Autopilot Design for an F16 Fighter Aircraft

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Outline

1. F16 Equation of Motion
2. Linearization and Longitudinal Motion Analysis
3. Control Law Development
4. Autopilot Design and Simulation
5. Future Work and Conclusion

F16 Equation of Motion

Total velocity and angle of attack and side slip angle

$$V_T = \sqrt{u^2 + v^2 + w^2}$$

$$\alpha = \arctan \frac{w}{u}$$

$$\beta = \arcsin \frac{v}{V_T}$$

Force equations

$$F_x = m(\dot{u} + qw - rv)$$

$$F_y = m(\dot{v} + ru - pw)$$

$$F_x = m(\dot{w} + pv - qu)$$

Aerodynamic forces

$$\overline{X} = \bar{q} S C_{X_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

$$\overline{Y} = \bar{q} S C_{Y_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

$$\overline{Z} = \bar{q} S C_{Z_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

Moment equations

$$M_x = \dot{p} I_x - \dot{r} I_{xz} + qr(I_z - I_y) - pq I_{xz}$$

$$M_y = \dot{q} I_y + pr(I_x - I_z) + (p^2 - r^2) I_{xz}$$

$$M_z = \dot{r} I_z - \dot{p} I_{xz} + pq(I_y - I_x) + qr I_{xz}$$

Aerodynamic moments

$$\overline{L} = \bar{q} S \bar{b} C_{l_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

$$\overline{M} = \bar{q} S \bar{c} C_{m_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

$$\overline{N} = \bar{q} S \bar{b} C_{n_T}(\alpha, \beta, p, q, r, \delta, \ldots)$$

Navigation equations

$$\dot{f} = p + \tan q (q \sin f + r \cos f)$$

$$\dot{q} = q \cos f - r \sin f$$

$$\dot{y} = \frac{q \sin f + r \cos f}{\cos q}$$

$$\dot{x}_E = u \cos y \cos q + v(\cos y \sin q \sin f - \sin y \cos f) + w(\cos y \sin q \cos f + \sin y \sin f)$$

$$\dot{y}_E = u \sin y \cos q + v(\sin y \sin q \sin f + \cos y \cos f) + w(\sin y \sin q \cos f - \cos y \sin f)$$

$$\dot{z}_E = -u \sin q + v \cos q \sin f + w \cos q \cos f$$

Linearization Around a Trim Point

System of equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$
$$\mathbf{y} = \mathbf{c}(\mathbf{x}, \mathbf{u})$$

At trim point

$$\mathbf{0} = \mathbf{f}(\mathbf{x_e}, \mathbf{u_e})$$

Taylor series expansion

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x_e}, \mathbf{u_e}) + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot (\mathbf{x} - \mathbf{x_e}) + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot (\mathbf{u} - \mathbf{u_e})$$
$$\mathbf{y} = \mathbf{c}(\mathbf{x_e}, \mathbf{u_e}) + \left[\frac{\partial \mathbf{c}}{\partial \mathbf{x}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot (\mathbf{x} - \mathbf{x_e}) + \left[\frac{\partial \mathbf{c}}{\partial \mathbf{u}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot (\mathbf{u} - \mathbf{u_e})$$

Perturbated variables

$$\delta\mathbf{x} = \mathbf{x} - \mathbf{x_e}$$
$$\delta\mathbf{y} = \mathbf{y} - \mathbf{y_e}$$
$$\delta\mathbf{u} = \mathbf{u} - \mathbf{u_e}$$

Jacobian

$$\delta\dot{\mathbf{x}} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot \delta\mathbf{x} + \left[\frac{\partial \mathbf{f}}{\partial \mathbf{u}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot \delta\mathbf{u}$$
$$\delta\mathbf{y} = \left[\frac{\partial \mathbf{c}}{\partial \mathbf{x}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot \delta\mathbf{x} + \left[\frac{\partial \mathbf{c}}{\partial \mathbf{u}}\right]_{\mathbf{x_e},\mathbf{u_e}} \cdot \delta\mathbf{u}$$

Linear system

$$\delta\dot{\mathbf{x}} = \mathbf{A} \cdot \delta\mathbf{x} + \mathbf{B} \cdot \delta\mathbf{u}$$
$$\delta\mathbf{y} = \mathbf{C} \cdot \delta\mathbf{x} + \mathbf{D} \cdot \delta\mathbf{u}$$

State space format

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

The controller

$$\delta\mathbf{u} = \mathbf{u} - \mathbf{ue} \Rightarrow \mathbf{u} = \mathbf{ue} + \delta\mathbf{u}$$

Trimming the F16 Aircraft

State for the longitudinal dynamics

$$\mathbf{x} = \begin{bmatrix} h & \theta & v & \alpha & q \end{bmatrix}^T$$

Trim point

$$\mathbf{x} = \begin{bmatrix} 15000 & 0.046492 & 600 & 0.046492 & 0 \end{bmatrix}^T$$

Stability derivative matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 600 & 2.9557\text{e-}013 & -600 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.00011042 & -32.17 & -0.011313 & 3.8334 & -0.65696 \\ 1.7359\text{e-}006 & -1.1714\text{e-}013 & -0.00017784 & -0.76788 & 0.9396 \\ 2.8715\text{e-}020 & 0 & -2.942\text{e-}018 & -2.251 & -1.0462 \end{bmatrix}$$

Control derivative matrix

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0015683 & 0.1448 \\ -1.2161\text{e-}007 & -0.0016222 \\ 0 & -0.16416 \end{bmatrix}$$

Eigen values

-0.90832+ 1.4472i
-0.90832- 1.4472i
-0.0043859+ 0.072077i
-0.0043859- 0.072077i
8.1796e-014

Analyzing the Longitudinal Motion

Short period mode

Eigen Value	Damping ratio	Natural freq
-0.90832 + 1.4472i	0.5325	1.70863
-0.90832 - 1.4472i	0.5325	1.70863

Phugoid mode

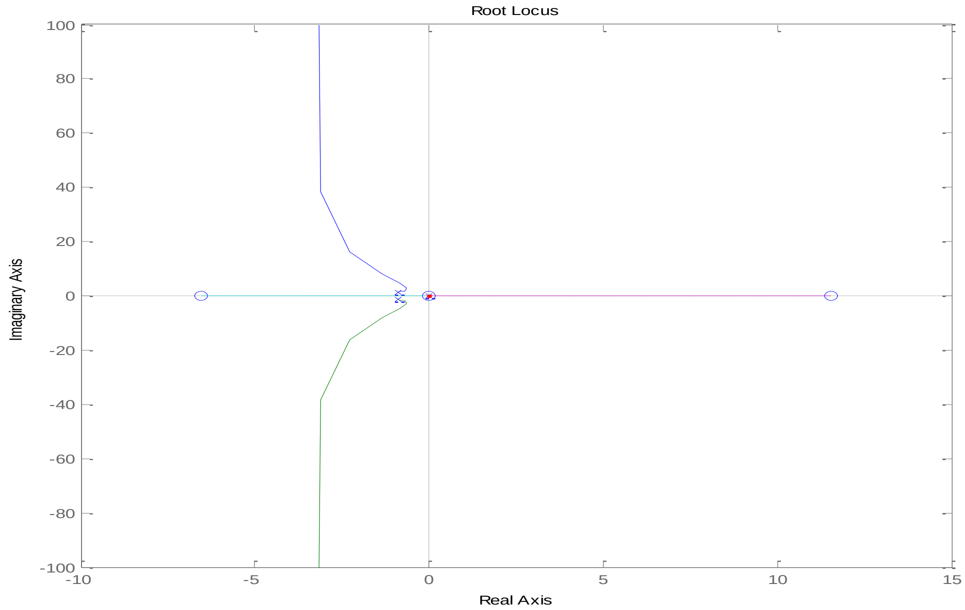
Eigen Value	Damping ratio	Natural freq
-0.0043859 + 0.072077i	6.07e -002	7.22e -002
-0.0043859 + 0.072077i	6.07e -002	7.22e -002

Altitude Characterization

Transfer function

$$\frac{h(s)}{\delta_e(s)} = \frac{4.441e-015\ s^4 + 0.9733\ s^3 - 4.905\ s^2 - 73.47\ s - 0.2997}{s^5 + 1.825\ s^4 + 2.941\ s^3 + 0.03508\ s^2 + 0.01522\ s - 1.245e-015}$$

Root locus



Poles and zero configuration

Poles

-0.90832+ 1.4472i

-0.90832- 1.4472i

-0.0043859+ 0.072077i

-0.0043859- 0.072077i

8.1796e-014

Zeros

-2.1918e+014

11.567

-6.5239

-0.0040803

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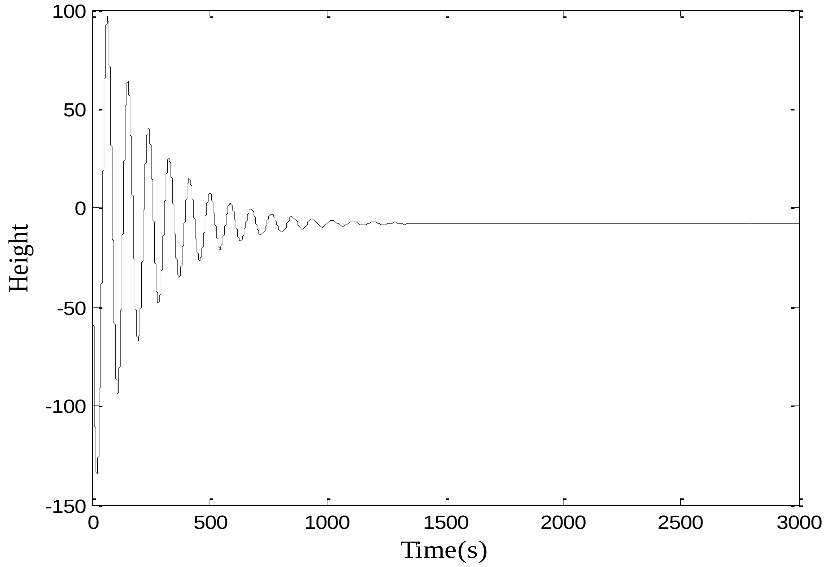
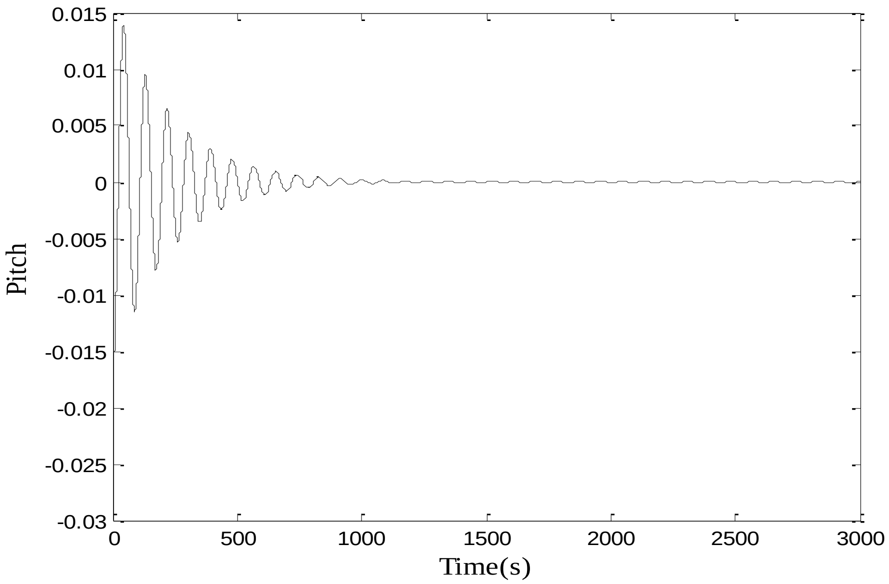
Altitude Characterization Contd: Impulse Response





Pitch Characterization

Transfer function

$$\frac{\theta(s)}{\delta_e(s)} = \frac{3.553e-015\ s^4 - 0.1642\ s^3 - 0.1243\ s^2 - 0.00161\ s + 9.121e-017}{s^5 + 1.825\ s^4 + 2.941\ s^3 + 0.03508\ s^2 + 0.01522\ s - 1.245e-015}$$

Root locus

Poles and zero configuration

Poles

-0.90832+ 1.4472i

-0.90832- 1.4472i

-0.0043859+ 0.072077i

-0.0043859- 0.072077i

8.1796e-014

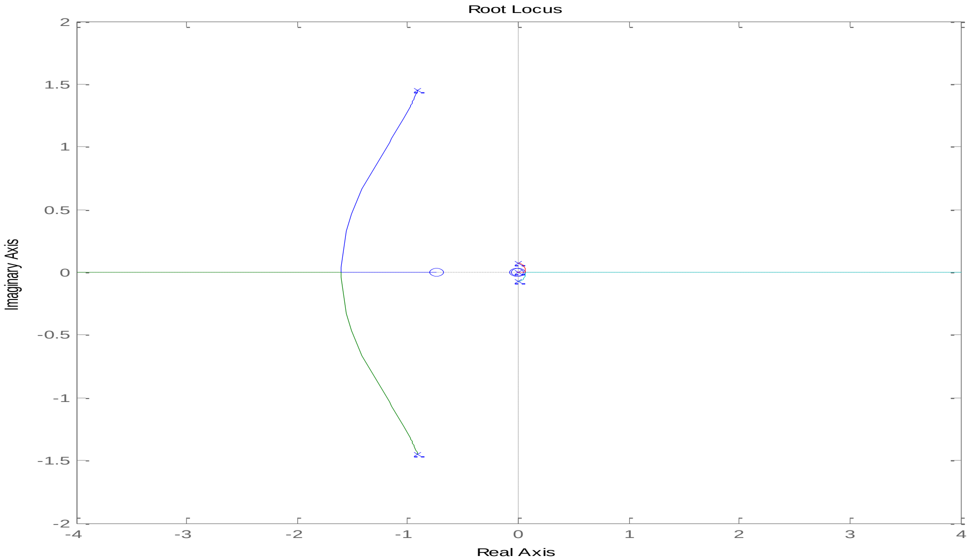
Zeros

4.6207e+013

-0.74377

-0.013184

5.6661e-014



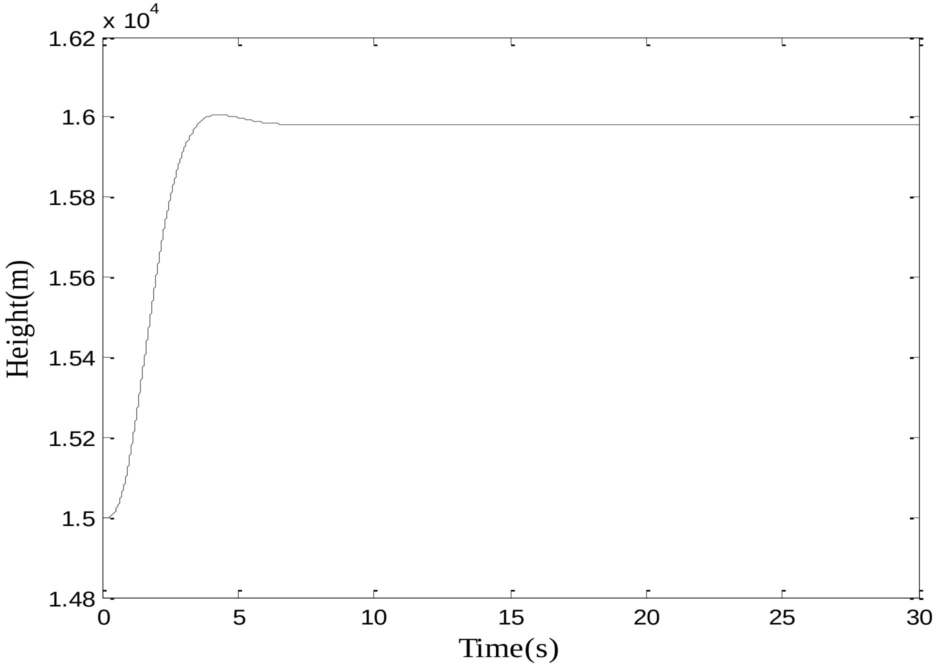
Controller Design Techniques: Pole Placement

Controller $\mathbf{u} = -\mathbf{K}\mathbf{x}$

Altitude tracking

Selected poles

$$\mathbf{p} = [-1+1j \quad -1-1j \quad -2 \quad -3 \quad -0.00407995 \;]$$



Pole Placement Contd.

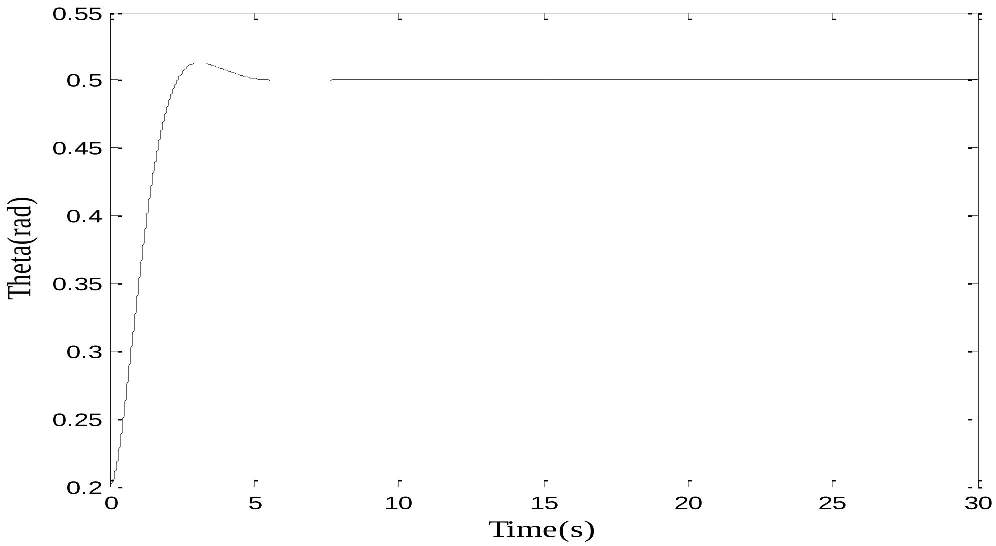
Controller design

$$\mathbf{u} = -\mathbf{K}\mathbf{x}$$

Pitch tracking

Selected poles

$$\mathbf{p} = [-1+1j \quad -1-1j \quad -0.013184 \quad -5.6661e-014 \quad -0.74377\ ]$$



State Feedback with Input Gain

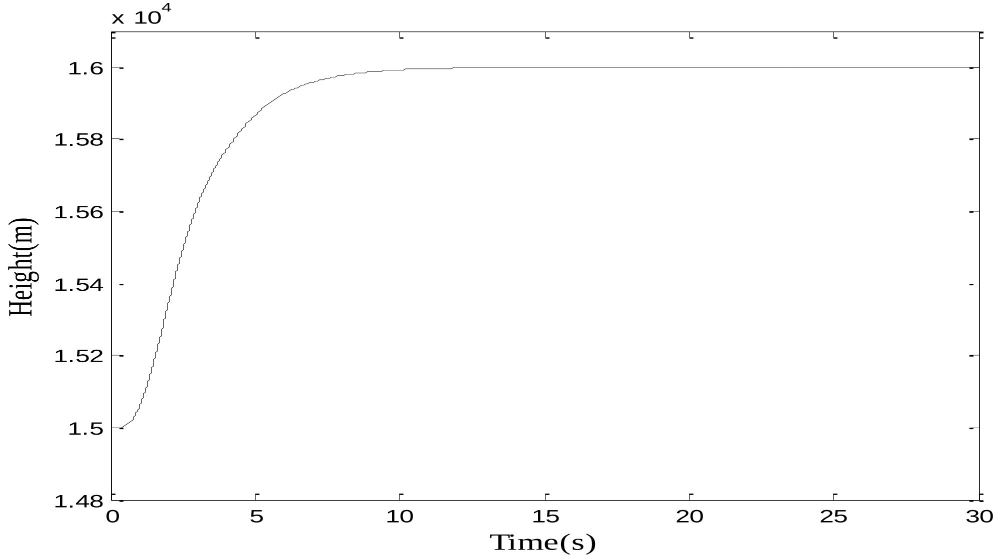
Gain

$$\mathbf{G} = -\left[\mathbf{C}\left(\mathbf{A} - \mathbf{B}\mathbf{K}\right)^{-1}\mathbf{B}\right]^{-1}$$

Altitude tracking

Selected poles

$$\mathbf{p} = [-2+1j \quad -2-1j \quad -2 \quad -3 \quad -0.00407995 \;]$$

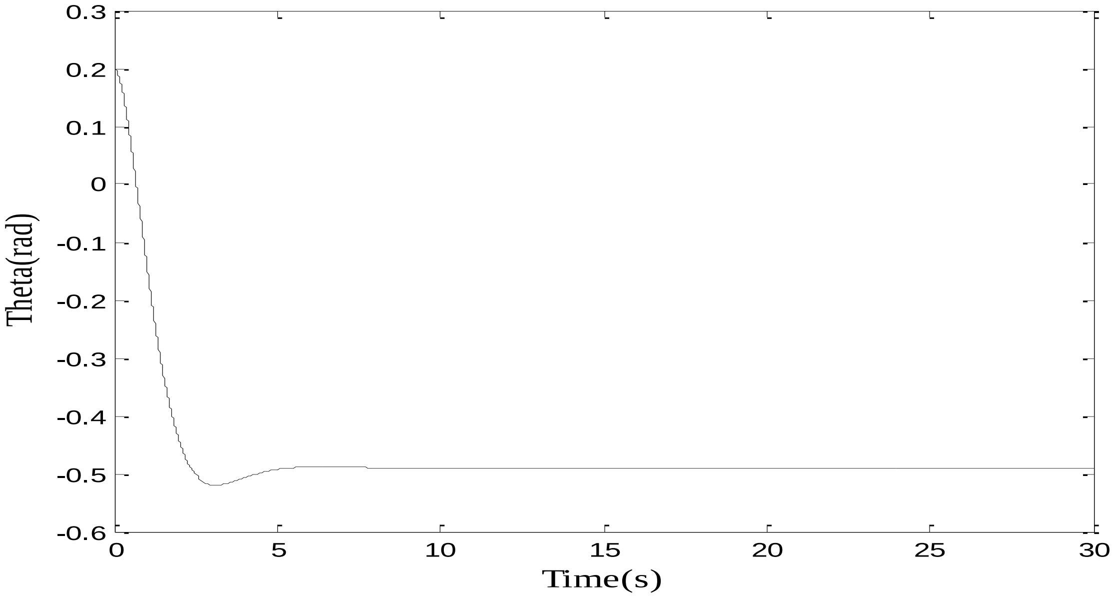


State Feedback with Input Gain Contd.

Gain

$$\mathbf{G} = -\left[\mathbf{C}\left(\mathbf{A} - \mathbf{B}\mathbf{K}\right)^{-1}\mathbf{B}\right]^{-1}$$

Pitch tracking



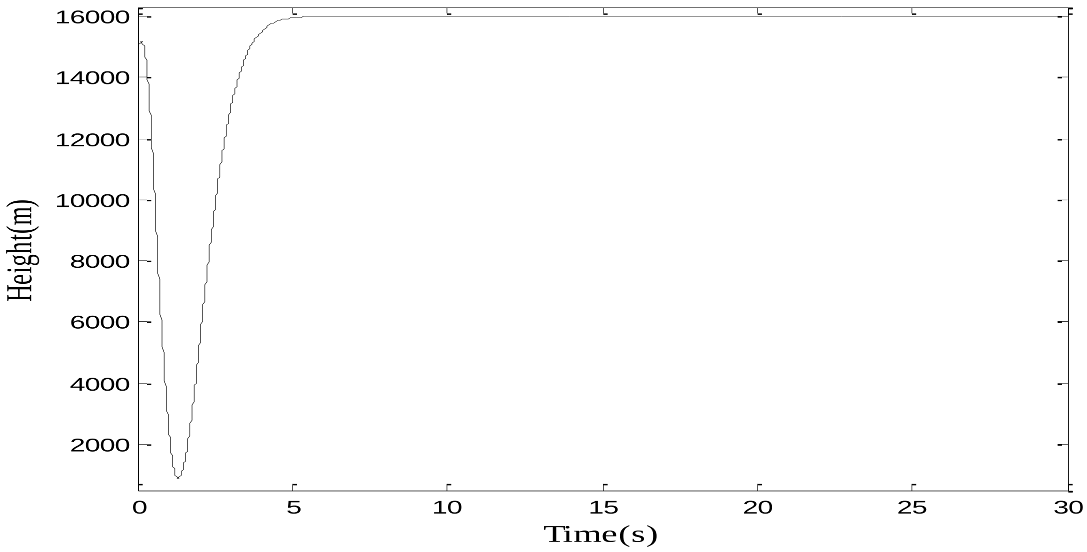
State Feedback with Integral Action

Controller $\mathbf{u} = -\mathbf{K}\mathbf{x}(t) + k_I \xi(t)$

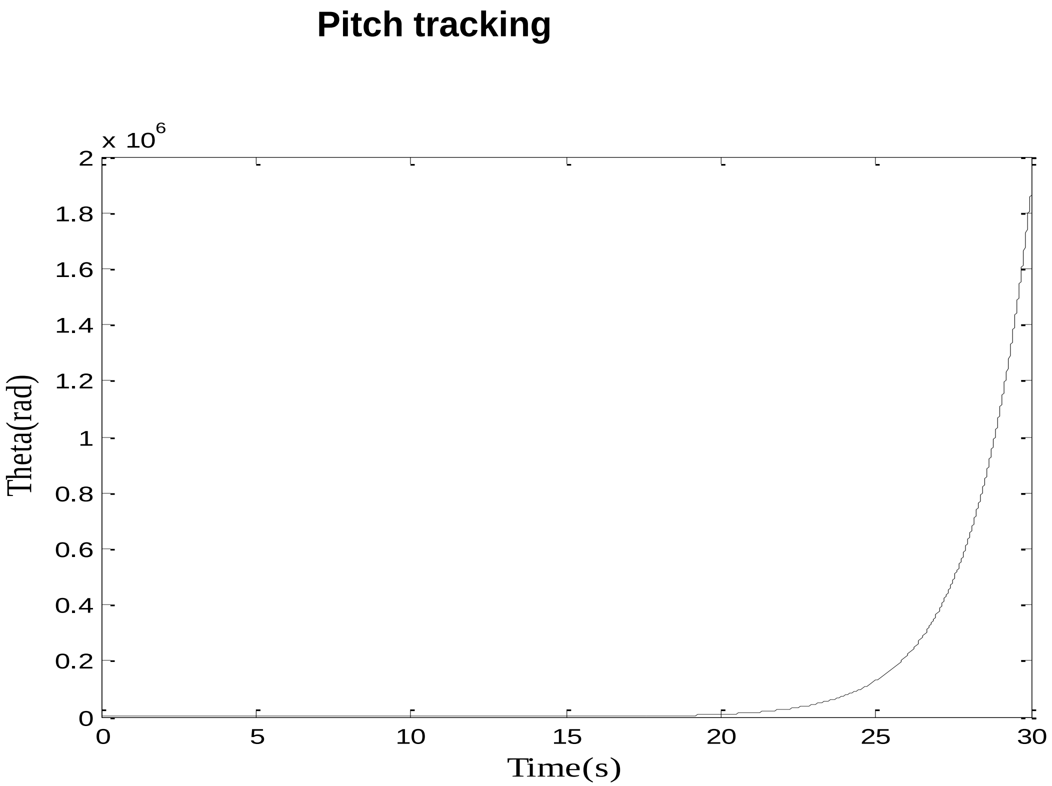
System $\begin{bmatrix} \dot{\mathbf{x}}(\mathbf{t}) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(\mathbf{t}) \\ \xi(t) \end{bmatrix} - \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} \begin{bmatrix} \mathbf{K} & -k_I \end{bmatrix}$

Gain $\mathbf{k}^* = \begin{bmatrix} \mathbf{K} & -k_I \end{bmatrix}$

Altitude tracking



State Feedback with Integral Action Contd.



LQR Controller Design with Integral Action

System

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)$$

Augmented state

$$\hat{\mathbf{x}} = [\mathbf{x(t)} \quad \omega(t)]^T$$

Augmented system

$$\dot{\hat{\mathbf{x}}}(\mathbf{t}) = \underbrace{\begin{bmatrix} \mathbf{A} & 0 \\ -\mathbf{C} & 0 \end{bmatrix}}_{\hat{\mathbf{A}}} \hat{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}}_{\hat{\mathbf{B}}} \mathbf{u} + \underbrace{\begin{bmatrix} 0 \\ r \end{bmatrix}}_{\mathbf{d}}$$

$$\mathbf{y} = \underbrace{[\mathbf{C} \quad 0]}_{\hat{\mathbf{C}}} \hat{\mathbf{x}}$$

Cost function

$$J = \frac{1}{2}(\mathbf{y}(\mathbf{t_f}) - \mathbf{r}(\mathbf{t_f}))^{\mathbf{T}} \mathbf{P} \mathbf{y}(\mathbf{y}(\mathbf{t_f}) - \mathbf{r}(\mathbf{t_f})) + \frac{\mathbf{1}}{\mathbf{2}} \mathbf{w}(\mathbf{t_f})^{\mathbf{T}} \mathbf{P_w} \mathbf{w}(\mathbf{t_f}) +$$
$$\frac{1}{2}\int_0^{t_f} \left[(\mathbf{y}-\mathbf{r})^{\mathbf{T}} \mathbf{Q_y} (\mathbf{y}-\mathbf{r}) + \mathbf{w}^{\mathbf{T}} \mathbf{Q_w} \mathbf{w} + \mathbf{u}^{\mathbf{T}} \mathbf{R} \mathbf{u} \right] dt$$

Control input

$$\mathbf{u(t)} = -\hat{\mathbf{K}}(\mathbf{t})\hat{\mathbf{x}}(\mathbf{t}) + \mathbf{K_r} \mathrm{r}$$

Ricatti equation

$$0 = \hat{\mathbf{Q}} + \mathbf{A}^{\mathbf{T}}\hat{\mathbf{S}} + \hat{\mathbf{S}}\hat{\mathbf{A}} - \hat{\mathbf{S}}\hat{\mathbf{B}}\hat{\mathbf{R}}^{-1}\mathbf{B}^{\mathbf{T}}\hat{\mathbf{S}}$$

All the parameters are defined as

$$\hat{\mathbf{K}}(\mathbf{t}) = \mathbf{R}^{-1}\hat{\mathbf{B}}^{\mathbf{T}}\hat{\mathbf{S}}(t)$$

$$\hat{\mathbf{Q}} = \begin{bmatrix} \mathbf{C}^{\mathbf{T}}\mathbf{Q_y}\mathbf{C} & 0 \\ 0 & \mathbf{Q_w} \end{bmatrix}$$

$$\mathbf{K_r} = \left[\mathbf{G}^2 \quad -\mathbf{W}^{(1)}\mathbf{C}^{\mathbf{T}}\mathbf{Q_y}\right]$$

$$\mathbf{W} = \mathbf{R}^{-1}\hat{\mathbf{B}}^{\mathrm{T}}\left[(\hat{\mathbf{A}} - \hat{\mathbf{B}}\hat{\mathbf{K}})^T\right]^{-1}$$
$$= \left[\mathbf{W}^{(1)}_{(n_u \times n_x)} \mathbf{W}^{(2)}_{(n_u \times n_x)}\right] n_u \times (n_x + n_y)$$

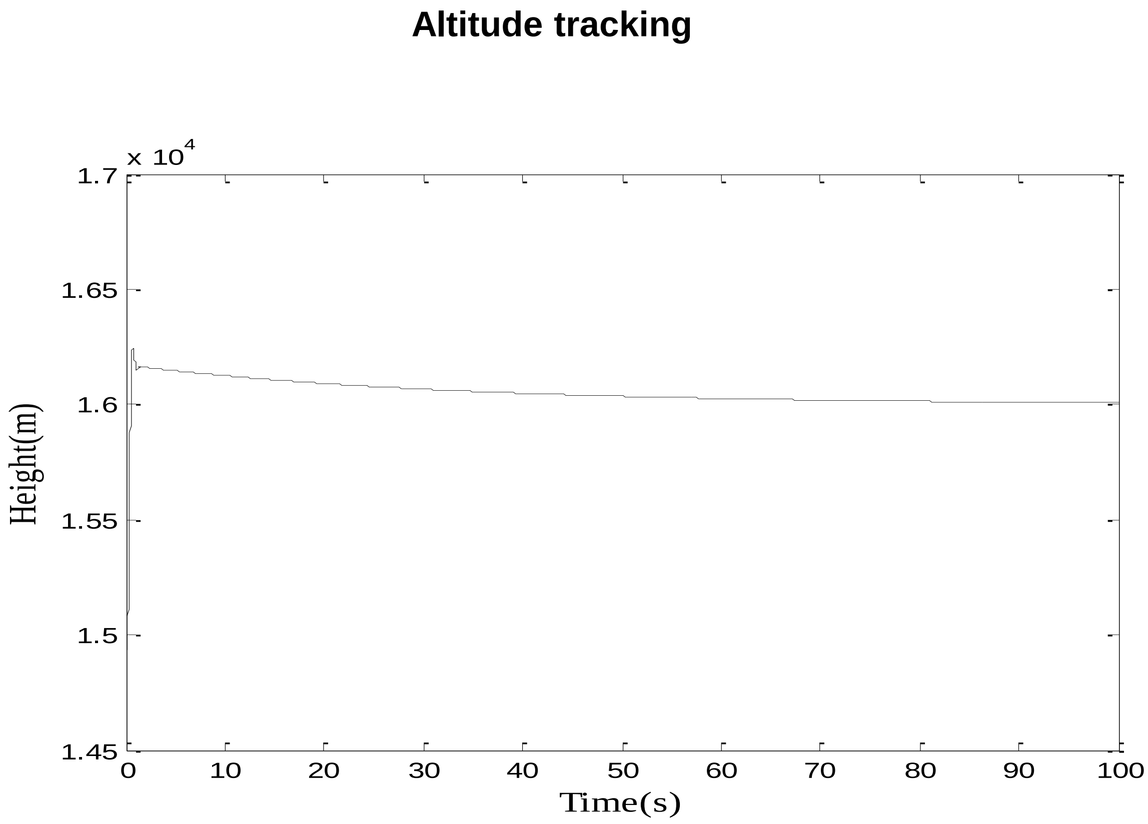
$$\mathbf{G} = \mathbf{R}^{-1}\hat{\mathbf{B}}^{\mathrm{T}}[(\hat{\mathbf{A}} - \hat{\mathbf{B}}\hat{\mathbf{K}})^T]^{-1}\hat{\mathbf{S}}$$
$$= \left[\mathbf{G}^{(1)}_{(n_u \times n_x)} \quad \mathbf{G}^{(2)}_{(n_u \times n_y)}\right] n_u \times (n_x + n_y)$$

$$\hat{\mathbf{K}} = [\mathbf{K_x} \quad \mathbf{K_w}]$$

Control input

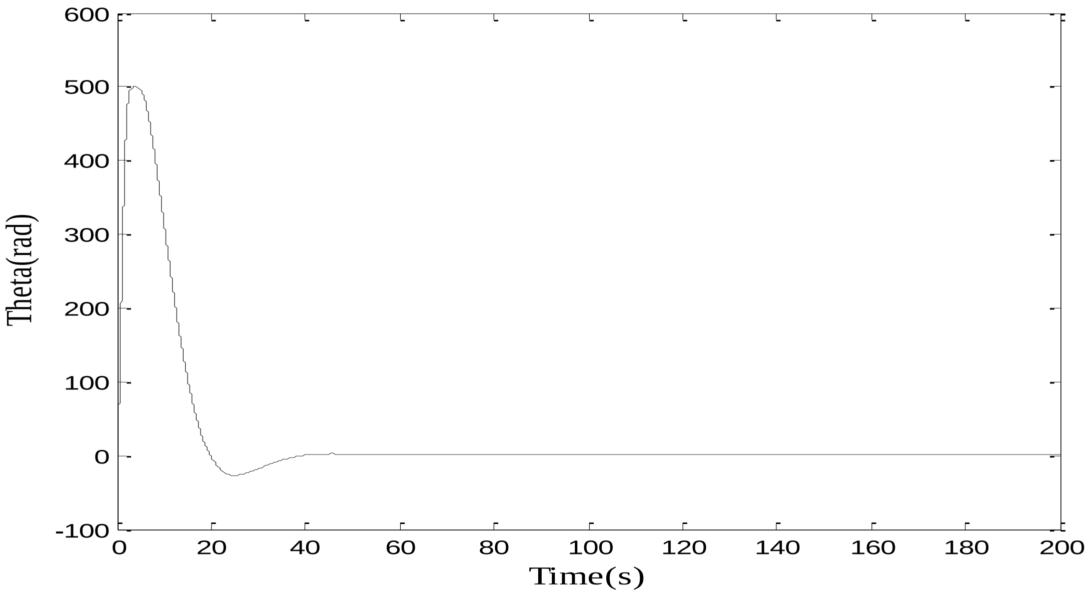
$$\mathbf{u} = -[\mathbf{K_x} \quad \mathbf{K_w}] \begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix} + \mathbf{K_r} r$$

LQR with Integral Action Contd.



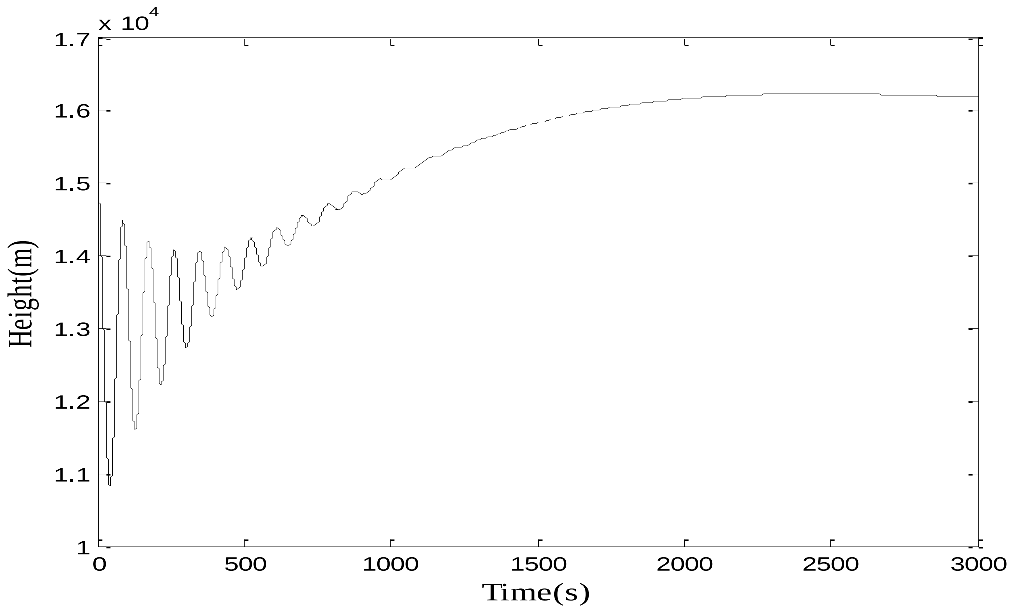
LQR with Integral Action Contd.

Pitch tracking

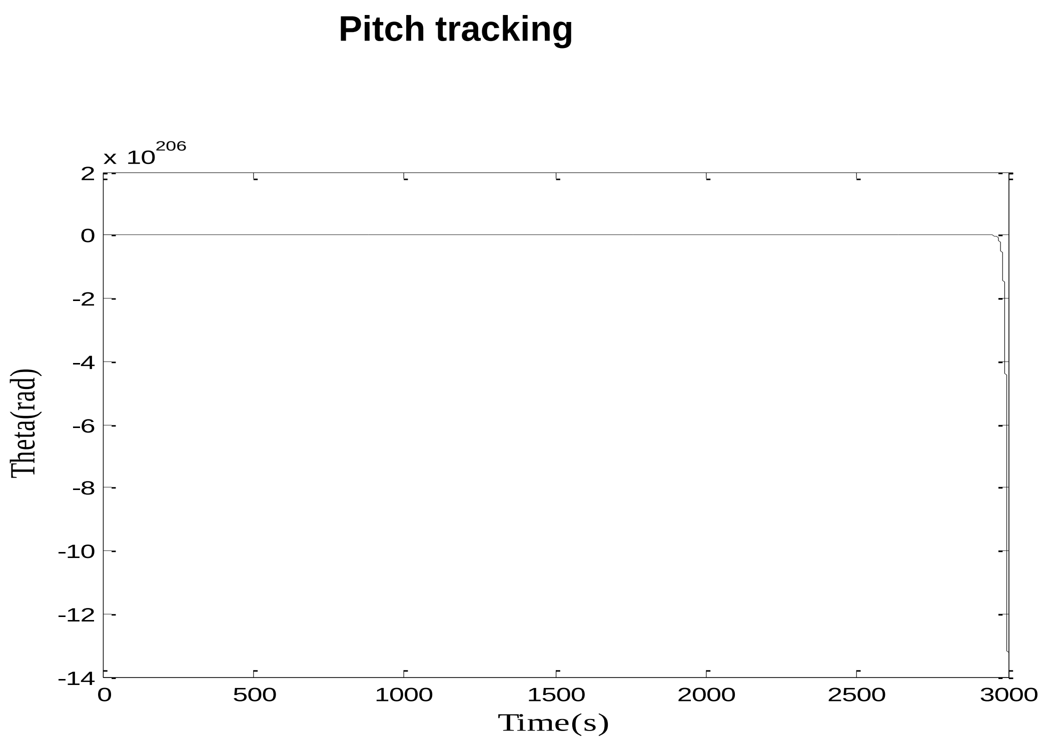


PID Controller

Altitude tracking



PID Controller



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Input Output Linearization

System

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \tilde{\mathbf{g}}(\mathbf{x})\mathbf{u}$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$

First integration

$$\dot{y}_i = \frac{\partial h_i(\mathbf{x})}{\partial \mathbf{x}}\dot{\mathbf{x}} = \frac{\partial h_i(\mathbf{x})}{\partial \mathbf{x}}\{\mathbf{f}(\mathbf{x}) + \tilde{\mathbf{g}}(\mathbf{x})\mathbf{u}\}$$
$$= \frac{\partial h_i(\mathbf{x})}{\partial \mathbf{x}}\mathbf{f}(\mathbf{x}) + \frac{\partial h_i(\mathbf{x})}{\partial \mathbf{x}}\tilde{\mathbf{g}}(\mathbf{x})\mathbf{u}$$
$$= L_{\mathbf{f}} h_i(\mathbf{x}) + \sum_{j=1}^{m} L_{\tilde{\mathbf{g}}_j} h_i(\mathbf{x}) u_j$$

Final output

$$\begin{bmatrix} y_1^{r_1} \\ \cdot \\ \cdot \\ y_p^{r_p} \end{bmatrix} = \begin{bmatrix} L_{\mathbf{f}}^{r_1} h_1(\mathbf{x}) \\ \cdot \\ \cdot \\ L_{\mathbf{f}}^{r_p} h_p(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{m} L_{\tilde{\mathbf{g}}_j} L_{\mathbf{f}}^{r_1-1} h_1(\mathbf{x}) \\ \cdot \\ \cdot \\ \sum_{j=1}^{m} L_{\tilde{\mathbf{g}}_j} L_{\mathbf{f}}^{r_p-1} h_p(\mathbf{x}) \end{bmatrix}_{p\times m} \begin{bmatrix} u_1 \\ \cdot \\ u_m \end{bmatrix}_{m\times 1}$$

$$\mathbf{y}^{(r)} = \mathbf{P}(\mathbf{x}) + \mathbf{E}(\mathbf{x})\mathbf{u}$$

Control input

$$\mathbf{u} = \left\{(\mathbf{E}^{\mathrm{T}}\mathbf{E})^{-1}\mathbf{E}^{\mathrm{T}}\right\}(\tilde{\mathbf{v}} - \mathbf{P}(\mathbf{x}))$$

Linearized equation

$$\mathbf{y}^{(r)} = \tilde{\mathbf{v}}$$

Tracking error

$$\mathbf{e} = \mathbf{y}_r - \mathbf{y}$$

Control law

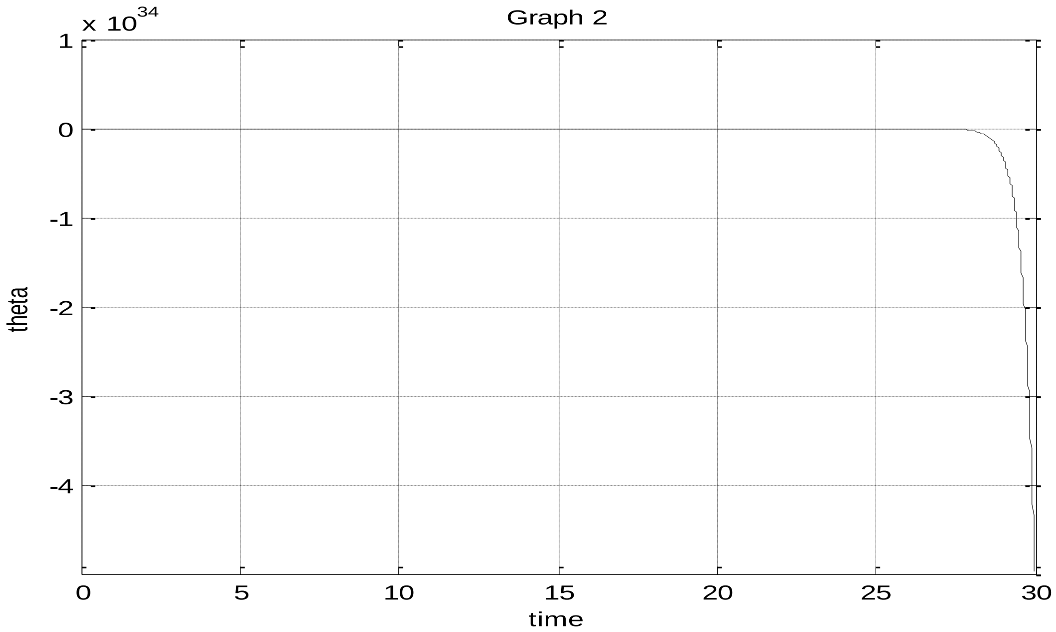
$$\tilde{\mathbf{v}} = \mathbf{y}_r^{(r)} + \mathbf{K}_1\mathbf{e}^{(r-1)} + \ldots\ldots + \mathbf{K}_{r-1}\dot{\mathbf{e}} + \mathbf{K}_r\mathbf{e}$$

Error dynamics

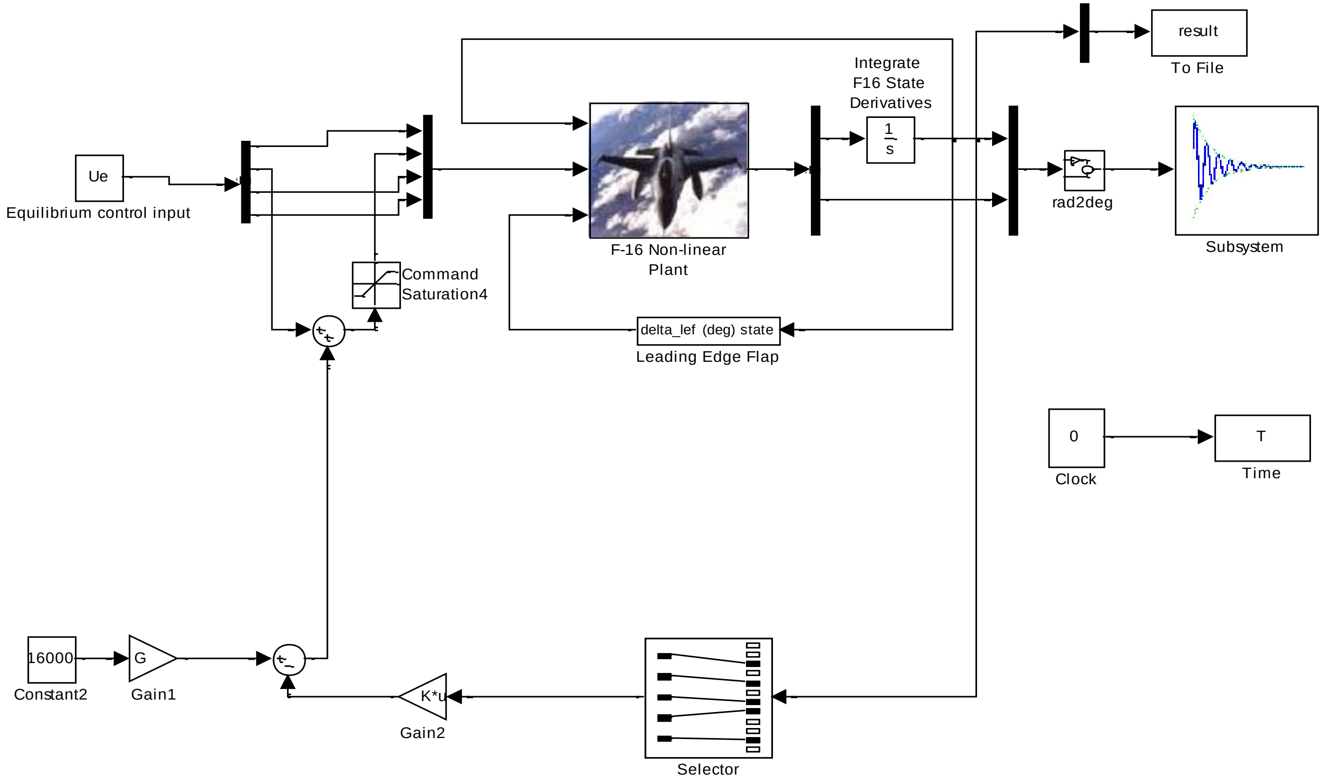
$$\mathbf{e}^{(r)} + \mathbf{K}_1\mathbf{e}^{(r-1)} + \ldots\ldots + \mathbf{K}_{r-1}\dot{\mathbf{e}} + \mathbf{K}_r\mathbf{e} = 0$$

Input Output Linearization Contd.

Pitch tracking



Auto Pilot Design: Altitude Tracking



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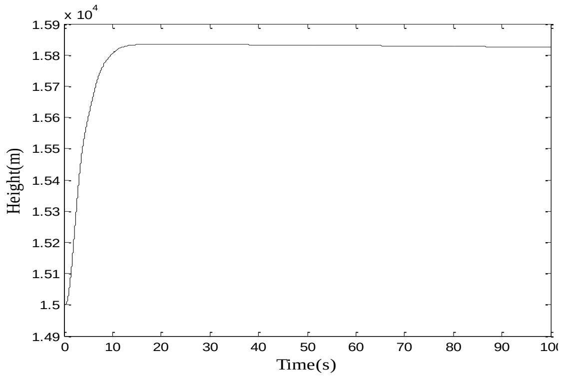
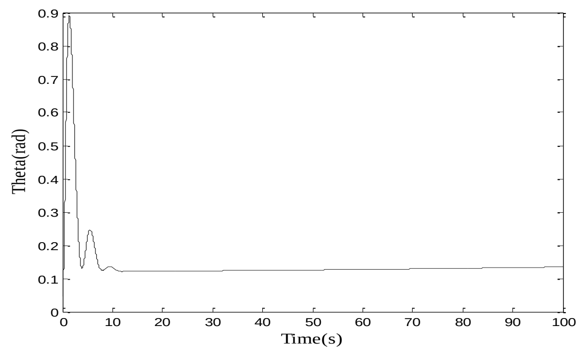
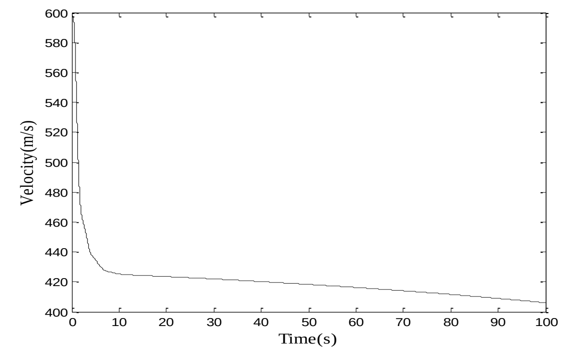
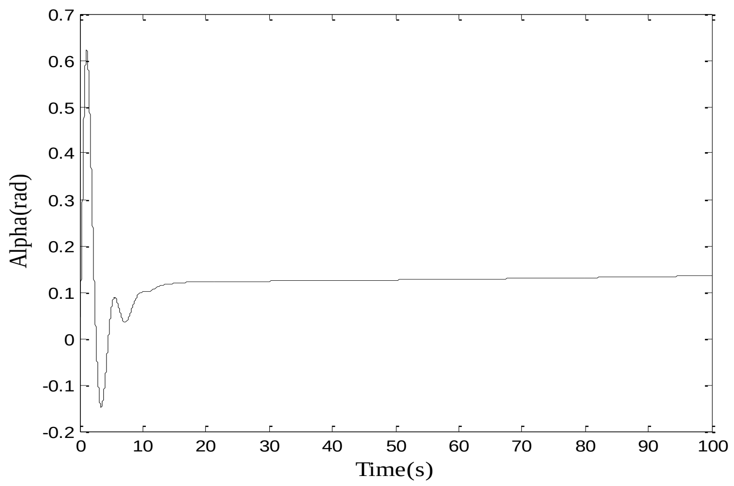
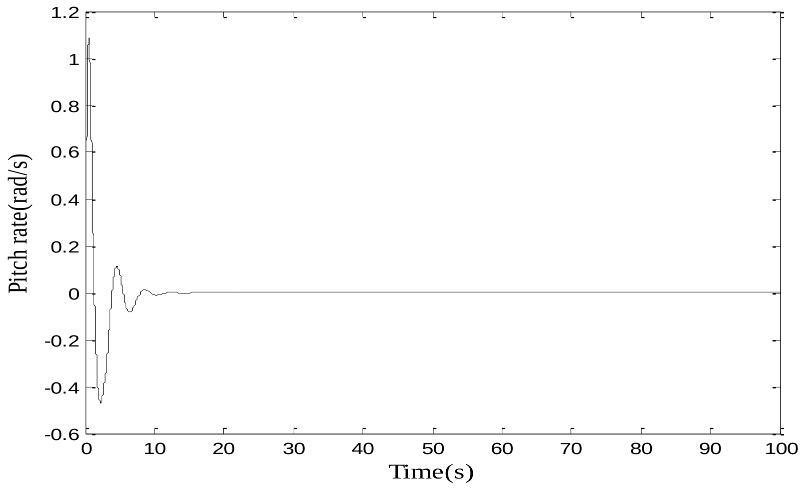
Results: Altitude Tracking











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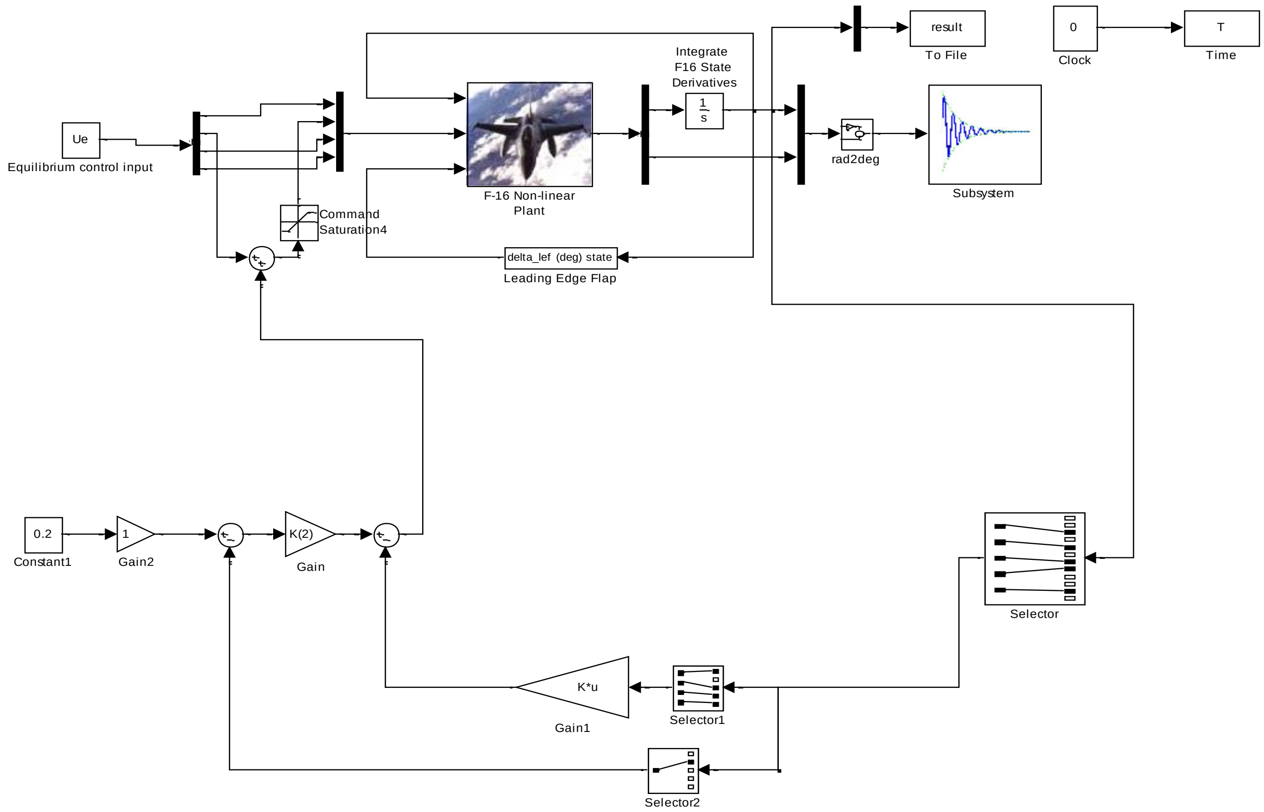
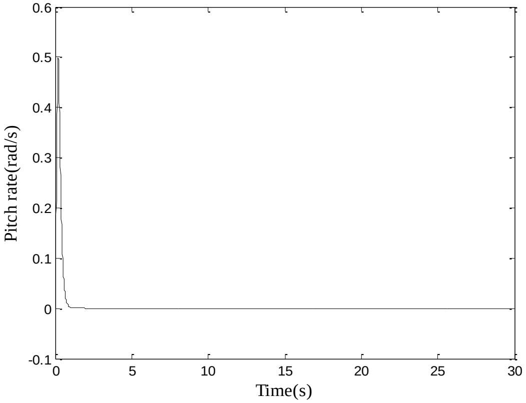
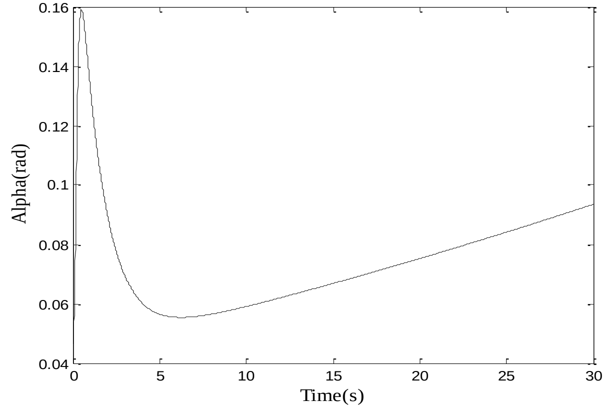
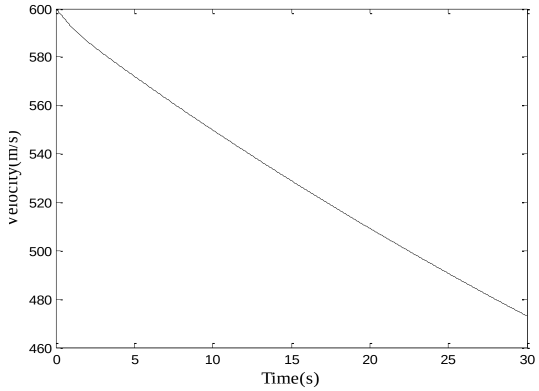
Auto Pilot Design: Pitch Tracking

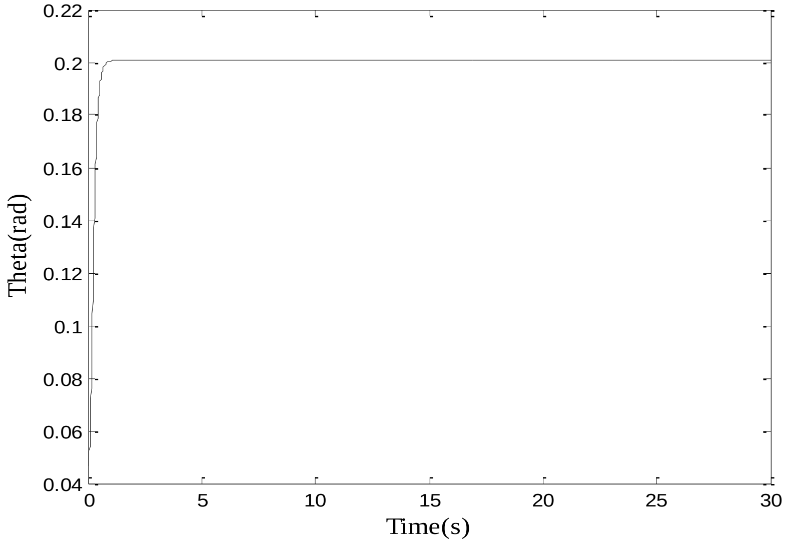


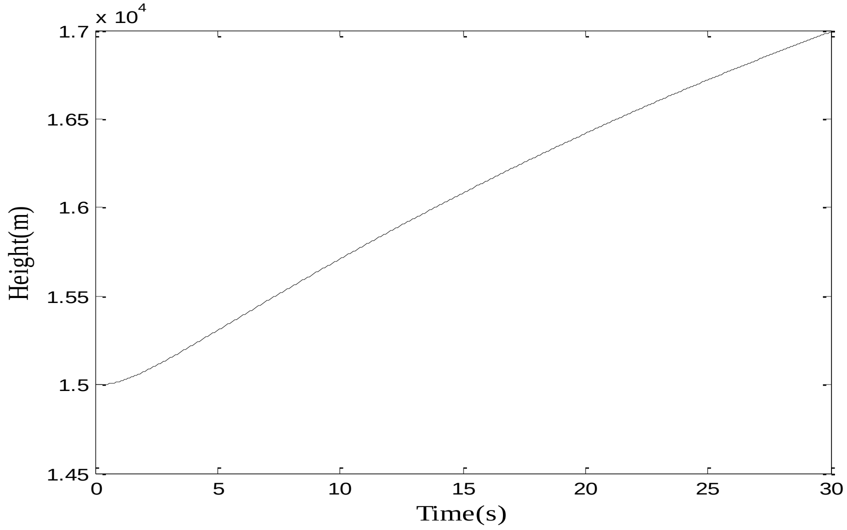
Results: Pitch Tracking











Conclusion

- Altitude tracking and pitch tracking is achieved with elevator input
- Six different controllers were tried to find the best controller that achieves the desired result when applied to the nonlinear F16 dynamics
- Altitude tracking is achieved with state feedback with input gain controller that eliminates steady state error
- Pitch hold is achieved with pole placement controller

Future Work

- Trim the system at various trim points and find various gains to increase the operating region (Gain Scheduling)

- Analyzing the motion in lateral direction

- Analyzing and develop controllers for the nonlinear system instead of linearizing the system at a trim point

  Ex: Dynamic feedback linearization(input- state feedback), Sliding mode controller, back stepping controller or adaptive robust controller

- Experimental verification and implementation of the F16 aircraft

Group Contribution

Gunneet Singh 100%

Manuel Vergara 100%

Sivaram Wijenddra 100%

Thank you