"Control Theory" - Exercise #2

E2.8, E2.9, E2.10, E2.14, E2.15, P2.18, P2.32, P2.34

The input $r(t)$ represents the desired position of the laser beam.

(a) If $r(t)$ is a unit step input, find the output $y(t)$.
(b) What is the final value of $y(t)$?

Answer: (a) $y(t) = 1 - 0.125e^{-50t} - 1.125e^{-10t}$, (b) $y_{ss} = 1$

E2.5 A noninverting amplifier uses an op-amp as shown in Fig. E2.5. Assume an ideal op-amp model and determine v_0/v_{in}.

Answer: $\frac{v_0}{v_{in}} = \left(1 + \frac{R_2}{R_1}\right)$

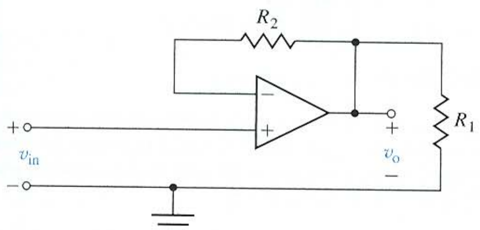

FIGURE E2.5 A noninverting amplifier using an op-amp.

E2.6 A nonlinear device is represented by the function

$$y = f(x) = x^{1/2}$$

where the operating point for the input x is $x_o = 1/2$. Determine the linear approximation in the form of Eq. (2.9).

Answer: $\Delta y = \Delta x/\sqrt{2}$

E2.7 A lamp's intensity stays constant when monitored by an optotransistor-controlled feedback loop. When the voltage drops, the lamp's output also drops, and optotransistor Q_1 draws less current. As a result, a power transistor conducts more heavily and charges a capacitor more rapidly [25]. The capacitor voltage controls the lamp voltage directly. A flow diagram of the system is shown in Fig. E2.7. Find the closed-loop transfer function, $I(s)/R(s)$ where $I(s)$ is the lamp intensity, and $R(s)$ is the command or desired level of light.

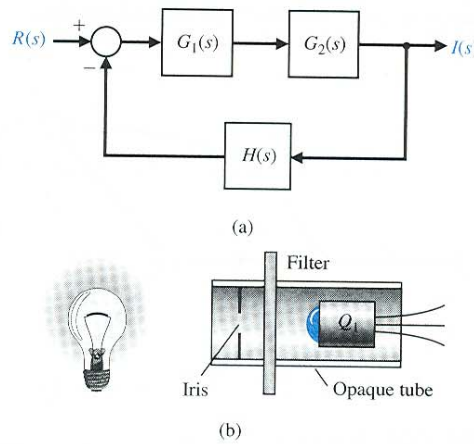

FIGURE E2.7 Lamp controller.

E2.8 A control engineer, N. Minorsky, designed an innovative ship steering system in the 1930s for the U.S. Navy. The system is represented by the signal-flow graph shown in Fig. E2.8 where $Y(s)$ is the ship's course, $R(s)$ is the desired course, and $A(s)$ is the rudder angle [17]. Find the transfer function $Y(s)/R(s)$.

Answer:

$$\frac{Y(s)}{R(s)} = \frac{K\,G_1(s)\,G_2(s)/s}{1 + G_1(s)H_3(s) + G_1(s)G_2(s)[H_1(s) + H_2(s)] + KG_1(s)G_2(s)/s}$$

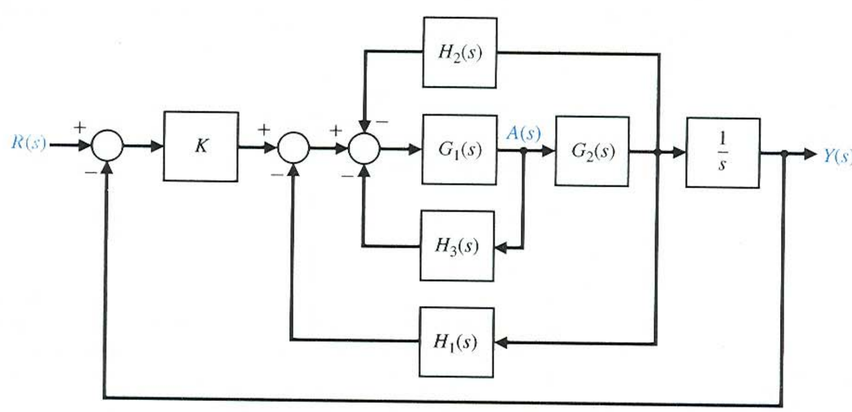

FIGURE E2.8 Ship steering system.

E2.9 A four-wheel antilock automobile braking system uses electronic feedback to control automatically the brake force on each wheel [16]. A simplified flow graph of a brake control system is shown in Fig. E2.9, where $F_f(s)$ and $F_R(s)$ are the braking force of the front and rear wheels, respectively, and $R(s)$ is the desired automobile response on an icy road. Find $F_f(s)/R(s)$.

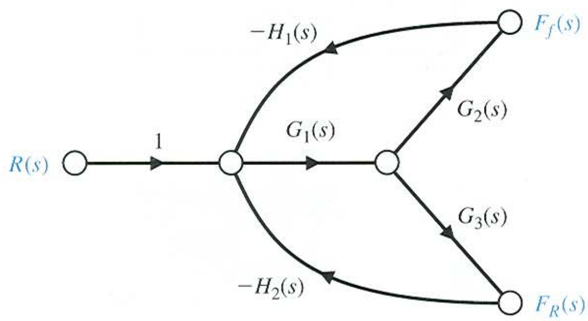

FIGURE E2.9 Brake control system.

E2.10 One of the most potentially beneficial applications of automotive control systems is the active control of the suspension system. One feedback control system uses a shock absorber consisting of a cylinder filled with a compressible fluid that provides both spring and damping forces [18]. The cylinder has a plunger activated by a gear motor, a displacement-measuring sensor, and a piston. Spring force is generated by piston displacement, which compresses the fluid. During piston displacement, the pressure imbalance across the piston is used to control damping. The plunger varies the internal volume of the cylinder. This feedback system is shown in Fig. E2.10. Develop a linear model for this device using a block diagram model.

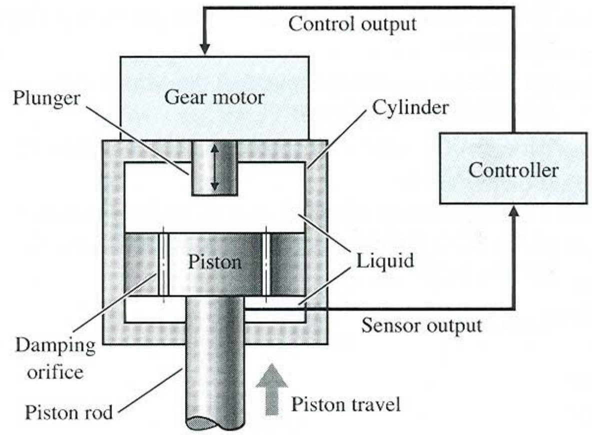

FIGURE E2.10 Shock absorber.

E2.11 A spring exhibits a force-versus-displacement characteristic as shown in Fig. E2.11. For small deviations from the operating point, find the spring constant when x_o is (a) -1.4, (b) 0, (c) 3.5.

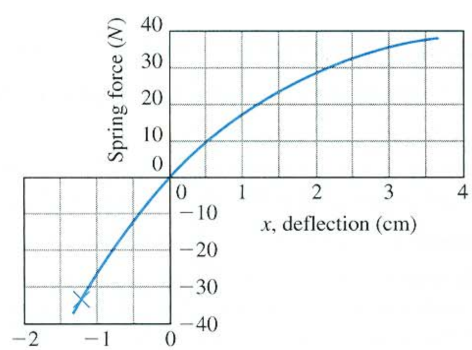

FIGURE E2.11 Spring characteristic.

E2.12 Off-road vehicles experience many disturbance inputs as they traverse over rough roads. An active suspension system can be controlled by a sensor that looks

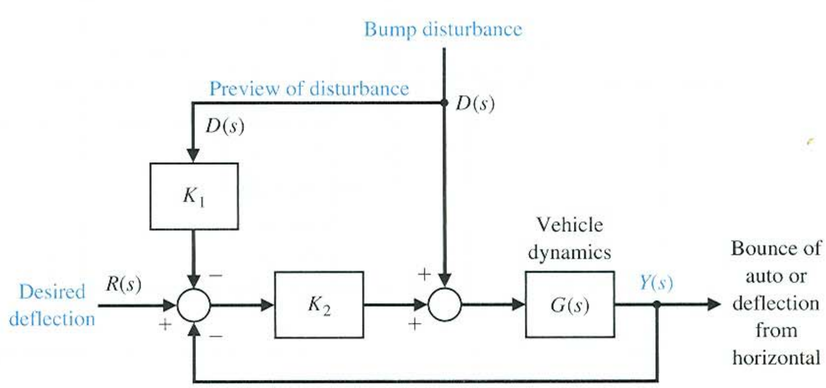

FIGURE E2.12 Active suspension system.

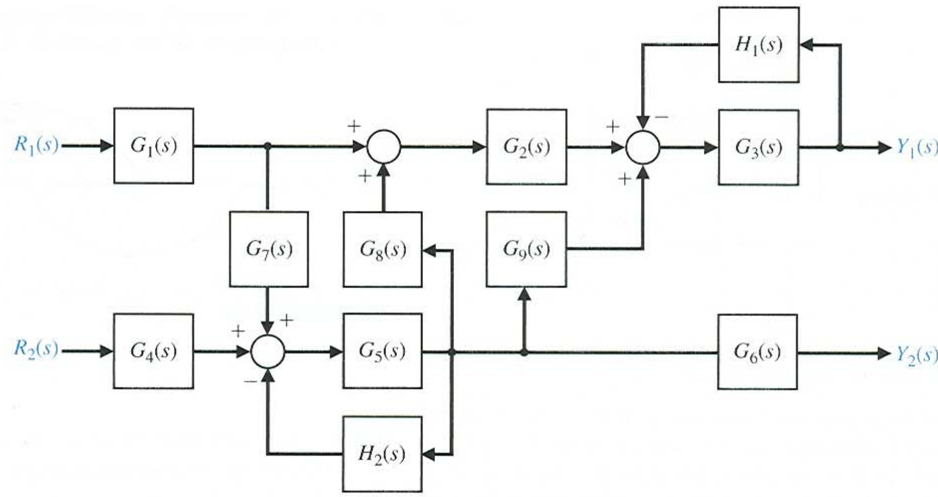

FIGURE E2.13 Multivariable system.

"ahead" at the road conditions. An example of a simple suspension system that can accommodate the bumps is shown in Fig. E2.12. Find the appropriate gain K_1 so that the vehicle does not bounce when the desired deflection is $R(s) = 0$ and the disturbance is $D(s)$.

Answer: $K_1K_2 = 1$

E2.13 Find the transfer function

$$\frac{Y_1(s)}{R_2(s)}$$

for the multivariable system in Fig. E2.13.

E2.14 Obtain the differential equations in terms of i_1 and i_2 for the circuit in Fig. E2.14.

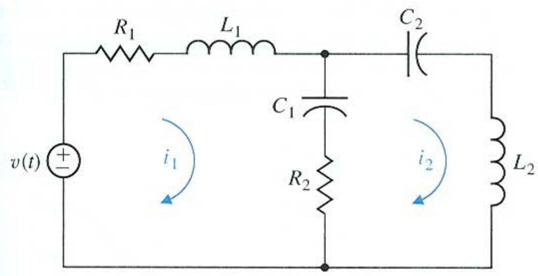

FIGURE E2.14 Electric circuit.

E2.15 The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p}{dt^2} + 2\frac{dp}{dt} + 4p = \theta$$

$$v_1 = r - p$$

$$\frac{d\theta}{dt} = 0.6v_2$$

$$v_2 = 7v_1.$$

The variables involved are as follows:

$r(t)$ = desired platform position
$p(t)$ = actual platform position
$v_1(t)$ = amplifier input voltage
$v_2(t)$ = amplifier output voltage
$\theta(t)$ = motor shaft position

Sketch a signal-flow diagram of the system, identifying the component parts and their transmittances; then determine the system transfer function $P(s)/R(s)$.

E2.16 A spring used in an auto shock absorber develops a force, f, represented by the relation

$$f = kx^4,$$

where x is the displacement of the spring. Determine a linear model for the spring when $x_o = 1$.

E2.17 The output, y, and input, x, of a device are related by

$$y = x + 0.79x^3.$$

(a) Find the values of the output for steady-state operation at the two operating points $x_o = 1$ and $x_o = 2$. (b) Obtain a linearized model for both operating points and compare them.

E2.18 The transfer function of a system is

$$\frac{Y(s)}{R(s)} = \frac{10(s+2)}{s^2 + 8s + 15}.$$

Determine $y(t)$ when $r(t)$ is a unit step input.

Answer: $y(t) = 1.33 + 1.67e^{-3t} - 3e^{-5t}, t \geq 0$

E2.19 Determine the transfer function $V_0(s)/V(s)$ of the operational amplifier circuit shown in Fig. E2.19. Assume an ideal operational amplifier. Determine the transfer function when $R_1 = R_2 = 100\ \text{k}\Omega$, $C_1 = 10\ \mu\text{f}$, and $C_2 = 5\ \mu\text{F}$.

P2.14 A rotating load is connected to a field-controlled dc electric motor through a gear system. The motor is assumed to be linear. A test results in the output load reaching a speed of 1 rad/s within ½ s when a constant 80 V is applied to the motor terminals. The output steady-state speed is 2.4 rad/s. Determine the transfer function of the motor, $\theta(s)/V_f(s)$ in rad/V. The inductance of the field may be assumed to be negligible (see Fig. 2.17). Also, note that the application of 80 V to the motor terminals is a step input of 80 V in magnitude.

P2.15 Consider the mass-spring system depicted in Fig. P2.15. Determine a differential equation to describe the motion of the mass, m. Obtain the system response to an initial displacement $x(0)=1$. Assume motion only in the vertical plane.

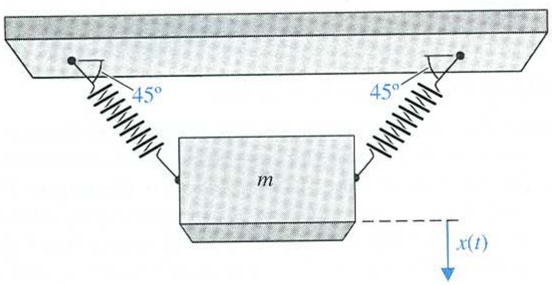

FIGURE P2.15 Suspended mass-spring system.

P2.16 Obtain a signal-flow graph to represent the following set of algebraic equations where x_1 and x_2 are to be considered the dependent variables and 6 and 11 are the inputs:

$$x_1 + 1.5x_2 = 6, \qquad 2x_1 + 4x_2 = 11.$$

Determine the value of each dependent variable by using the gain formula. After solving for x_1 by Mason's signal-flow gain formula, verify the solution by using Cramer's rule.

P2.17 A mechanical system is shown in Fig. P2.17, which is subjected to a known displacement $x_3(t)$ with respect to the reference. (a) Determine the two independent equations of motion. (b) Obtain the equations of motion in terms of the Laplace transform, assuming that the initial conditions are zero. (c) Sketch a signal-flow graph representing the system of equations. (d) Obtain the relationship between $X_1(s)$ and $X_3(s)$, $T_{13}(s)$, by using Mason's signal-flow gain formula. Compare the work necessary to obtain $T_{13}(s)$ by matrix methods to that using Mason's signal-flow gain formula.

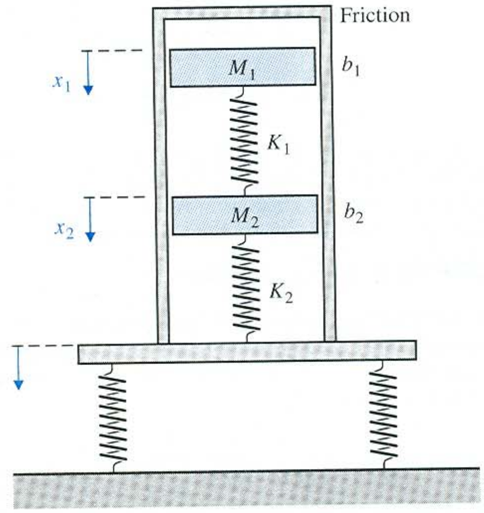

FIGURE P2.17 Mechanical system.

P2.18 An LC ladder network is shown in Fig. P2.18. One may write the equations describing the network as follows:

$$I_1 = (V_1 - V_a)Y_1, \qquad V_a = (I_1 - I_a)Z_2,$$
$$I_a = (V_a - V_2)Y_3, \qquad V_2 = I_aZ_4.$$

Construct a flow graph from the equations and determine the transfer function $V_2(s)/V_1(s)$.

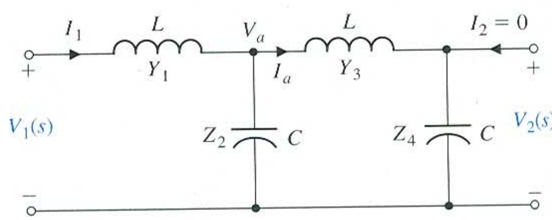

FIGURE P2.18 *LC* Ladder network.

P2.19 A voltage follower (buffer amplifier) is shown in Figure P2.19. Show that $T = v_0/v_{in} = 1$. Assume an ideal op-amp.

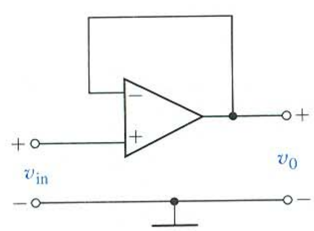

FIGURE P2.19 A buffer amplifier.

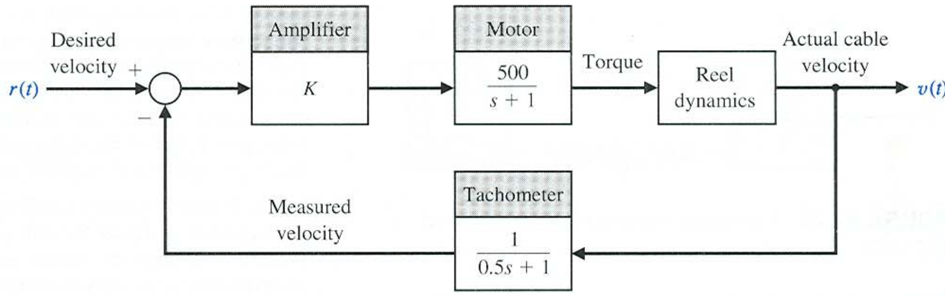

FIGURE P2.31 Cable reel control system.

sired cable speed is 50 m/sec. Develop a digital computer simulation of this system and obtain the response of the speed over 20 seconds for the three values of gain $K = 0.2, 0.4$, and 0.6. The reel angular velocity $\dot{\omega} = d\theta/dt$ is equal to $1/I$ times the integral of the torque. Note that the inertia changes with time as the reel is unwound. However, an equation for I within the simulation will account for this change. Select the gain K to limit the overshoot to less than 9% and yet attain the fastest response. Assume $W = 2.0$, $D = 0.1$, and $R = 3.5$ at $t = 0$.

P2.32 An interacting control system with two inputs and two outputs is shown in Fig. P2.32. Solve for $Y_1(s)/R_1(s)$ and $Y_2(s)/R_1(s)$, when $R_2 = 0$.

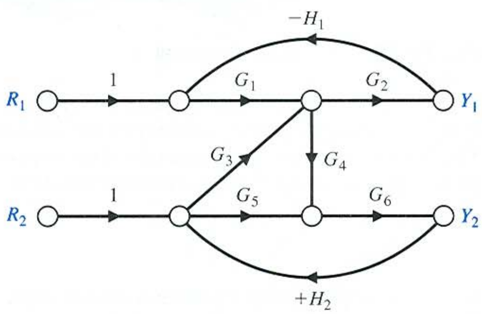

FIGURE P2.32 Interacting system.

P2.33 A system consists of two electric motors that are coupled by a continuous flexible belt. The belt also passes over a swinging arm that is instrumented to allow measurement of the belt speed and tension. The basic control problem is to regulate the belt speed and tension by varying the motor torques.

An example of a practical system similar to that shown occurs in textile fiber manufacturing processes when yarn is wound from one spool to another at high speed. Between the two spools the yarn is processed in a way that may require the yarn speed and tension to be controlled to within defined limits. A model of the system is shown in Fig. P2.33. Find $Y_2(s)/R_1(s)$. Determine a relationship for the system that will make Y_2 independent of R_1.

P2.34 Find the transfer function for $Y(s)/R(s)$ for the idle speed control system for a fuel injected engine as shown in Fig. P2.34.

P2.35 The suspension system for one wheel of an old-fashioned pickup truck is illustrated in Fig. P2.35. The mass of the vehicle is m_1 and the mass of the wheel is m_2. The suspension spring has a spring constant k_1, and the tire has a spring constant k_2. The damping constant of the shock absorber is b. Obtain the transfer function $Y_1(s)/X(s)$, which represents the vehicle response to bumps in the road.

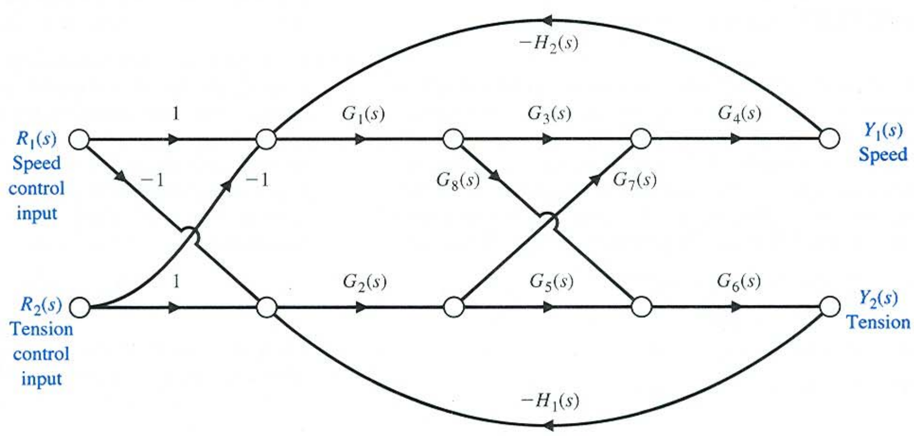

FIGURE P2.33 A model of the coupled motor drives.