LATERAL FLIGHT CONTROL
MECH 6091

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Project overview

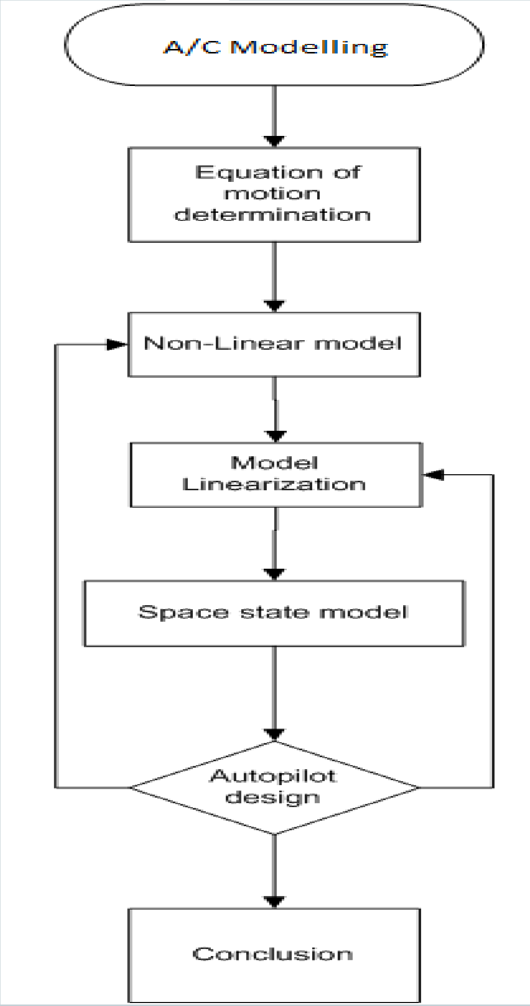
- Introduction
- Equations of Motion
- Non-Linear and Linear Modeling
- Autopilot design and Simulink Demonstration
- Results and Discussion

Introduction

- Objective:

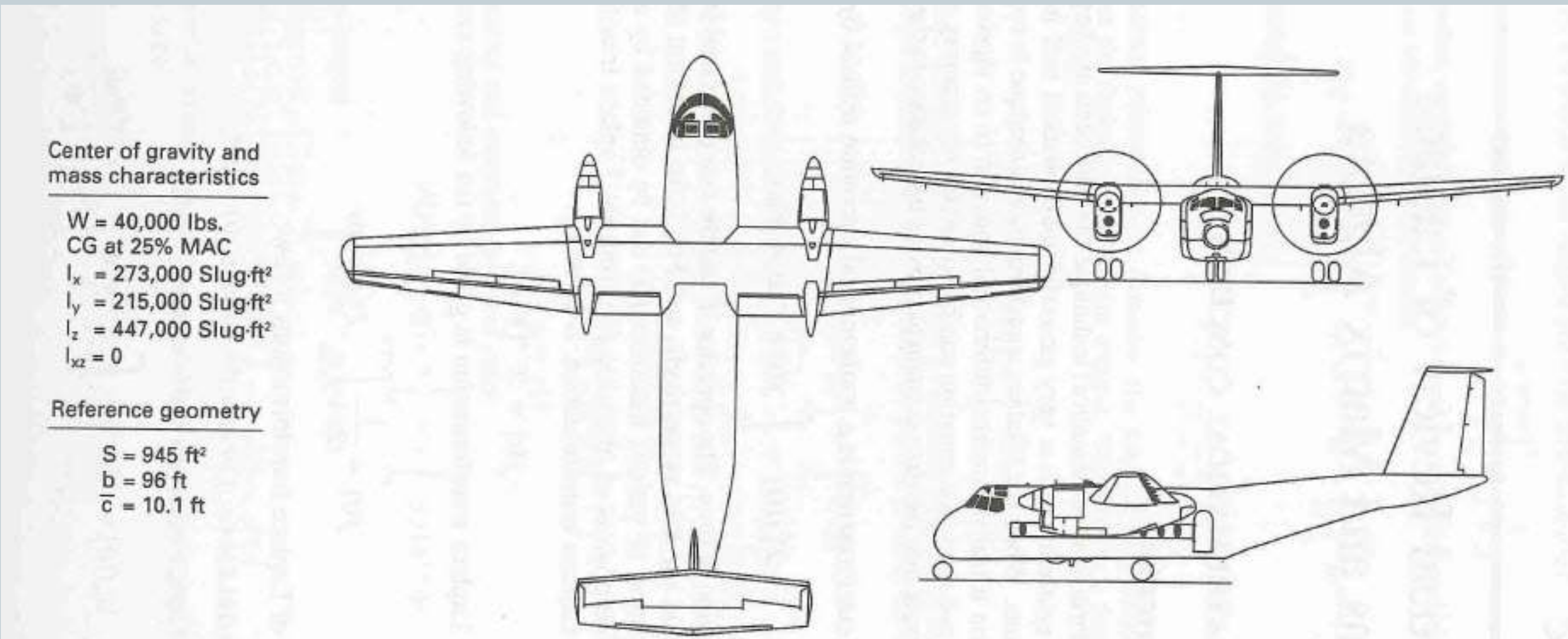
  Design a control system for an existing aircraft for lateral motion. Matlab/Simulink software is used to implement design and test for the designed autopilot control system.

Introduction



Introduction

- STOL transport data was used from R.Nelson *Flight Stability and Automatic Control* textbook to test the control system.



Assumptions

Assumptions	Neglect
1. Flat Earth	Earth's curvature
2. Non-rotating earth	Coriolis acceleration
	Centripetal force
3. Constant mass	Mass variation, shift
4. Rigid body	Aero-elasticity
5. No rotating masses	Propellers/turbines
6. Symmetric aircraft	Exotic configurations
7. Constant wind	Wind shear, turbulence

Equations of Motion

- Equations of motion are found applying Newton's second law:

$$\sum F = \frac{d}{dt}(mV) = \frac{dm}{dt}\vec{V} + m\frac{d}{dt}\vec{V} = m\frac{d}{dt}\vec{V}$$

$$\sum \vec{M} = \frac{d}{dt}(\vec{H}) = \frac{d}{dt}(\vec{I}\vec{\omega})$$

- The forces considered for lateral motion are:

- Fa: **Aerodynamic forces** acting on the vertical tail.
- T : **Thrust** pushes forward along the length of the aircraft
- D : **Drag** pulls back along the length of the aircraft
- W : **Weight**

- The Moments considered for lateral motion are:
- Rolling moment L about the C.G.
- Yawing moment N about the C.G.

Equations of Motion

Dynamic equations:

$$\begin{bmatrix}\dot{u}\\ \dot{v}\\ \dot{w}\end{bmatrix} = \begin{bmatrix} -gsin\theta + \frac{X}{m} - qw + rv \\ gsin\theta cos\phi + \frac{Y}{m} - ru + pw \\ gsin\phi cos\theta + \frac{Z}{m} - pv + qu \end{bmatrix}$$

$$\begin{bmatrix}\dot{P}\\ \dot{Q}\\ \dot{R}\end{bmatrix} = I^{-1}\left(\begin{bmatrix}L\\ M\\ N\end{bmatrix} - {}^{SI}\vec{\omega} \times \vec{H}\right)$$

Kinematic equations:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ (-c\phi s\psi + s\phi s\theta s\psi) & c\phi c\psi + s\phi s\theta s\psi & s\phi c\theta \\ (s\phi s\psi + c\phi s\theta c\psi) & (-s\phi c\psi + c\phi s\theta s\psi) & c\phi c\theta \end{bmatrix} \times \begin{bmatrix}u\\ v\\ w\end{bmatrix}$$

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix} c\alpha & c\alpha \tan(\theta)\, s(\phi) - s\alpha\ s(\phi)/c\theta & \tan(\theta)\, c(\phi) c\alpha - s\alpha c(\phi)/c\theta \\ 0 & c(\phi) & 0 \\ s\alpha & s\alpha \tan(\theta)\, s(\phi) + c\alpha s(\phi)/c\theta & s\alpha \tan(\theta)\, c(\phi) + c\alpha c(\phi)/c\theta \end{bmatrix} \begin{bmatrix}P\\ Q\\ R\end{bmatrix}$$

Nonlinear Model

$$f_1 = \dot{\beta} = \frac{1}{mV} + \left[(mg\sin(\phi)\cos(\beta) - mg\alpha\sin(\beta)) - T sin(\beta) - F_{y\beta} \right] - \dot{\psi} + \dot{\phi}\alpha$$

$$f_{2=} \dot{P} = \left(1 + \frac{I_{xx}^2}{I_{zz}I_{xx} - I_{xz}^2} \right) \frac{L}{I_{xx}} + \left(\frac{I_{xz}}{I_{zz}I_{xx} - I_{xz}^2} \right) N$$

$$f_{3=} \dot{R} = \left(\frac{I_{xx}^2}{I_{zz}I_{xx} - I_{xz}^2} \right) N + \left(\frac{I_{xz}}{I_{zz}I_{xx} - I_{xz}^2} \right) L$$

$$f_{4=} \dot{\phi} = P$$

$$f_{5=} \dot{\psi} = R cos(\phi)$$

$$f_{6=} \dot{y} = V sin(\beta + \psi)$$

The above Equations are derived from the dynamic and kinematic equations based On the following assumptions:

- Angle of attack (α) is small and constant.
- Pitch angle (θ) and the rate of the change of pitch angle (Q) are zeros.

Linear Model

- For the linearization the **Jacobian matrix** is used and the states and inputs control of the model is specified as:

$$X = [\beta \; P \; R \; \phi \; \psi \; y]', U = [\delta_a \; \delta_r]'$$

- However the controller should keep the constant the velocity, the thrust is not included as input control since it is assumed enough to get constant velocity and all initial conditions are zeros.
- State space representation of the linear model as this form

$$\dot{X} = A\,X + B\,U$$

$$Y = C\,X + DU$$

Linear Model

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \cdots & \frac{\partial f_1}{\partial X_6} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_6}{\partial X_1} & \cdots & \frac{\partial f_6}{\partial X_6} \end{bmatrix}_{6\times 6}, B = \begin{bmatrix} \frac{\partial f_1}{\partial U_1} & \frac{\partial f_1}{\partial U_2} \\ \vdots & \vdots \\ \frac{\partial f_6}{\partial U_1} & \frac{\partial f_6}{\partial U_2} \end{bmatrix}_{6\times 2}$$

A and B are Jacobian Matrices

$$\begin{bmatrix} A_{11} & \alpha & -1 & \frac{g}{Vo} & 0 & 0 \\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ Vo & 0 & 0 & 0 & Vo & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{1}{2}\rho Vo^2 S_w \bar{c}((I3)C_{L\delta a} + (I4)C_{N\delta a}) & \frac{1}{2}\rho Vo^2 S_w \bar{c}((I3)C_{L\delta r} + (I4)C_{N\delta r}) \\ \frac{1}{2}\rho Vo^2 S_w \bar{c}((I1)C_{N\delta a} + (I2)C_{L\delta a}) & \frac{1}{2}\rho Vo^2 S_w \bar{c}((I1)C_{N\delta r} + (I2)C_{L\delta r}) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Linear Model

$$A_{11} = -\frac{To}{mVo} - \frac{\rho VoS_cC_{y\beta}}{mVo}$$

$$A_{21} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I3)C_{L\beta} + (I4)C_{N\beta}$$

$$A_{22} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I3)C_{LP} + (I4)C_{NP})$$

$$A_{23} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I3)C_{Lr} + (I4)C_{Nr})$$

$$A_{31} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I1)C_{N\beta} + (I4)C_{L\beta})$$

$$A_{32} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I1)C_{NP} + (I2)C_{LP})$$

$$A_{33} = \frac{1}{2}\rho Vo^2S_w\bar{c}((I1)C_{Nr} + (I2)C_{Lr})$$

$$I1 = 1 + \frac{I_{xx}^2}{I_{zz}I_{xx} - I_{xz}^2}$$

$$I2 = I4 = \frac{I_{xz}}{I_{zz}I_{xx} - I_{xz}^2}$$

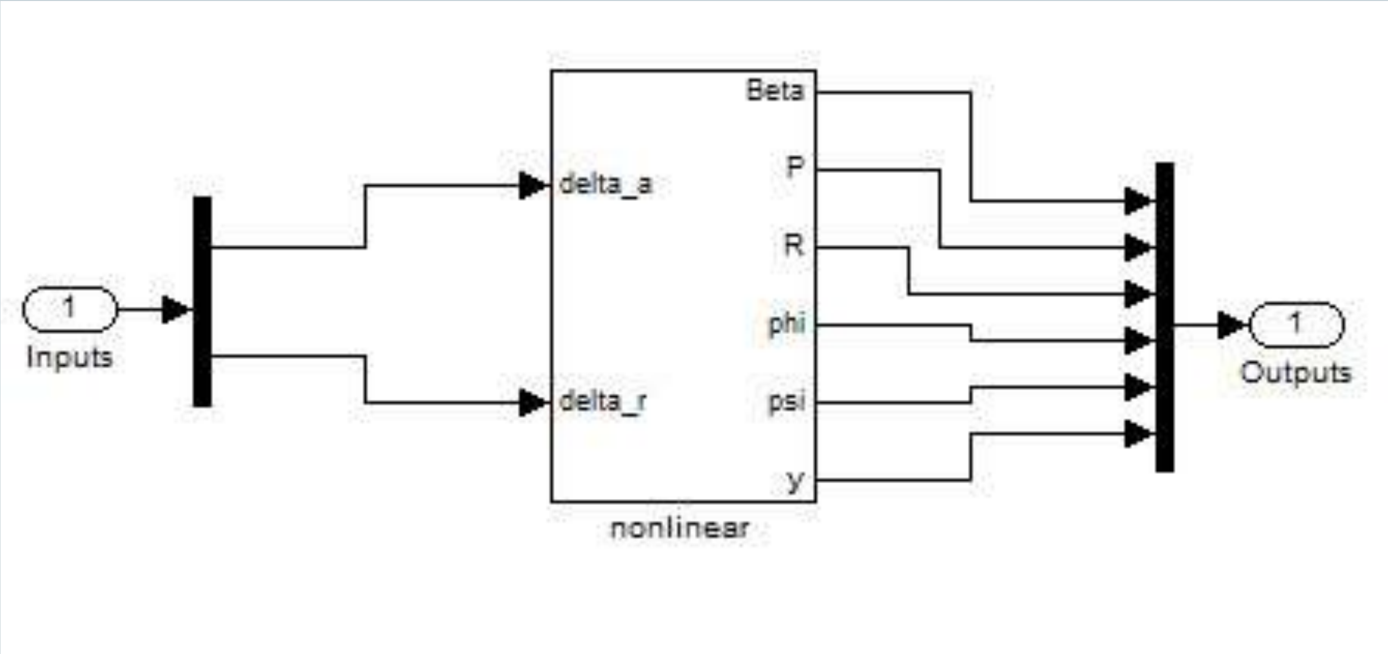
$$I3 = \frac{I_{xx}^2}{I_{zz}I_{xx} - I_{xz}^2}$$

LQR Controller

- Linear-quadratic Regular LQR controller was used for the lateral control system.
- Linear quadratic Regulator Controller is the best controller signal to bring the system from an initial state to the steady state . As we know the choosing of weighting matrix (Q and R) are very important and to minimize the cost function according to this function:

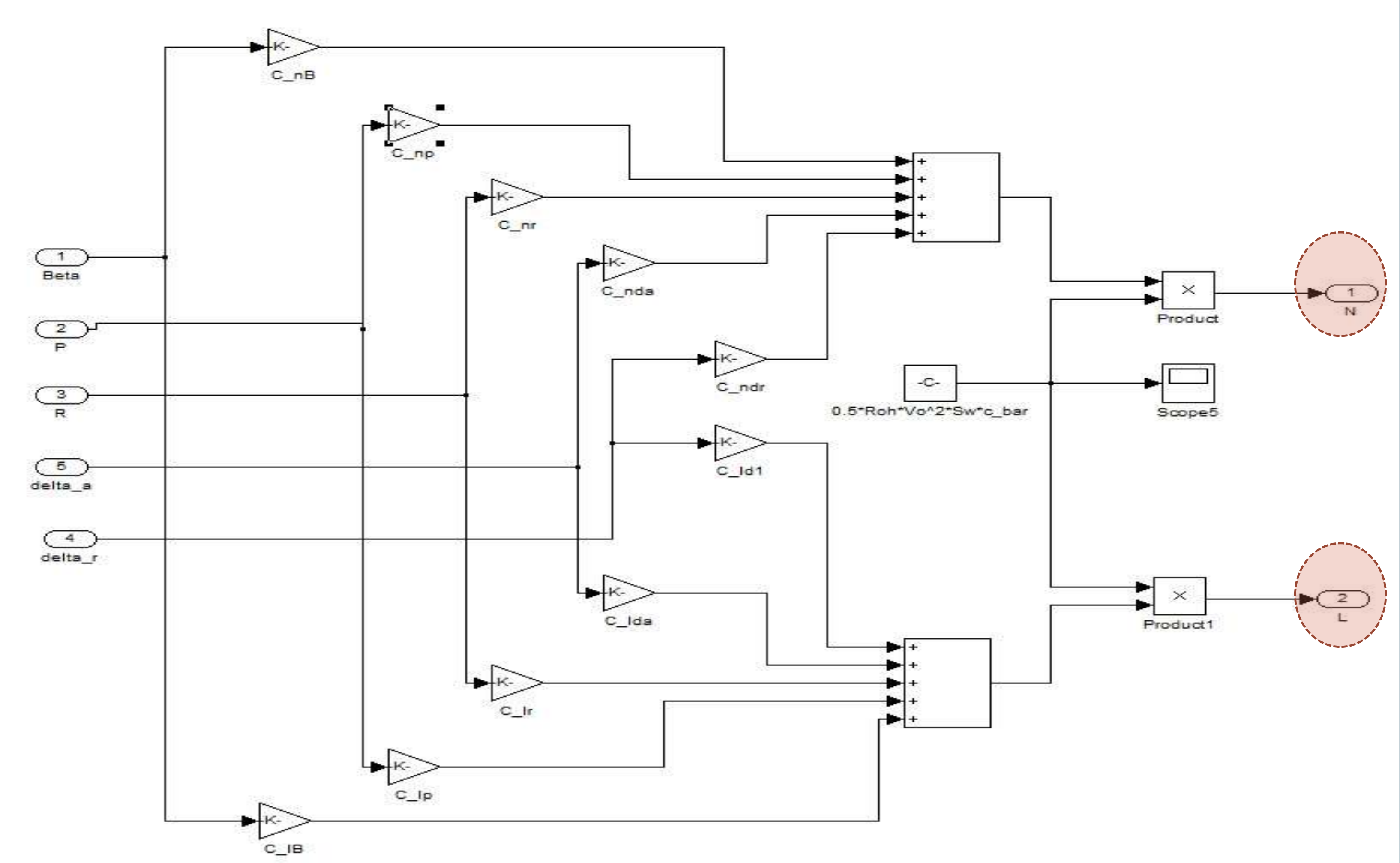
$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt$$

Simulink Model



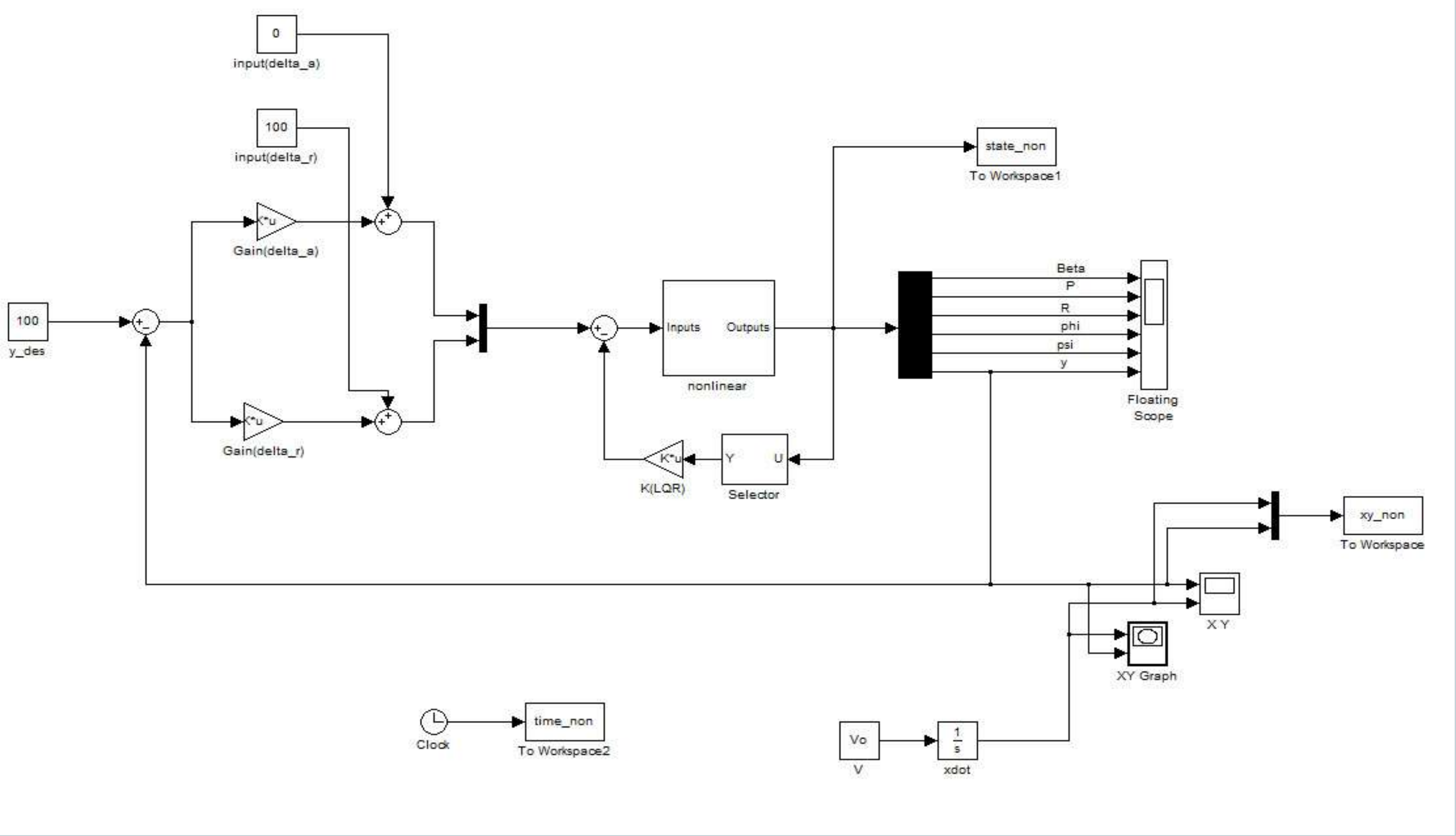
The block contains the nonlinear aircraft dynamics

Simulink Model



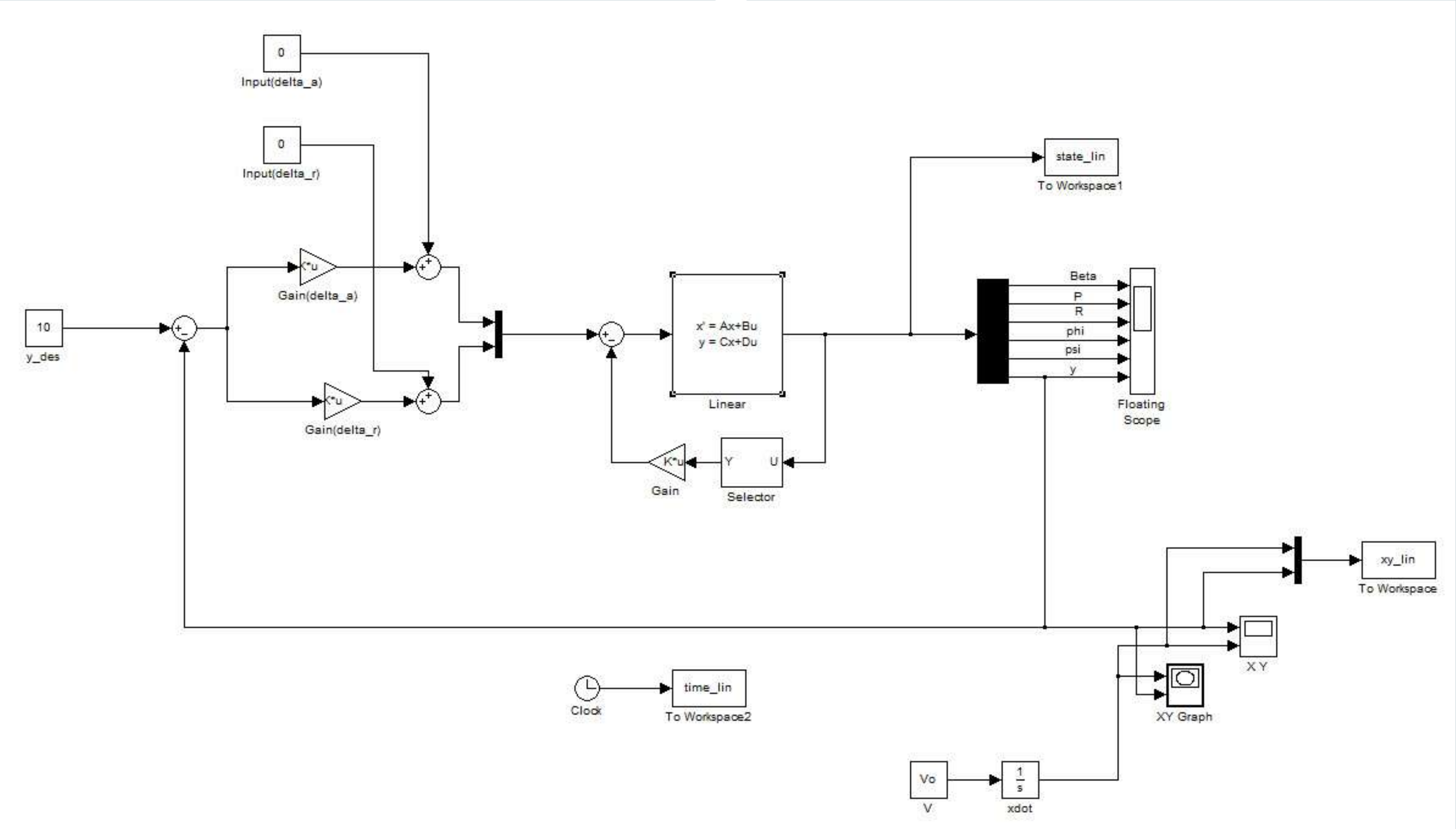
Subsystem produces the Moments N,L

Simulink Model



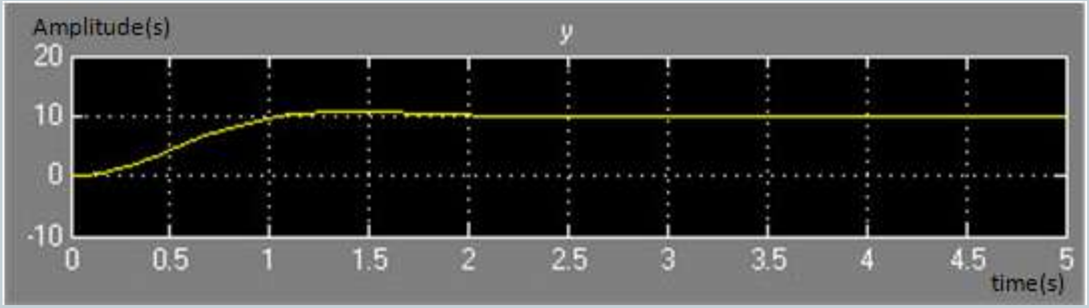
NonLinear simulink model of the Autopilot system

Simulink Model

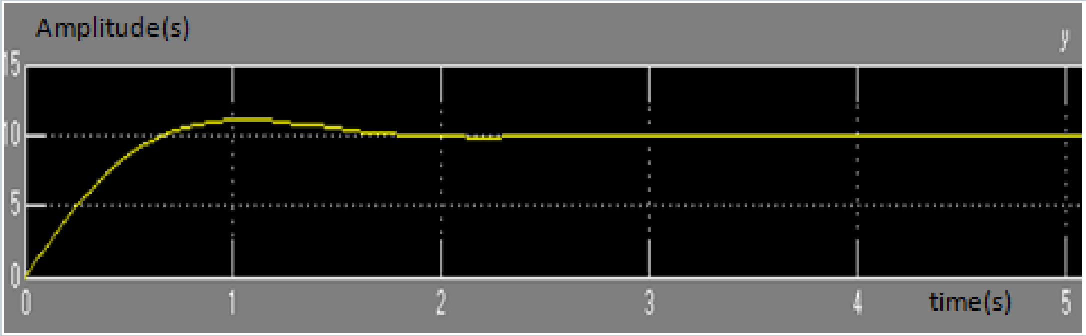


Linear simulink model of the Autopilot system

Time Response: linear vs nonlinear



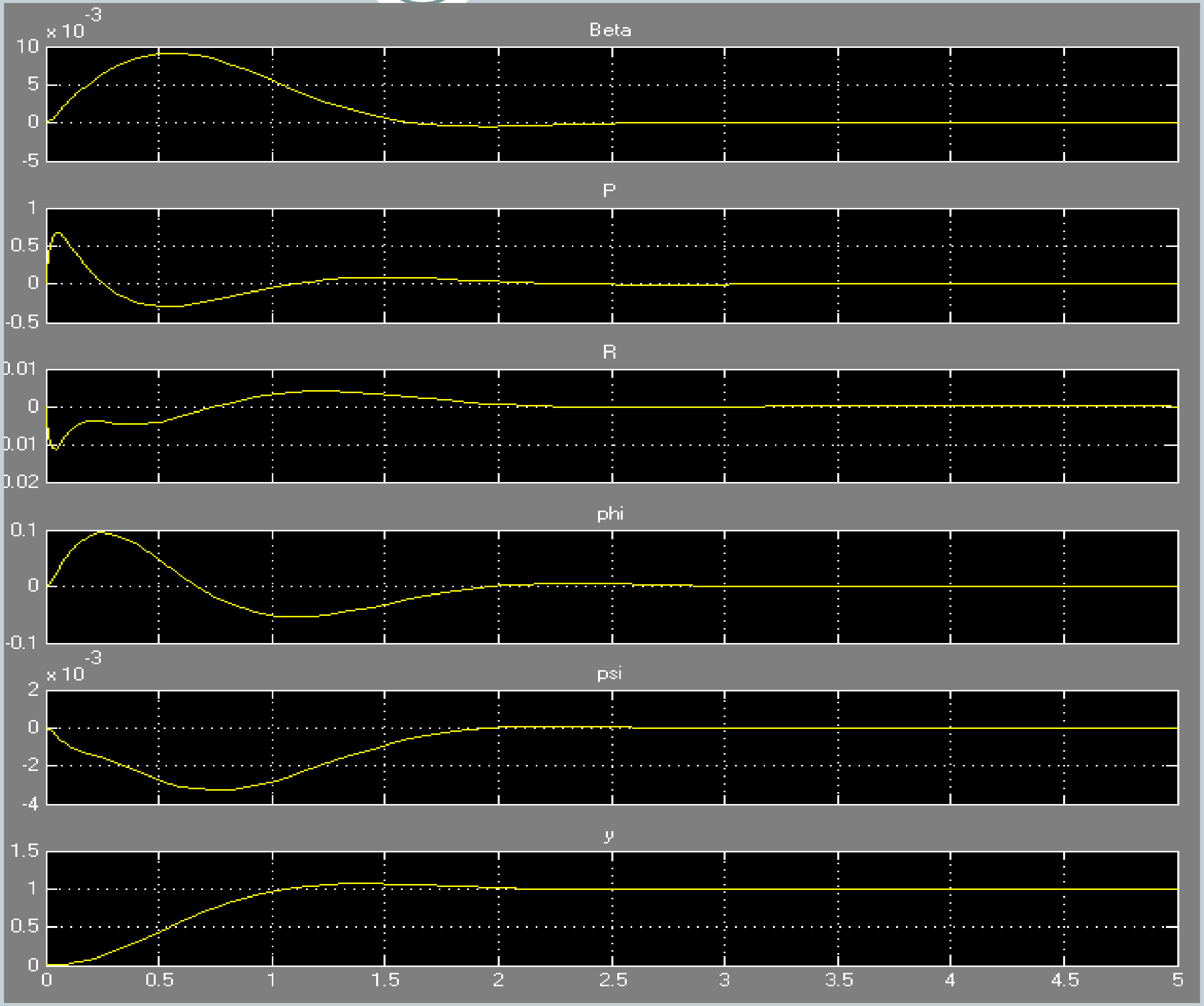
Linearized model time reponse for y0=10



Non-Linear model time reponse for y0=10

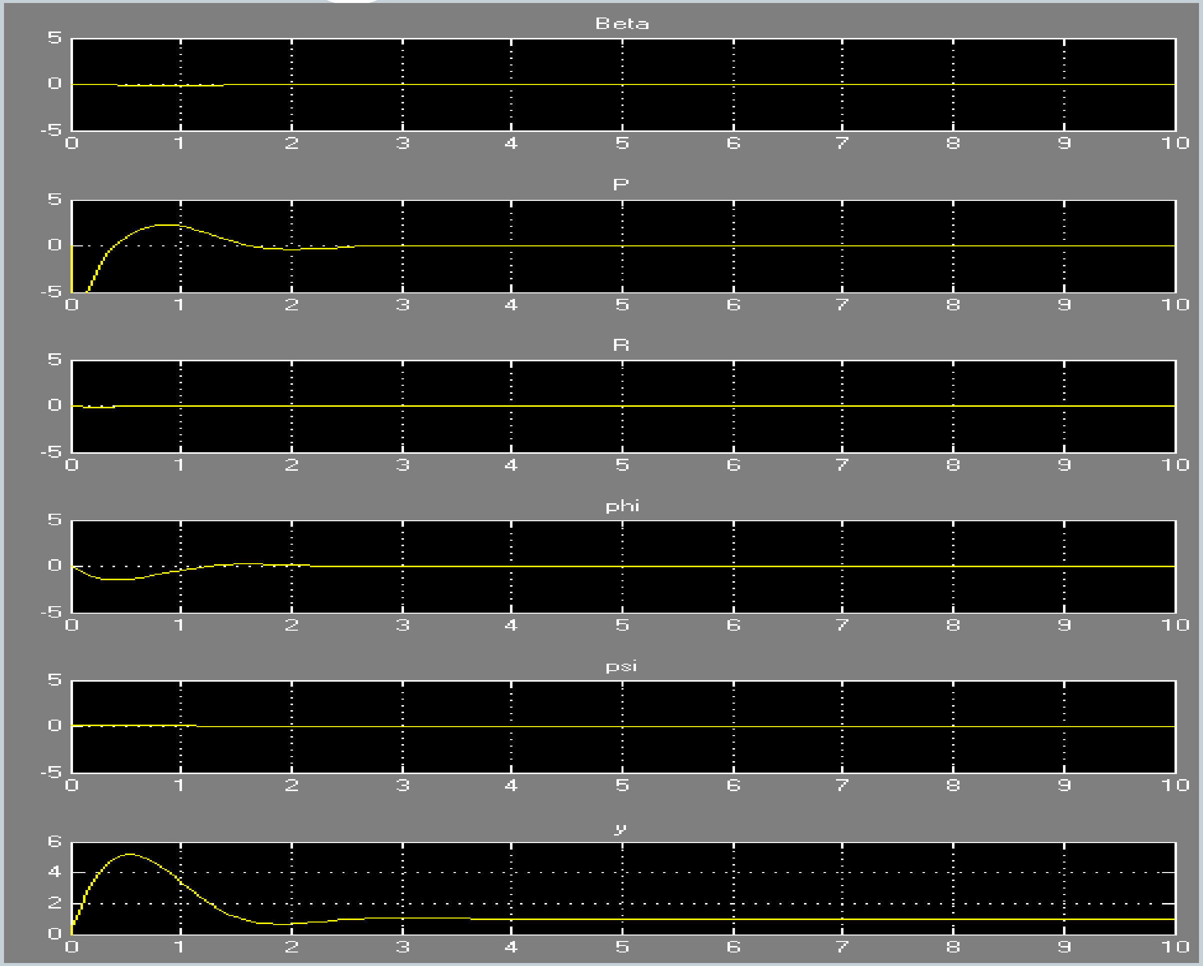
Linear System Results

- For y0=1 and δa=0.005

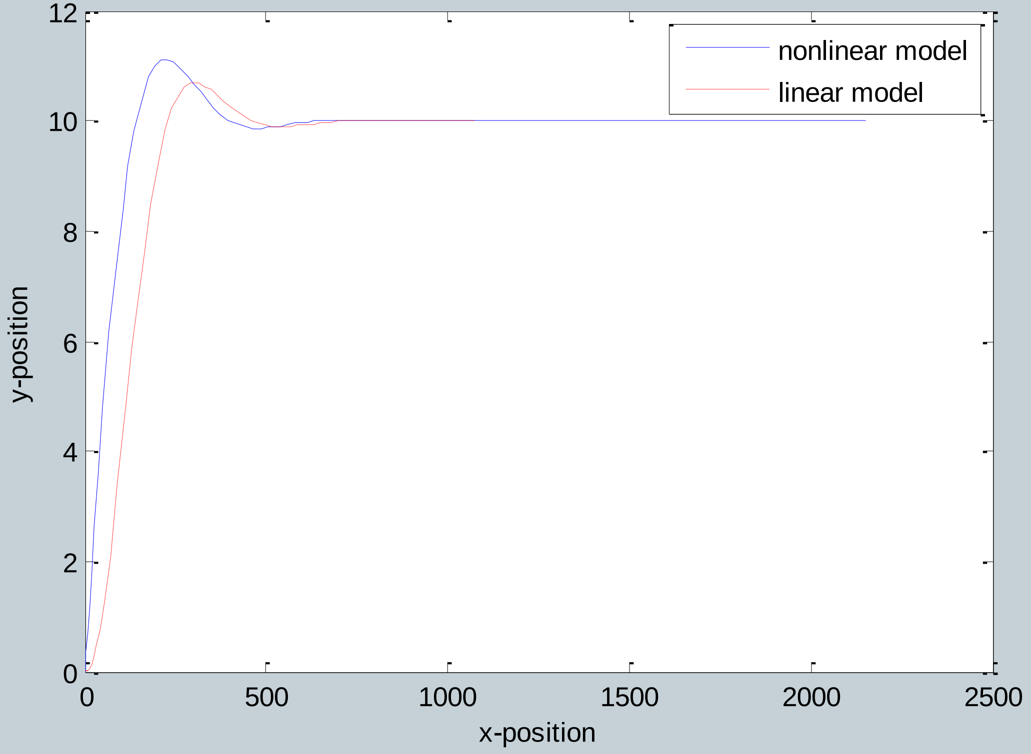


Non-Linear System Results

- For y0=1 and δa=0.005



linear vs nonlinear control



Conclusion

- The general equations of motion were developed for the lateral motion of an aircraft.
- The equations were linearized.
- Simulink models were built for both linear and non-linear models of the autopilot control system.
- Comparing the response of the reference input y for both linear and non-linear has shown that the controller works well for both systems.

REFERENCES

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