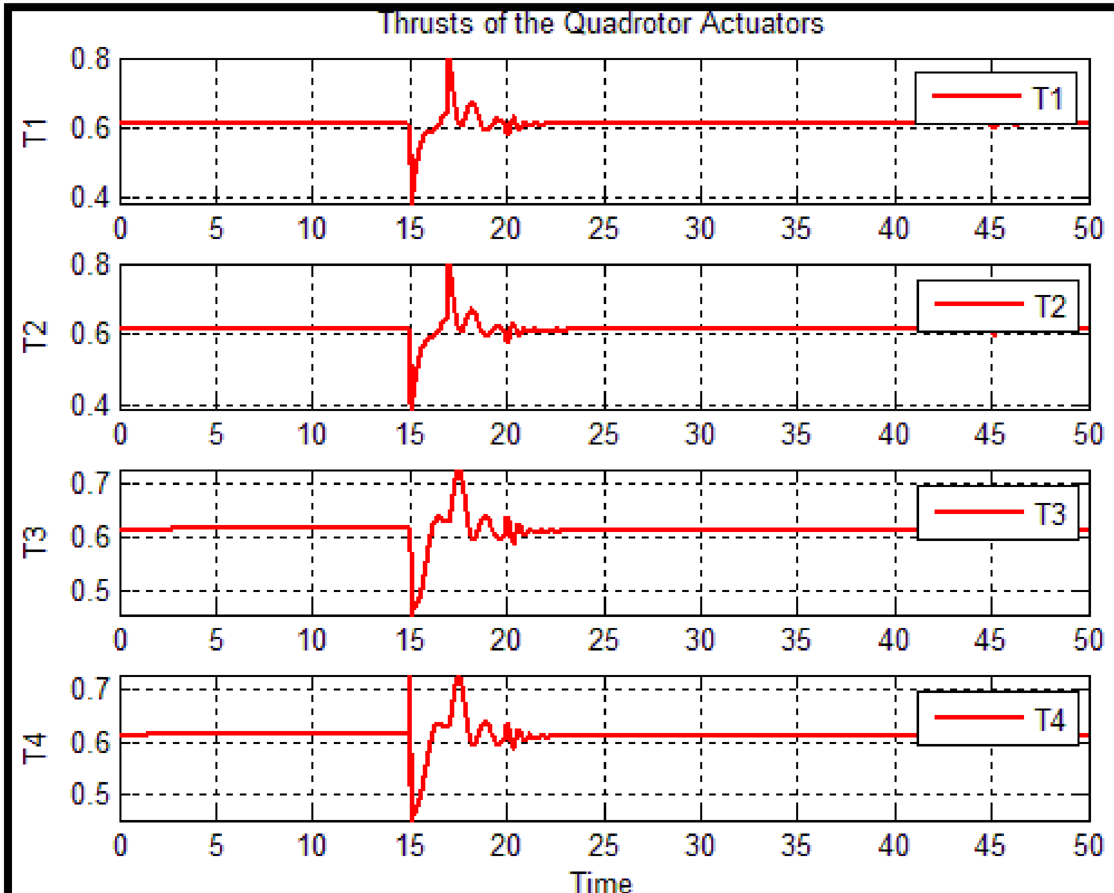


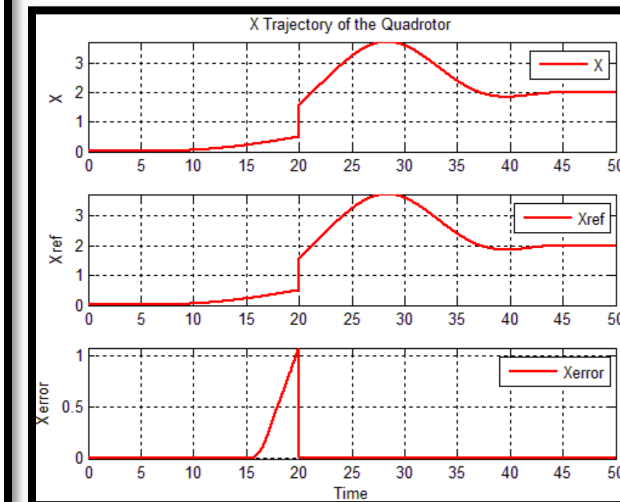
Fault Scenario 2

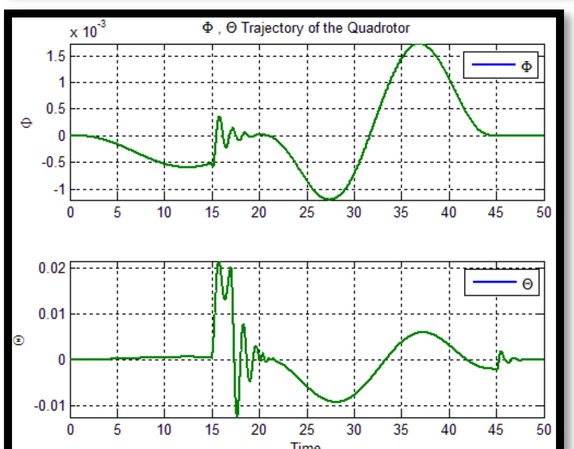
- Fault: unsymmetrical 20% in Actuator 1 & Actuator 2
- Noise: No noise
- Final time: 40
- Delay: 30 steps
- Time Steps:0.01
- Time of fault occurrence: 15
- Detection time: 15.32
- Isolation Time: 16.01

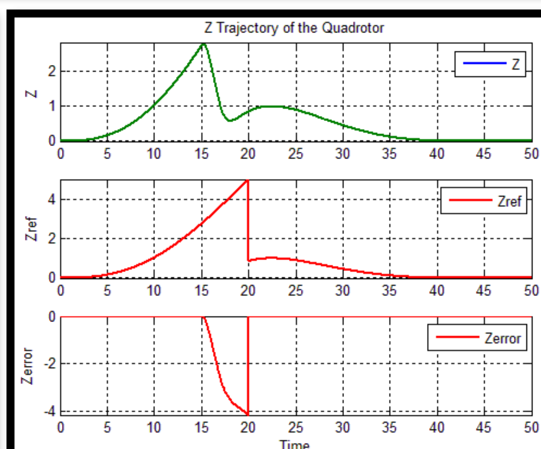
	Initial Con.	Final Con.	Recon Traj.
X	0	1	2
Y	0	1	2
Z	0	10	0
ψ	0	0	0

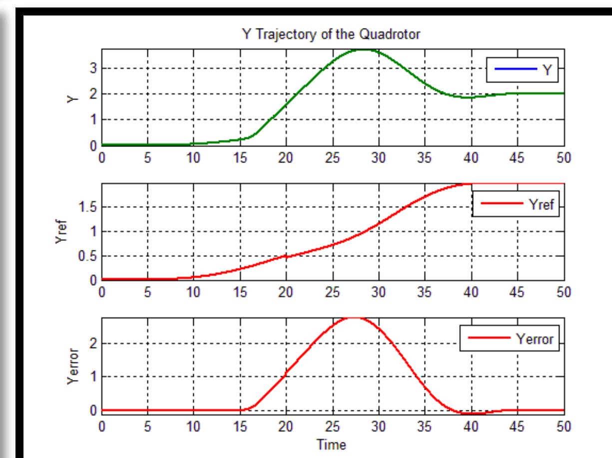
Scenario 2 Results











Thanks For Your Attention