Mech 6091 Project
F16 dynamics and control

Introduction

- Dynamics model
- Linearization techniques
- Control

Dynamics

- Sonneveldt F16 Model
- Rigid body 6Dof non-linear equations of motion
- External forces:
  - $F(\alpha, \beta, \delta e)$
  - Determined from NASA wind tunnel tests

Equations of motion

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big]_B + \boldsymbol{\omega} \times m\mathbf{V}$$

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}\Big]_B + \boldsymbol{\omega} \times \mathbf{H}$$

$$\mathbf{H} = I\omega$$

Equations of motion - expanded

$$\dot{u} = rv - qw - g\sin\theta + \frac{1}{m}(\bar{X} + F_T)$$
$$\dot{v} = pw - ru + g\sin\phi\cos\theta + \frac{1}{m}\bar{Y}$$
$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{1}{m}\bar{Z}$$

$$\dot{p} = (c_1 r + c_2 p)q + c_3\bar{L} + c_4(\bar{N} + h_E q)$$
$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7(\bar{M} + F_T z_T - h_E r)$$
$$\dot{r} = (c_8 p - c_2 r)q + c_4\bar{L} + c_9(\bar{N} + h_E q)$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi)$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta}$$

$$\dot{x}_E = u\cos\psi\cos\theta + v(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) + w(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi)$$
$$\dot{y}_E = u\sin\psi\cos\theta + v(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi) + w(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi)$$
$$\dot{z}_E = -u\sin\theta + v\cos\theta\sin\phi + w\cos\theta\cos\phi$$

where the moment of inertia components are given by [Lewis and Stevens, 1992

$$\Gamma c_1 = (I_y - I_z)I_z - I_{xz}^2 \qquad \Gamma c_4 = I_{xz} \qquad c_7 = \frac{1}{I_y}$$
$$\Gamma c_2 = (I_x - I_y + I_z)I_{xz} \qquad c_5 = \frac{I_z - I_x}{I_y} \qquad \Gamma c_8 = I_x(I_x - I_y) + I_{xz}^2$$
$$\Gamma c_3 = I_z \qquad c_6 = \frac{I_{xz}}{I_y} \qquad \Gamma c_9 = I_x$$

with $\Gamma = I_x I_z - I_{xz}^2$.

External forces

$$
\begin{aligned}
\bar{L} &= \bar{q}SbC_{l_T}(\alpha, \beta, p, q, r, \delta, \ldots) \\
\bar{M} &= \bar{q}S\bar{c}C_{m_T}(\alpha, \beta, p, q, r, \delta, \ldots) \\
\bar{N} &= \bar{q}SbC_{n_T}(\alpha, \beta, p, q, r, \delta, \ldots)
\end{aligned}
$$

$$
\begin{aligned}
\bar{X} + F_T - mg\sin\theta &= m(\dot{u} + qw - rv) \\
\bar{Y} + mg\sin\phi\cos\theta &= m(\dot{v} + ru - pw) \\
\bar{Z} + mg\cos\phi\sin\theta &= m(\dot{w} + pv - qu)
\end{aligned}
$$

Force coefficients

$$
\begin{aligned}
C_{m_T} \quad = \quad & C_m(\alpha, \beta, \delta_e) + C_{Z_T}\left[x_{cg_r} - x_{cg}\right] + \delta C_{m_{LEF}}\left(1 - \frac{\delta_{LEF}}{25}\right) \\
+ \quad & \frac{q\bar{c}}{2V_T}\left[C_{m_q}(\alpha) + \delta C_{m_{q_{LEF}}}(\alpha)\right]\left(1 - \frac{\delta_{LEF}}{25}\right) + \delta C_m(\alpha) + \delta C_{m_{ds}}(\alpha, \delta_e)
\end{aligned}
$$

$$
\text{where} \quad \delta C_{m_{LEF}} = C_{m_{LEF}}(\alpha, \beta) - C_m(\alpha, \beta, \delta_e = 0^o)
$$

NOTE: Cm(α, β, ...) are simply lookup table functions

Linearization

- Want to determine [A, B, C, D] state matrices
- Note:
  - State matrices operate on *perturbations* from equilibrium/trimmed conditions

$$\underline{\dot{\tilde{X}}} = \underline{\underline{A}}\,\underline{\tilde{X}} + \underline{\underline{B}}\,\underline{\tilde{U}}$$

$$Y = \underline{\underline{C}}\,\underline{\tilde{X}} + \underline{\underline{D}}\,\underline{\tilde{U}}$$

Linearization techniques

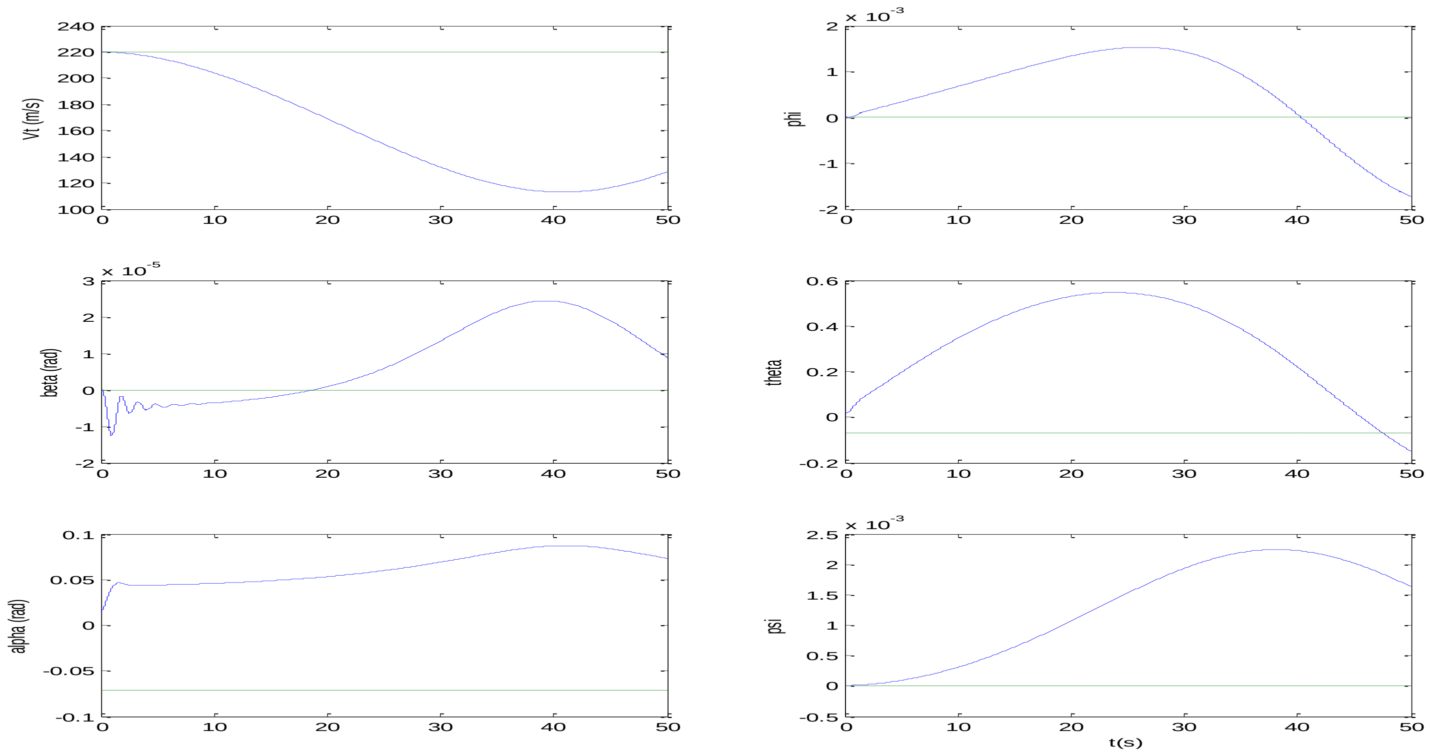
- LINMOD
- Small perturbation expansion
- Numerical linearization: Finite Difference Method

LINMOD

- Simple to implement
- Good results for simple systems (ex: spring-mass-damper)
- Not appropriate for complex systems (i.e. Aircraft)

Linmod open loop response – Step elevator input

Sonneveldt vs LINMOD



LINMOD results

B matrix

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
64.94	0	0	0

Control vector

dth: thrust lever deflection
de: elevator deflection
dr: rudder deflection
da: aileron deflection

i.e. Control inputs have no impact on Aircraft motion!

Small perturbations

- Common linearization technique
- Assume aircraft will only have small deviations from some equilibrium position

Small perturbations

Assume small deviations

$$U = U_1 + u \qquad V = V_1 + v \qquad W = W_1 + w$$
$$P = P_1 + p \qquad Q = Q_1 + q \qquad R = R_1 + r$$
$$\Psi = \Psi_1 + \psi \qquad \Theta = \Theta_1 + \theta \qquad \Phi = \Phi_1 + \phi$$

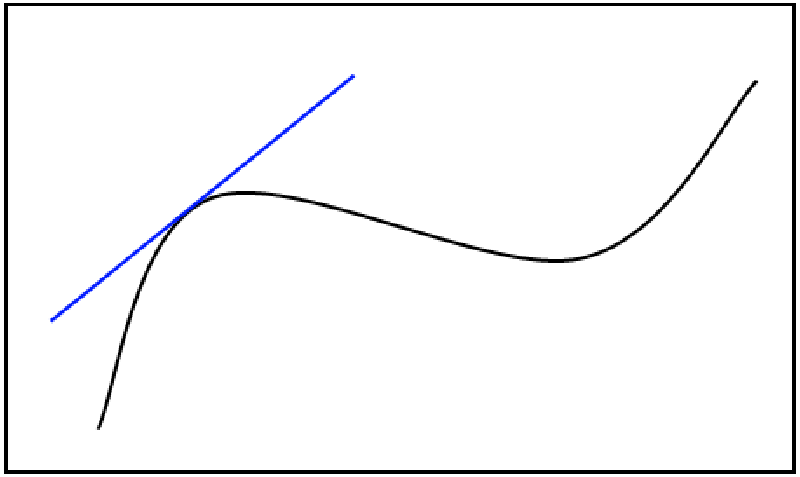
Derive linear motion equations

$$m\dot{u} = -mg\theta \cos\Theta_1 + f_{A_x} + f_{T_x}$$
$$I_{yy}\dot{q} = m_A + m_T$$
$$m(\dot{w} - U_1 q) = -mg\theta \sin\Theta_1 + f_{A_z} + f_{T_z}$$

$$I_{xx}\dot{p} - I_{xz}\dot{r} = l_A + l_T$$
$$m(\dot{v} + U_1 r) = mg\phi \cos\Theta_1 + f_{A_y} + f_{T_y}$$
$$I_{zz}\dot{r} - I_{xz}\dot{p} = n_A + n_T$$

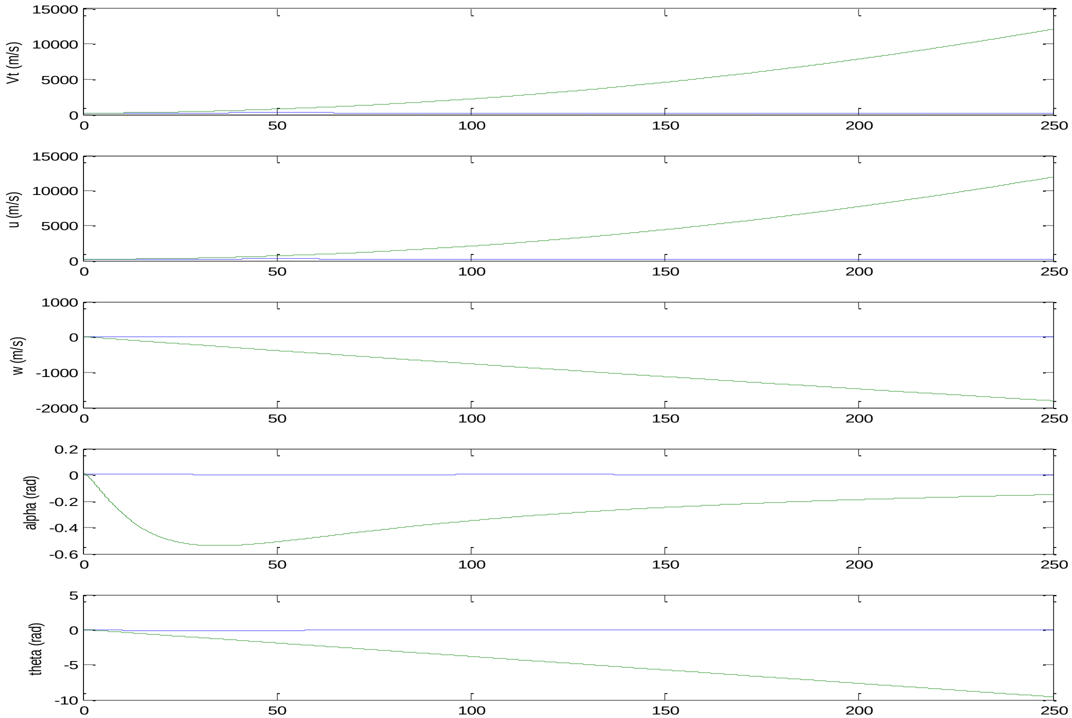
Small perturbations

- Recall: Force, moment coefficients are lookup table functions
- Use linear interpolation about equilibrium condition to determine coefficient



Small perturbations open loop response

Response due to small elevator deflection



Legend

- Linear
- Sonneveldt

Small perturbations results

- Technique is inappropriate
- Force/Moment coefficients are 3 & 4 dimensional surfaces. Approximating is difficult

Finite difference method

- Numerically determines [A,B,C,D] state matrices
- Relatively simple to implement

Finite difference method

State matrix

$$\mathbf{A} = \frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{bmatrix} \frac{\partial f_1}{\partial \dot{x}_1} & \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial \psi} \\ \frac{\partial f_2}{\partial \dot{x}_1} & \frac{\partial f_2}{\partial x_1} & \cdots & \frac{\partial f_2}{\partial \psi} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{6n+6}}{\partial \dot{x}_1} & \frac{\partial f_{6n+6}}{\partial x_1} & \cdots & \frac{\partial f_{6n+6}}{\partial \psi} \end{bmatrix}$$

State matrix element approximation

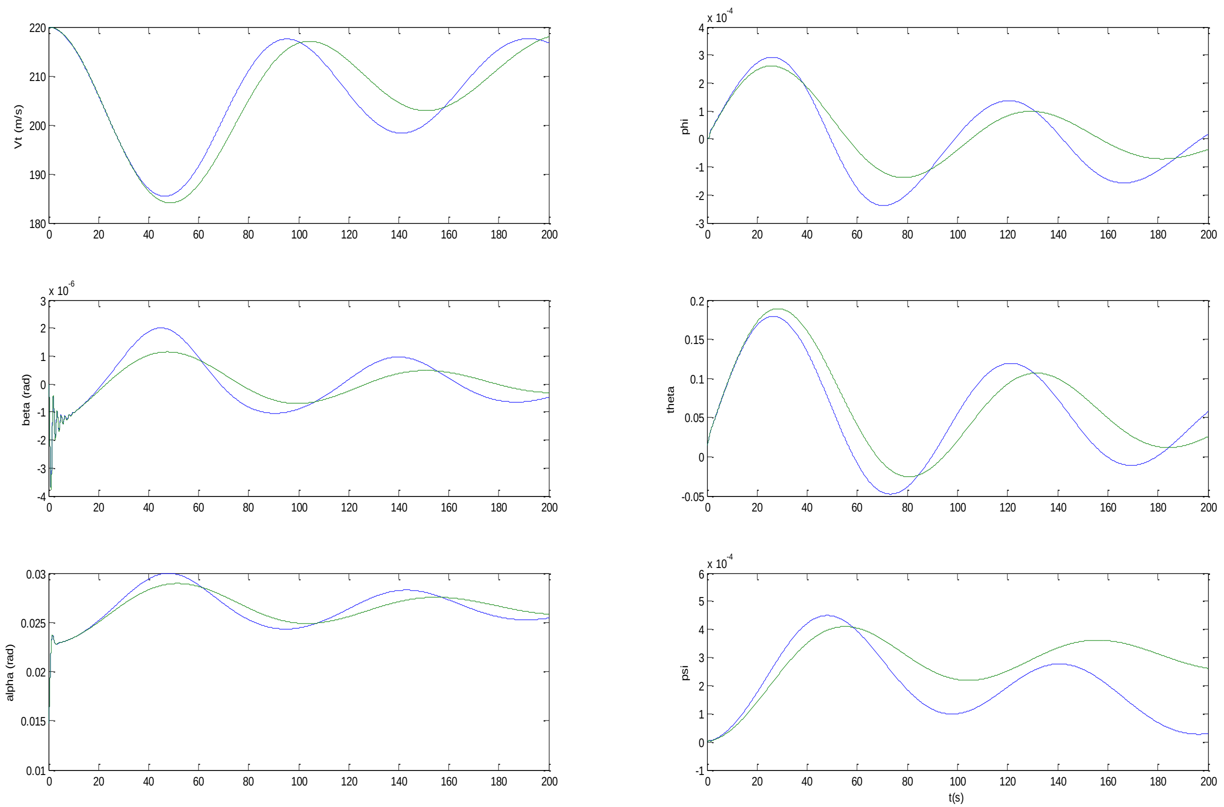
$$\frac{\partial f_1}{\partial \dot{x}_1} \approx \frac{f_1' - f_{1_0}}{\Delta \dot{x}_1}$$

State f1 derivative = [(State f1+delta) – State f1@equilibrium]/delta

where State f1 = u, v, w, phi, q, etc.

Finite difference open loop response

Response due to small elevator deflection

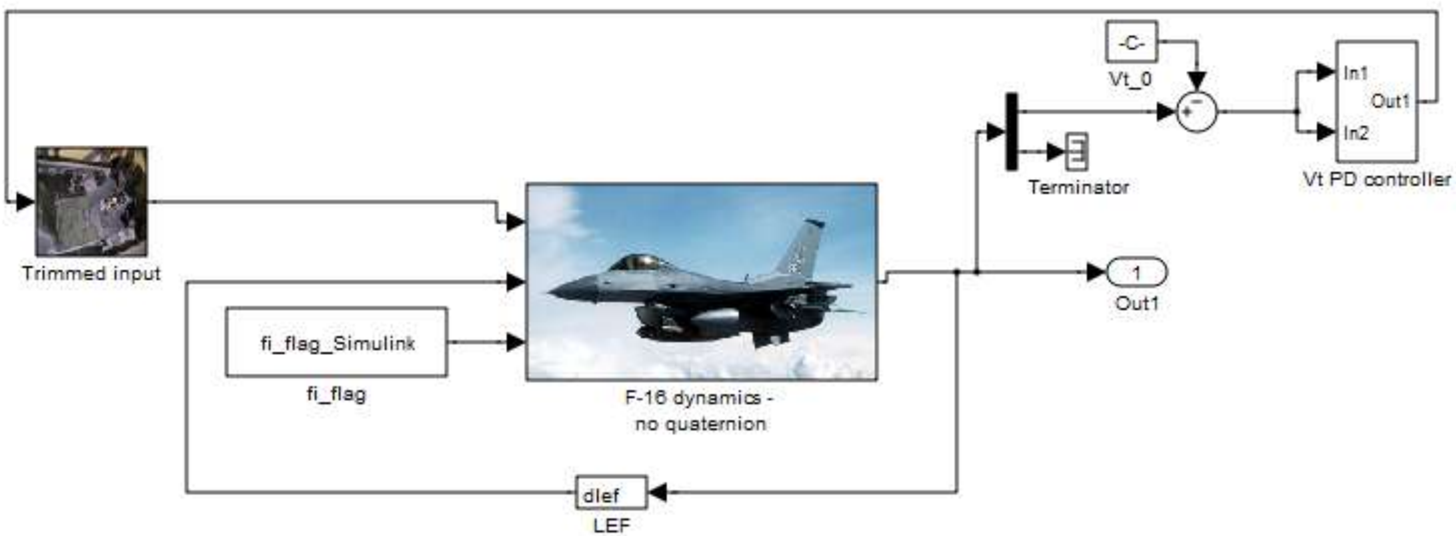


Legend

- Linear
- Sonneveldt

- Vo=220m/s
- Altitude= 2000m

Closed Loop Control



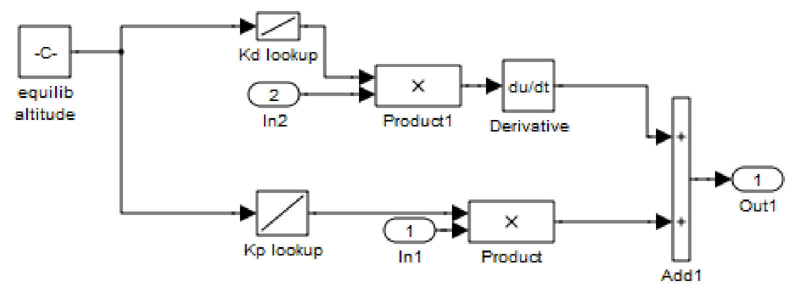
Closed Loop Control

- Longitudinal motion only
- Reponses to elevator inputs (step, impulse)
- Scheduled PD controller

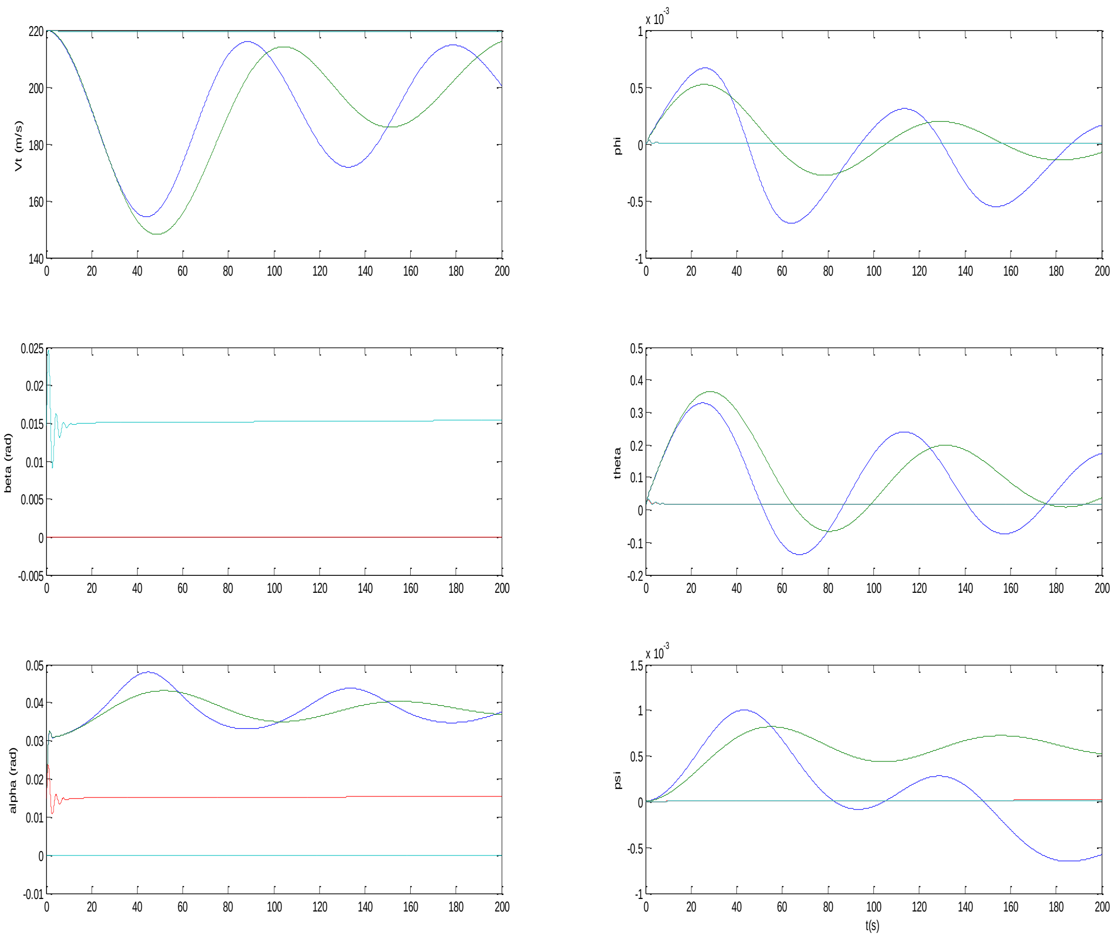
$$G(s) = K_p + K_d s$$

Scheduled PD controller

- Use root locus diagram of linearized model to determine appropriate controller gains
- Interpolate gains when deviate from operating condition
- Apply controller to non-linear model to validate



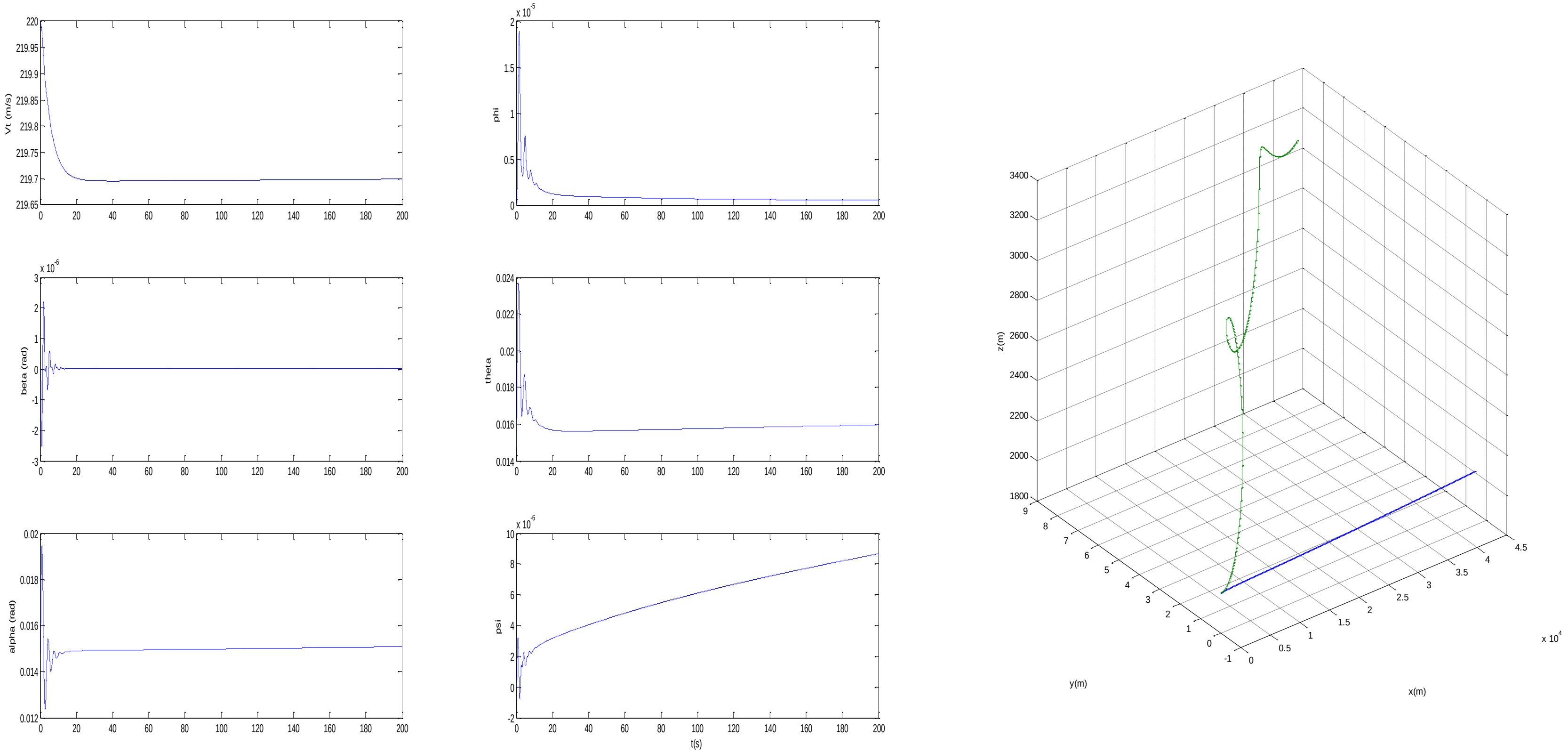
Open vs closed loop response to step input



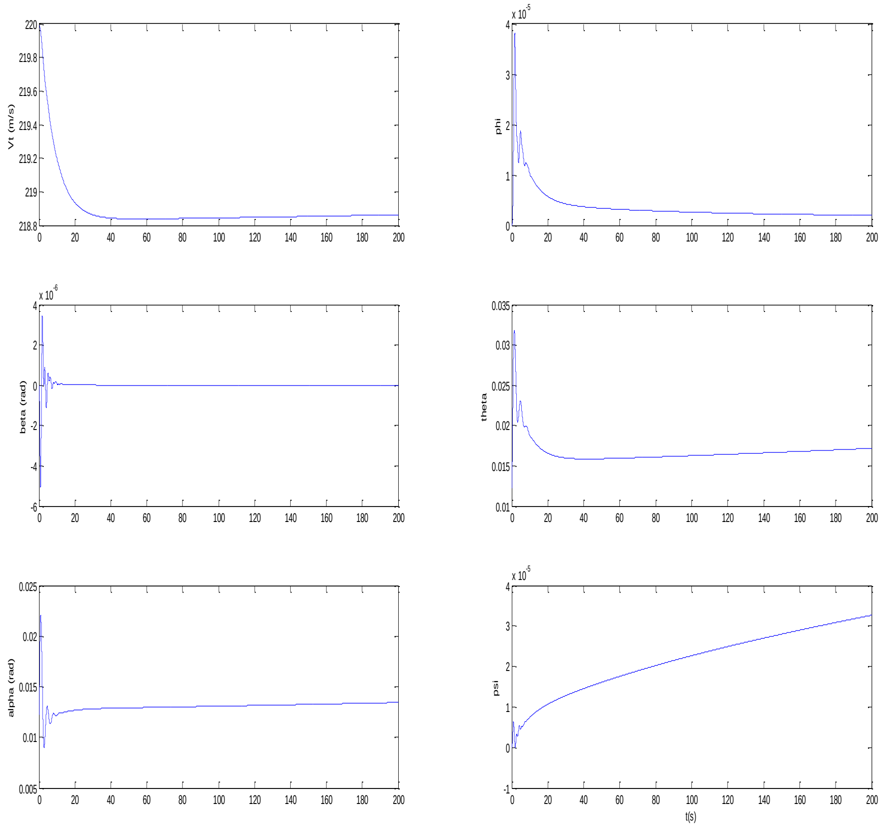
- Open loop Linear
- Open loop Sonneveldt
- Closed loop Sonneveldt

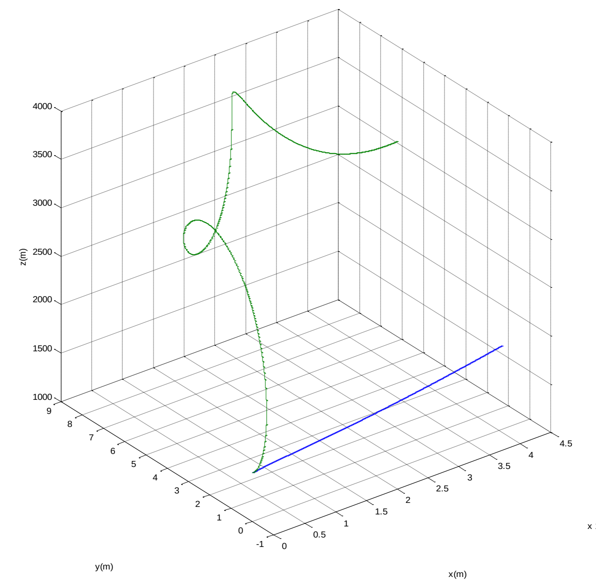
- Vo=220m/s
- Altitude= 2000m

Open vs closed loop response to step input Vo=220m/s, Altitude= 2000m



Open vs closed loop response to step input Vo=220m/s, Altitude= 1500m





Conclusions

- Linearization technique should be chosen appropriately for the design task.
- Gain scheduling is an appropriate controller technique for most applications.