Quad-Rotor Helicopter Active Fault Tolerant Control

Supervisor: Dr. Youmin Zhang

Amin Salar **6032761**
Zahra Gallehdari **1309102**
Narges Roofigari **8907926**

Fault Diagnosis and Fault Tolerant Control Systems
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Contents

Introduction

- ✓ Conventional Feedback and PID Controllers
  - Very good response in normal situation
  - Unable to tolerate the fault

- ✓ Passive and Active FTCS
  - Passive FTCS
    - Capable of tolerating one or more system component faults
    - Without reconfiguring the control system structure or the parameters
  - Active FTCS
    - Reconfigurable controller
    - FDD part
    - Reconfiguration mechanism
    - Command governor

Quad-Rotor Modeling

- ✓ Test bench of the proposed methods
- ✓ Nonlinear Model
- ✓ With six degrees of freedom: yaw, pitch, roll, x (longitudinal motion), y (lateral motion) and z (altitude)
- ✓ In most of the studies altitude, yaw, pitch and roll are controlled with thrust of the four rotors
- ✓ x and y are controlled by choosing appropriate values for other variables
- ✓ The equations of motion:

$$\ddot{x} = \frac{u_1(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi)}{m}$$

$$\ddot{y} = \frac{u_2(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi)}{m} - \frac{K_{d2}\dot{y}}{m}$$

$$\ddot{z} = \frac{u_3 \cos\theta \cos\phi}{m} - \frac{k_{d3}\dot{z}}{m} - g$$

$$\ddot{\phi} = \frac{1}{J_x}\left[u_2 l - k_{d4}\dot{\phi} - \dot{\theta}\dot{\psi}(J_z - J_y)\right]$$

$$\ddot{\theta} = \frac{1}{J_y}\left[u_3 l - k_{d5}\dot{\theta} - \dot{\phi}\dot{\psi}(J_x - J_z)\right]$$

$$\ddot{\psi} = \frac{1}{J_z}\left[u_4 l - k_{d6}\dot{\psi} - \dot{\theta}\dot{\phi}(J_y - J_x)\right]$$

Quad-Rotor Linearized Model

- ✓ Assumptions:
  - The body inertia in the axis direction is the same. The gyroscopic effect is negligible.
  - No disturbance affects the system or the rate of yaw angle is zero.
  - Drag terms are neglected

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \\ \dot{\Phi} \\ \ddot{\Phi} \\ \dot{\theta} \\ \ddot{\theta} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ \Phi \\ \dot{\Phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{l}{J_x} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{l}{J_y} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{l}{J_z} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

Combined Model Reference Adaptive Control

- ✓ Concept:
  - Combining direct and indirect model reference adaptive control (MRAC) architectures for generic dynamical systems
  - Using Prediction errors in addition to tracking errors in formulating adaptive law dynamics
  - Gaining better (smoother than MRAC) transient characteristics

- ✓ Novelties (Lavretsky, 2009)
  - Applicable to a generic class of MIMO dynamical systems with matched uncertainties
  - Does not require online measurements of the system state derivative
  - Designed to augment a baseline linear controller

Direct Model Reference Adaptive Control

- A class of MIMO uncertain dynamical systems
- $\dot{x}_p = A_p x_p + B_p \Lambda \left(u + \Theta_d^T \Phi_d(x_p) \right)$
- $y = C_p x_p$
- $e_y(t) = y(t) - r(t)$
- $\dot{e}_{yI} = e_y = y - r$
- $\dot{x} = Ax + B\Lambda \left(u + d(x_p) \right) + B_c r$

$$A = \begin{bmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_p \end{bmatrix}, \quad B = \begin{bmatrix} 0_{m\times m} \\ B_p \end{bmatrix}, \quad B_c = \begin{bmatrix} -I_{m\times m} \\ 0_{n_p\times m} \end{bmatrix}$$

$$y = \begin{pmatrix} 0_{m\times m} & C_p \end{pmatrix} x = Cx$$

- $A_{ref} = A + B\Lambda K_x^T$
- $\dot{x} = A_{ref} x + B\Lambda \left(u + \begin{bmatrix} K_x^T & \Theta_d^T \end{bmatrix} \begin{bmatrix} -x \\ \Phi_d(x_p) \end{bmatrix} \right) + B_c r$
- $\dot{x}_{ref} = A_{ref} x_{ref} + B_c r$

Unknown Parameters

Direct Model Reference Adaptive Control

- $e = x - x_{ref}$
- $\dot{e} = A_{ref}e - B\Lambda[\Delta K_x^T \quad \Delta\Theta_d^T]\begin{bmatrix} -x \\ \Phi_d(x_p) \end{bmatrix}\left(\widehat{\Theta}_d - \Theta_d\right)^T$
- $A_{ref}^T P_{ref} + P_{ref} A_{ref} = -Q_{ref}$
- $u_{ad} = \widehat{K}_x^T x - \widehat{\Theta}_d^T \Phi_d(x_p)$

✓ Direct MRAC Laws based on above equations and Lyapunov arguments

$$\begin{cases} \dot{\widehat{K}}_x = -\Gamma_x x e^T P_{ref} B \\ \dot{\widehat{\Theta}}_d = -\Gamma_{\Phi_d} \Phi_d e^T P_{ref} B \end{cases}$$

Indirect Model Reference Adaptive Control

- $\dot{x} + \lambda_f x = \lambda_f x + A_{ref} x + B\Lambda\left(u + [K_x^T \quad \Theta_d^T]\begin{bmatrix} -x \\ \Phi_d(x_p) \end{bmatrix}\right) + B_c r$
- $\dot{x}_f = \lambda_f (x - x_f)$ **Stable Filter Dynamics**
- $Y(t) = (B^T B)^{-1} B^T \left(\lambda_f (x - x_f) - A_{ref} x_f - B_c r_f\right) = \Lambda\left(u_f + [K_x^T \quad \Theta_d^T]\begin{bmatrix} -x \\ \Phi_d(x_p) \end{bmatrix}\right)$
- $\hat{Y}(t) = \hat{\Lambda}\left(u_f - \hat{K}_x^T x_f + \hat{\Theta}_d^T \Phi_{df}\right)$
- $e_Y = \Lambda[\Delta K_x^T \quad \Delta\Theta_d^T]\begin{bmatrix} -x_f \\ \Phi_{df}(x_p) \end{bmatrix} + \Delta\Lambda\left(u_f - \hat{K}_x^T x_f + \hat{\Theta}_d^T \Phi_{df}\right)$

Predictor Output Estimation Error
+
Lyapunov Arguments
=
Parameter Estimation Laws

$$\begin{cases} \dot{\hat{K}}_x = \Gamma_x x_f e_Y^T \\ \dot{\hat{\Theta}}_d = -\Gamma_{\Phi_d} \Phi_{df} e_Y^T \\ \dot{\hat{\Lambda}}^T = -\Gamma_\Lambda \left(u_f - \hat{K}_x^T x_f + \hat{\Theta}_d^T \Phi_{df}\right) e_Y^T \end{cases}$$

Combined Model Reference Adaptive Control

CMRAC Laws by combining direct MRAC laws with parameter estimation laws

$$\begin{cases} \dot{\hat{K}}_x = -\Gamma_x\left(x e^T P_{ref} B - x_f \gamma_c e_Y^T\right) \\ \dot{\hat{\Theta}}_d = \Gamma_{\Phi_d}\left(\Phi_d e^T P_{ref} B - \Phi_{df} \gamma_c e_Y^T\right) \\ \dot{\hat{\Lambda}}^T = -\Gamma_\Lambda\left(u_f - \hat{K}_x^T x_f + \hat{\Theta}_d^T \Phi_{df}\right) \gamma_c e_Y^T \end{cases}$$

MRAC and CMRAC Design

Three types of matched uncertainties:

- ✓ Linear-in-state uncertainty $K_{x_p}^T x$
- ✓ Control effectiveness constant uncertainty $\Lambda > 0$
- ✓ Nonlinear-in-state uncertainty in the form of $d(x_p) = \Theta_d^T \Phi_d(x_p)$

```
Kxp =

        0         0         0         0
        0         0         0         0
        0         0         0         0
        0         0         0         0
  -0.2384    0.3224    0.0049    0.3898
  -0.2041   -0.1723   -0.0002    0.0142
   0.4973   -0.1162    0.4385    0.4639
  -0.0160    0.1897   -0.0823   -0.1354
  -0.1715    0.1382   -0.0899   -0.0201
  -0.1652   -0.1072   -0.1796    0.0264
   0.0294   -0.2016    0.4247    0.1009
   0.2541    0.0765   -0.1297   -0.1651
```

```
Landa =

   0.4000         0         0         0
        0    0.4000         0         0
        0         0    0.4000         0
        0         0         0    0.4000
```

$d(x_p)$: Gaussian function

MRAC and CMRAC Design

- $\dot{x}_p = \left(A_{pBL} + B_p \Lambda K_{x_p}^T\right) x_p + B_p \Lambda \left(u + d(x_p)\right)$

$$x_p = [z \quad \dot{z} \quad \phi \quad \dot{\phi} \quad \theta \quad \dot{\theta} \quad \Psi \quad \dot{\Psi}]^T$$

$$A_{BL} = \begin{bmatrix} 0_{m\times m} & C_p \\ 0_{n_p\times m} & A_{p_{BL}} \end{bmatrix} \qquad B = \begin{bmatrix} 0_{m\times m} \\ B_p \end{bmatrix}$$

LQR Baseline Control Gain

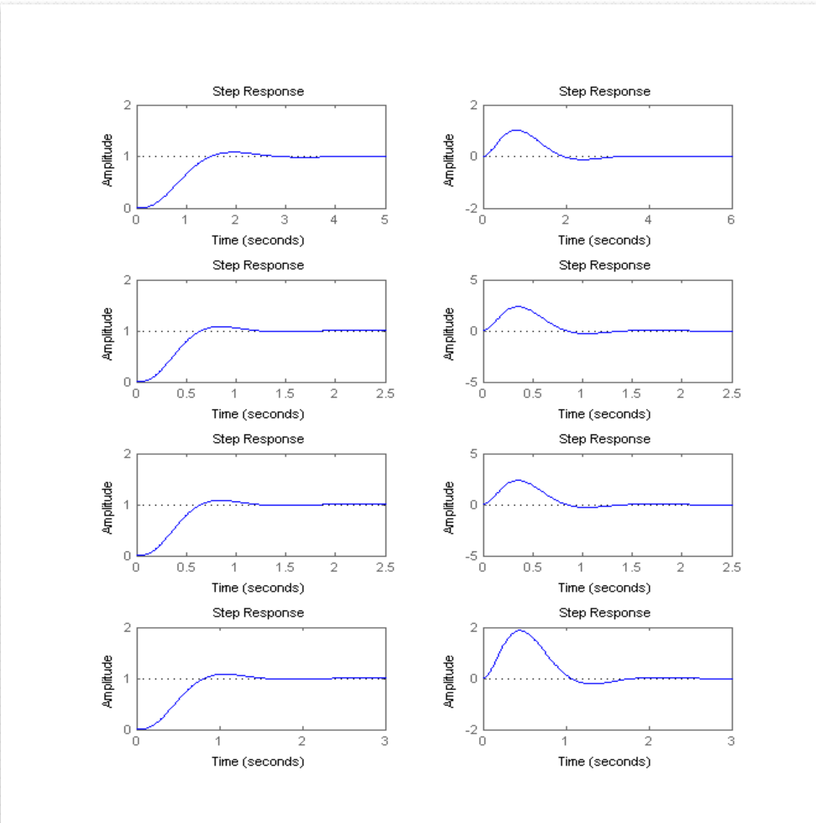
Kx_BL =

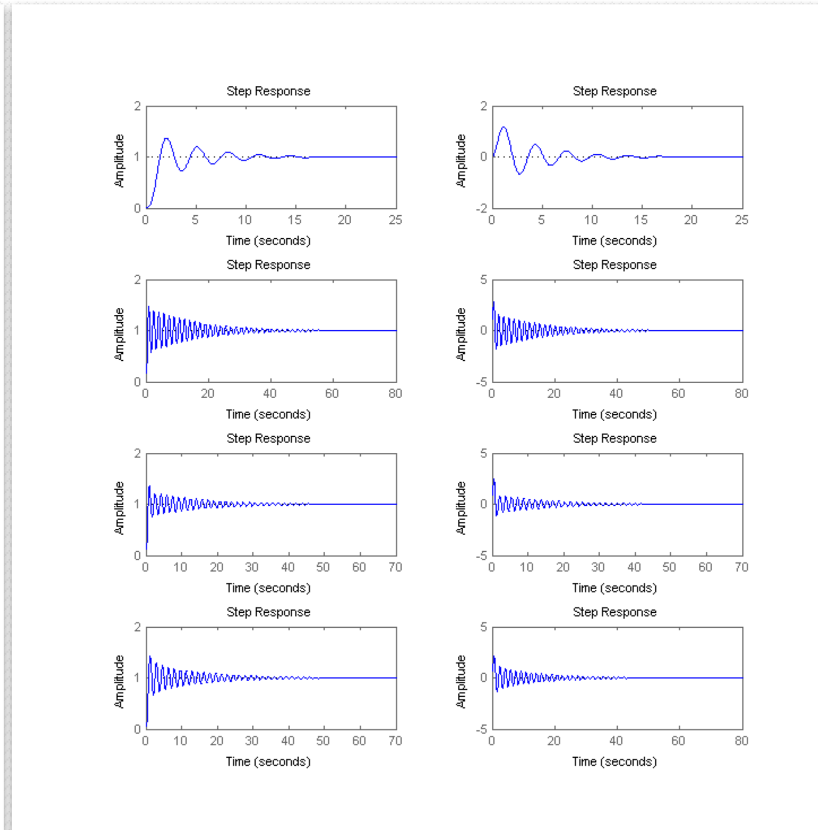
0	0	0	-10.0000
-10.0000	0	0	0
0	-10.0000	0	0
0	0	-10.0000	0
0	0	0	-7.8944
0	0	0	-3.1161
-3.4311	0	0	0
-0.5886	0	0	0
0	-3.4311	0	0
0	-0.5886	0	0
0	0	-4.3276	0
0	0	-0.9364	0

Baseline LQR Simulation Results

- ✓ $A_{ref} = A_{BL} + BK_{xBL}^T$
- ✓ $A_{new} = A_{BL} + \Lambda B(K_{xp} + K_{xBL})^T$

As Modeling Uncertainty

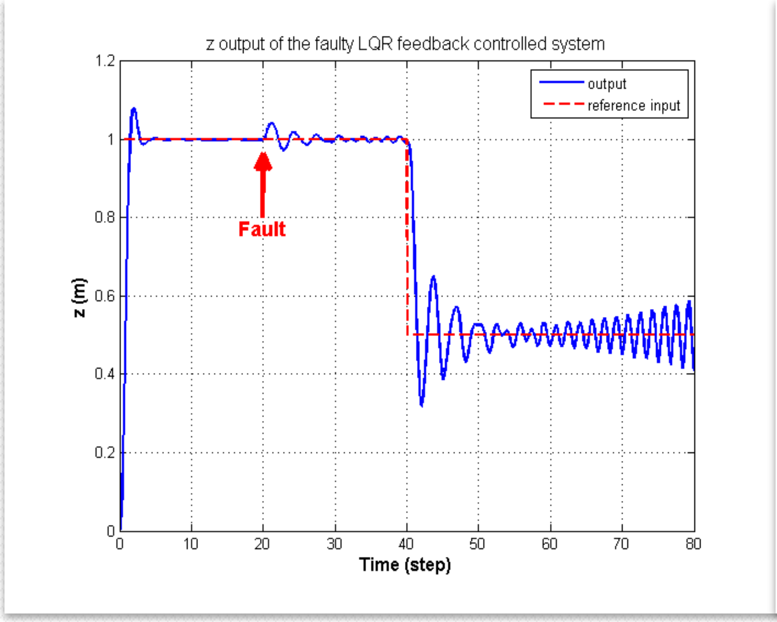


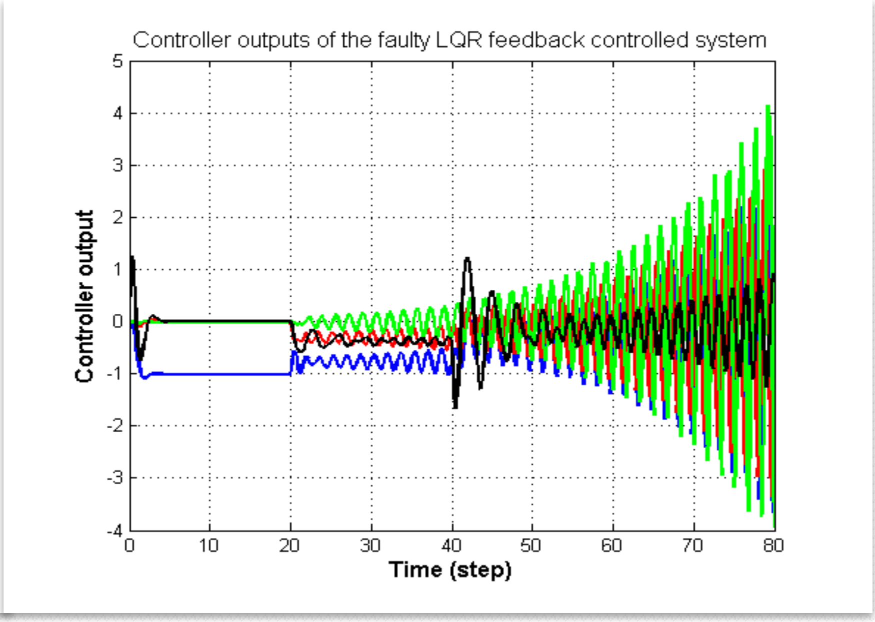


Baseline LQR Simulation Results

- ✓ Change in system dynamics in step 20
  - By introducing $K_{x_p}^T x$ and Λ to the system
  - System begins to oscillate but damped the oscillations
- ✓ A change of reference input in step 40
  - Faulty system becomes unstable

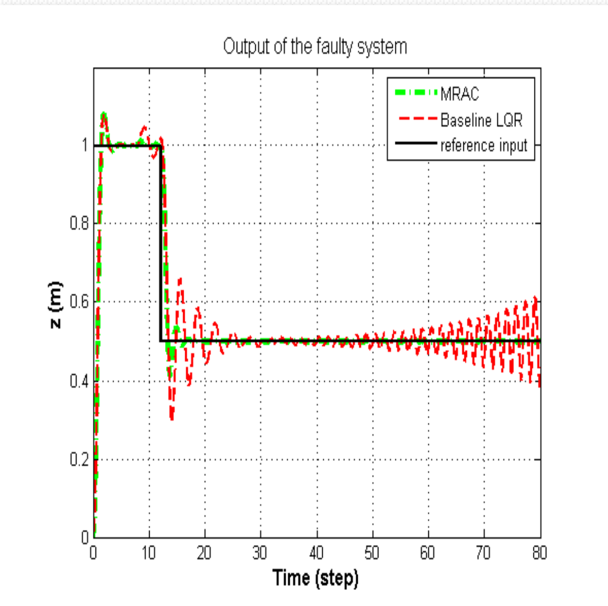
As Fault

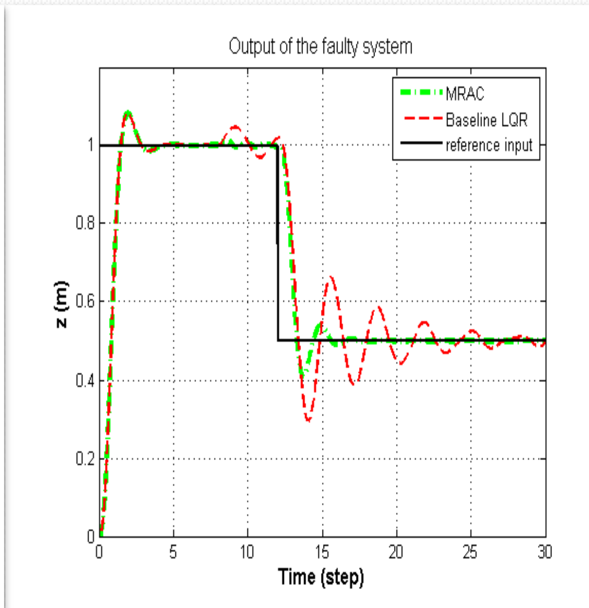


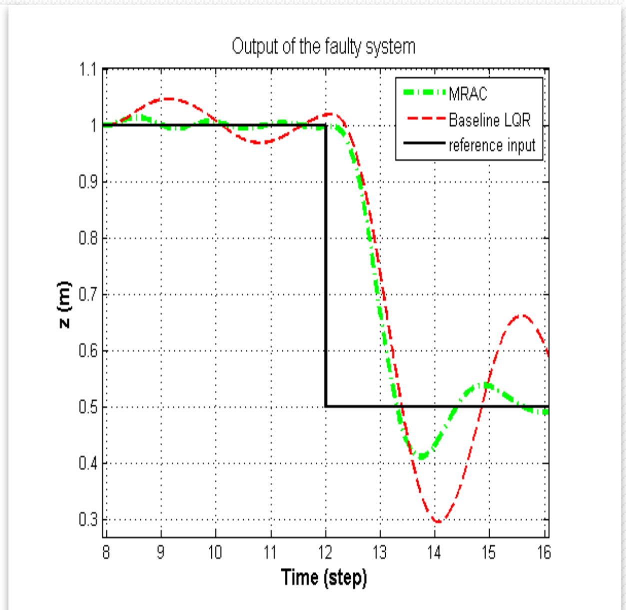


Implementing MRAC Architecture

- ✓ Adding Direct MRAC to the baseline LQR controller
  - Improved system recovery from fault induced in step 8
- ✓ Changing the reference input in step 12
  - System <u>did not</u> become unstable as oppose to the LQR controlled system

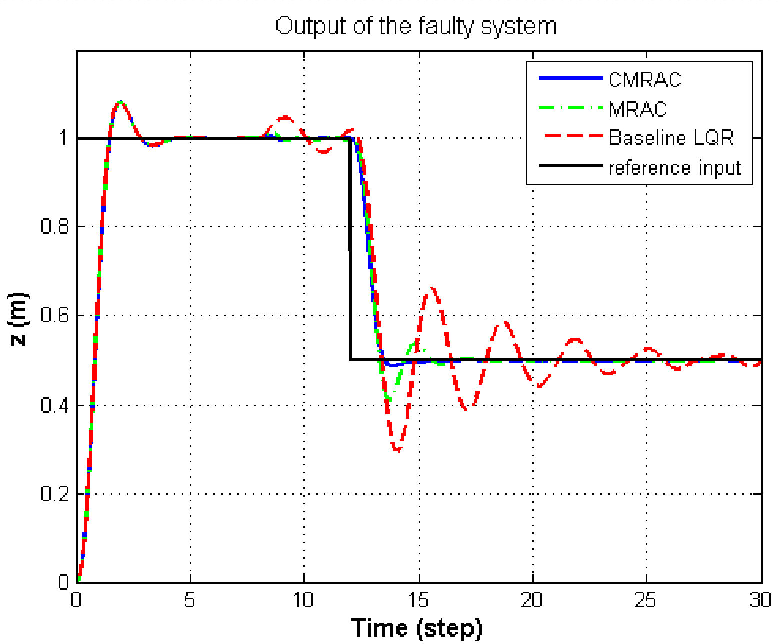


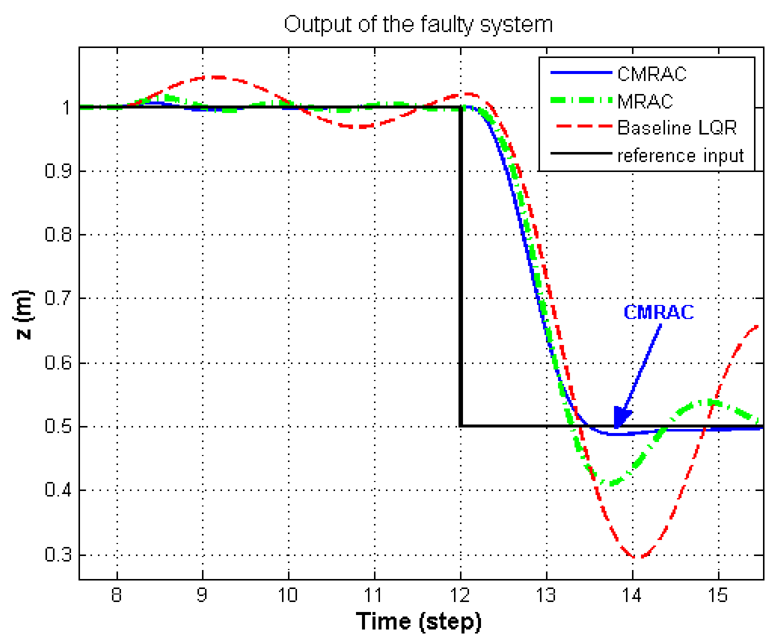




Implementing CMRAC Architecture

- ✓ Combined/Composite model reference adaptive controller (CMRAC)
- ✓ Combined direct adaptive control system with its indirect counterpart
- ✓ Improving the performance of the fault tolerant control system
- ✓ Test the results using the quad-rotor linear model
- ✓ Clearly better transient responses
- ✓ More reliable method for fault recovery than the other two



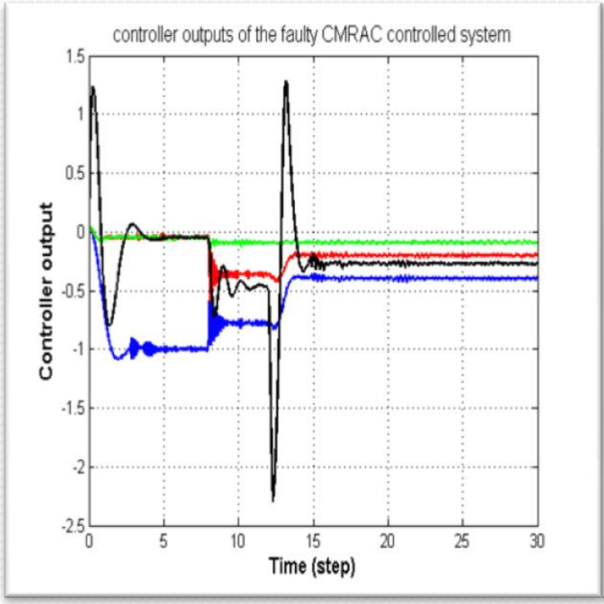


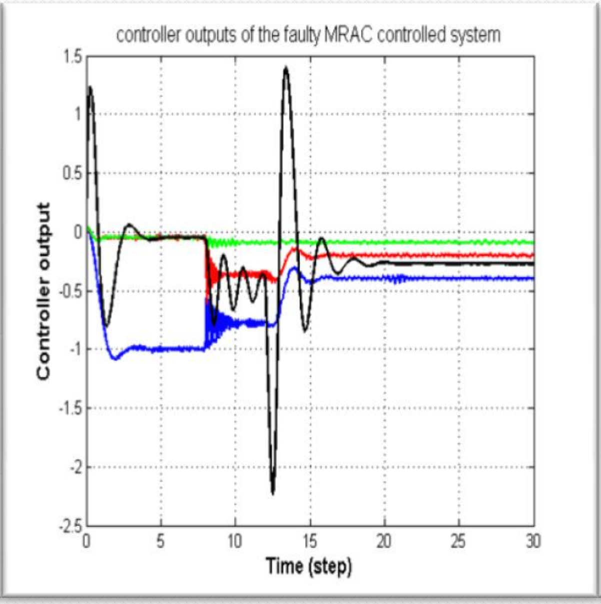
Implementing CMRAC Architecture

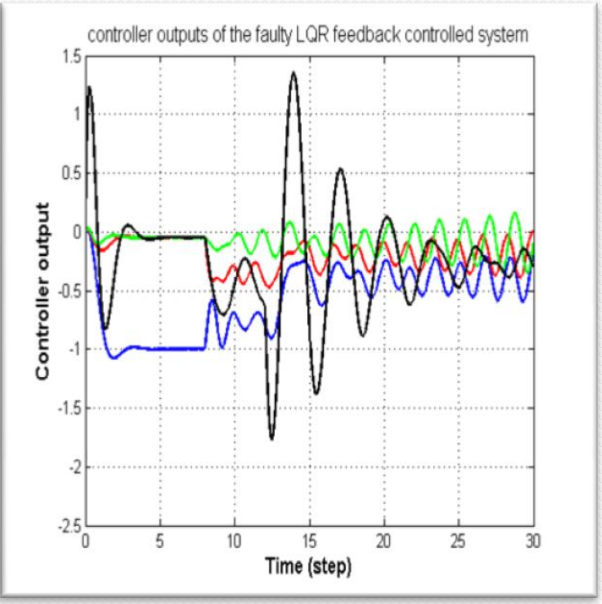
- ✓ Choosing correct values for symmetric matrices Q_{ref}, Γ_x, Γ_{Φ_d}, Γ_Λ and γ_c are very important
- ✓ If "rates of adaptation" matrices Γ_x and Γ_{Φ_d} have large singular values unwanted oscillations may happen

Controller Output Signal Comparison

- ✓ A drawback for those methods
  - Unwanted high frequency oscillations in controller output signal of MRAC and CMRAC (Lavretsky, 2009)







Adaptive Lyapunov Based Control

- ✓ How design the control law that forces the system to track desired trajectories?
- ✓ Motion equation of the altitude

$$\ddot{Z} = -\mathrm{g} + \mathrm{b}_1^* \cos\theta \cos\phi \,/ m U_1$$

- ✓ How to design U_1 in a way $z \to z_r, \dot{z} \to \dot{z}_r as\ t \to \infty$?

$$u_1 = \hat{\alpha}_1 \left(\frac{m}{cos\theta cos\varphi}(-c_{12} y_{12-} y_{11} + g + \ddot{z}_r + \dot{\beta}_1)\right)$$

$$y_{11}(t) = z(t) - z_r(t) = \mathrm{e(t)}$$
$$y_{12}(t) = \dot{z}(t) - \dot{z}_r(t) - \beta_1$$
$$\beta_1(t) = -c_{11} y_{11}(t)$$
$$\dot{\hat{\alpha}}_1 = -\gamma_1 \frac{u_{c1}}{m} cos\theta cos\varphi y_{12}$$

$$V(t) = \frac{1}{2} y_{11}^2 + \frac{1}{2} y_{12}^2 + \frac{b_1^*}{2\gamma} \quad (\hat{\alpha}_1 - \alpha_1)^2, V(t) \geq 0, \dot{V}(t) \leq 0$$

Adaptive Lyapunov Based Control

- ✓ Roll angle

$$\ddot{\varphi} = l.\frac{b_2^*}{J_x}u_2 - \frac{\dot{\theta}\dot{\psi}(J_z - J_y)}{J_x}$$

$$u_2 = \hat{\alpha}_2 u_{c2},$$

$$u_{c2} = \frac{J_x}{l}\left(-c_{22}y_{22} - y_{21} + \dot{\beta}_2 + \ddot{\varphi}_r + \frac{\dot{\theta}\dot{\psi}(J_z - J_y)}{J_x}\right)$$

- ✓ Pitch angle

$$\ddot{\theta} = l.\frac{b_3^*}{J_y}u_3 - \frac{\dot{\varphi}\dot{\psi}(J_x - J_z)}{J_y}$$

$$u_3 = \hat{\alpha}_3 u_{c3},$$

$$u_{c3} = \frac{J_y}{l}\left(-c_{32}y_{32} - y_{31} + \dot{\beta}_3 + \ddot{\theta}_r + \frac{\dot{\varphi}\dot{\psi}(J_x - J_z)}{J_y}\right)$$

Adaptive Lyapunov Based Control

- ✓ Yaw angle

$$\ddot{\psi} = l.\frac{b_4^*}{J_z}u_4 - \frac{\dot{\varphi}\dot{\theta}(J_y - J_x)}{J_z}$$

$$u_4 = \hat{\alpha}_4 u_{c4}$$

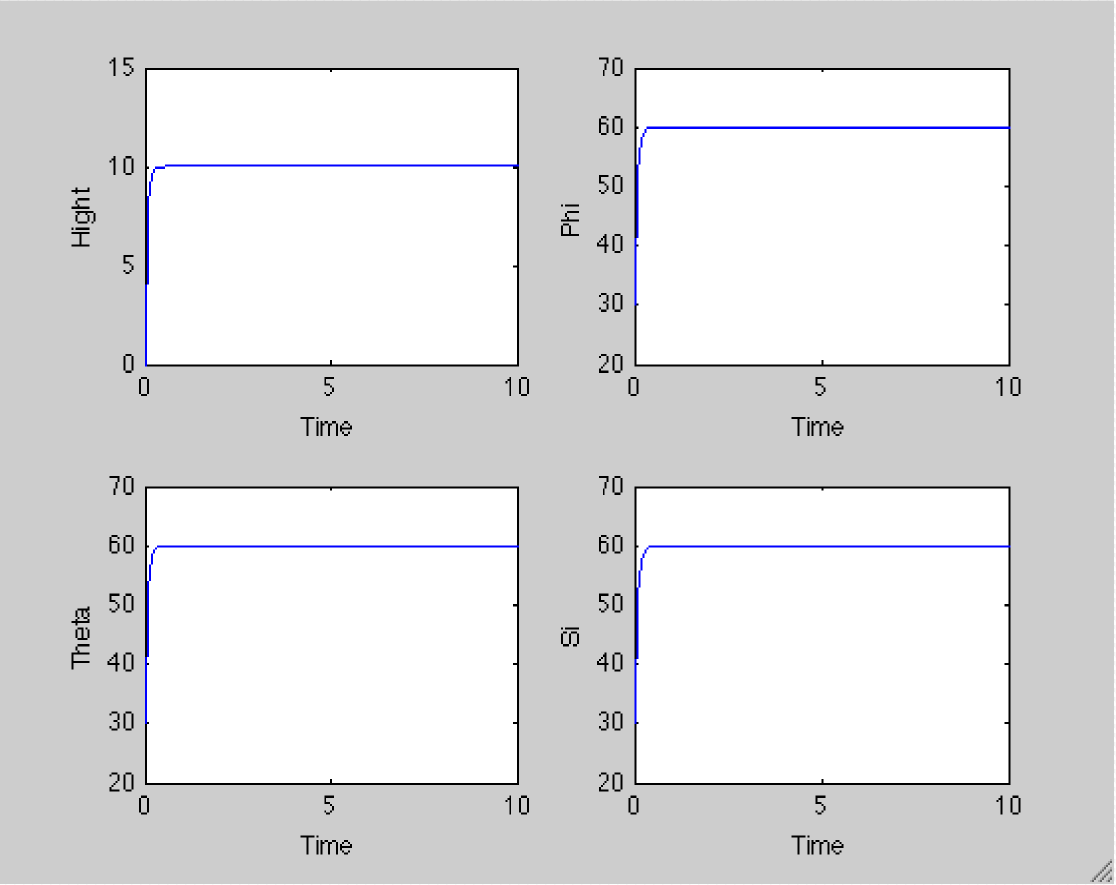
$$u_{c4} = \frac{J_y}{l}(-c_{42}y_{42-}y_{41} + \dot{\beta}_4 + \ddot{\theta}_r + \frac{\dot{\varphi}\dot{\psi}(J_x - J_z)}{J_y}$$

- ✓ How choose controller parameters?!
- ✓ How these parameters affect transient response?!
- ✓ Nothing provided in the reference!

Adaptive Lyapunov Based Control

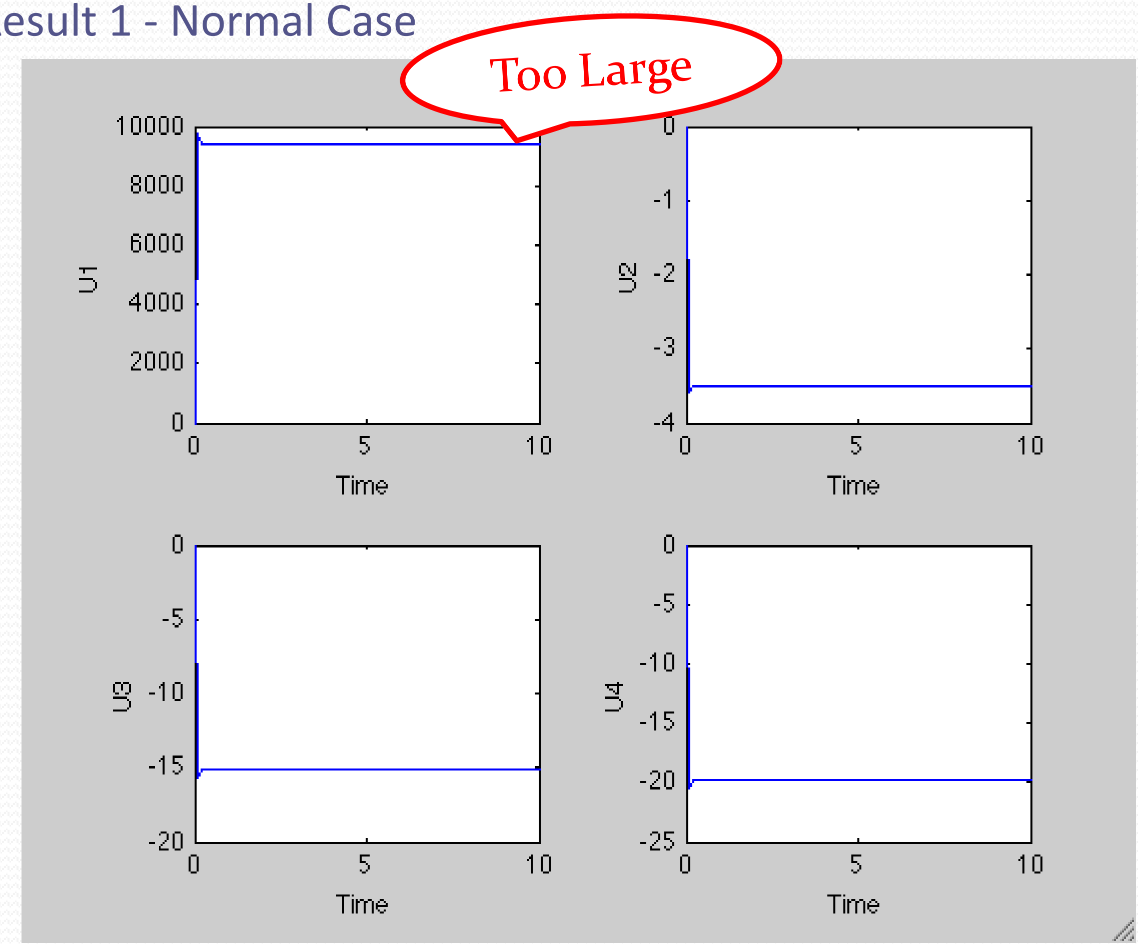
Simulation Result 1 - Normal Case

$$c_{11} = 302,$$
$$c_{12} = 18,$$
$$\gamma_1 = 0.01$$
$$c_{21} = 416,$$
$$c_{22} = 14,$$
$$\gamma_2 = 0.0015$$
$$c_{31} = 408,$$
$$c_{32} = 15,$$
$$\gamma_3 = 0.0146$$
$$c_{41} = 390,$$
$$c_{42} = 12.8,$$
$$\gamma_4 = 0.013$$



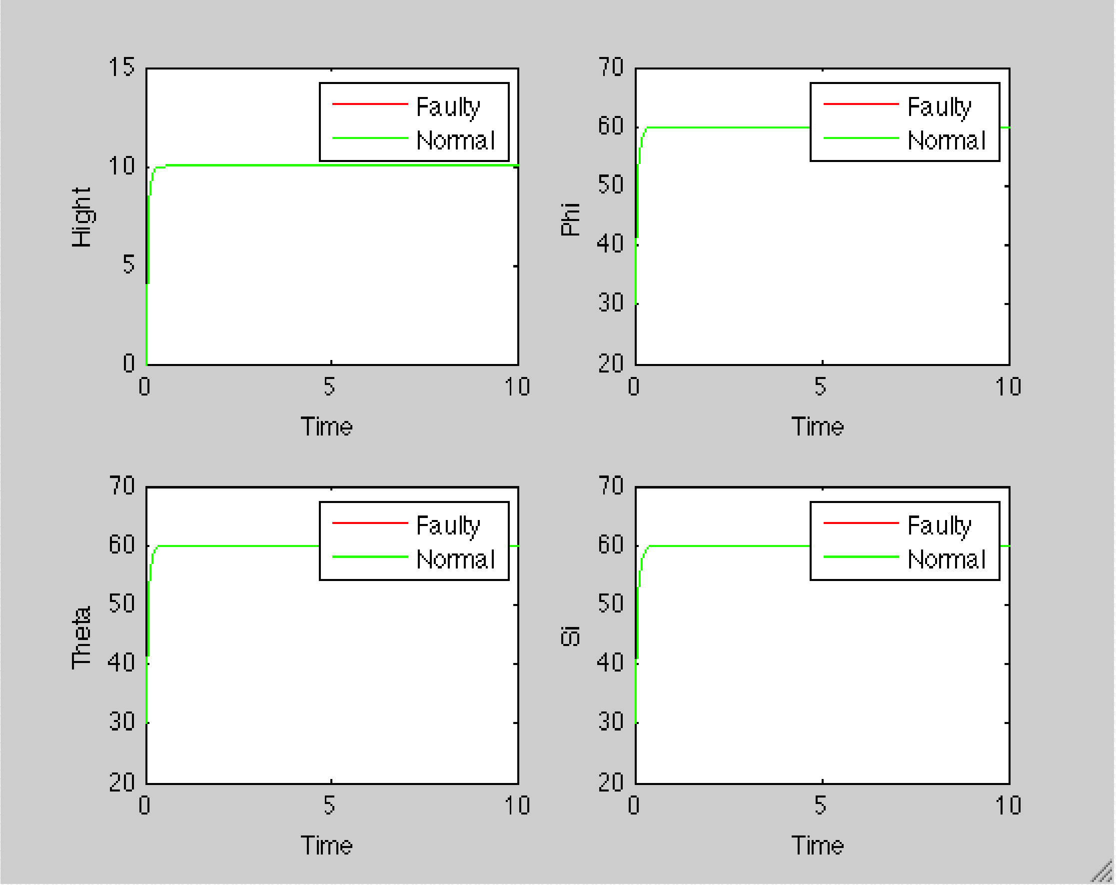
Adaptive Lyapunov Based Control

Simulation Result 1 - Normal Case



Adaptive Lyapunov Based Control

Simulation Result 1 - 80% loss of effectiveness of actuator1 in t=5 sec



Adaptive Lyapunov Based Control

Parameter Selection

Put u1 in motion equation :

$$\ddot{z} = -c_{12}y_{12-}y_{11} + \ddot{z}_r + \dot{\beta}_1$$

$$(\ddot{z} - \ddot{z}_r) + (c_{12}c_{11})\big(\dot{z}(t) - \dot{z}_r(t)\big) + (1 + c_{12}c_{11})(z(t) - z_r(t)) = 0$$

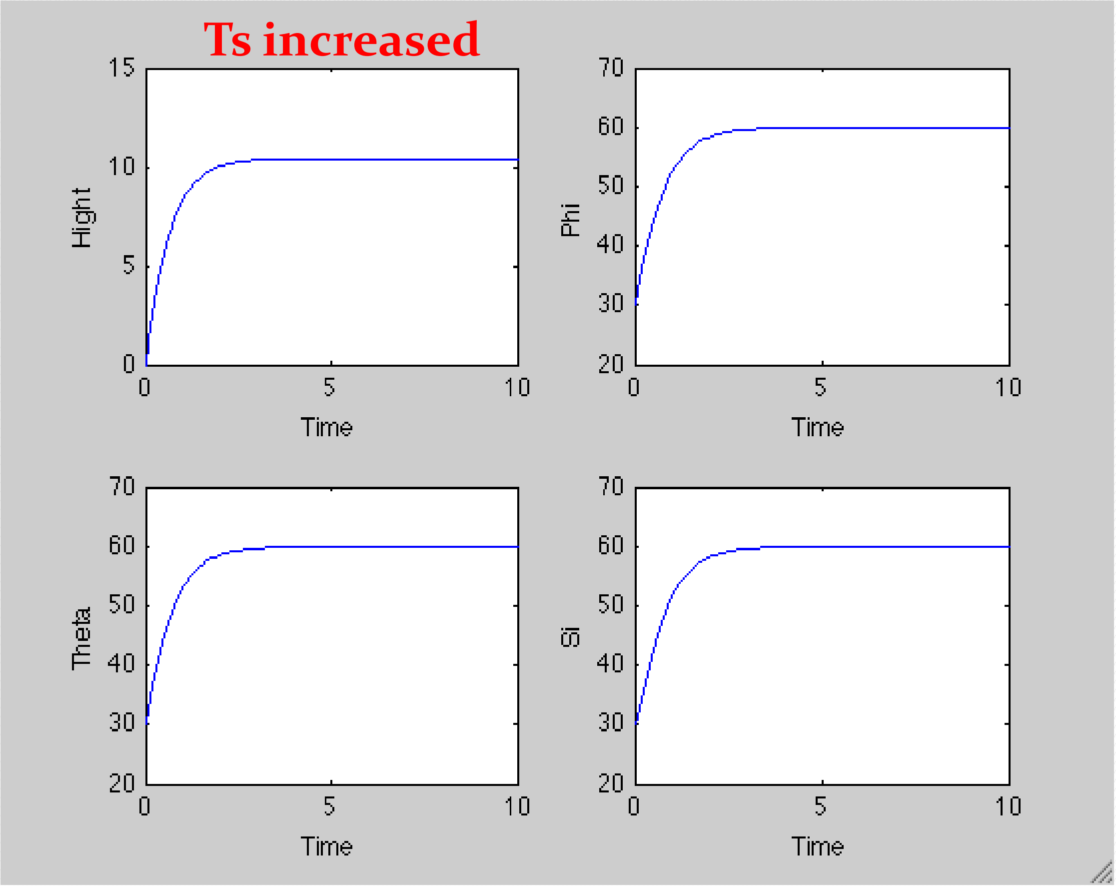
Characteristic Equation

$$\lambda^2 + (c_{12}c_{11})\lambda + (1 + c_{12}c_{11}) = 0$$

Adaptive Lyapunov Based Control

Simulation Result 2 - Normal Case Slowing the system 10 times

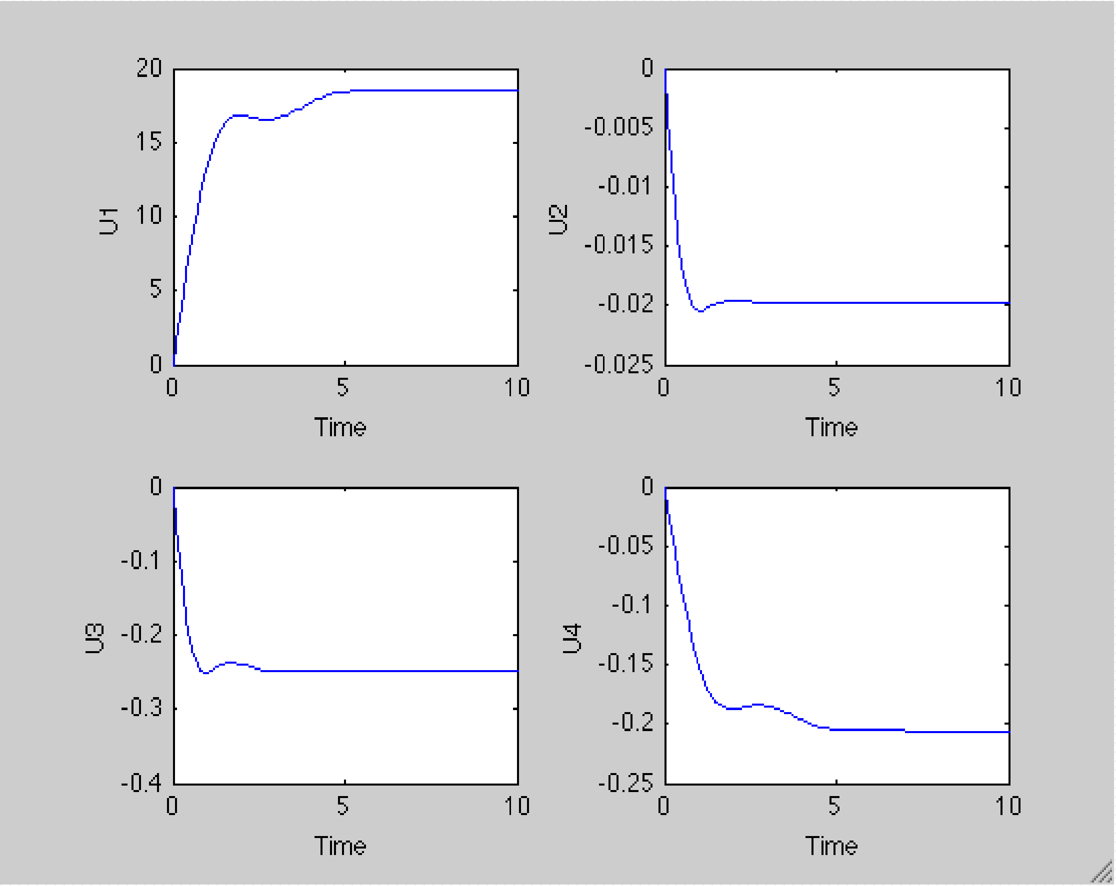
$c_{11} = 30.2,$
$c_{12} = 1.8,$
$\gamma_1 = 0.01$
$c_{21} = 41.6,$
$c_{22} = 1.4,$
$\gamma_2 = 0.0015$
$c_{31} = 40.8,$
$c_{32} = 15,$
$\gamma_3 = 0.0146$
$c_{41} = 39.0,$
$c_{42} = 1.28,$
$\gamma_4 = 0.013$



System states

Adaptive Lyapunov Based Control

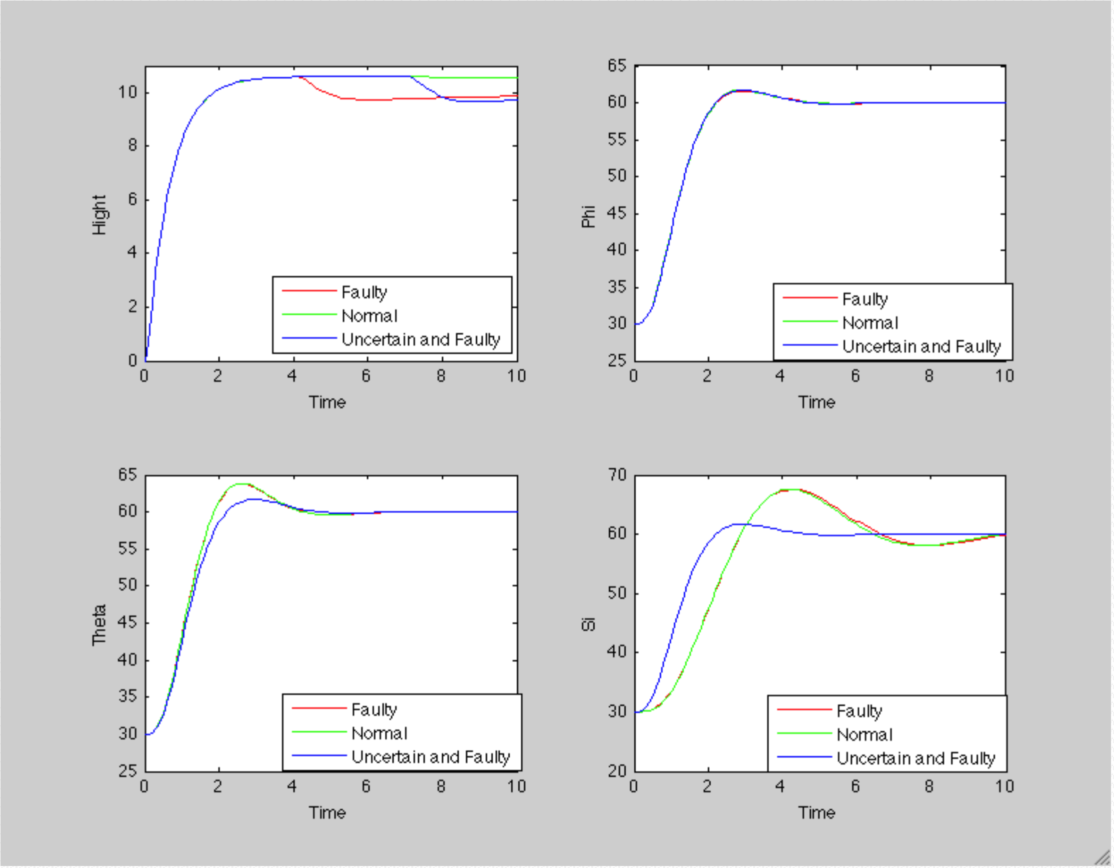
Simulation Result 2 - Normal Case Slowing the system 10 times



Actuator signals

Adaptive Lyapunov Based Control

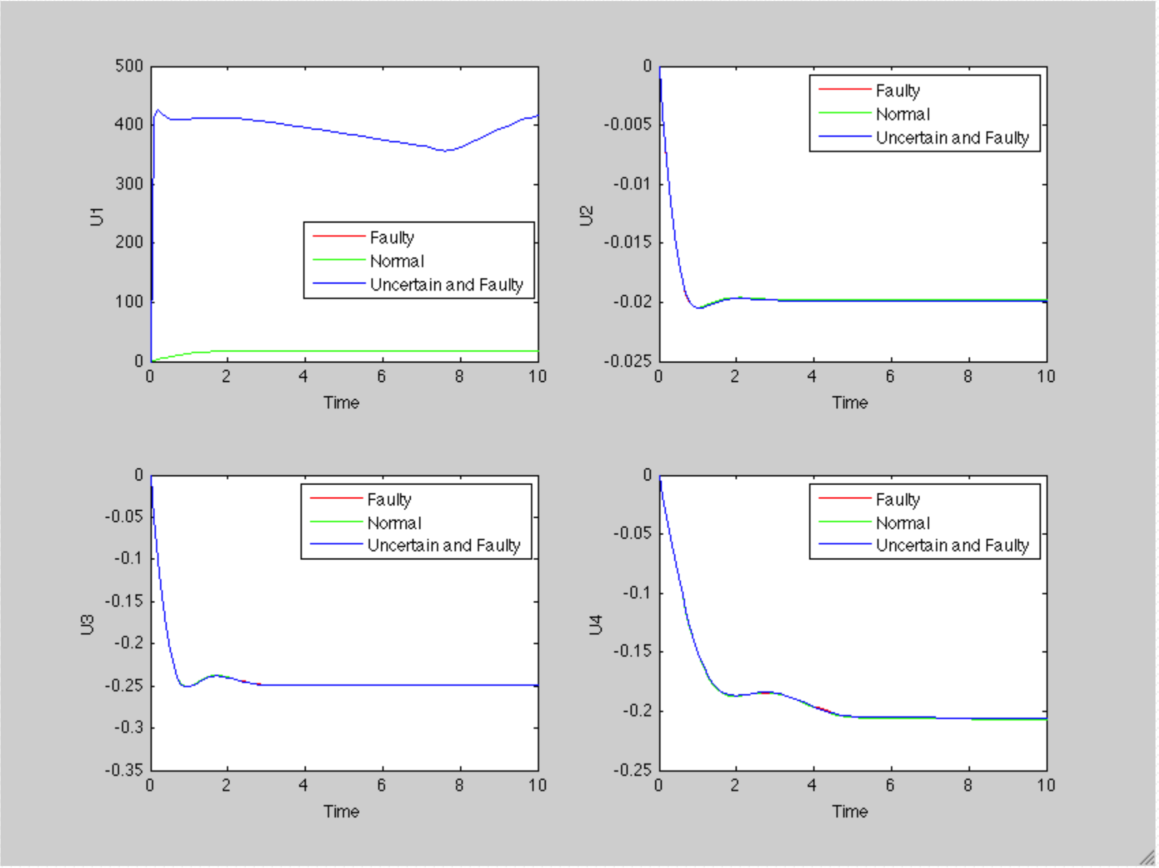
Simulation Result 2 - 80% loss of effectiveness and 80% uncertainty in t=5 sec



System states

Adaptive Lyapunov Based Control

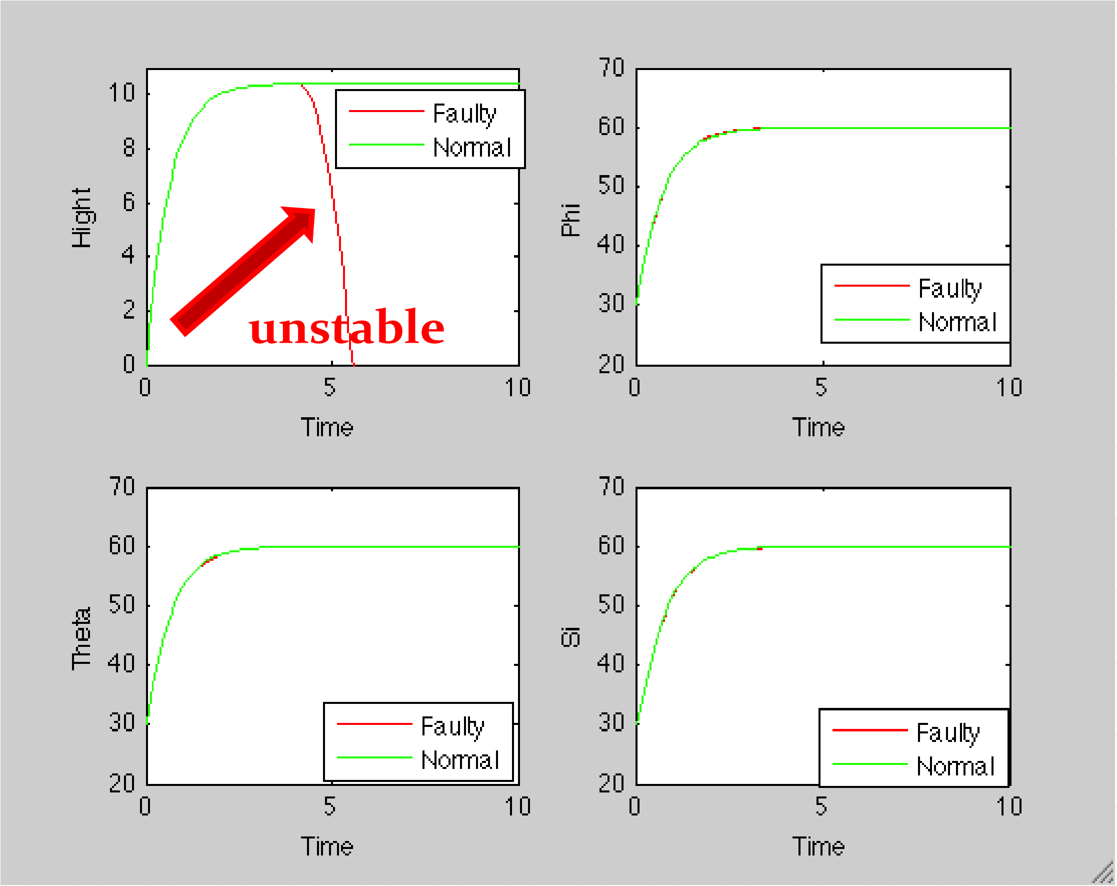
Simulation Result 2 - 80% loss of effectiveness and 80% uncertainty in t=5 sec



Actuator signals

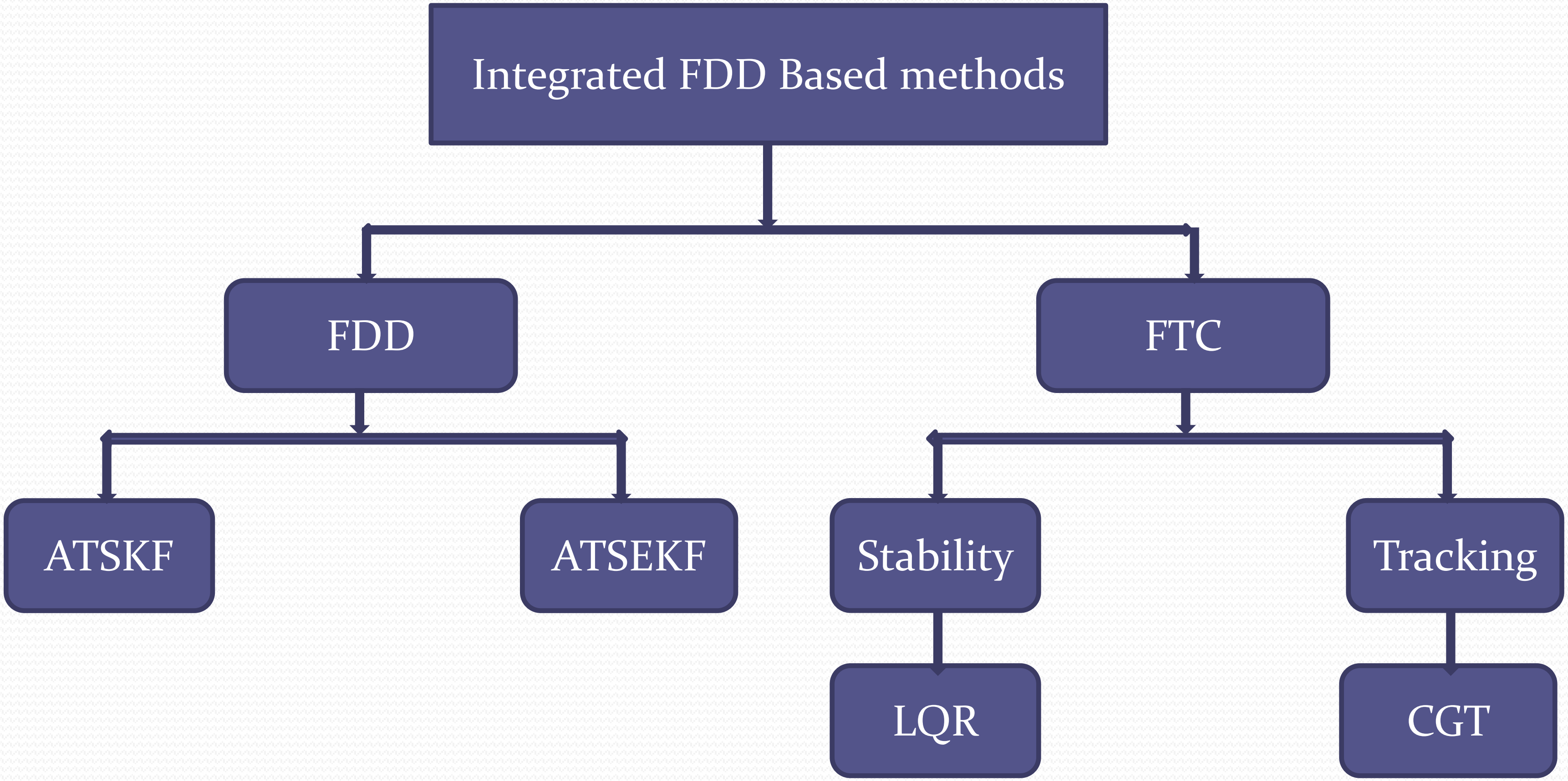
Adaptive Lyapunov Based Control

Simulation Result 2 - Total loss of effectiveness in t=5 sec



System states

Integrated FDD Based Methods



Adaptive Two Stages Extended Kalman Filter

- ✓ Biased augmented nonlinear discrete-time system

$$X(k+1) = f\big(x(k)\big) + B(k)u(k) + D(k)\gamma(k) + w^x(k)$$

$$\gamma(k+1) = \gamma(k) + w^\gamma(k)$$

$$y(k+1) = h\big(x(k+1)\big) + v(k+1)$$

$$D = -B\begin{bmatrix} u_1^k & 0 & \dots & 0 \\ 0 & & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & \dots & u_l^k \end{bmatrix}$$

$w^x(k), w^\gamma(k)$ and $v(k)$: white noise

Adaptive Two Stages Extended Kalman Filter

- ✓ Jacobians in the Taylor's series expansion of f and h

$$\mathrm{F}\left(\tilde{x}(k|k)\right) \triangleq \left.\frac{\partial f\left(x(k), u(k), k\right)}{\partial x(k)}\right|_{x(k)=x(k|k)}$$

$$\mathrm{H}\left(\tilde{x}(k|k)\right) \triangleq \left.\frac{\partial h\left(x(k), u(k), k\right)}{\partial x(k)}\right|_{x(k)=x(k|k)}$$

- ✓ Bias Free State Estimator

$$\tilde{x}(k+1|k) = f(\tilde{x}(k|k)) + B(k)u(k) + (M(k) - V(k+1|k)\hat{\gamma}(k|k)$$

$$\tilde{P}^{x}(k+1|k) = F\left(\tilde{x}(k|k)\right)\tilde{P}^{x}(k|k)F\left(\tilde{x}(k|k)\right)^{T}(k) + Q^{x} + M(K)P^{\gamma}(k|k)M^{T}(K) - V(K+1|k)P^{\gamma}(k+1|k)V^{T}(K+1|k)$$

$$\tilde{x}(k+1|k+1) = \tilde{x}(k+1|k) + \widetilde{K}^{x}(k+1)(y(k+1) - h(\tilde{x}(k+1|k)))$$

$$\widetilde{K}^{x}(k+1) = \tilde{P}^{x}(k+1|k)H^{T}(k+1)(H(k+1)\tilde{P}^{x}(k+1|k)H^{T}(k+1) + R(k+1))^{-1}$$

$$\tilde{P}^{x}(k+1|k+1) = (I - \widetilde{K}^{x}(k+1)H(k+1))\tilde{P}^{x}(k+1|k)$$

Adaptive Two Stages Extended Kalman Filter

- Optimal Bias estimator

$$\hat{\gamma}(k+1|k) = \hat{\gamma}(k|k)$$

$$P^{\gamma}(k+1|k) = P^{\gamma}(k|k) + Q^{\gamma}$$

$$\hat{\gamma}(k+1|k+1) = \hat{\gamma}(k+1|k) + K^{\gamma}(k+1)\big(\tilde{r}(k+1) - N(k+1|k)\hat{\gamma}(k|k)\big)$$

$$K^{\gamma}(k+1) = P^{\gamma}(k+1|k)\, N^{T}(k+1|k)(N(k+1|k)P^{\gamma}(k+1|k)N^{T}(k+1|k) + \tilde{S}(k+1))^{-1}$$

$$P^{\gamma}(k+1|k+1) = (I - K^{\gamma}(k+1)N(k+1|k))P^{\gamma}(k+1|k)$$

- the filter residual and its covariance can be calculated as below:

$$\tilde{r}(k+1) = (y(k+1) - h(\tilde{x}(k+1|k)))$$

$$\tilde{S}(k+1) = H(k+1)\tilde{P}^{x}(k+1|k)H^{T}(k+1) + R(k+1)$$

- Coupling Equations

$$M(K) = F\big(\tilde{x}(k|k)\big)V(K|k) + D(k)$$

$$V(K+1|k) = M(K)P^{\gamma}(k|k)(P^{\gamma}(k+1|k))^{-1}$$

$$N(k+1|k) = H(K+1)V(K+1|k)$$

$$V(K+1|k+1) = V(K+1|k) - \widetilde{K}^{x}(k+1)N(k+1|k)$$

- Compensated State Error Covariance Estimation

$$\hat{x}(k+1|k) = \tilde{x}(k+1|k+1) + V(k+1|k+1)\hat{\gamma}(k+1|k+1)$$

$$P(k+1|k+1) = \tilde{P}^{x}(k+1|k+1) + V(k+1|k+1)\tilde{P}^{\gamma}(k+1|k+1)V^{T}(k+1|k+1)$$

Reconfiguration

- ✓ Linearizing the model in the vicinity of the current operating point and update system matrices: F(k), and H(k)
- ✓ Estimate system states and actuator effectiveness factor with TSEKF
- ✓ Use the estimated effectiveness factor and update generate faulty matrix $(B^f = B(I - \hat{\gamma}(k|k))$
- ✓ Update controller gains with system matrices (F, B^f, H)
  - Stability: Feedback gain - LQR (F, B^f, Q, R)
    Q and R positive semi-definite and positive definite matrices
  - Tracking: Feed-forward gain- CGT

$$K_{feedforward} = [\phi_{22} - K_{feedback}\phi_{12}]$$

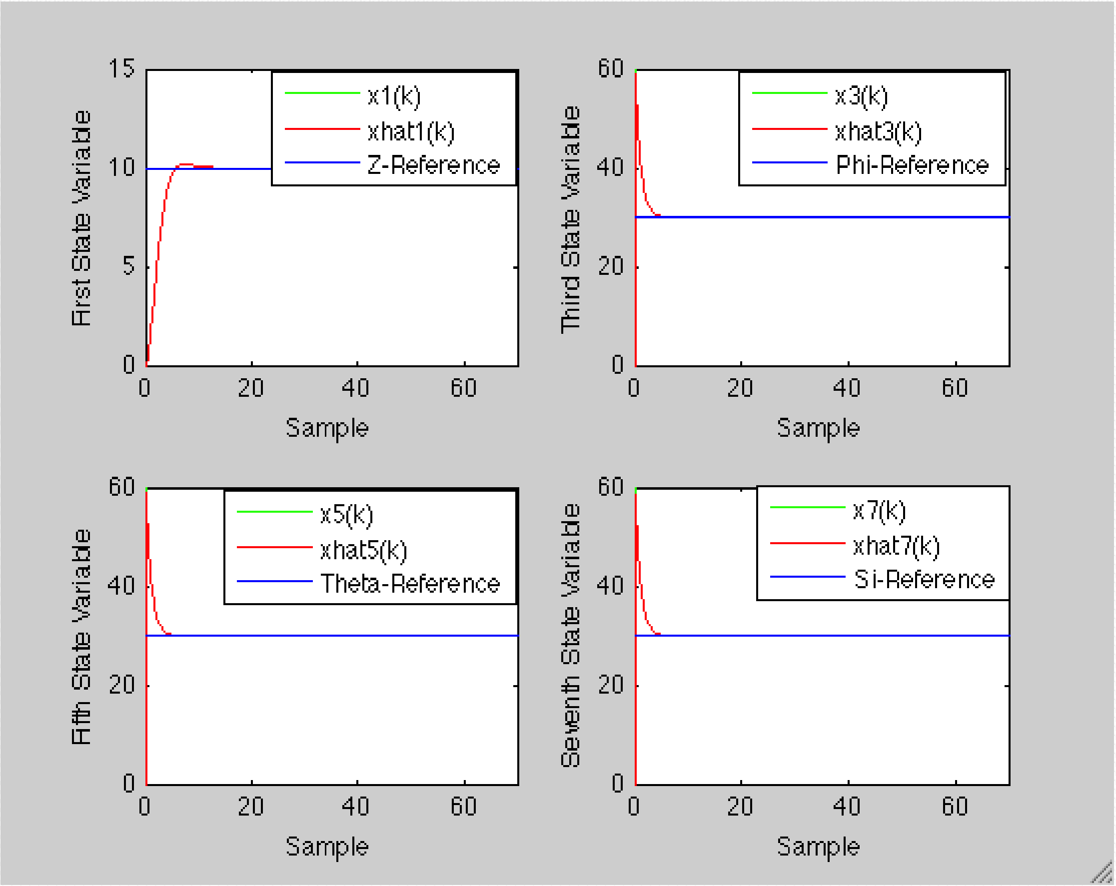
$$\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{F} - I & B^f \\ \mathbf{H} & 0 \end{bmatrix}^{-1}$$

- ✓ Control Law:

$$u(k) = K_{feedforward}r(k) + K_{feedback}\hat{x}(k)$$

Adaptive Two Stages Kalman Filter

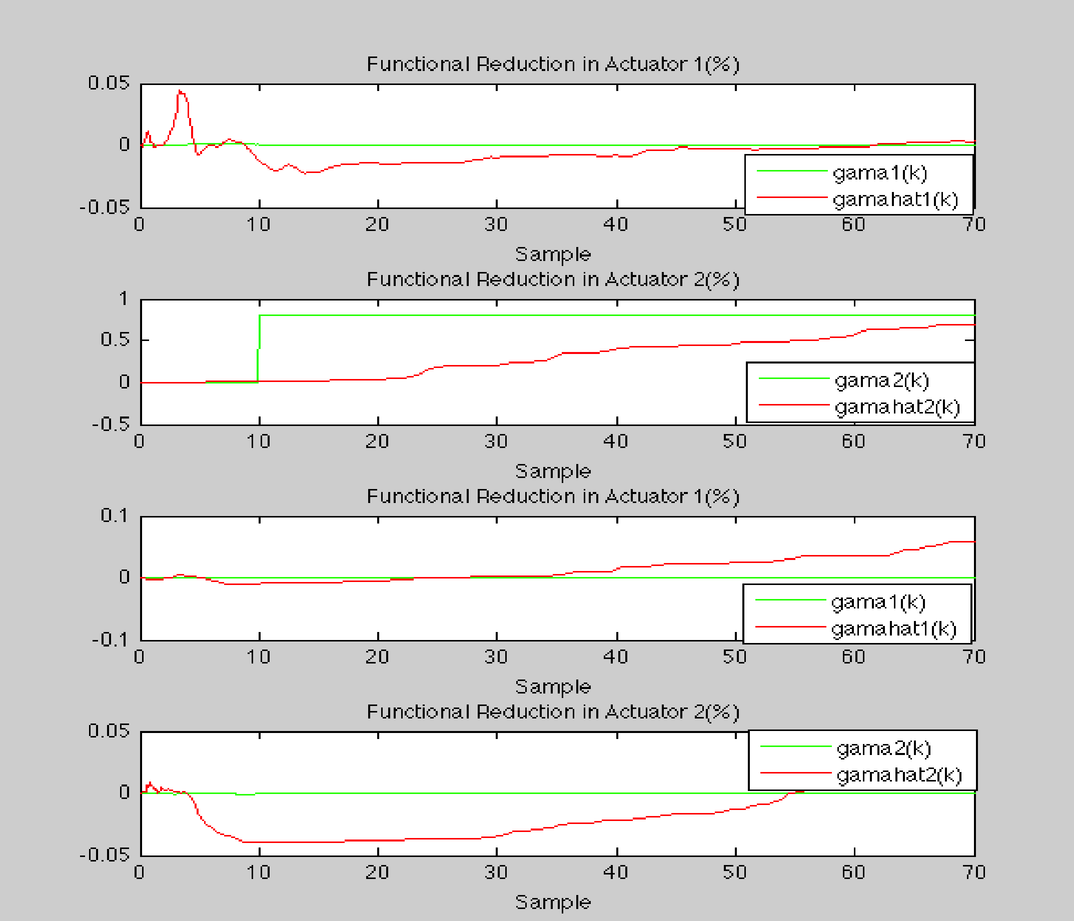
Simulation Result - 80% loss of effectiveness - t=10s



System states

Adaptive Two Stages Kalman Filter

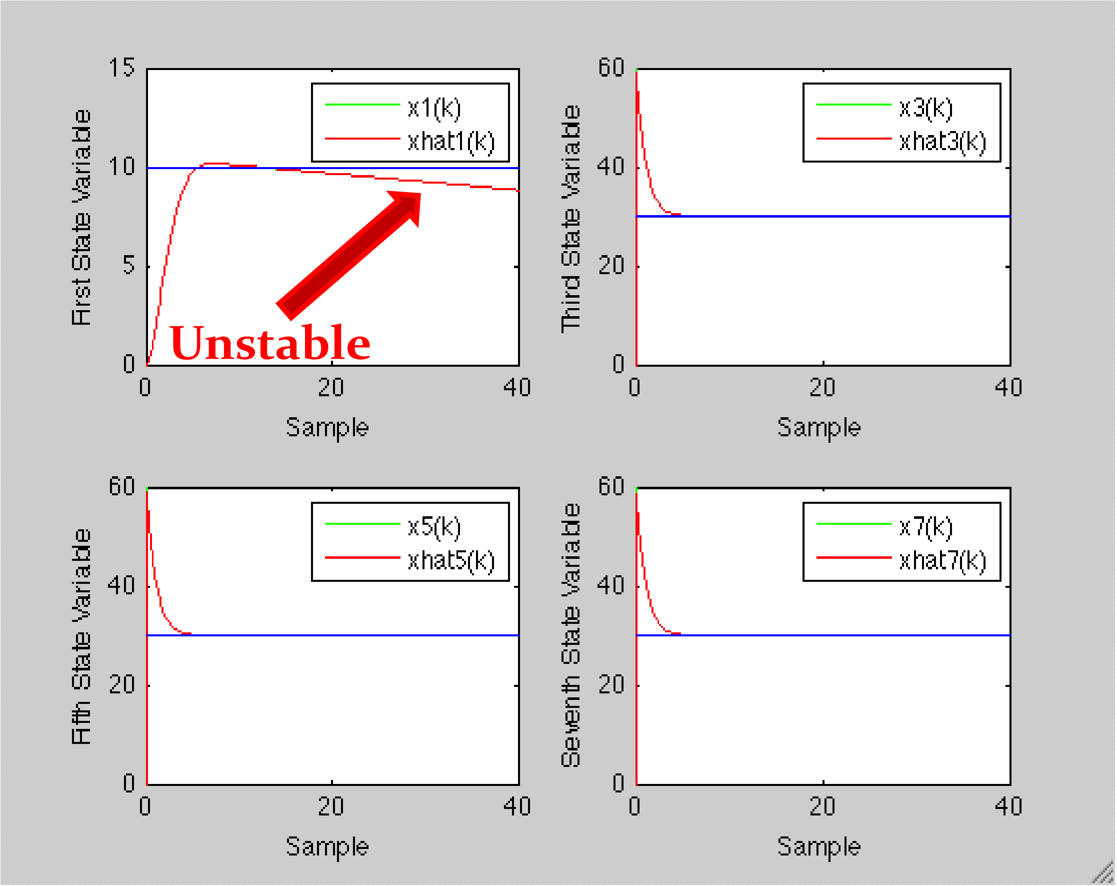
Simulation Result - 80% loss of effectiveness - t=10s



Effectiveness factor

Adaptive Two Stages Kalman Filter

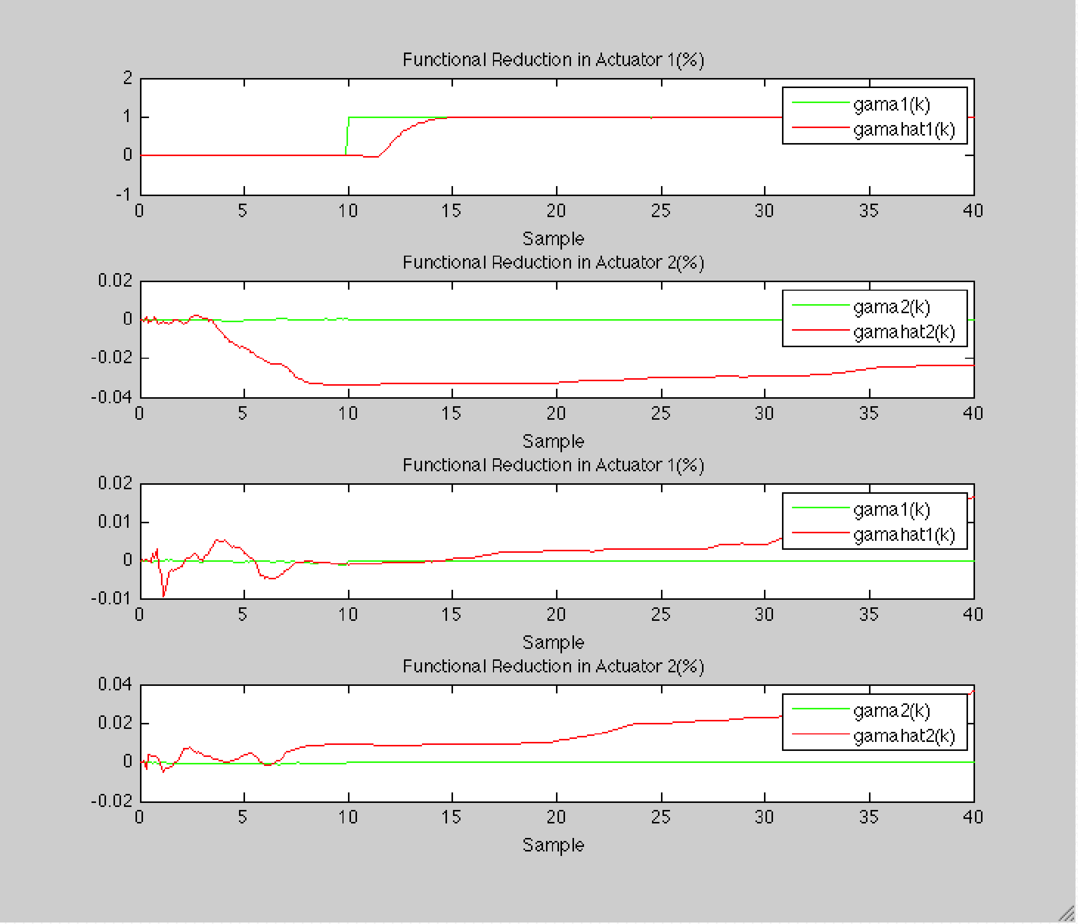
Simulation Result - Total loss of effectiveness - t=10s



System states

Adaptive Two Stages Kalman Filter

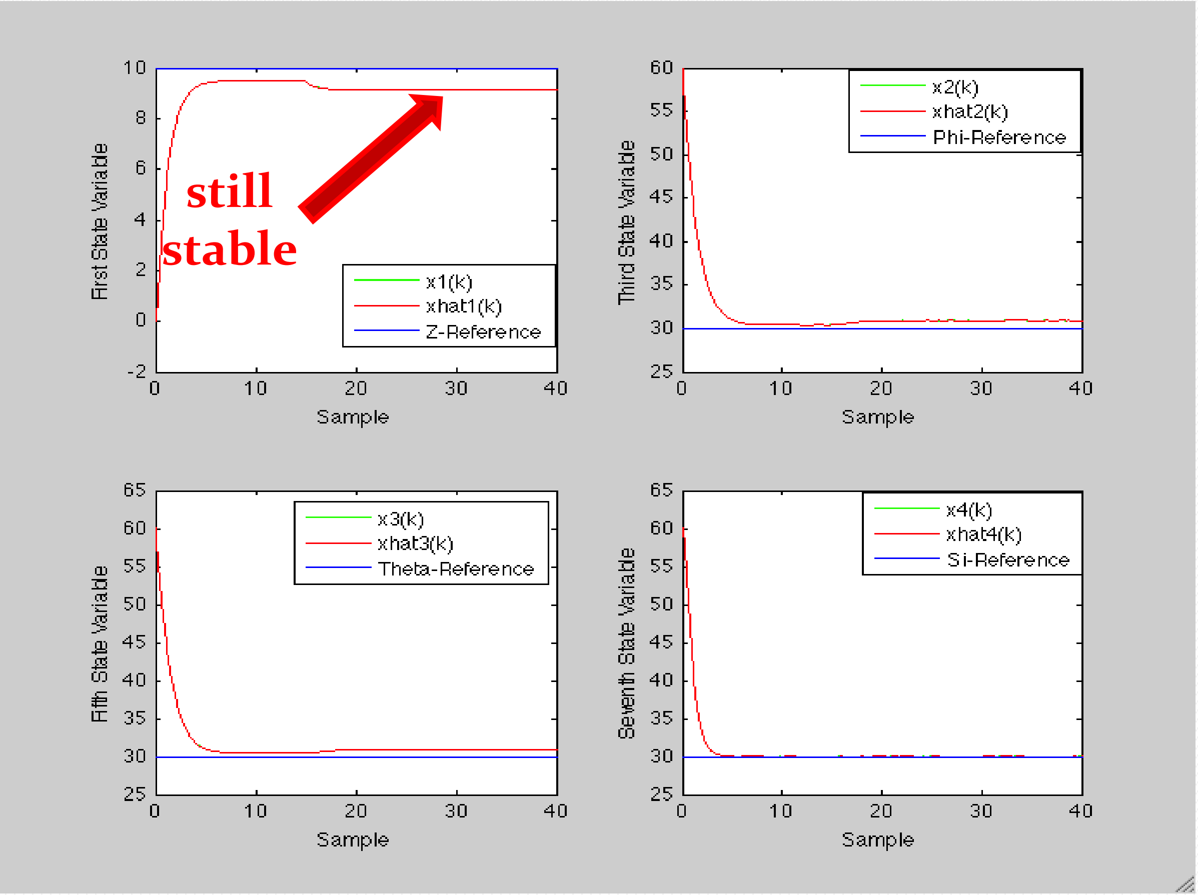
Simulation Result - Total loss of effectiveness - t=10s



Effectiveness factor

Adaptive Two Stages Extended Kalman Filter

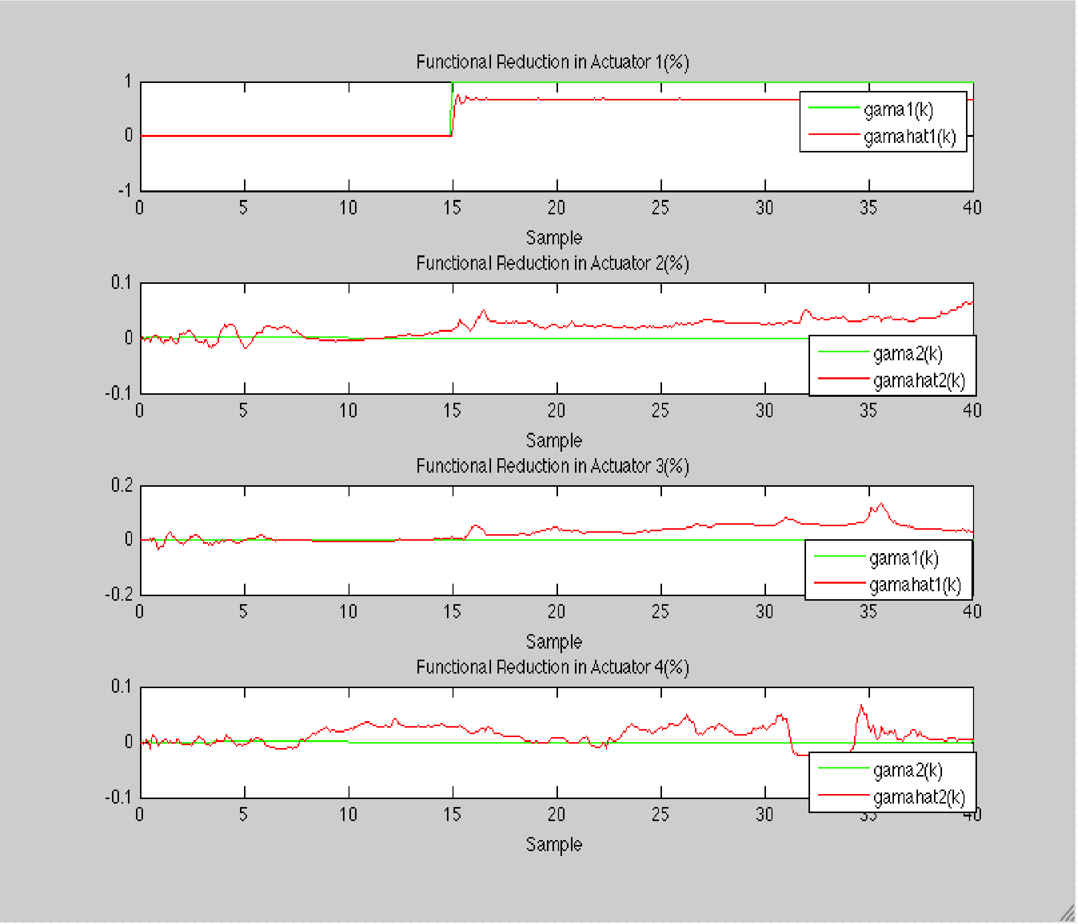
Simulation Result - Total loss of effectiveness - t=15s



System states

Adaptive Two Stages Extended Kalman Filter

Simulation Result - Total loss of effectiveness - t=10s



Effectiveness factor

Accepted performance degradation

- ✓ Why?

Avoiding faulty actuator or other healthy actuators (depending on the structure of the system) to work beyond their capacity.

- ✓ How?

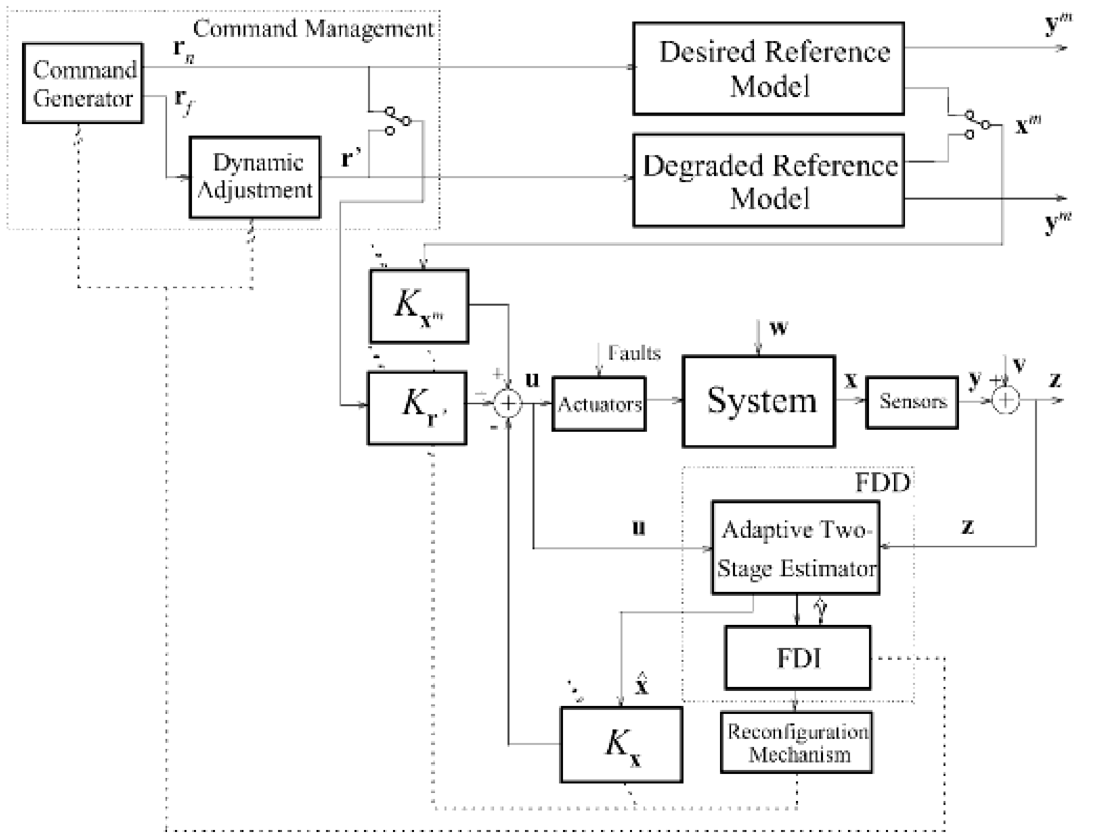
Incorporating an accepted performance degradation for post fault mode.

- ✓ Approach:

Reconfigurable model following control

Control Policy and Overall Structure

- ✓ Based on model following method



Reference model

✓ Desired reference model of the system with no actuator fault :

$$\dot{x} = A_d x + B_d u$$

$$y = C_d x$$

✓ Mode degradation matrix:

$$\psi = diag\,[\alpha_1, \alpha_2, \ldots\, \alpha_n] \qquad \alpha_j \geq 1 \quad \forall\, j = 1, \ldots n.$$

✓ Degraded reference model:

$$\dot{x} = A_f x + B_f u$$

$$y = C_f x$$

$$A_f = \psi^{-1} A_d \qquad , B_f = \psi^{-1} B_d \quad , C_f = C_d$$

Dynamic tapering of inputs

- ✓ Input adjustment for post failure mode:
  - Static
  - Dynamic
- ✓ Dynamic:

$$r'_k = r'_{k-1} + k_k \left[r_k - r_{k-1} \right]$$

r'_k= modified command input

$$r_k = \begin{cases} r_n & k < k_d \\ r_f & k \geq k_d \end{cases}$$

$$k_k = 1 - \sigma e^{-\tau (k - K_D)}, \qquad k \geq K_D$$

Model Following Reconfigurable Controller

$$\begin{cases} x_{k+1} = Fx_k + Gu_k + w_k^x & k < k_F \quad \text{system during normal operation} \\ x_{k+1} = Fx_k + G^f u_k + w_k^x & k \geq k_F \quad \text{system with actuator fault} \end{cases}$$

$$y_k = H_r x_k$$

$$z_k = Hx_k + v_k$$

- ✓ Desired reference model:

$$\begin{cases} x_{k+1}^m = F_n^m x_{k+1}^m + G_n^m r_k \\ y_k^m = H_n^m x_k^m \end{cases} \qquad k < k_f$$

- ✓ Degraded reference model:

$$\begin{cases} x_{k+1}^m = F_f^m x_{k+1}^m + G_f^m r'_k \\ y_k^m = H_f^m x_k^m \end{cases} \qquad k < k_f$$

Model Following Reconfigurable Controller

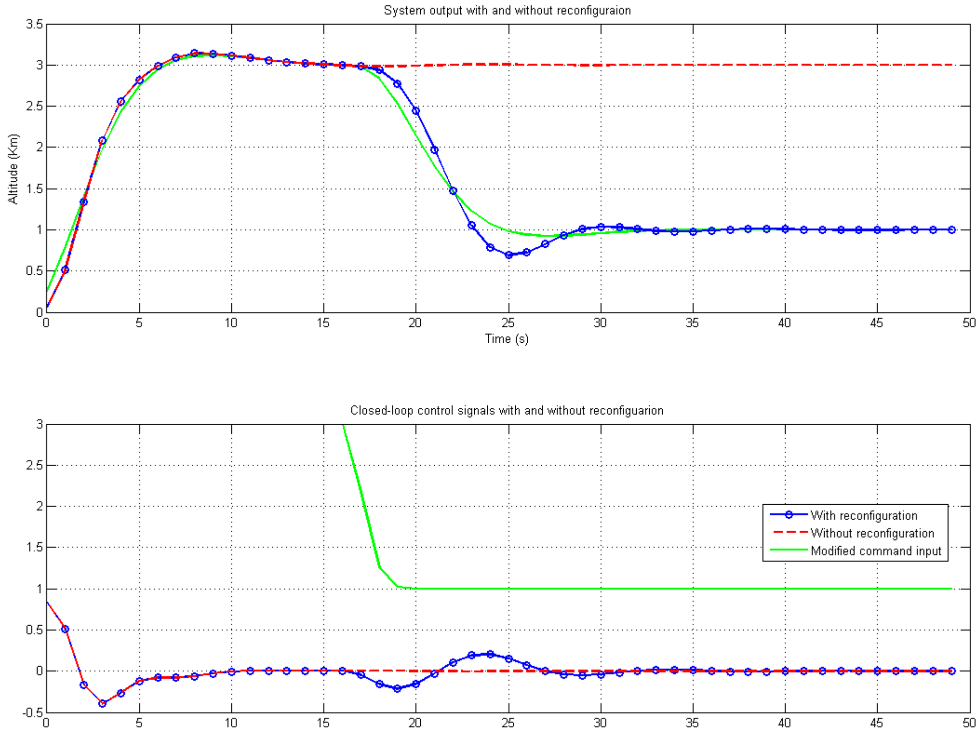
- ✓ $u_k^n = -K_x^n x_n + K_{x^m}^n x_k^m + K_r^n r_k$
- ✓ $u_k^f = -K_x^f x_n + K_{x^m}^f x_k^m + K_r^f r_k \qquad k \geq k_R$

- ✓ Main objective
  - $e_k \rightarrow 0$
  - $e_k = y_k - y_k^m = H_r x_k - H^m x_k^m \Rightarrow$
  - $u_k = -K_x x_k + (S_{21} + K_x S_{11}) x_k^m + (S_{22} + K_x S_{12}) r'_k$

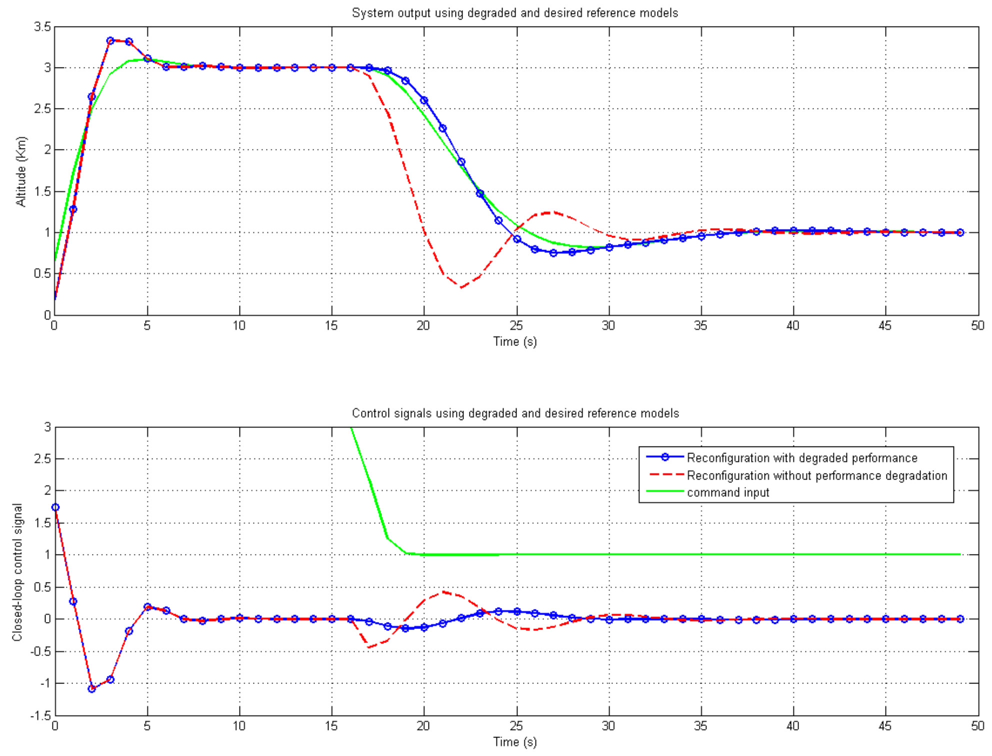
$$S_{11} = \phi_{11} S_{11} (F^m - I) + \phi_{12} H^m$$
$$S_{12} = \phi_{11} S_{11} G^m$$
$$S_{21} = \phi_{21} S_{11} (F^m - I) + \phi_{22} H^m$$
$$S_{22} = \phi_{21} S_{11} G^m$$

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{cases} \begin{bmatrix} F - I & G \\ H_r & 0 \end{bmatrix}^{-1} \\ \begin{bmatrix} F - I & \hat{G}_k^f \\ H_r & 0 \end{bmatrix}^{-1} \end{cases}$$

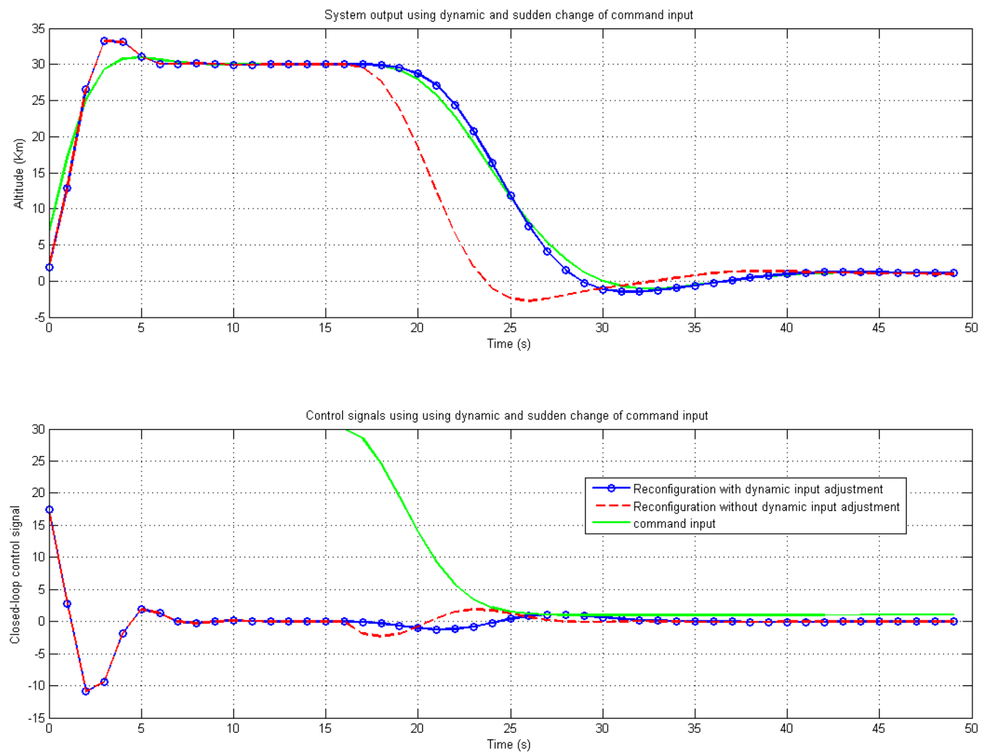
System Signals with and without Reconfiguration



System Signals Using Degraded and Desired Reference Models



System Signals Using Dynamic and Sudden Change of Command Input



Conclusions

Method	Advantages	Disadvantages
Adaptive CMRAC	1. Acceptable performance in presence of linear & non-linear in-state uncertainties and control effectiveness uncertainties. 2. Smooth transient behavior in presence of fault	High frequency oscillations in control signal
Adaptive Lyapunov	Fast reconfiguration capabilities in presence of fault	1. Not having the capability to tolerate total loss of control effectiveness 2. Demand too much control effort if a strict tracking performance is required
FDD-based LQR (TSKF)	Fast diagnosis by the FDD part	Not having the capability to tolerate total loss of control effectiveness
FDD-based LQR (ETSKF)	the capability to tolerate total loss of control effectiveness	Slow convergence rate for parameter estimation
FDD-based Model following method	1. Can incorporate graceful performance degradation in controller design in presence of fault 2. Guaranteeing the value of control signals within the actuator limitations	1. Not having the capability to handle total loss of effectiveness in actuators 2. Failing to incorporate a degraded performance for fault situation causes the output to lose track of the desired reference model in some sever fault scenarios

Thanks for Your Attention