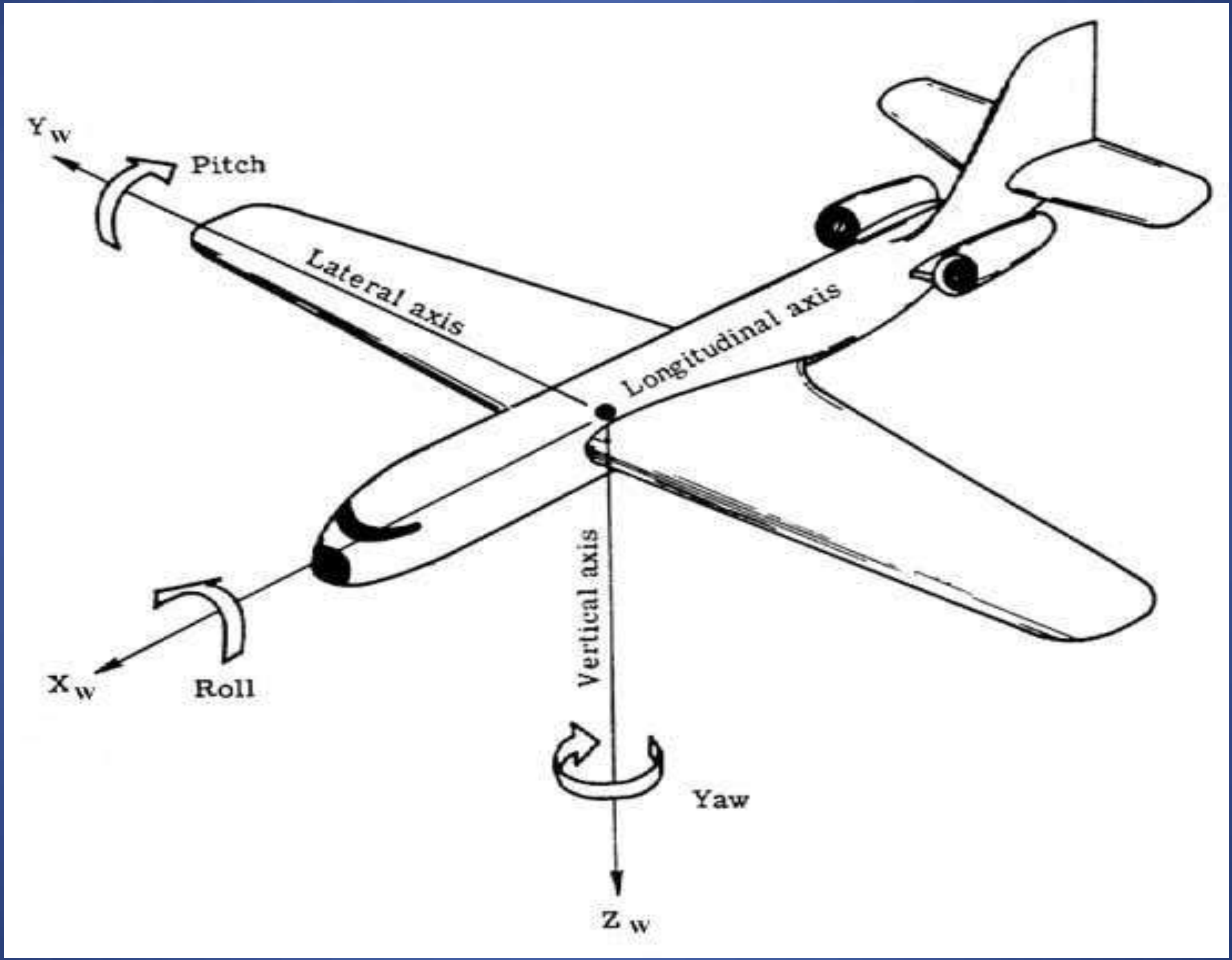
Modeling and Simulation of a Three Degree of Freedom Longitudinal Aero plane System

Figure 1: Boeing 777 and example of a two engine business jet



Y w
Pitch
Lateral axis
Longitudinal axis
Vertical axis
X w
Roll
Yaw
Z w

Nonlinear dynamic equations of motion for the longitudinal direction of aircraft

$$\dot{u} = X/m - g\sin\theta + rv - qw$$

$$\dot{w} = Z/m + g\cos\phi\cos\theta + qu - pv$$

$$\dot{x}_1 = (\cos\theta\cos\psi)u + (-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi)v$$
$$+ (-\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)w$$

$$\dot{z}_1 = (-\sin\theta)u + (\sin\phi\cos\theta)v + (\cos\phi\cos\theta)w$$

$$\dot{q} = \left[M - (I_{xx} - I_{zz})pr - I_{xz}(p^2 - r^2)\right] \div I_{yy}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi$$

Nonlinear dynamic equations of motion for the longitudinal direction of aircraft

- The longitudinal equations of motions are considered for the aircraft model with the following assumptions
- V=0
- Y=0
- p=0
- r=0
- Ø=0
- Ψ=0
- Ixx=0
- Izz=0
- Ixz=0

Nonlinear dynamic equations of motion for the longitudinal direction of aircraft

The equations after assumptions become:

$$\dot{u} = X/m - g\sin\theta - qw$$

$$\dot{w} = Z/m + g\cos\theta + qu$$

$$\dot{x}_1 = (\cos\theta)u + (\sin\theta)w$$

$$\dot{z}_1 = (-\sin\theta)u + (\cos\theta)w$$

$$\dot{q} = [M] \div I_{yy}$$

$$\dot{\theta} = q$$

The state vector can be shown as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = X_{long} = \begin{bmatrix} u \\ w \\ x \\ z \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} Axial\ \ Velocity \\ Vertical\ \ Velocity \\ Range \\ Altitude(-) \\ Pitch\ \ Rate \\ Pitch\ \ Angle \end{bmatrix}$$

The control input can be shown as:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = U = \begin{bmatrix} \delta e \\ \delta t \\ \delta s \end{bmatrix} = \begin{bmatrix} Elevator\ \ Command \\ Throttle,\% \\ stabulator \end{bmatrix}$$

Methods used to stabilize our model

- The Linear Quadratic Regulator (LQR)controller.
- The Proportional Integral Deferential (PID) controller.

Gain Calculations
(Ziegler and Nichols Method)

$$P \quad control \quad k_p = 0.5 k_{pu}$$

$$PI \quad control \quad \begin{cases} k_p = 0.45 k_{pu} \\ k_i = 0.45 k_{pu} / (0.83 T_u) \end{cases}$$

$$PID \quad control \quad \begin{cases} k_p = 0.6 k_{pu} \\ k_i = 0.6 k_{pu} / (0.5 T_u) \\ k_d = 0.6 k_{pu} / (0.125 T_u) \end{cases}$$

Gain Calculations (Transfer Functions)

$$\frac{X}{\delta e} = \frac{-7.501e-006\ s^4 + 2757\ s^3 + 4246\ s^2 + 281.7\ s + 0.002549}{s^6 + 3.531\ s^5 + 17.14\ s^4 + 2.067\ s^3 + 0.1181\ s^2 + 1.071e-006\ s}$$

$$\frac{Z}{\delta e} = \frac{-27.83\ s^3 - 83.67\ s^2 + 3947\ s + 67.91}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

$$\frac{\theta}{\delta e} = \frac{-18.81\ s^3 - 29.05\ s^2 - 0.6134\ s - 1.455e-006}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

$$\frac{X}{\delta T} = \frac{4.674\ s^4 + 16.42\ s^3 + 78.44\ s^2 - 0.09065\ s + 6.172e-017}{s^6 + 3.531\ s^5 + 17.14\ s^4 + 2.067\ s^3 + 0.1181\ s^2 + 1.071e-006\ s}$$

$$\frac{Z}{\delta T} = \frac{-0.1001\ s^3 - 0.7965\ s^2 - 2.517\ s - 8.463}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

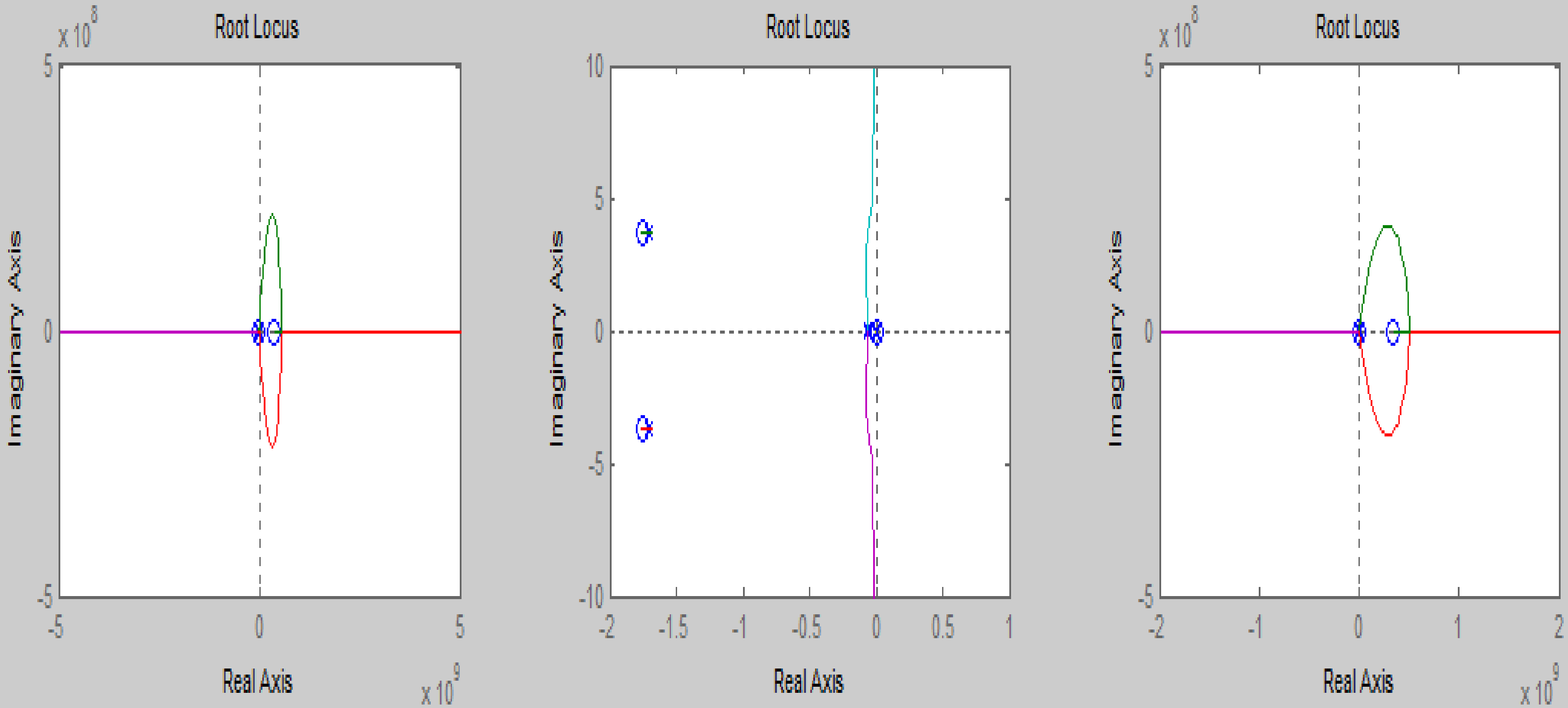
$$\frac{\theta}{\delta T} = \frac{0.009251\ s^2 + 0.05644\ s + 5.875e-019}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

$$\frac{X}{\delta S} = \frac{-1.393e-005\ s^4 + 4629\ s^3 + 7058\ s^2 + 468.3\ s + 0.004237}{s^6 + 3.531\ s^5 + 17.14\ s^4 + 2.067\ s^3 + 0.1181\ s^2 + 1.071e-006\ s}$$

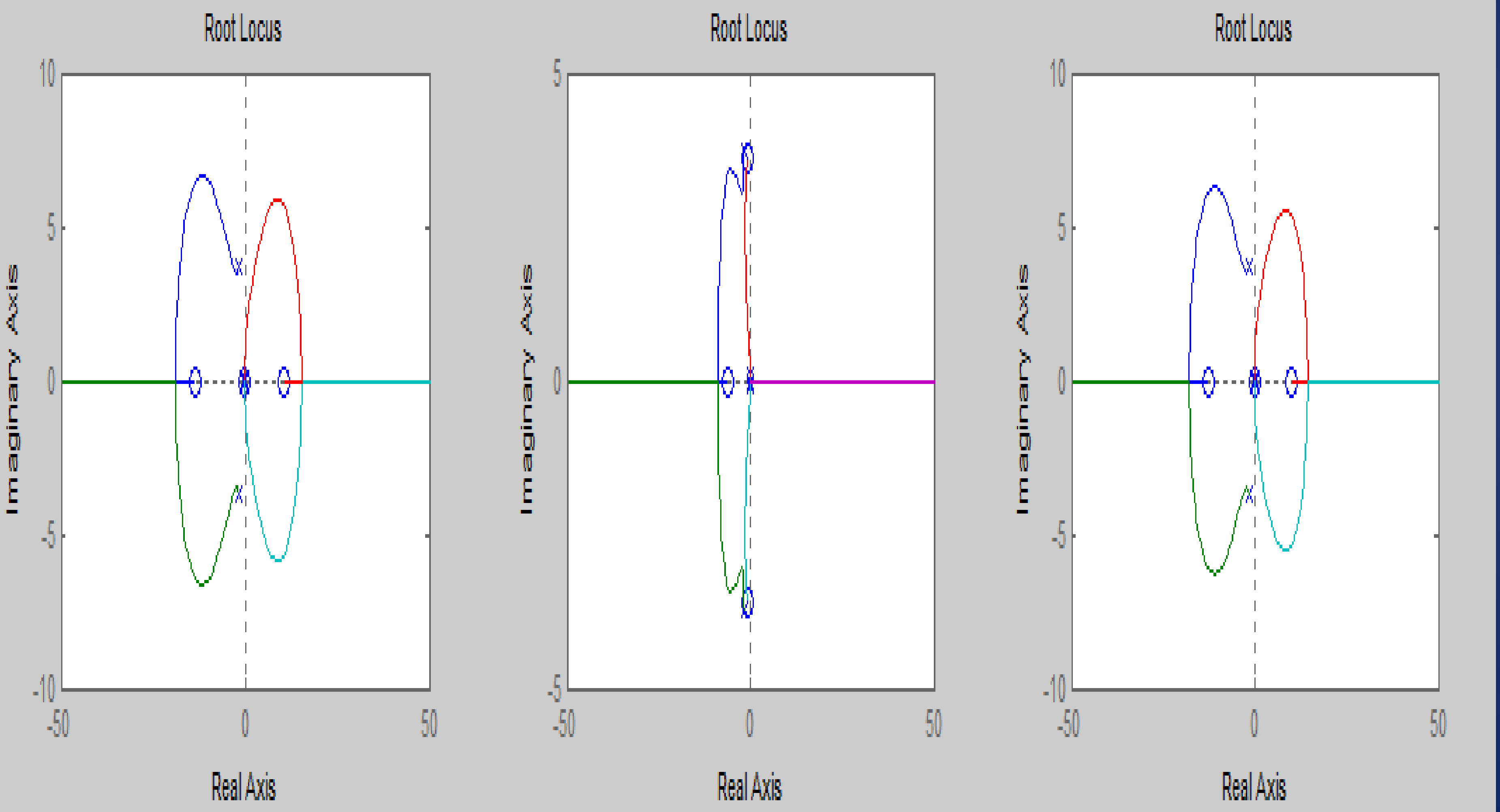
$$\frac{Z}{\delta S} = \frac{-52.06\ s^3 - 150.4\ s^2 + 6553\ s + 112.8}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

$$\frac{\theta}{\delta S} = \frac{-31.58\ s^3 - 48.29\ s^2 - 1.022\ s - 2.444e-006}{s^5 + 3.531\ s^4 + 17.14\ s^3 + 2.067\ s^2 + 0.1181\ s + 1.071e-006}$$

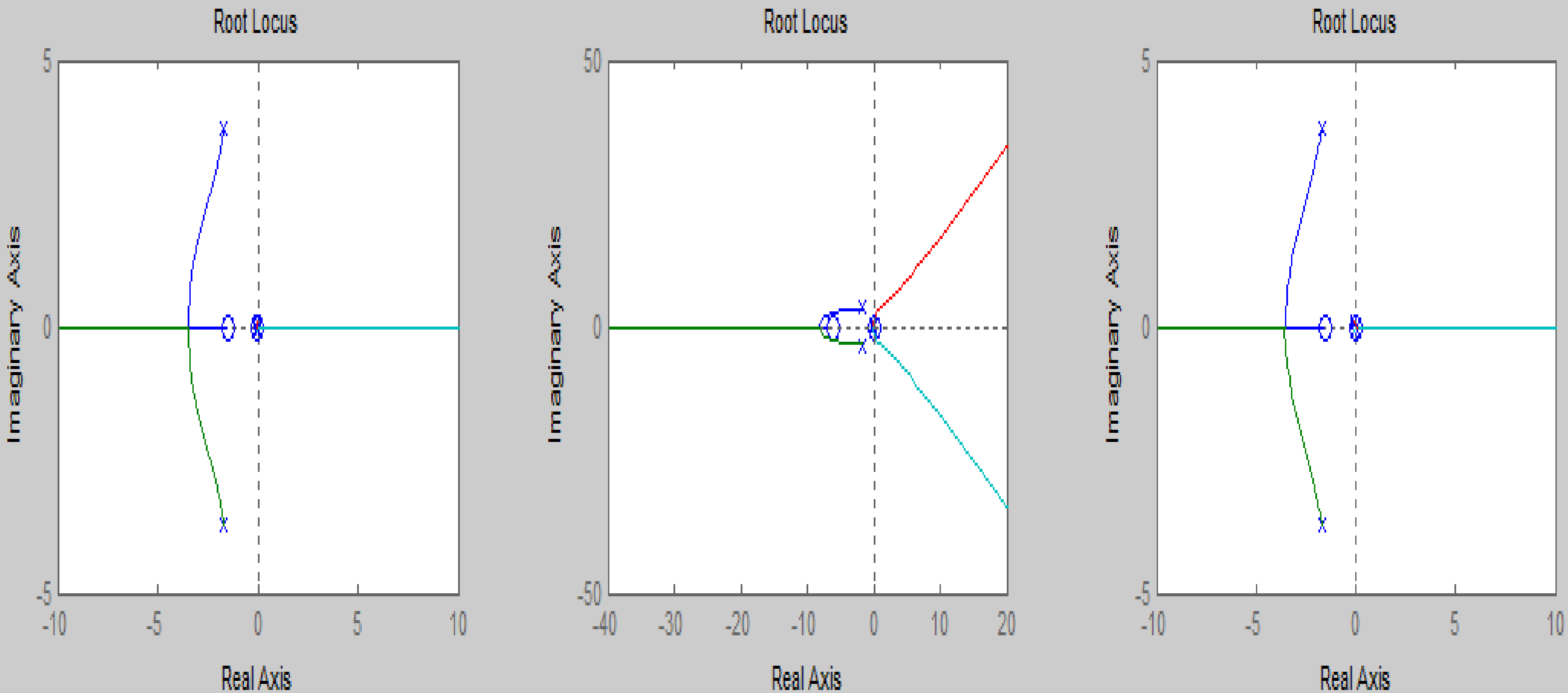
Root Locus Plots



Root Locus Plots



Root Locus Plots



Cost Function Theory

$$J = \int_{to}^{t} e^2 + \lambda u^2 dt$$

$$e \approx (X - Xcomm)$$

J is the energy spent by the actuators in order to regulate the system towards equilibrium

The error vector (**e) is defined as the difference between the actual state vector, and the commanded value**

- λ is a Lagrange multiplier

Cost Function Theory

If a different value of weighting is required on each of the elements of the error vector and input u:

- A square matrices, denoted here as Q, R (identity matrices), are used to ensure that J is non-negative for all values of e, and u, but is zero when X and Xcomm (no inputs) are equal .
- For each choice of Q, and R, minimization of J corresponds to a unique choice of x, using specific inputs.
- Essentially, the ratio between Q and R matrices represents the effort on actuators .

Cost Function Theory

Q >> R

The error is penalized, therefore the performances are maximized at the cost of an important effort on the actuators

Q << R

The control effort is penalized, therefore the energy used to compensate is reduced at the cost of lower performances.

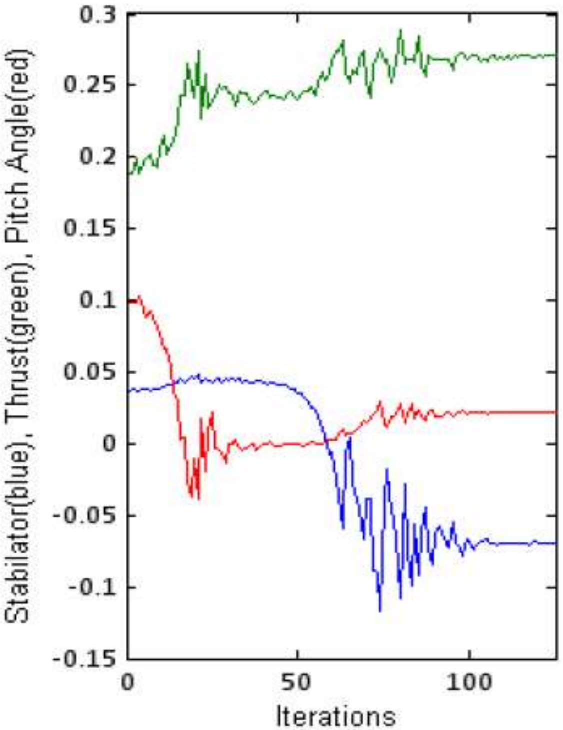
Cost Function Theory

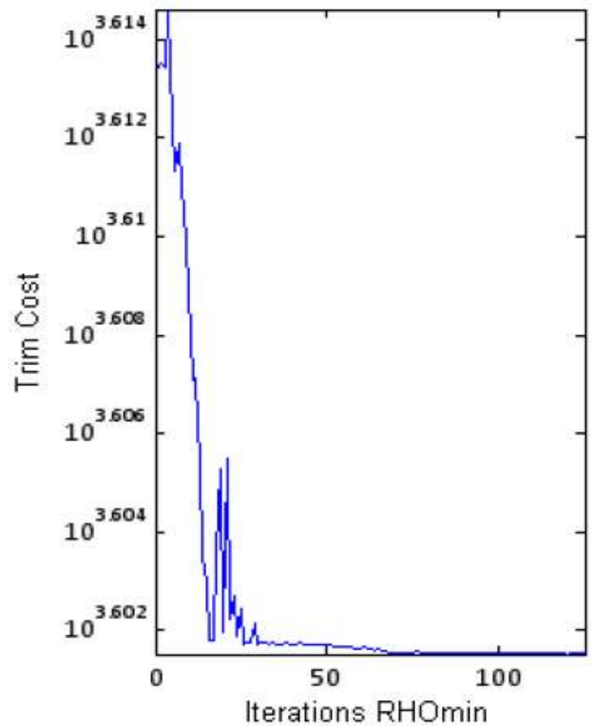
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{n \times n} \qquad R = \rho \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{r \times r}$$

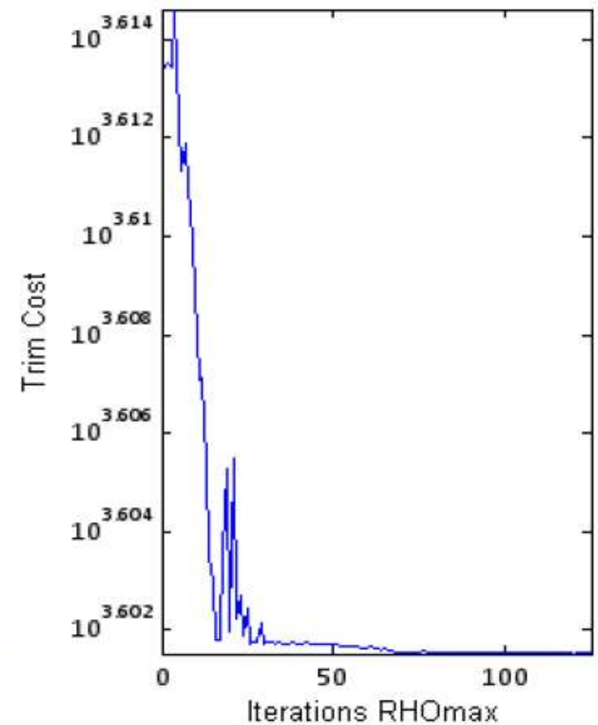
RESULTS

Cost Function Results

$$J_1 = \int_{to}^{t} \dot{x}' Q \dot{x} dt$$



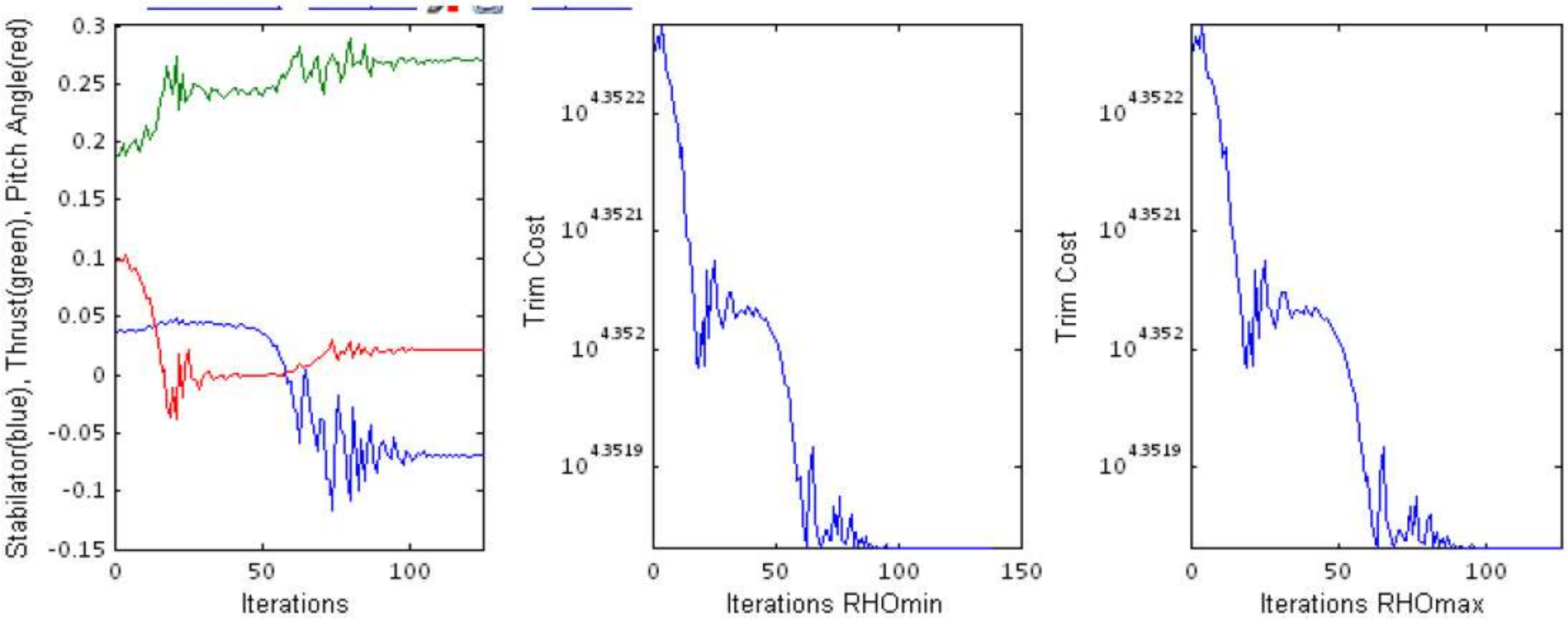




Highest performance, lowest cost, all state variables, and no inputs were used

RESULTS

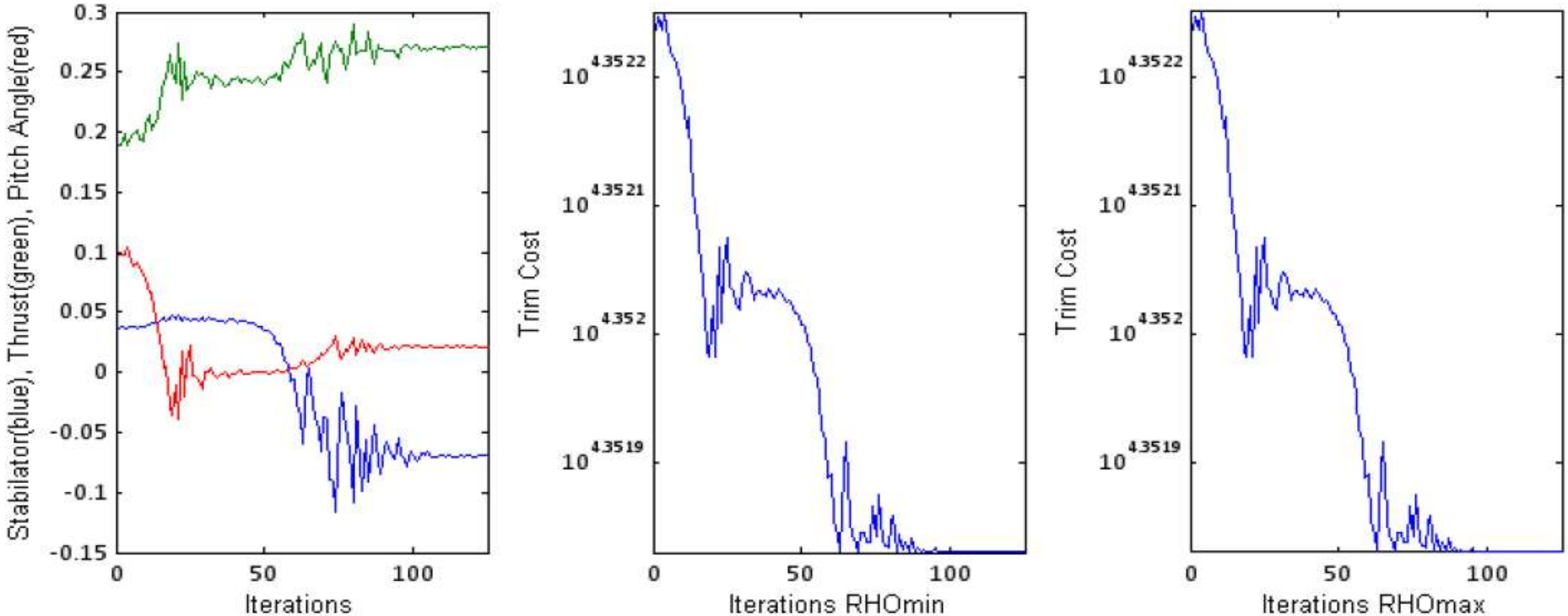
$$J_2 = \int_{to}^{t} \dot{x}' Q \dot{x}$$



Lower performance, higher cost, all state variables, no inputs, but integration without dt

RESULTS

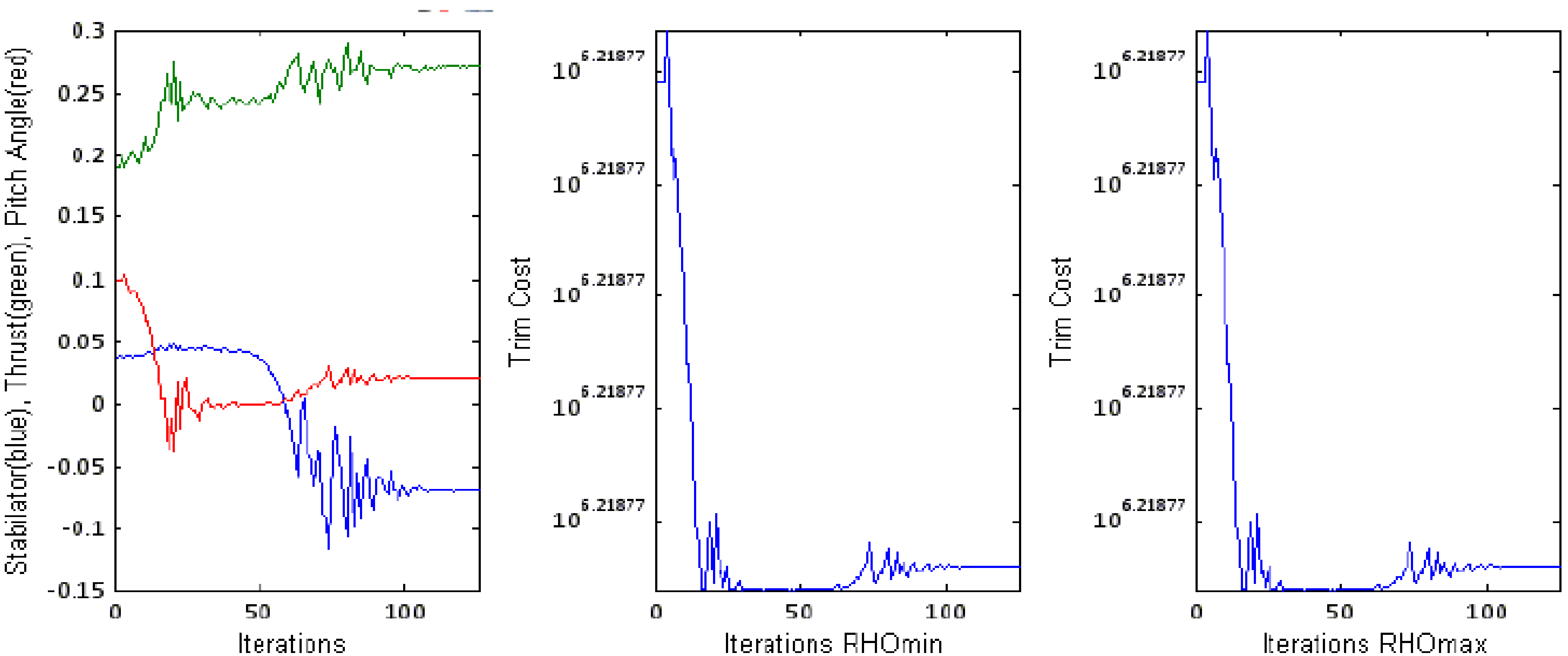
$$J_3 = \int_{to}^{t} \dot{x}' Q \dot{x} \, dt$$



Lower performance, higher cost, not all state variables, no inputs, and specific state variables were used for compromise between cost and performance

RESULTS

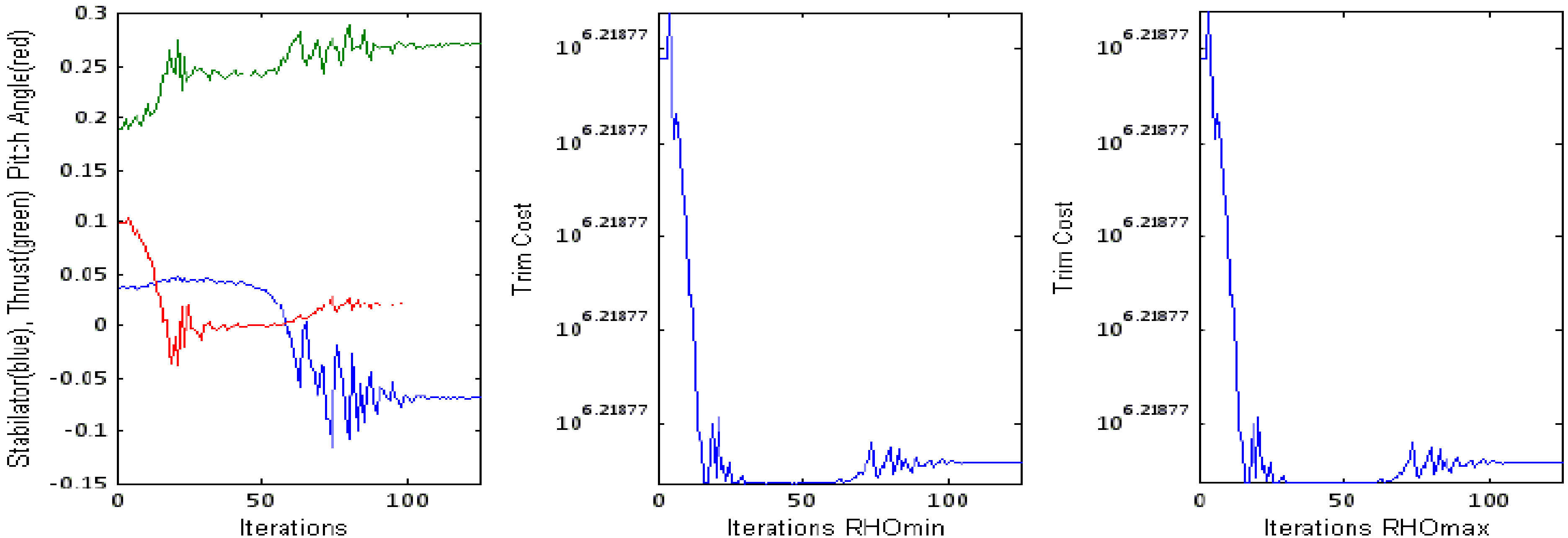
$$J = \int_{to}^{t} x'Qx\,dt$$



High performance, higher cost, all state variables, and no inputs were used for compromise between cost and performance

RESULTS

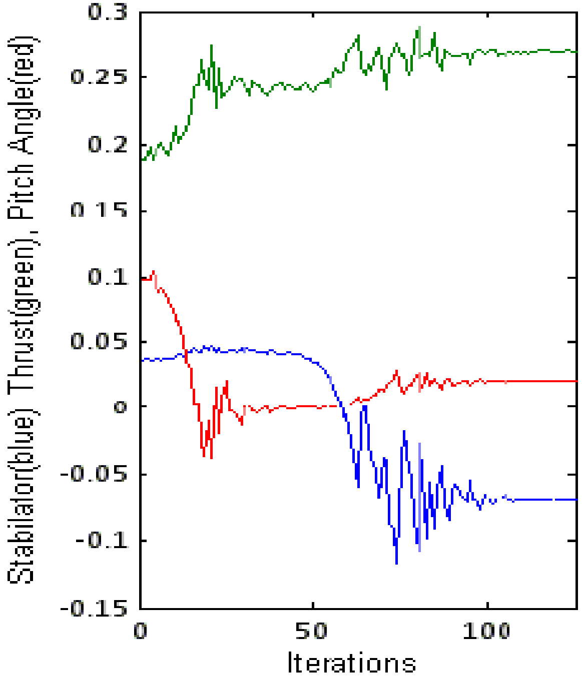
$$J_5 = \int_{to}^{t} (x' Q x + u' \rho R_{elevator} u) dt$$

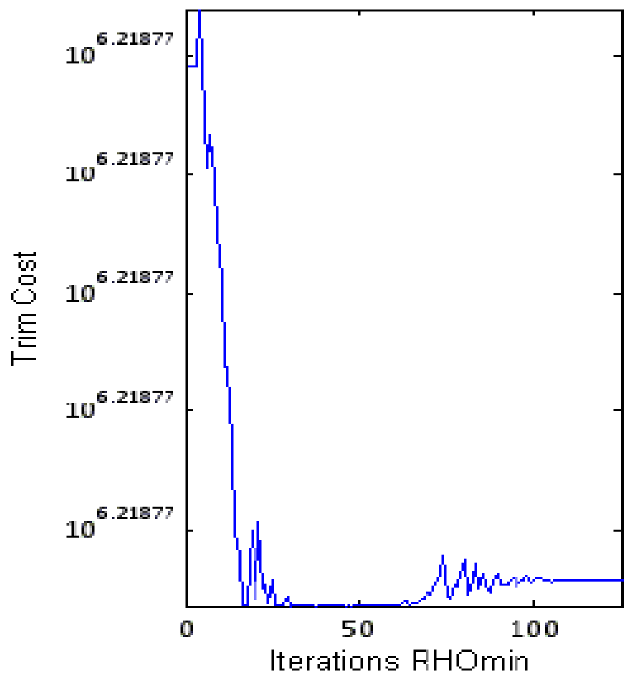


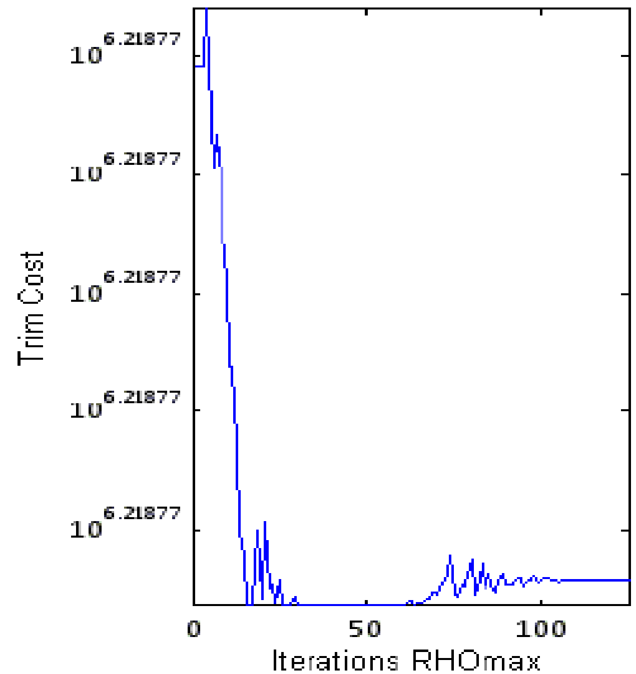
High performance, higher cost, all state variables, and elevator input is used only for reducing cost

RESULTS

$$J = \int_{to}^{t} (x'Qx + u'\rho R_{throttle} u)dt$$



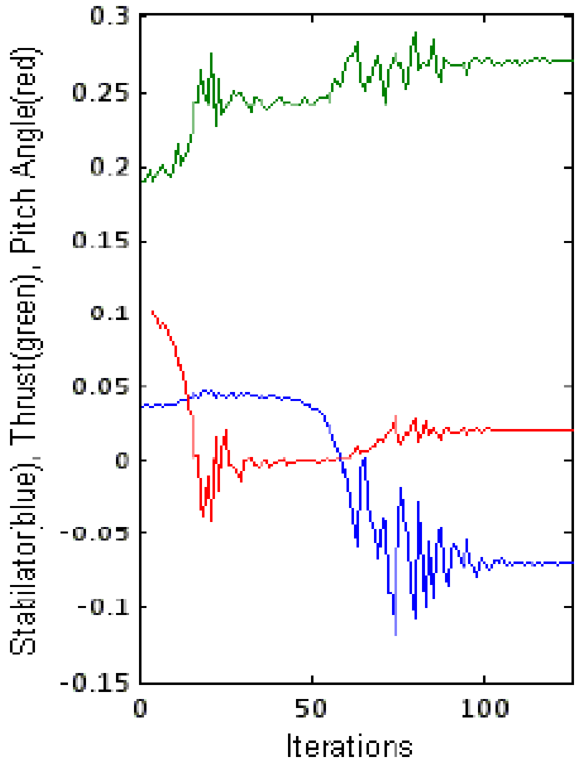


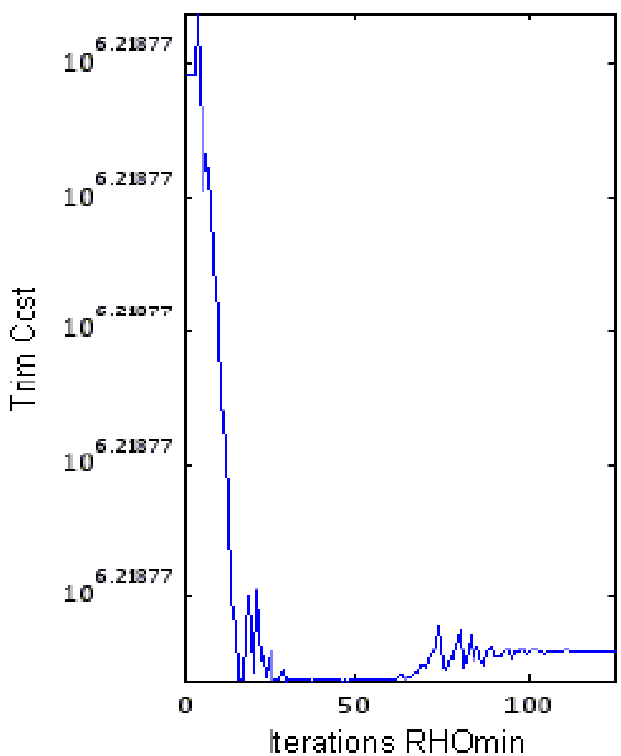


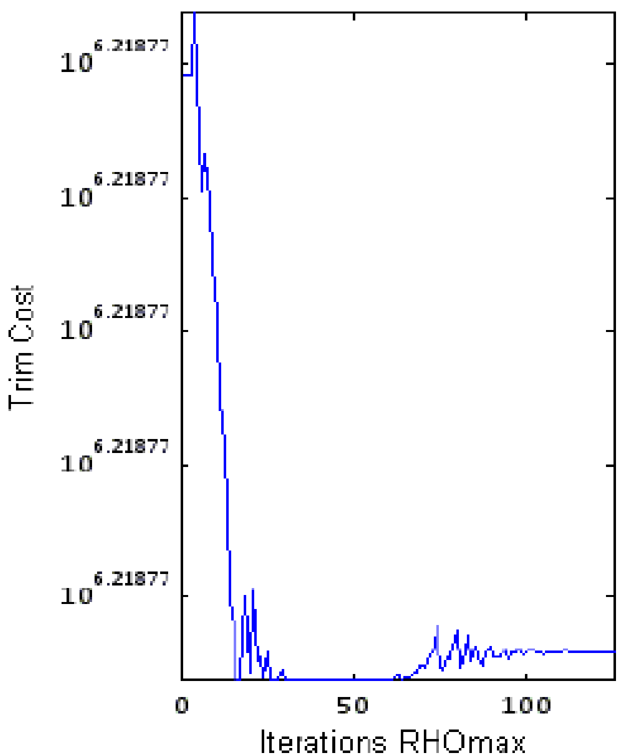
High performance, higher cost, all state variables, and throttle input is used only for reducing cost

RESULTS

$$J_7 = \int_{to}^{t} (x'Qx + u'\rho R_{stabilator}u)dt$$



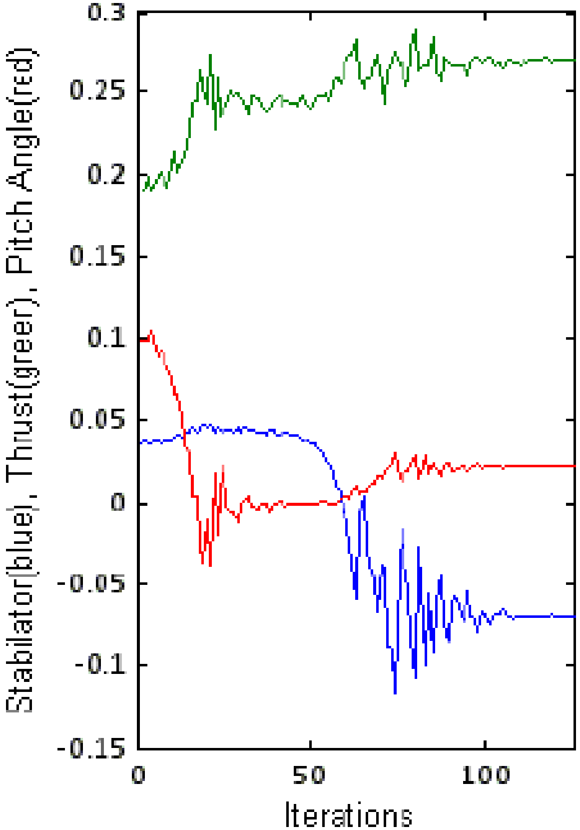


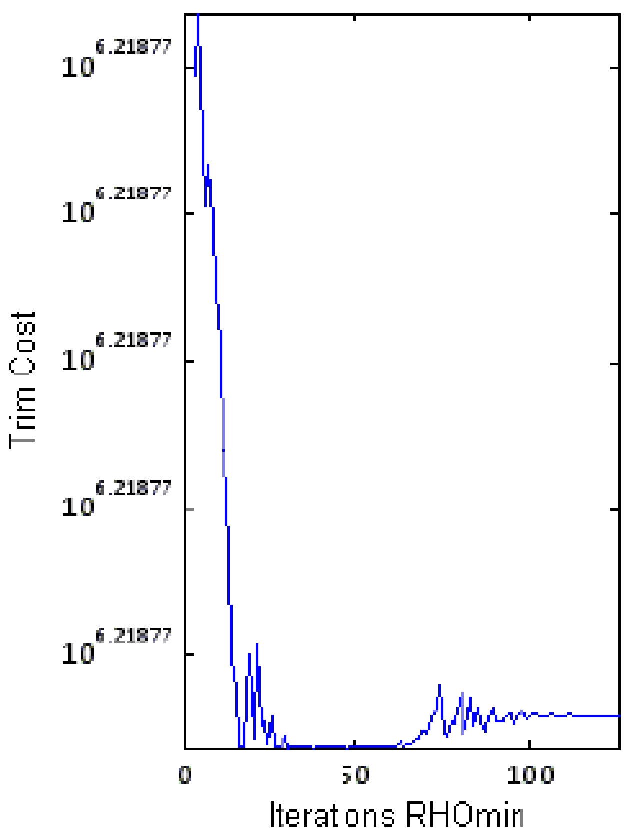


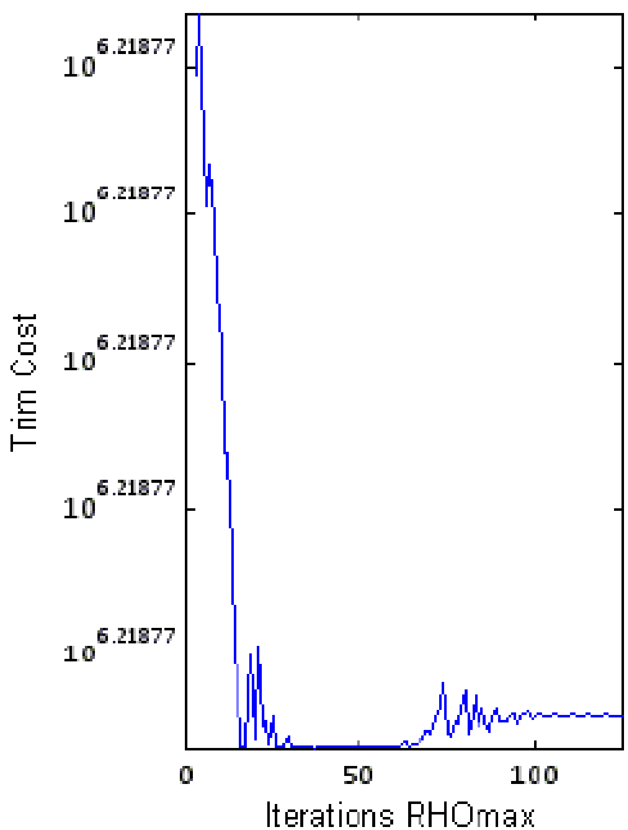
High performance, higher cost, all state variables, and stabilator input is used only for reducing cost

RESULTS

$$J_8 = \int_{to}^{t} (x' Qx + u' \rho R_{all\_Inputs} u) dt$$







High performance, highest cost, all state variables, and all inputs were used (which is our real case)

Cost Function Conclusion

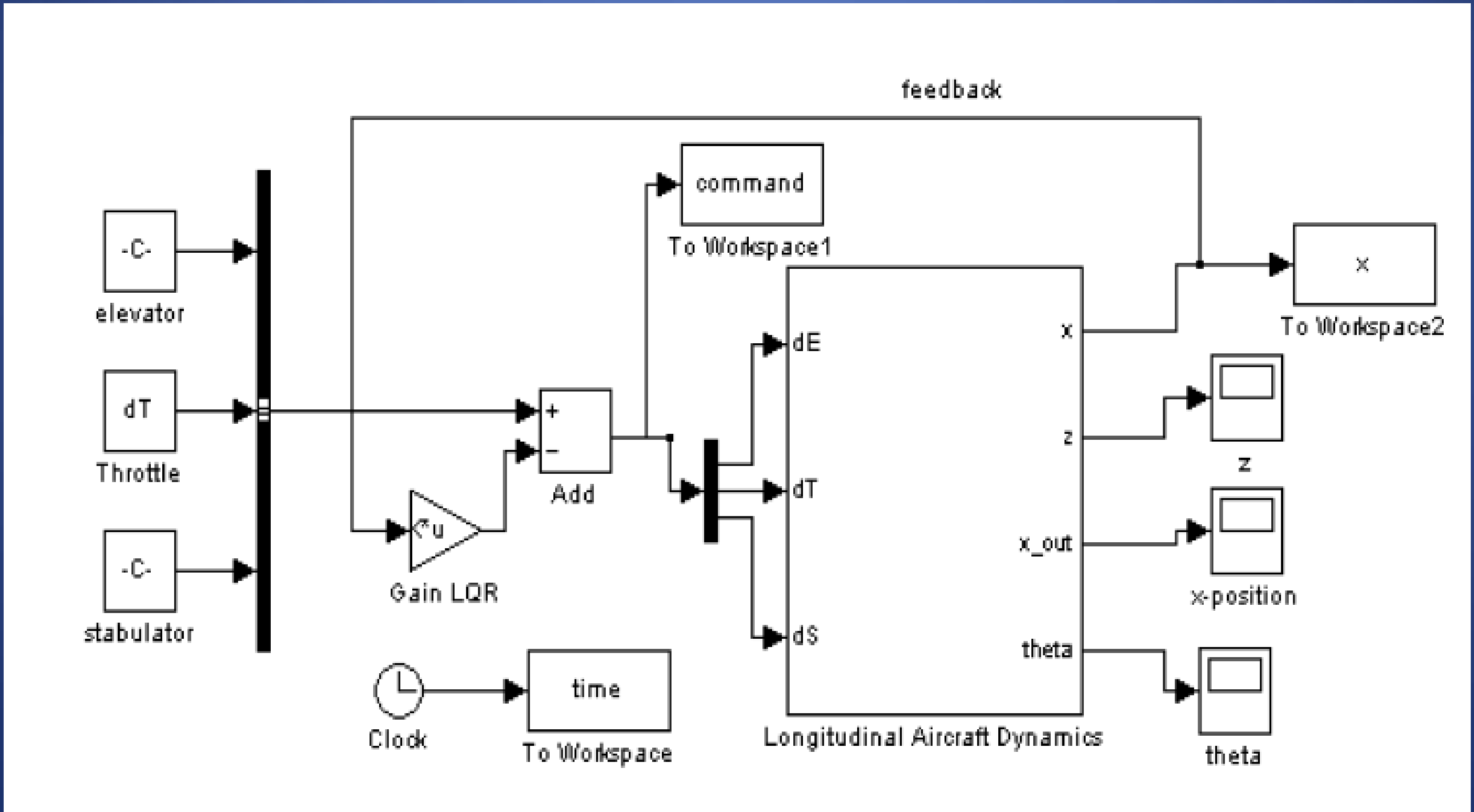
No much difference in results due to ρ that is very small.
- From J1 to J8 (cost is getting higher), and from J1 to J3 (performance is getting better), but from J4 to J8 (no change in performance).

-J8 is our choice for controlling all the inputs with all state variables of the system with high performance and high cost.

- For future work, we can compromise between J 3and J 4 to have J9 with specific state variables (not all state variables) and also specific inputs for compromising between performance and cost.

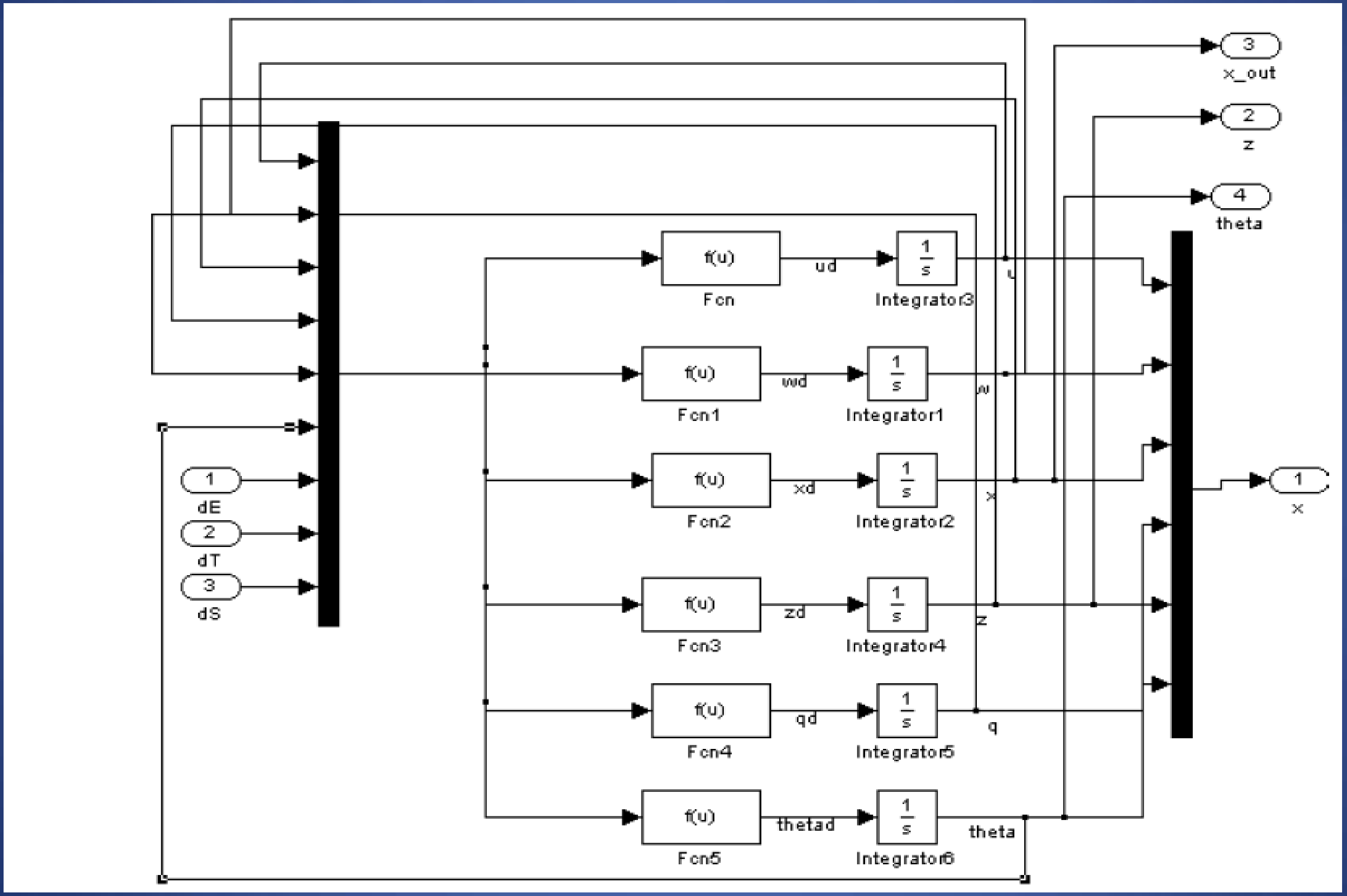
OVERALL SYSTEM LAYOUT

LQR Layout:



Feedback linearization of the model for LQR layout

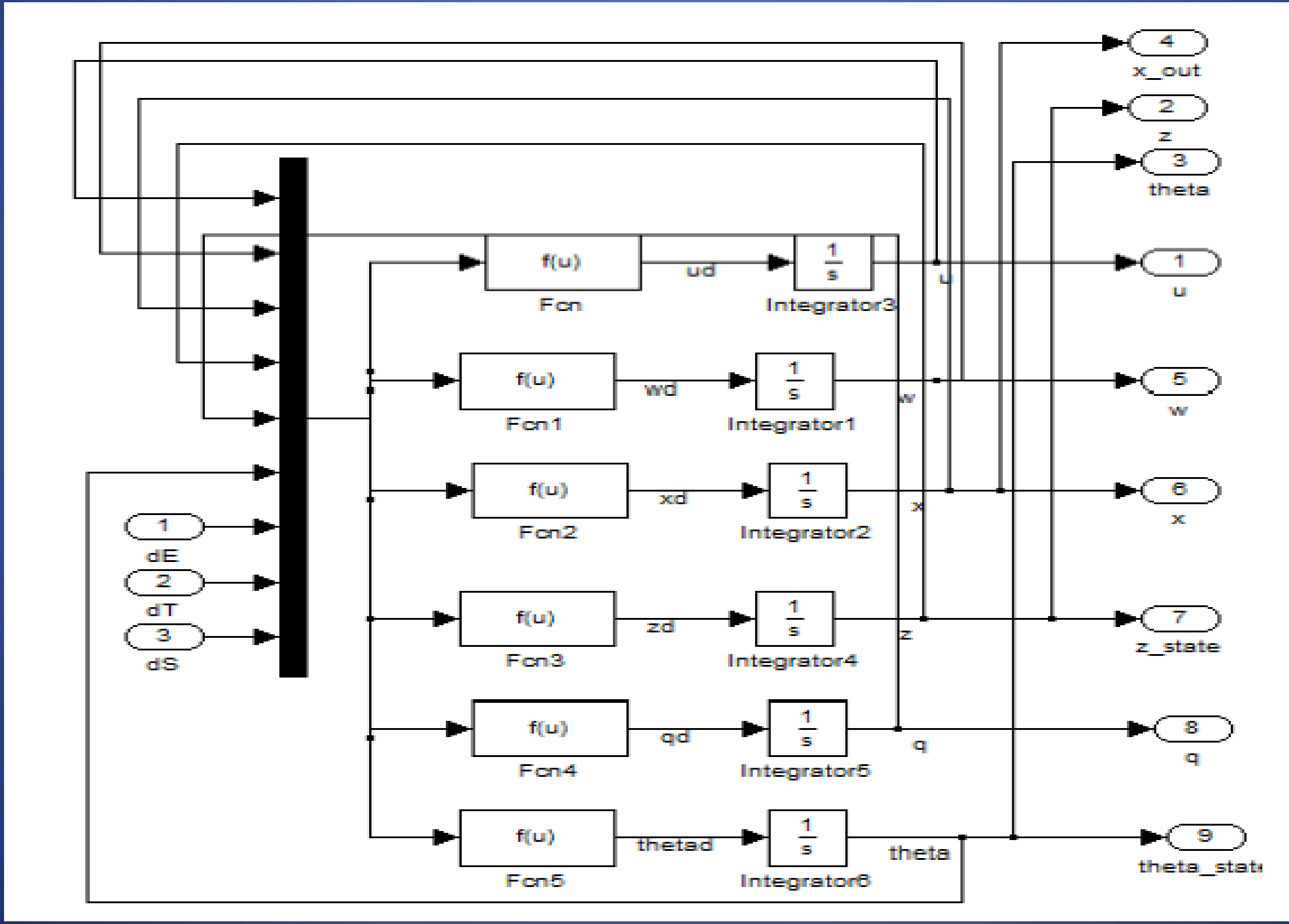
OVERALL SYSTEM LAYOUT



Dynamic model of longitudinal aircraft for LQR layout

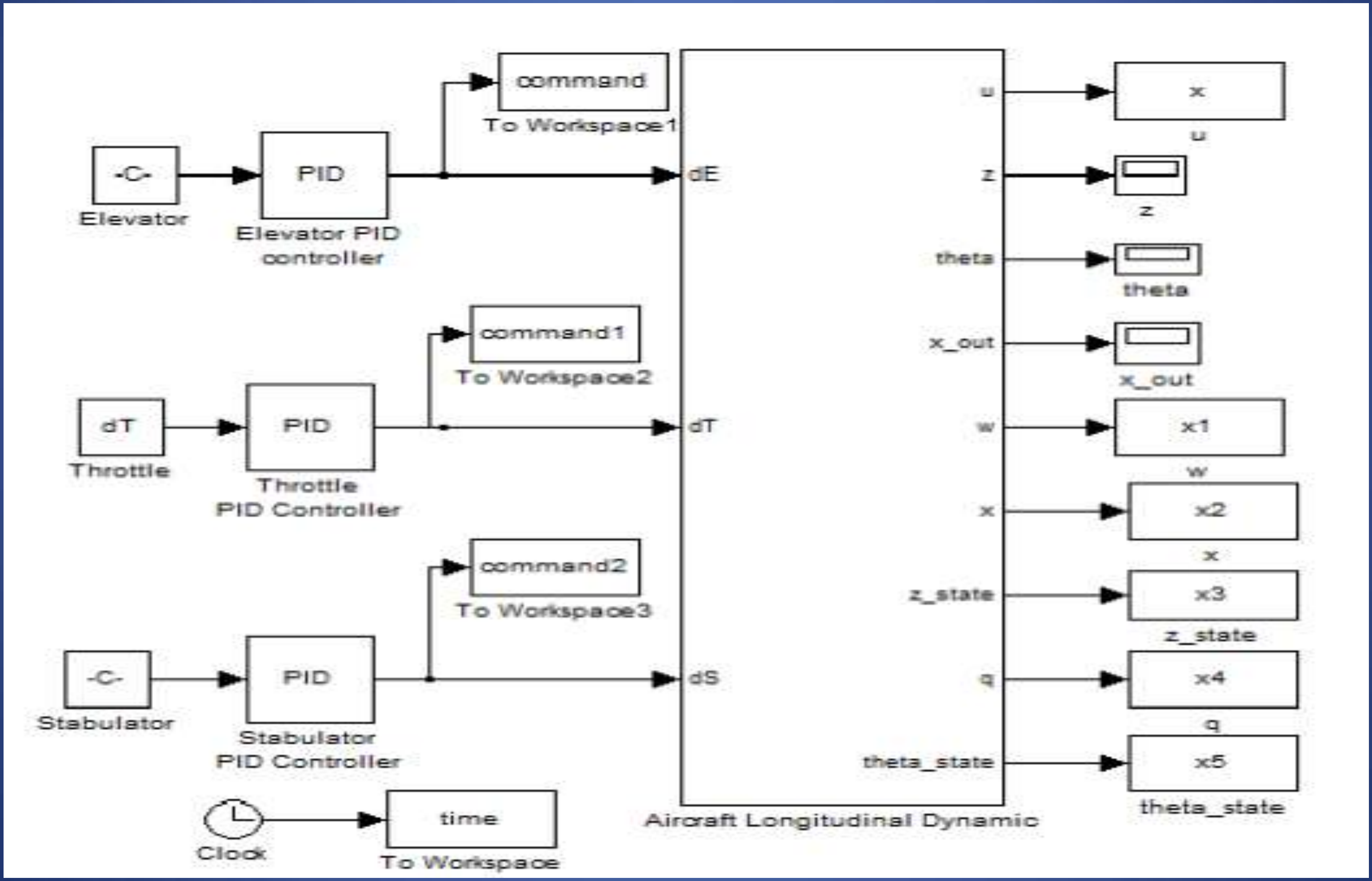
OVERALL SYSTEM LAYOUT

PID Layout:



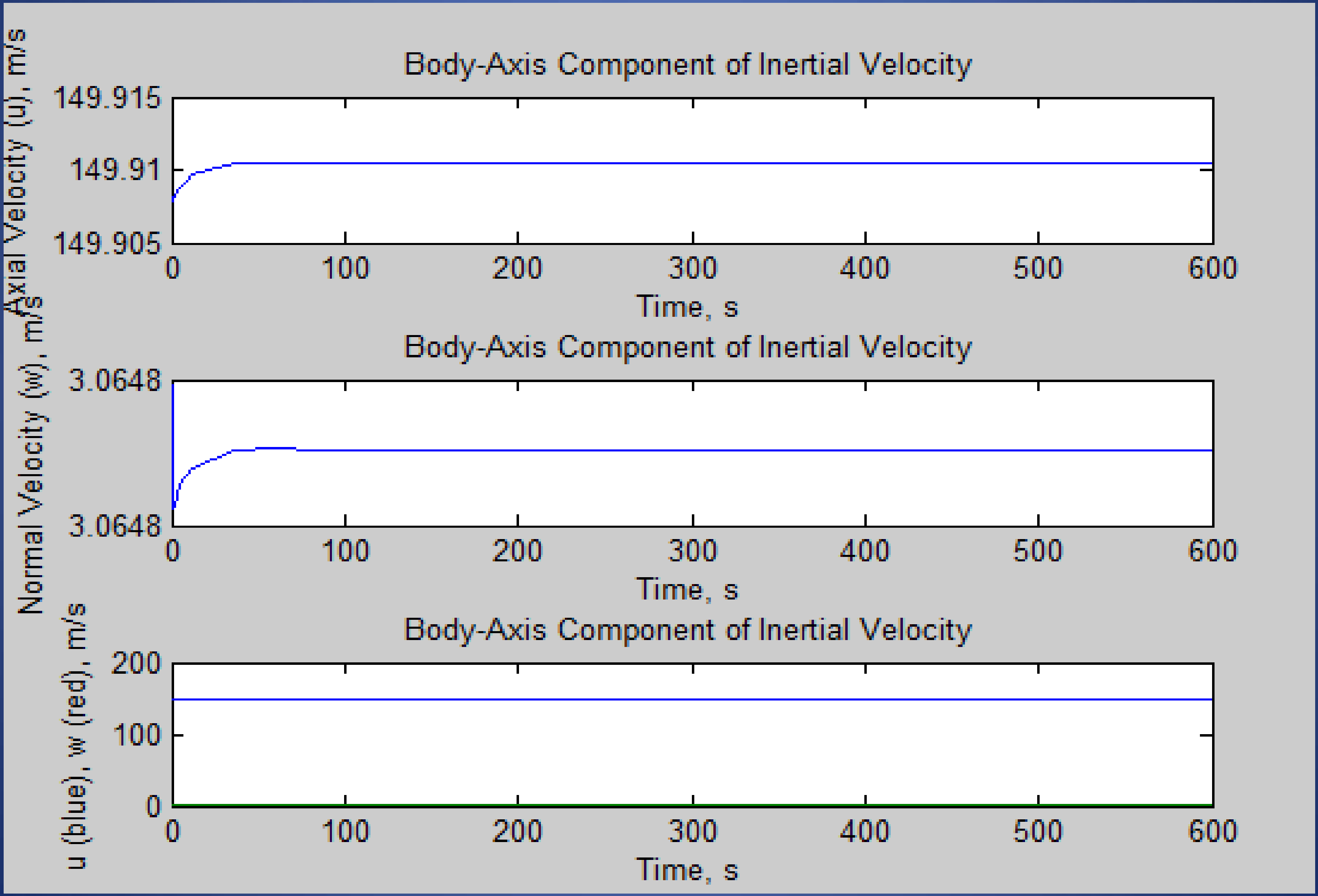
Dynamic model of longitudinal aircraft for PID layout

OVERALL SYSTEM LAYOUT

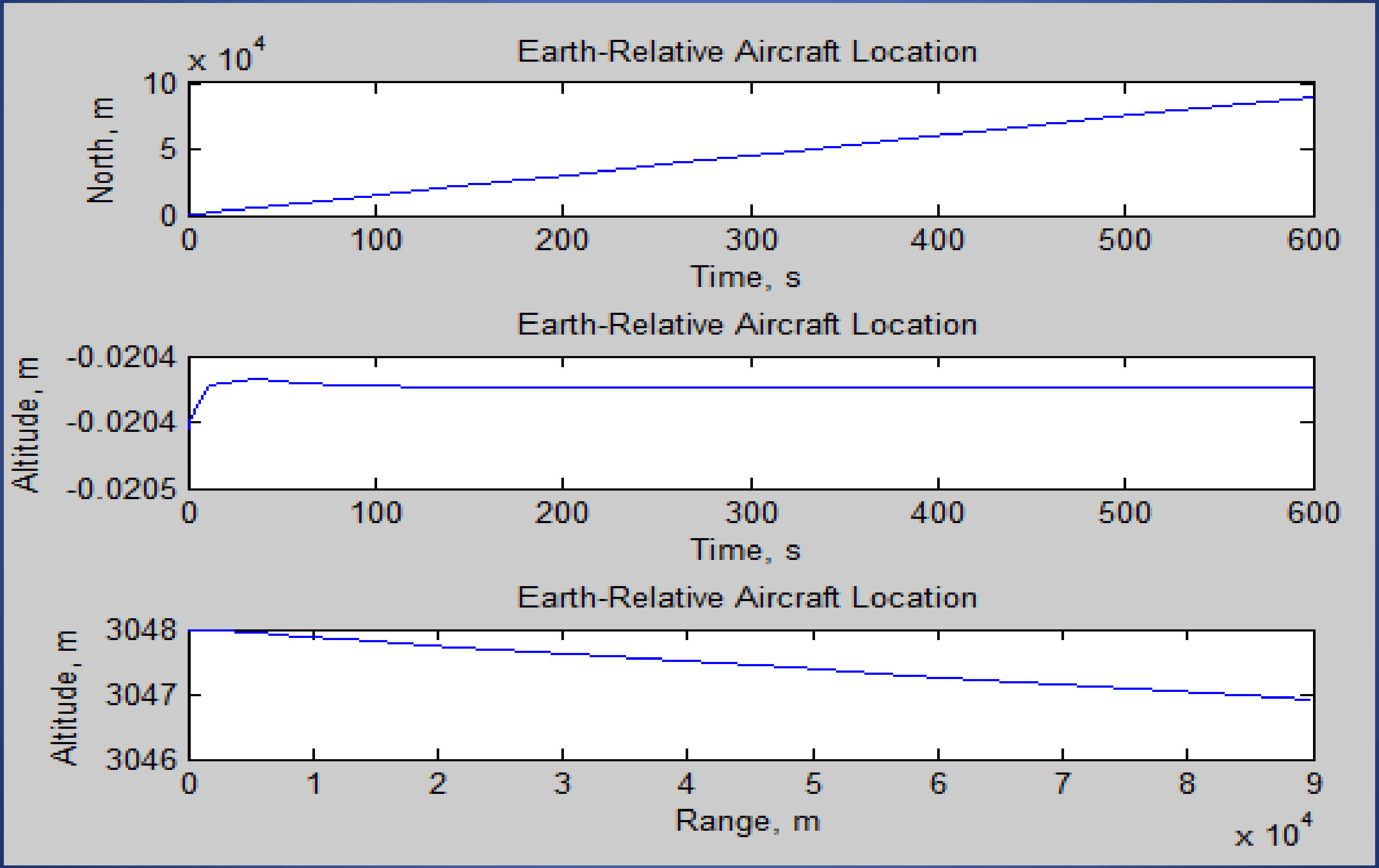


PID layout for the longitudinal aircraft model

RESULTS

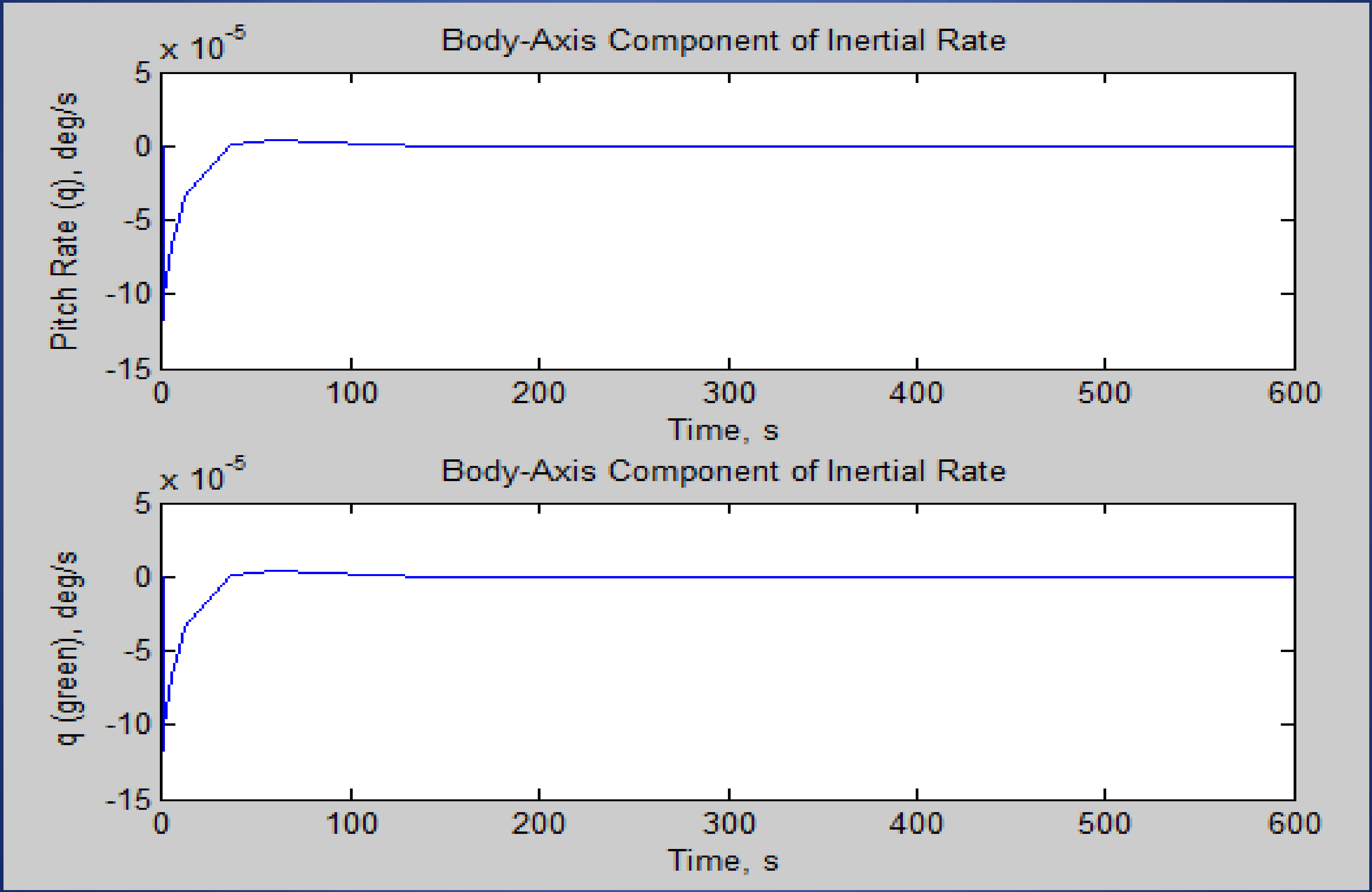

Body-Axis Component of Inertial Velocity
Axial Velocity (u), m/s
149.915
149.91
149.905
0
100
200
300
400
500
600
Time, s
Body-Axis Component of Inertial Velocity
Normal Velocity (w), m/s
3.0648
3.0648
0
100
200
300
400
500
600
Time, s
Body-Axis Component of Inertial Velocity
u (blue), w (red), m/s
200
100
0
0
100
200
300
400
500
600
Time, s

RESULTS



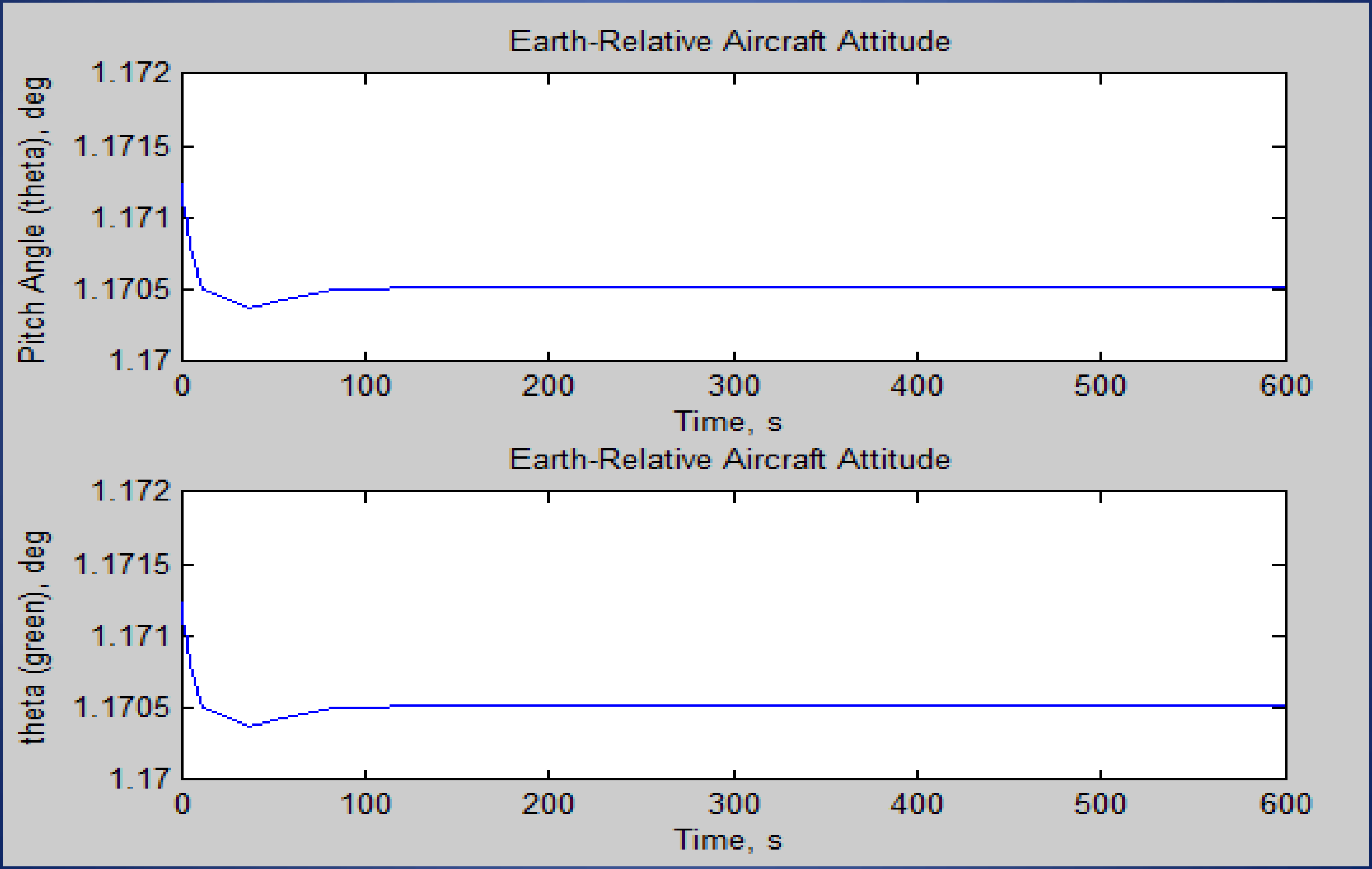
Earth-Relative Aircraft Location
x 10^4
North, m
10
5
0
0 100 200 300 400 500 600
Time, s
Earth-Relative Aircraft Location
Altitude, m
-0.0204
-0.0204
-0.0205
0 100 200 300 400 500 600
Time, s
Earth-Relative Aircraft Location
Altitude, m
3048
3047
3046
0 1 2 3 4 5 6 7 8 9
Range, m
x 10^4

RESULTS



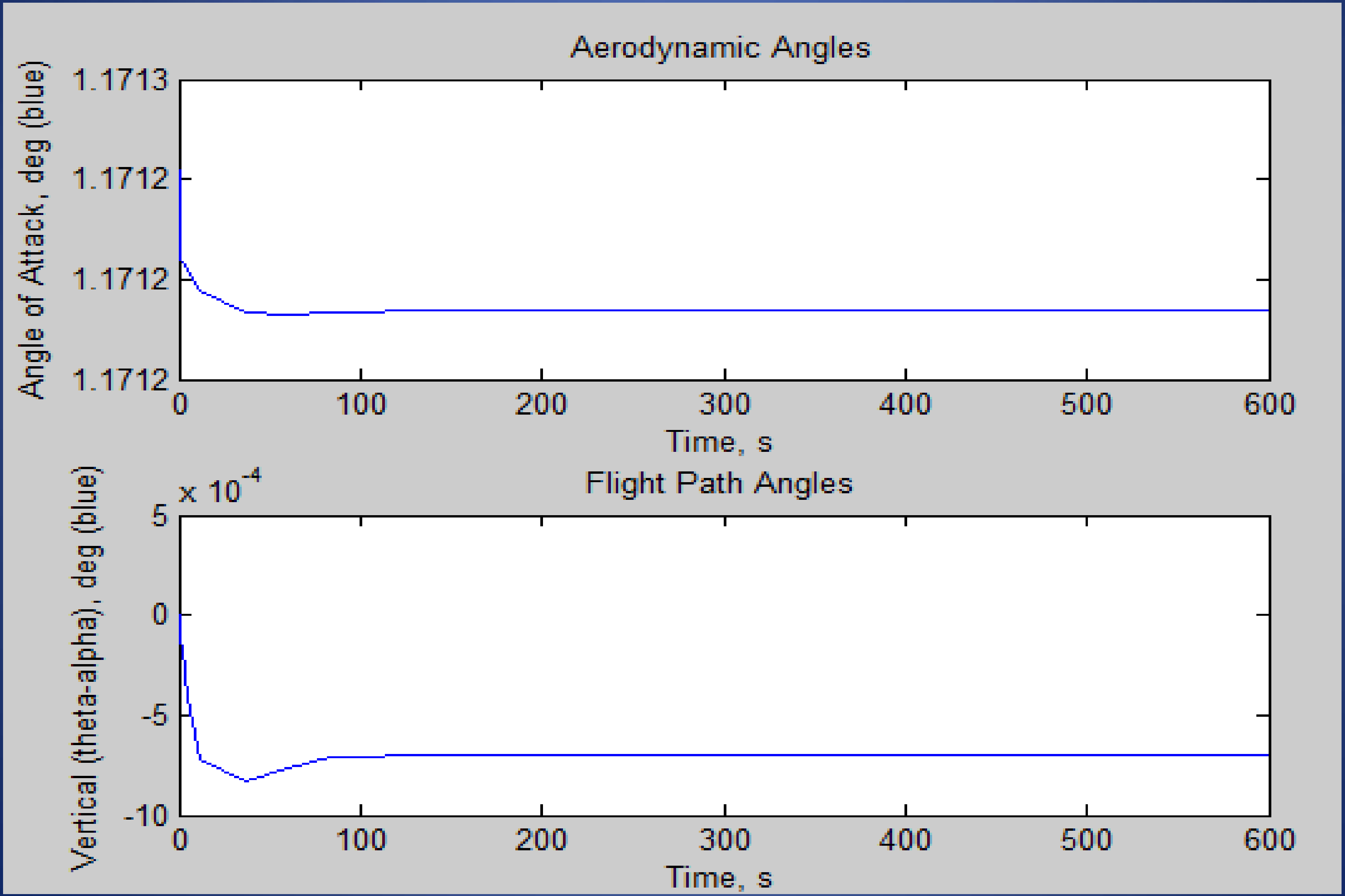
Body-Axis Component of Inertial Rate
x 10^-5
5
0
-5
-10
-15
Pitch Rate (q), deg/s
0
100
200
300
400
500
600
Time, s
Body-Axis Component of Inertial Rate
x 10^-5
5
0
-5
-10
-15
q (green), deg/s
0
100
200
300
400
500
600
Time, s

RESULTS



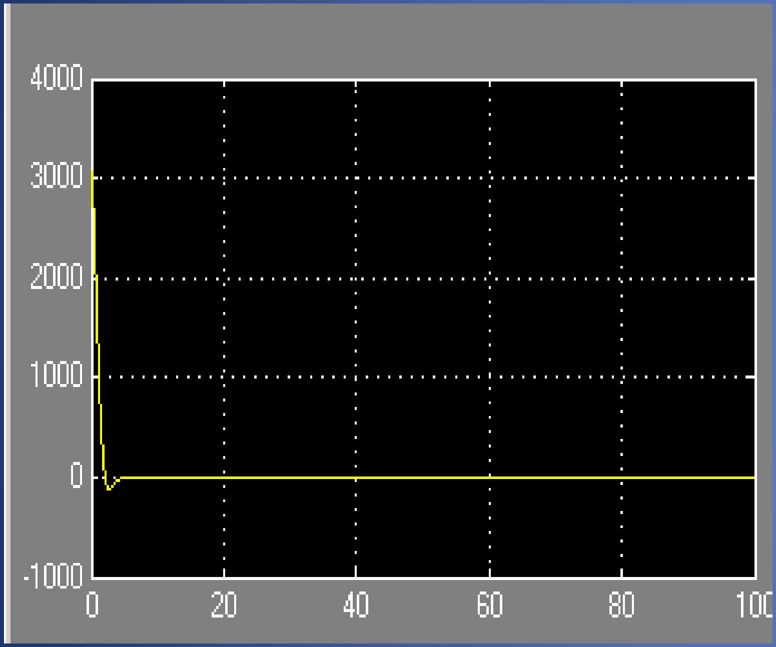
Earth-Relative Aircraft Attitude
Pitch Angle (theta), deg
1.172
1.1715
1.171
1.1705
1.17
0
100
200
300
400
500
600
Time, s
Earth-Relative Aircraft Attitude
theta (green), deg
1.172
1.1715
1.171
1.1705
1.17
0
100
200
300
400
500
600
Time, s

RESULTS

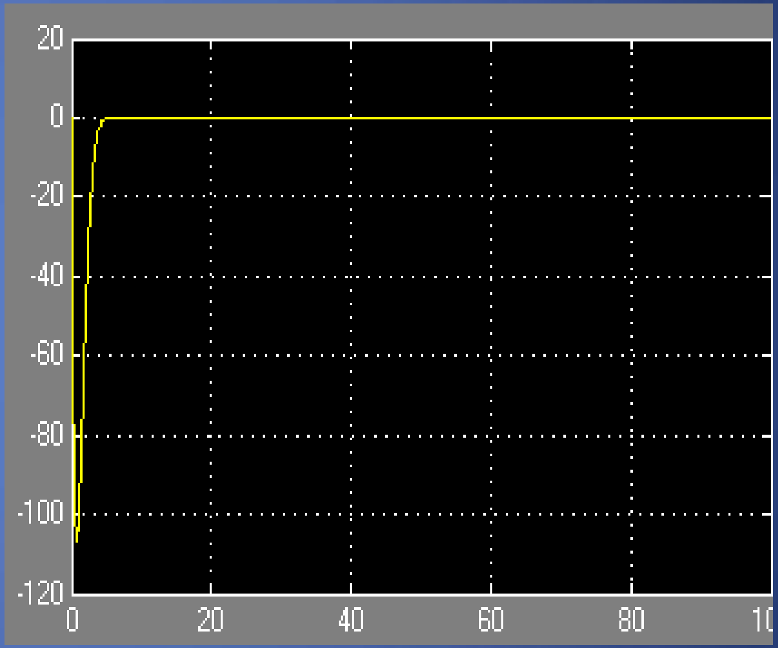

Aerodynamic Angles
Angle of Attack, deg (blue)
1.1713
1.1712
1.1712
1.1712
0
100
200
300
400
500
600
Time, s
Flight Path Angles
x 10^-4
Vertical (theta-alpha), deg (blue)
5
0
-5
-10
0
100
200
300
400
500
600
Time, s

RESULTS

LQR Feedback Results :

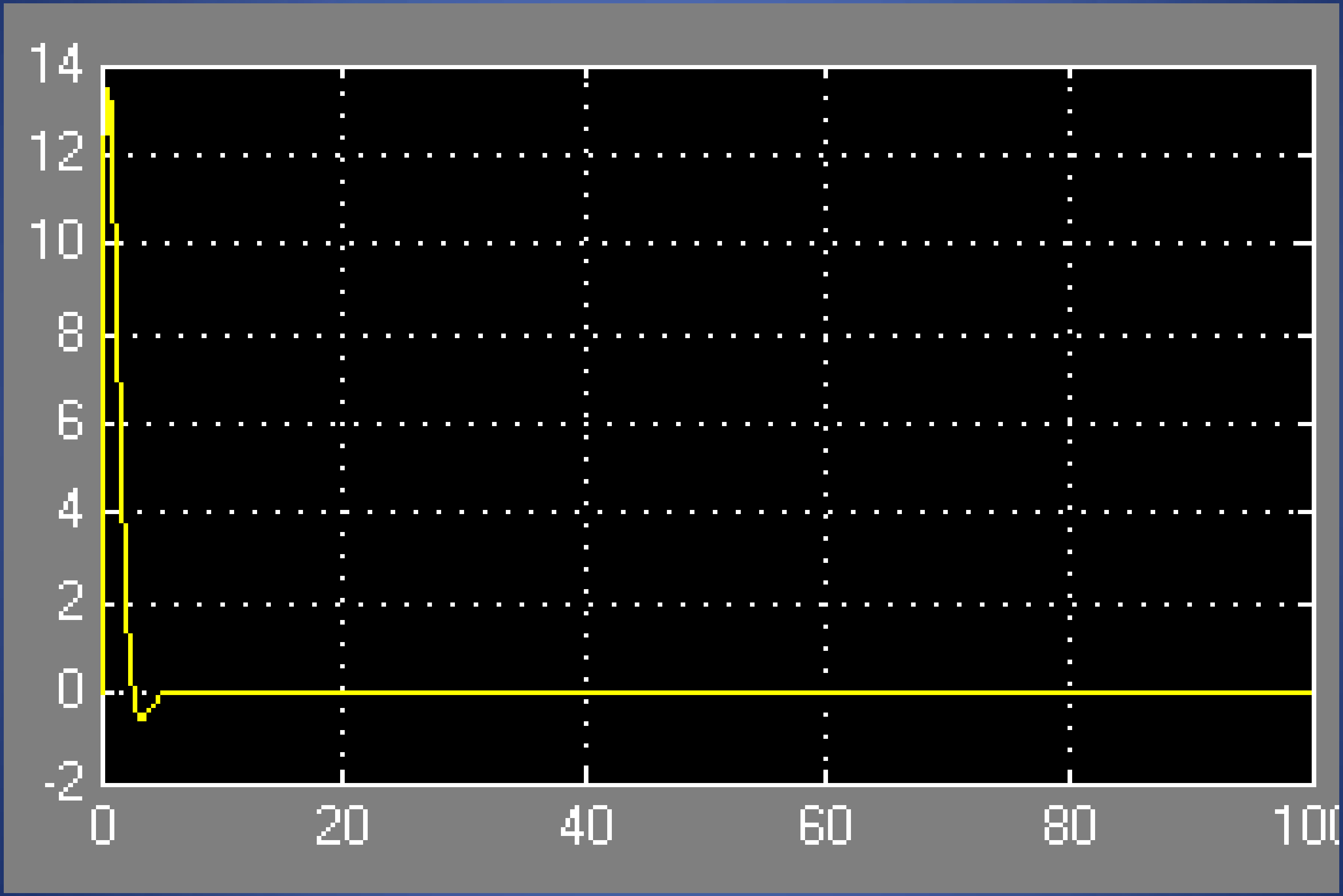


Altitude stabilization



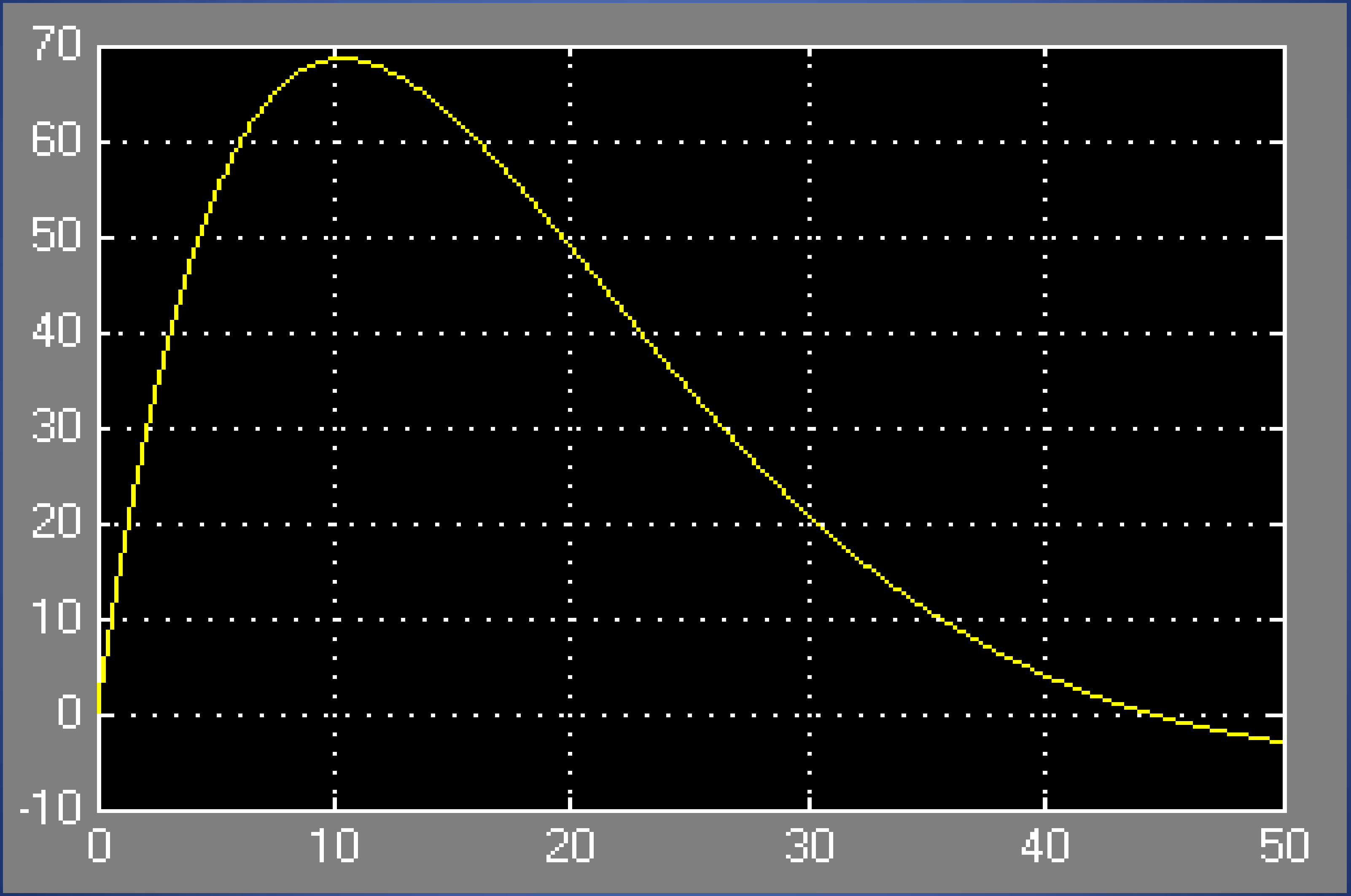
Range stabilization

RESULTS



Pitch angle stabilization

RESULTS



Range stabilization

THANK YOU

Done By: Abbas Chamas
Fadi Al Fara
Bilal Jaroudi
Moustafa Moustafa