Concordia University

Title: Flight Control Systems

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Date: 16-12-2011

1. **Combined control method**

2. **LQG/LTR method**

3. **Their comparison for two example (longitudinal landing and flare phase landing)**

Combined method

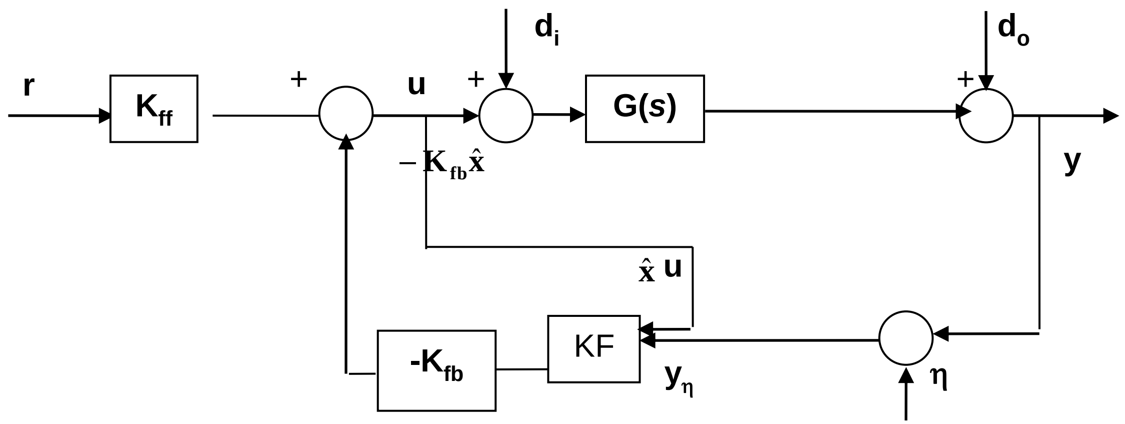
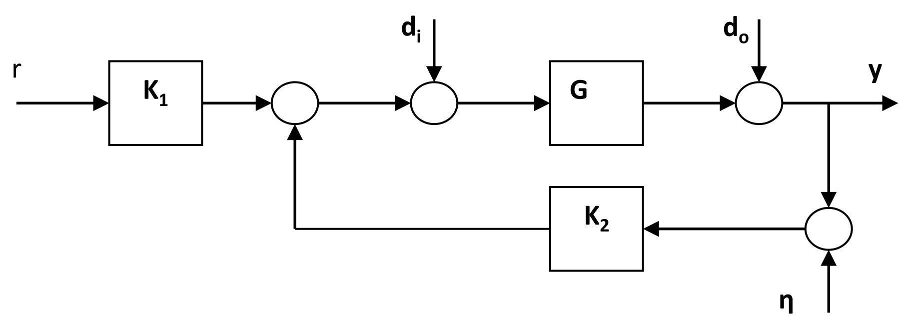


Figure 1. Combination LQT with Kalman Filter (KF)



$$\mathbf{K}_1(s) = \left(\mathbf{I} - \mathbf{K}_{fb}(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}_{fb} + \mathbf{L}\mathbf{C})^{-1}\mathbf{B}\right)\mathbf{K}_{ff}$$

$$\mathbf{K}_2(s) = \mathbf{K}_{fb}(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}_{fb} + \mathbf{L}\mathbf{C})^{-1}\mathbf{L}$$

$$\mathbf{y} = (\mathbf{I} + \mathbf{G}\mathbf{K}_2)^{-1}[\mathbf{G}\mathbf{K}_1\mathbf{r} - \mathbf{G}\mathbf{K}_2\boldsymbol{\eta} + \mathbf{G}\mathbf{d}_i + \mathbf{d}_o]$$

LQG/LTR method

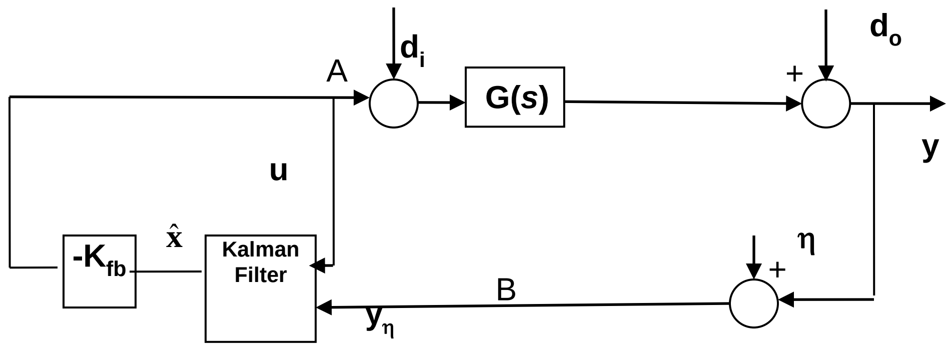
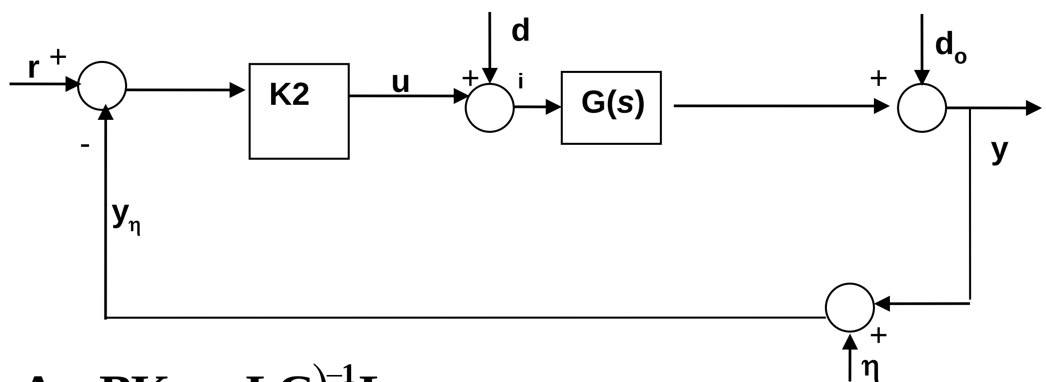


Figure 2. Combination LQR with Kalman Filter (KF).



$$\mathbf{K}_2(s) = \mathbf{K}_{\mathbf{fb}}\left(s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}_{\mathbf{fb}} + \mathbf{L}\mathbf{C}\right)^{-1}\mathbf{L}$$

$$\mathbf{y} = \left(\mathbf{I} + \mathbf{G}\mathbf{K}_2\right)^{-1}\left[\mathbf{G}\mathbf{K}_2\mathbf{r} - \mathbf{G}\mathbf{K}_2\boldsymbol{\eta} + \mathbf{G}\mathbf{d}_{\mathbf{i}} + \mathbf{d}_{\mathbf{o}}\right]$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}.\mathbf{x}(t) + \mathbf{B}.\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) = \mathbf{r}(t) - \mathbf{C}.\mathbf{x}(t)$$

$\mathbf{x}(t) \in R^n$ $\quad$ $\mathbf{u}(t) \in R^m$

$\mathbf{x}(t_0) = \mathbf{x_0}$ $\quad$ $t_0 \leq t \leq t_f$

$\mathbf{y}(t) \in R^p$ $\quad$ $m \geq p$

Minimisation Policy in combined méthod

$$J = \frac{1}{2}\mathbf{e}^{\mathbf{T}}(t_f)\mathbf{F}\mathbf{e}(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left[\mathbf{e}^{\mathbf{T}}(t)\mathbf{Q}\mathbf{e}(t) + \mathbf{u}^{\mathbf{T}}(t)\mathbf{R}\mathbf{u}(t)\right]dt$$

Minimisation Policy in LQG/LTR

$$J = \frac{1}{2}x^{\mathbf{T}}(t_f)\mathbf{F}x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\left[x^{\mathbf{T}}(t)\mathbf{Q}x(t) + \mathbf{u}^{\mathbf{T}}(t)\mathbf{R}\mathbf{u}(t)\right]dt$$

Calculating feed forward and feedback gain.

Rank [**co**] = *n* $$[\mathbf{co}] = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \dots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

$$\mathbf{u}(t) = -\mathbf{K_{fb}}\mathbf{x}(t) + \mathbf{u_c}(t)$$

$$\dot{\mathbf{P}}(t) = -\mathbf{PA} - \mathbf{A^T P} - \mathbf{C' QC} + \mathbf{PBR^{-1}B^T P} = \mathbf{0}$$

$$\mathbf{K_{fb}} = \mathbf{R^{-1}B^T P}$$

Calculating Feedback gain for both methods are the same

$$\dot{\mathbf{h}}(t) = -[\mathbf{A} - \mathbf{BK_{fb}}]^{\mathbf{T}}\mathbf{h}(t) - \mathbf{C^T Q r}(t) = 0$$

$$\mathbf{h}(t) = \left(\mathbf{A} - \mathbf{BK_{fb}}\right)^{\mathbf{T}^{-1}}\left(-\mathbf{C^T Q}\,\mathbf{r}(t)\right)$$

$$\mathbf{K_{ff}} = \mathbf{R^{-1}B^T}\left[-\left(\mathbf{A} - \mathbf{BK_{fb}}\right)^{\mathbf{T}^{-1}}\mathbf{C^T Q}\right]$$

$$\mathbf{u_c}(t) = \mathbf{R^{-1}B^T h}(t)$$

$$\mathbf{u}(t) = -\mathbf{K_{fb}}\mathbf{x}(t) + \mathbf{K_{ff}}\mathbf{r}(t)$$

Kalman Filter Dynamics and its gain calculation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{H}\boldsymbol{\xi}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \boldsymbol{\theta}(t)$$

$$\boldsymbol{\varepsilon}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$$

$$J = \mathrm{E}\left\{\sum_{i=1}^{n} \varepsilon_i^{\,2}(t)\right\} = \mathrm{E}\left\{\boldsymbol{\varepsilon}^{\mathbf{T}}(t)\boldsymbol{\varepsilon}(t)\right\} = \mathrm{trace}\left[\mathrm{E}\left\{\boldsymbol{\varepsilon}(t)\boldsymbol{\varepsilon}^{\mathbf{T}}(t)\right\}\right]$$

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{L}[\mathbf{y}(t) - \mathbf{C}\hat{\mathbf{x}}(t)]$$

$$\boldsymbol{\Sigma} = \mathrm{cov}\left\{\boldsymbol{\varepsilon};\boldsymbol{\varepsilon}\right\} = \mathrm{E}\left\{\boldsymbol{\varepsilon}\,\boldsymbol{\varepsilon}^{\mathbf{T}}\right\}$$

$$\mathbf{A}\boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{A}^{\mathbf{T}} + \mathbf{G}\boldsymbol{\Xi}\boldsymbol{\Xi}^{\mathbf{T}} - \boldsymbol{\Sigma}\mathbf{C}^{\mathbf{T}}\boldsymbol{\Theta}^{-1}\mathbf{C}\boldsymbol{\Sigma} = \mathbf{0}$$

$$\mathbf{L} = \boldsymbol{\Sigma}\mathbf{C}^{\mathbf{T}}\boldsymbol{\Theta}^{-1}$$

LTR Procedure for both methods:

Consider: $\mathbf{G}_p(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$

Is minimum phase. We have to select an appropriate **Q** and **R** for the riccati equation to find feedback gain so that $\mathbf{K_{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ Has good step respond ie.

In low frequency $\underline{\sigma}\left[\mathbf{K_{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\right] \succ\succ 1$

In high frequency $\overline{\sigma}\left[\mathbf{K_{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\right] \prec\prec 1$

$$\mathbf{\Xi} = \mu\mathbf{I} + \mathbf{BB'}$$

$$\mathbf{\Theta} = \mu\mathbf{I}$$

$$\mathbf{K}_2(s)\mathbf{G}_\mathbf{p}(s) \longrightarrow \mathbf{K_{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

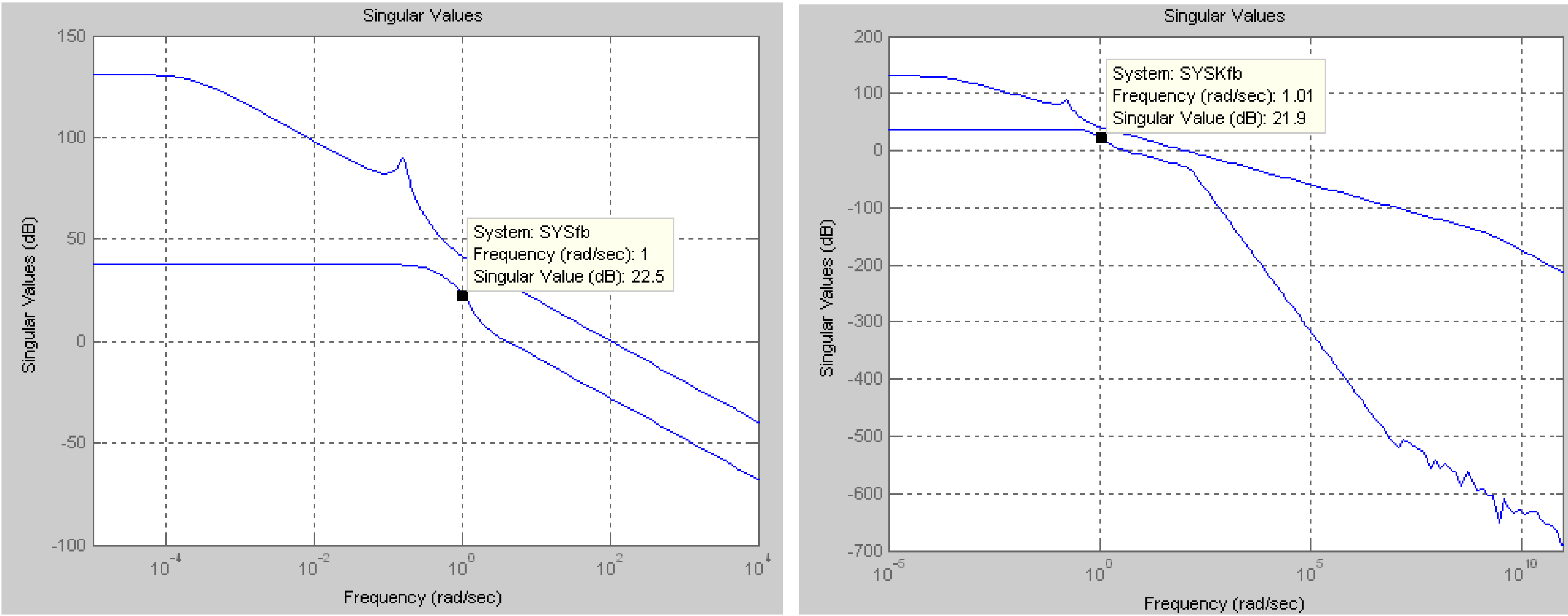
Example: The example is about Longitudinal Control for automatic landing. The matrices **A, B** and **C** describe the longitudinal dynamics for a transport aircraft in the flare phase when its speed is 250 ft/s.

$$\mathbf{V}(t) = \sin(t) \qquad \mathbf{h}(t) = 29.1 \times \exp\left(-\frac{t}{2.667}\right)$$

$$\mathbf{A} = \begin{pmatrix} -0.0386 & 18.9840 & -32.1930 & 0 & 0.0001 \\ -0.0010 & -0.6325 & 0.0056 & 1.0000 & 0.0000 \\ 0 & 0 & 0 & 1.0000 & 0 \\ 0.0001 & -0.7591 & -0.0008 & -0.5138 & -0.0000 \\ -0.0436 & -249.7600 & 249.7600 & 0 & 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 10.1000 & 0 \\ -0.0002 & 0 \\ 0 & 0 \\ 0.0247 & -0.0108 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$-1 < \delta_{th} < 1 \qquad -25^0 < \delta_e < 25^0$$



(a) (b)

Figure 3: (a) Minimum and maximum singular values diagram for $\mathrm{SYSfb} = \mathbf{K}_{\mathbf{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ (dB). (b) Minimum and maximum singular values diagram for $\mathrm{SYS\ Kbf} = \mathbf{K}_2(s)\mathbf{G}_{\mathbf{p}}(s)$ (dB).

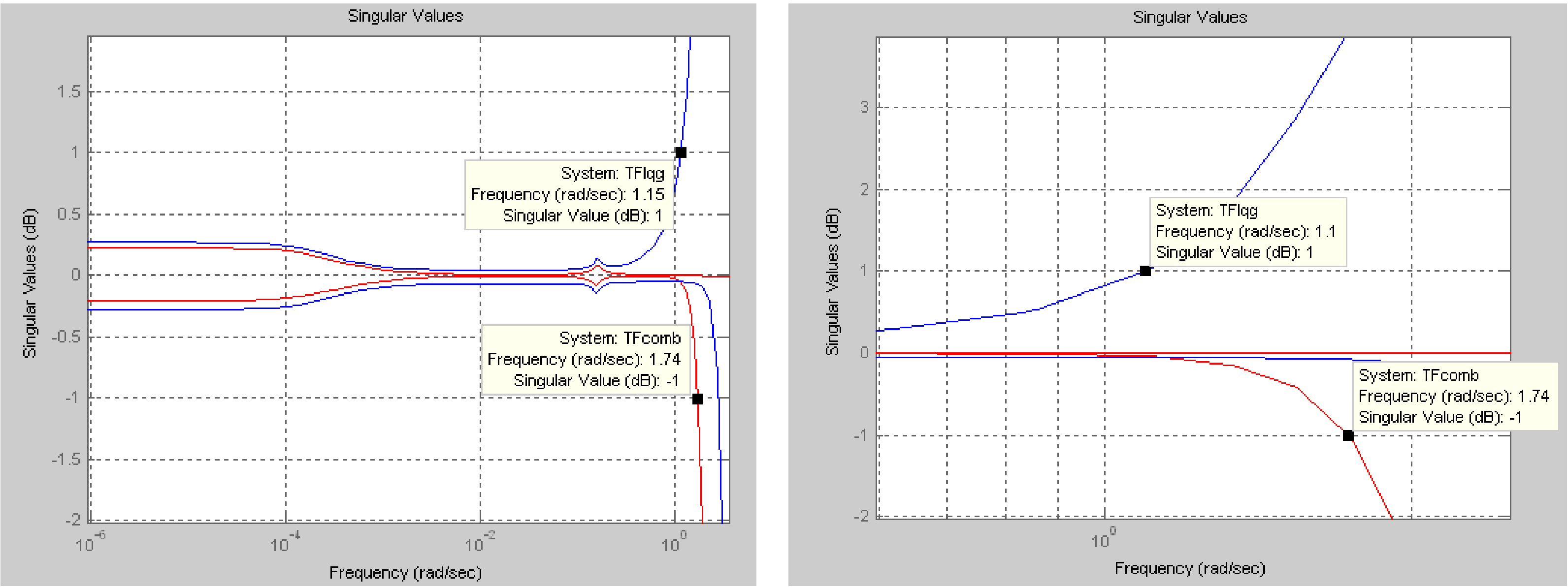
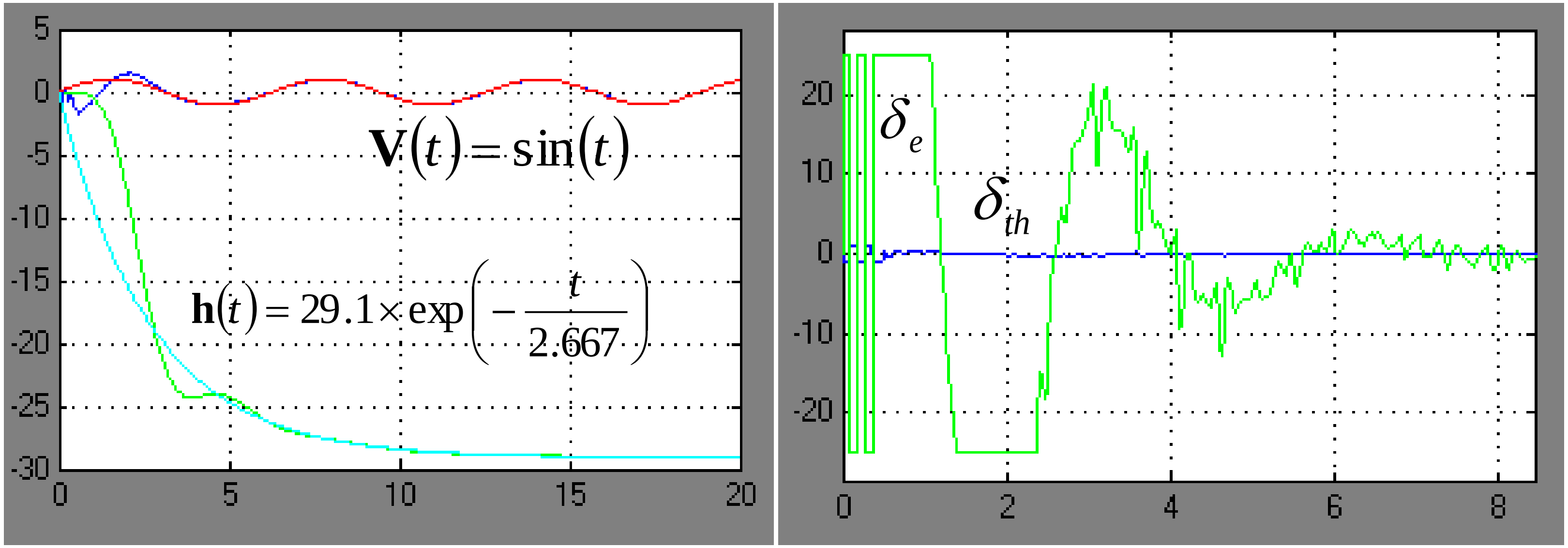


Figure 4: LQG/LTR minimum and maximum singular values diagram for input to output transfer function ($(\mathbf{I}+\mathbf{GK}_2)^{-1}\mathbf{GK}_2$)

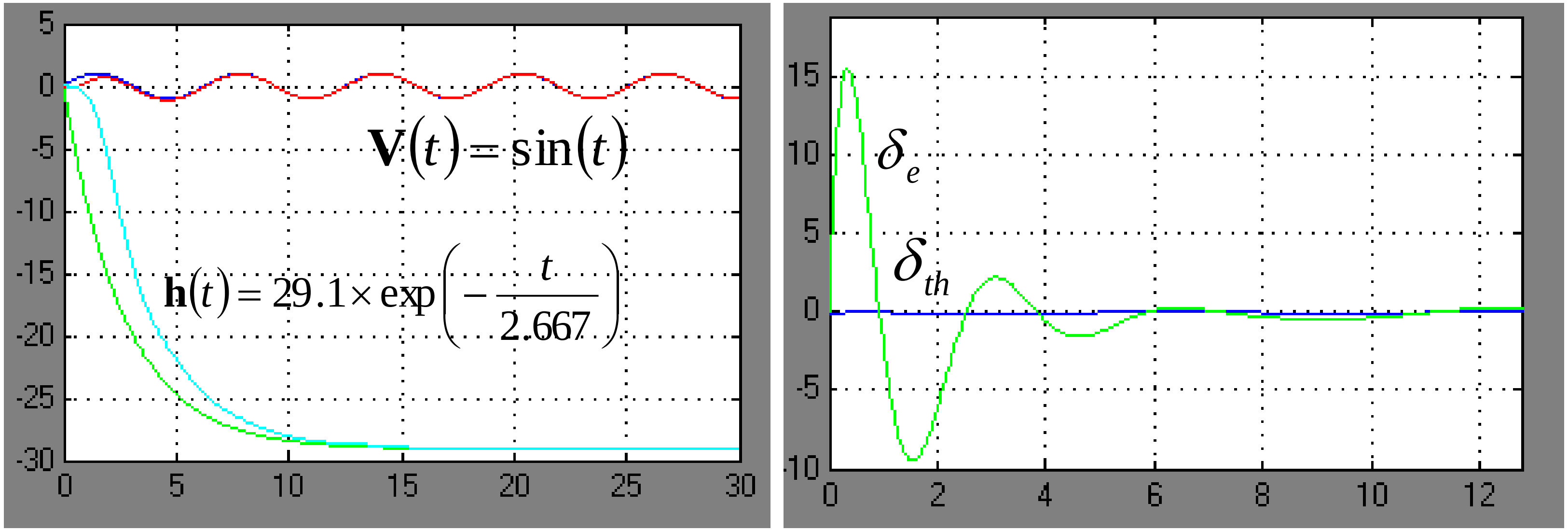
and combined method minimum and maximum singular values diagram for input to output transfer function ($(\mathbf{I}+\mathbf{GK}_2)^{-1}\mathbf{GK}_1$) (dB).



(a) (b)

Fifure 5: (a) LQG/LTR’s tracking performance

(b) Control vector



(a) (b)

Fifure 6: (a) combined method’s tracking performance

(b) Control vector

Example 2: The example is about Longitudinal Control for automatic landing. The matrices A, B and C describe the longitudinal dynamics for a medium size transport aircraft in the flare phase when its speed is 250 ft/s.

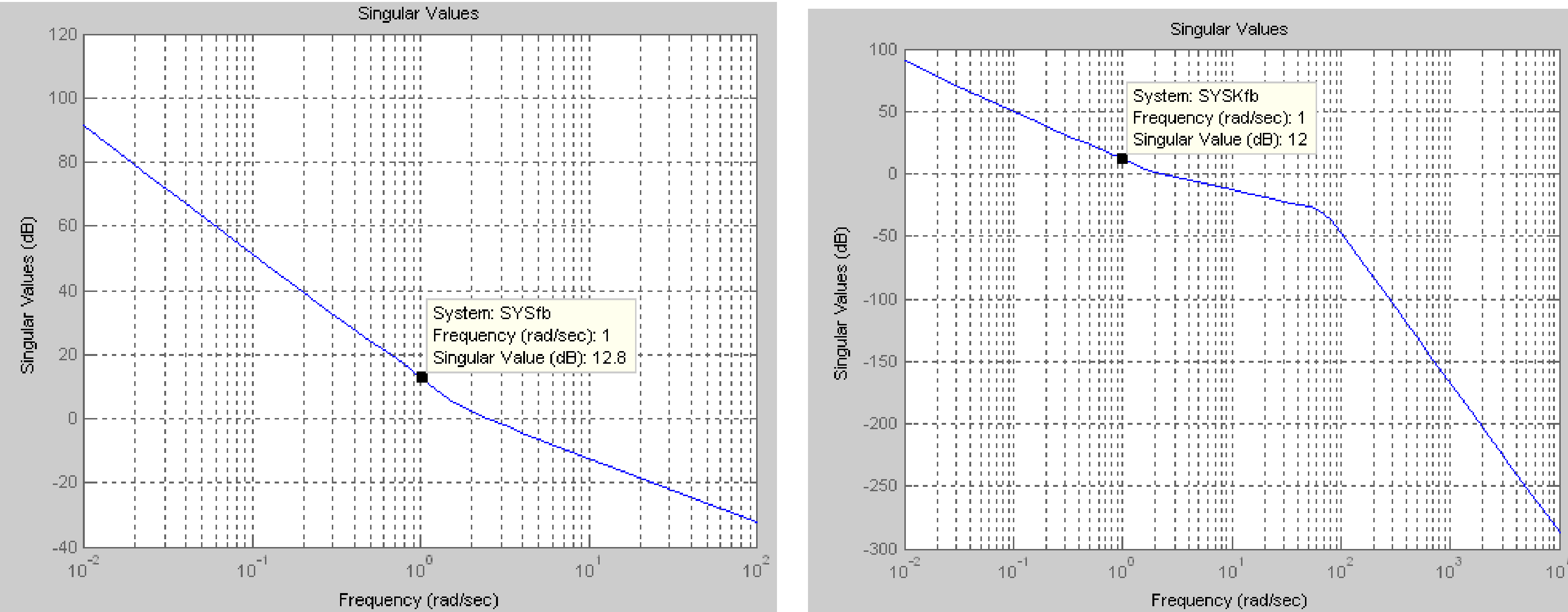
$$\mathbf{A} = \begin{pmatrix} -0.6463 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \\ -0.7739 & 0 & -0.5298 & 0 & -0.0110 \\ -250.0000 & 250.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10.0000 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{pmatrix}$$

$$-25^0 < \delta_{te} < 25^0$$

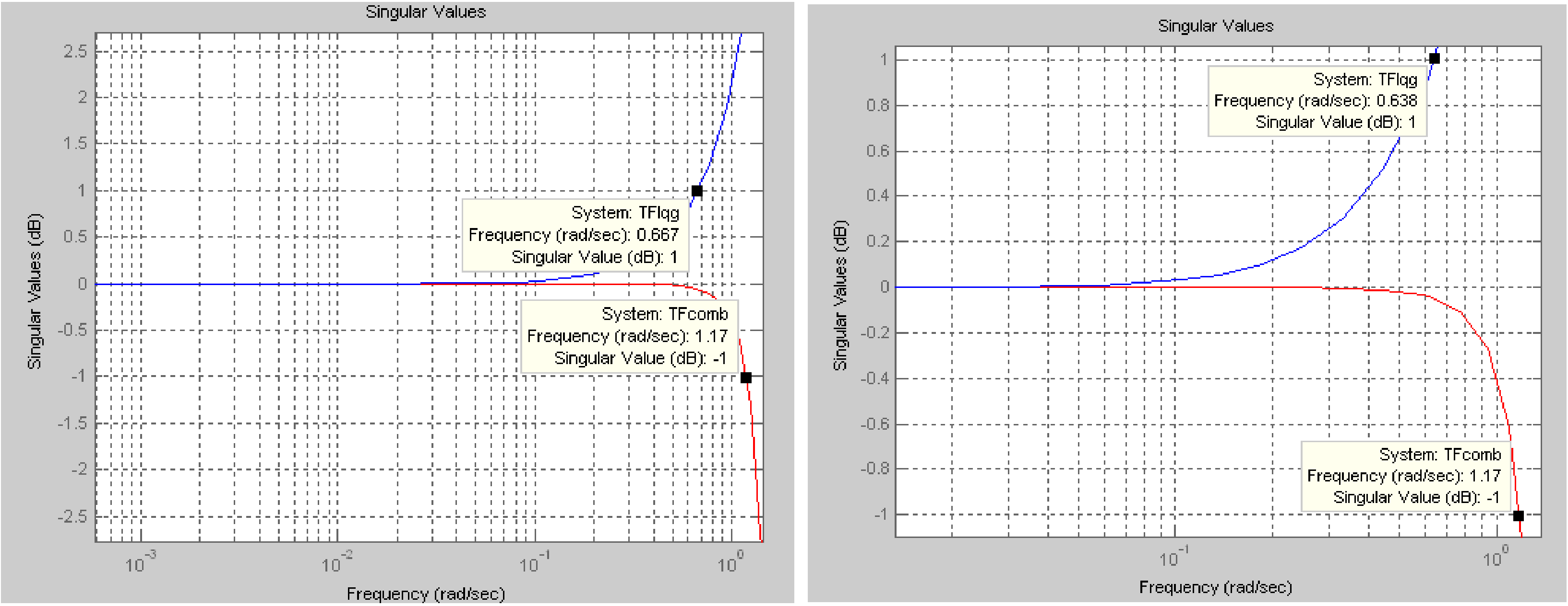
$$\mathbf{h}(t) = 36.37 \times \exp\left(-\frac{t}{3.333}\right)$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$



(a) (b)

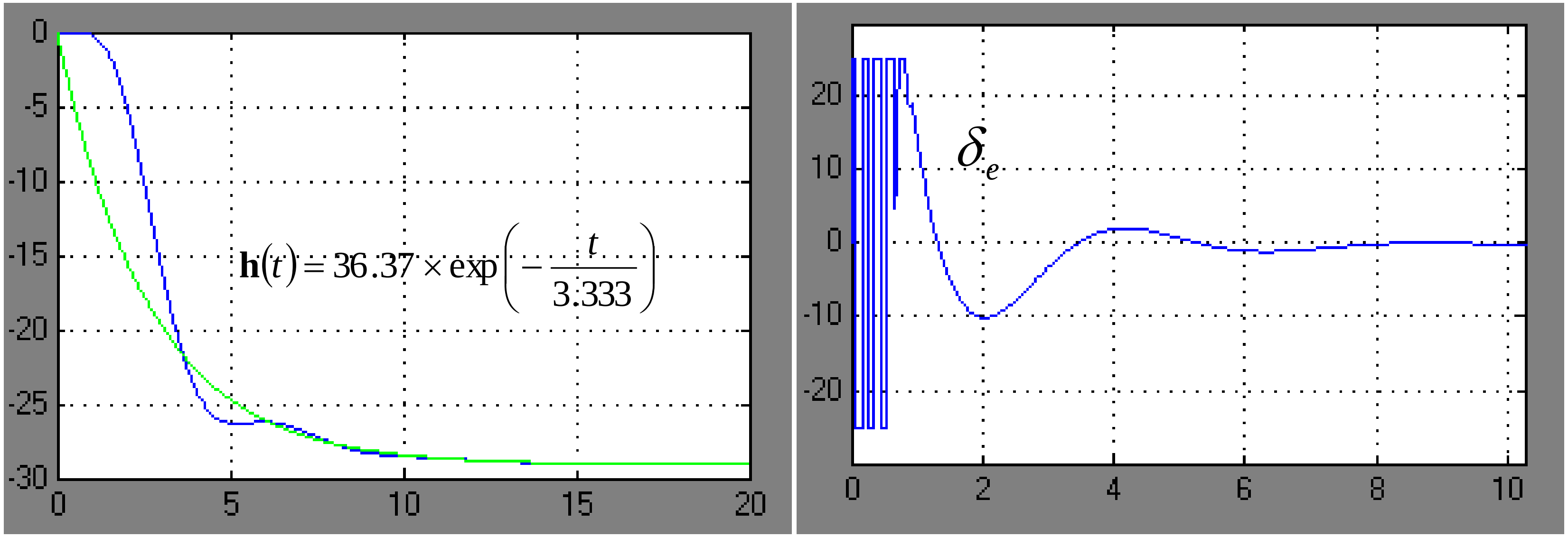
Figure 7: (a) Minimum and maximum singular values diagram for $\mathrm{SYSfb} = \mathbf{K_{fb}}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ (dB). (b) Minimum and maximum singular values diagram for $\mathrm{SYS\ Kbf} = \mathbf{K}_2(s)\mathbf{G}_p(s)$ (dB).



(a) (b)

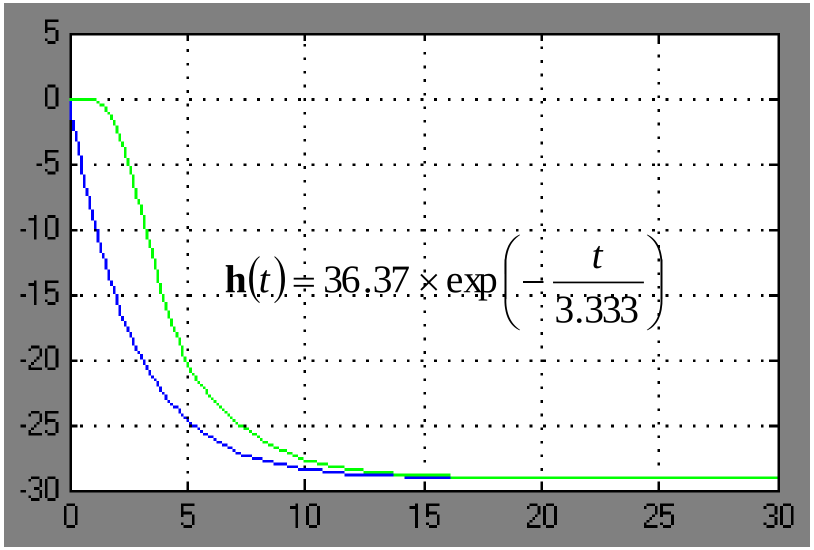
Figure 8 : (a) LQG/LTR's minimum and maximum singular values diagram for input to output transfer function ($(\mathbf{I}+\mathbf{GK}_2)^{-1}\mathbf{GK}_2$)

(b) combined method's minimum and maximum singular values diagram for input to output transfer function ($(\mathbf{I}+\mathbf{GK}_2)^{-1}\mathbf{GK}_1$) (dB).

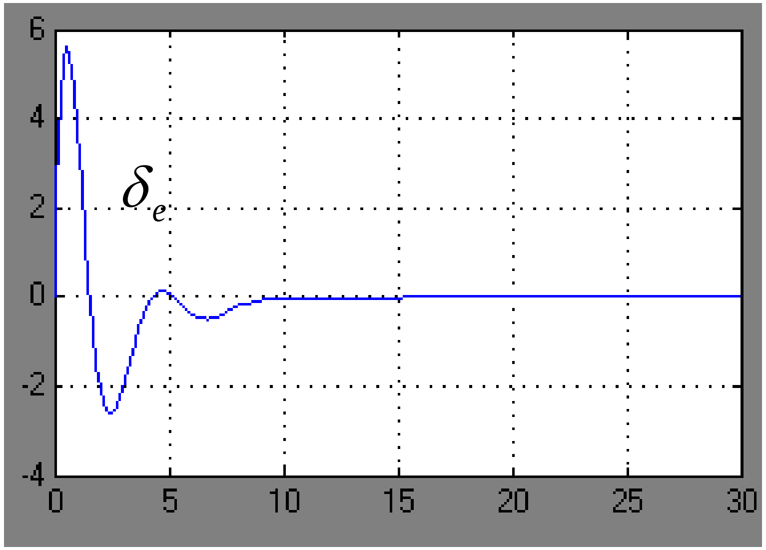


(a) (b)

Fifure 9: (a) LQG/LTR's tracking performance
(b) Control

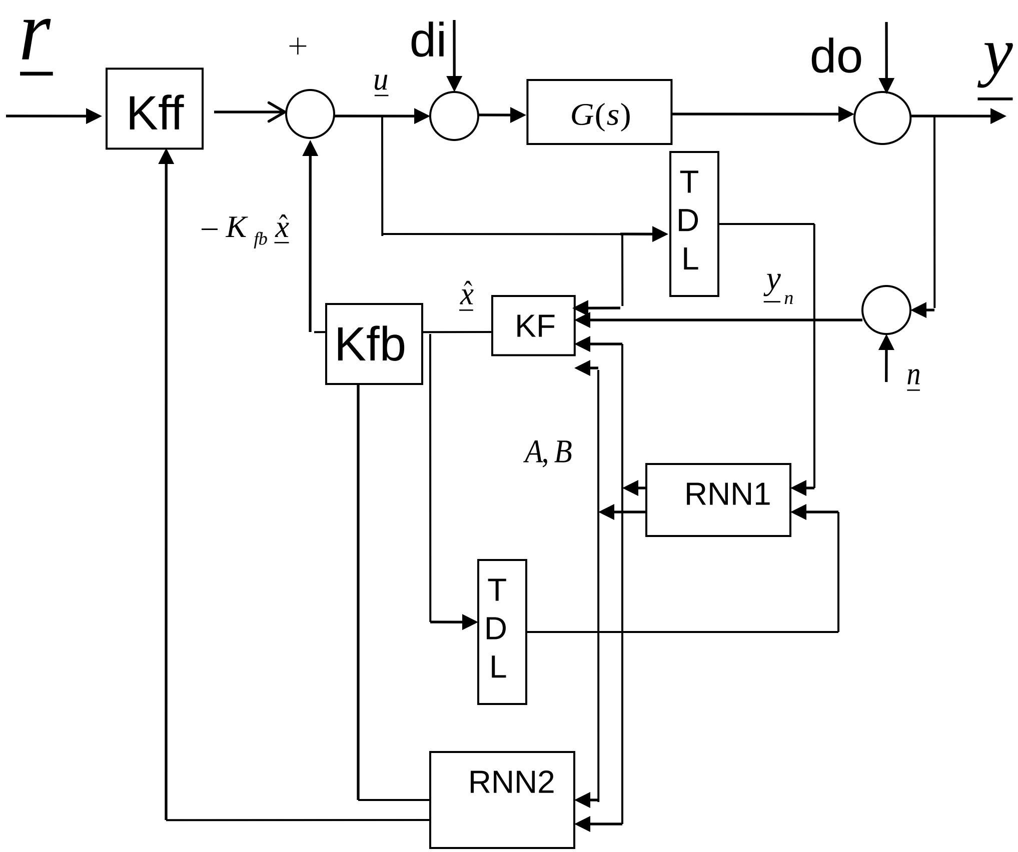


(a)



(b)

Fifure 10: (a) combined method's tracking performance
(b) Control vector



Fifure 11: A new idea for Online Controller system

End

Thank you for your attention