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Detection, estimation, and accommodation of loss of control effectiveness

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SUMMARY

In this paper, an adaptive Kalman filtering algorithm is developed for use to estimate the reduction of control effectiveness in a closed-loop setting. Control effectiveness factors are used to quantify faults entering control systems through actuators. A set of covariance-dependent forgetting factors is introduced into the filtering algorithm. As a result, the change in the control effectiveness is accentuated to help achieve a more accurate estimate more rapidly. A weighted sum-squared bias estimate is defined for the change detection. The state estimate is fed back to achieve the steady-state regulation, while the control effectiveness estimate is used for the on-line tuning of the control law. A stability analysis is performed for the adaptive regulator. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: fault detection and isolation; adaptive estimation; fault tolerant control

1. INTRODUCTION

Due to the increasing complexity of modern engineering systems, reliability has become an increasingly important issue. One way to improve reliability is to enhance the fault tolerance of the systems. Many researchers have focused on the development of methodologies to detect and isolate faults [1–7], so that measures could be taken to accommodate their effects. The prompt accommodation of certain critical faults is of paramount importance in some applications. Therefore, the timely identification of a faulty model with sufficient accuracy may be necessary. The faulty model can then be used in a subsequent effort to restructure the system for fault accommodation. The attempt to achieve the self-repairing feature in flight control systems [8]

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and recent developments on the fault tolerant control [9–12] are examples of the endeavor along this direction. In Reference [8], the effect of impaired control surfaces is measured by the amount of deviation of control effectiveness factors from their normal values. The deviation is estimated so that the control action in the flight control system can be adjusted accordingly. The remaining authority of the impaired surfaces may have to be fully utilized. In this regard, the information needed is often beyond the reach of many existing fault detection and isolation methods.

With the application of fault tolerant control in mind, the estimation of control effectiveness is formulated in this paper as an augmented state Kalman filter problem in which control effectiveness factors are modelled as the augmented random bias states. The bias states enter the original state-space model additively, and therefore preserves the linearity assumed for the fault-free model. This is a property peculiar to the faults affecting the control effectiveness. For historical reasons [13], the term bias will be abused to describe the control effectiveness in this paper whenever convenient. To deal with the fact that abrupt changes in control effectiveness factors are in fact not biases, but step functions of random magnitudes stepping down at random times, an additive noise term is introduced in the bias state equation, and an individual forgetting factor [14] is introduced for each effectiveness factor estimate. The purpose is to manifest the change of the effectiveness factor in its estimate.

Since the additive noise introduced into the bias evolution equation bears no relation to either the process noise or the measurement noise in the dynamic system model, the two-stage filtering algorithm by Keller and Darouach [15] can be applied with some modification to obtain the bias estimates. This algorithm takes advantage of the fact that noises enter different equations are uncorrelated, the augmented state Kalman filter is decoupled into a modified bias-free state estimator, an optimal bias estimator, and a set of update equations dealing with the state-bias coupling. The implication with regard to our control effectiveness estimation problem is that the dimensions of the individual filters are no greater than the larger dimension between the state space and the input space. This two-stage filtering algorithm however, is not designed for tracking biases that are subject to abrupt changes at random times. We propose to insert forgetting factors which are neither temporally nor spacially uniform into the decoupled bias estimator to ensure an effective tracking of the changes in the control effectiveness factor. A hypothesis test using weighted sum-squared bias estimates (WSSBE) is proposed to warrant the safety of the system in case some severe loss of control effectiveness has been detected. It also serves to validate the bias estimates because the estimates are to be used in on-line control law adaptation as soon as they are obtained.

The above estimation problem was solved first in an open-loop setting in Reference [16]. It is shown in this paper that the same solution applies in the closed-loop setting. A regulator problem is considered in this paper which uses the state estimates as feedback variables. A version of the separation principle is invoked to allow a separate regulator design process. As a result the closed-loop system can be arranged to have the unity rank perturbation structure for which the mapping theorem [17] can be applied to conclude the stability of the regulator. The control law is designed to adapt to the change of the control effectiveness, and therefore provides fault tolerance.

The paper is organized as follows. In Section 2, the control effectiveness estimation is formulated as an augmented state Kalman filter problem, and the regulator design as a partial estimate feedback problem. The estimation solution is presented in the form of the two-stage Kalman filter [15]. Section 3 is focused on the modification of the two-stage Kalman filter so that changes in the control effectiveness factors can be tracked more rapidly. Section 4 discusses the regulator design issues, in particular, the regulator stability subject to the control effectiveness estimation error. In Section 5, some simulations are carried out for the estimate of reduction of control effectiveness and the accommodation of the control effectiveness loss through the re-design of the feedback law. Section 6 gives a brief summary of the simulation results, and a brief discussion of a few related issues.

2. BACKGROUND AND PROBLEM FORMULATION

In this section, a dynamic system subject to the reduction of control effectiveness is described by a linear time-varying discrete state-space model augmented with a bias evolution equation. Then the state and bias decoupled filtering algorithm developed by Keller and Darouach [15] is slightly modified and applied to obtain an adaptive Kalman filter solution. A further modification to the algorithm is made in the next section so that the estimate is more responsive to the changes in the biases. The state estimate is then fed back to regulate the output of the system. The discussion on the regulator design and stability analysis is deferred to a later section.

Consider a linear discrete model of the form

$$\begin{aligned} \mathbf{x}_{k+1} &= A_k \mathbf{x}_k + B_k \mathbf{u}_k + \mathbf{w}_k^x \\ \mathbf{y}_{k+1} &= C_k \mathbf{x}_{k+1} + \mathbf{v}_{k+1} \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in R^n$, $\mathbf{u}_k \in R^l$ and $\mathbf{y}_{k+1} \in R^m$ are the state, control input and output variables, respectively. $\mathbf{w}_k^x$ and $\mathbf{v}_{k+1}$ denote the white noise sequences of uncorrelated Gaussian random vectors with zero means and covariance matrices Q_k^x and R_{k+1}, respectively. The initial state $\mathbf{x}_0$ is specified as a random Gaussian vector with mean $\tilde{\mathbf{x}}_0$ and covariance $\tilde{P}_0$.

To include the possible loss of control effectiveness in the model, l control effectiveness factors, $-1 \leqslant \gamma_{ik} \leqslant 0$, $i = 1, \ldots, l$, are introduced as functions of discrete time k. The state equation that reflects the situation is

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + [\mathbf{b}_1 \gamma_k^1 \;\; \mathbf{b}_2 \gamma_k^2 \ldots \mathbf{b}_l \gamma_k^l] \begin{bmatrix} u_k^1 \\ u_k^2 \\ \vdots \\ u_k^l \end{bmatrix} + \mathbf{w}_k^x \tag{2}$$

or, more compactly,

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u}_k + E_k \boldsymbol{\gamma}_k + \mathbf{w}_k^x \tag{3}$$

where E_k is defined by

$$E_k = B_k U_k \tag{4}$$

and

$$U_k = \begin{bmatrix} u_k^1 & 0 & \cdots & 0 \\ 0 & u_k^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & u_k^l \end{bmatrix}, \qquad \boldsymbol{\gamma}_k = \begin{bmatrix} \gamma_k^1 \\ \gamma_k^2 \\ \vdots \\ \gamma_k^l \end{bmatrix} \tag{5}$$

Since our ultimate goal is to gain the knowledge of the amount of loss in the control effectiveness so that some adaptive control mechanism can ensue, estimates in a recursive form are most desirable. In the absence of the knowledge on the evolution of the effectiveness factors, a description in the form of a random bias with a large additive noise covariance is appropriate. Thus

$$\boldsymbol{\gamma}_{k+1} = \boldsymbol{\gamma}_k + \mathbf{w}_k^{\gamma} \tag{6}$$

The covariance for γ_0 should be small because the effectiveness factors are assumed to start at the normal values ($=0$), and the additive noise covariance should be large so that the monotonous evolution of the bias state will largely be disregarded in the estimates. On the other hand, the constant nature of the bias state, whenever it holds true, can help reduce the steady-state estimation error if the covariance of the additive noise is sufficiently small. Therefore, the covariance of $\mathbf{w}_k^{\gamma}$ can play a significant role as a design parameter in trading off between the convergence speed and the steady-state accuracy for the faulty parameter estimation. Since there is no *a priori* information on the times when the effectiveness factors change, the noise covariance is set to be time invariant and diagonal. When information is available with regard to the probability of fault occurrence in each control channel, the corresponding variance in the additive noise covariance matrix should be assigned proportionally. Further discussion on this covariance as a design parameter is given in the next section in relation to the design of forgetting factors.

Thus the bias augmented model has the following form:

$$\begin{aligned} \mathbf{x}_{k+1} &= A_k \mathbf{x}_k + B_k \mathbf{u}_k + E_k \gamma_k + \mathbf{w}_k^x \\ \gamma_{k+1} &= \gamma_k + \mathbf{w}_k^{\gamma} \\ \mathbf{y}_{k+1} &= C_{k+1} \mathbf{x}_{k+1} + \mathbf{v}_{k+1} \end{aligned} \tag{7}$$

where the noise sequences $\mathbf{w}_k^x$, $\mathbf{w}_k^{\gamma}$, and $\mathbf{v}_k$ are assumed to be zero mean uncorrelated white Gaussian noise sequences with

$$E = \left\{ \begin{bmatrix} \mathbf{w}_k^x \\ \mathbf{w}_k^{\gamma} \\ \mathbf{v}_k \end{bmatrix} [\mathbf{w}_j \;\; \mathbf{w}_j^{\gamma} \;\; \mathbf{v}_j] \right\} = \begin{bmatrix} Q^x & 0 & 0 \\ 0 & Q^{\gamma} & 0 \\ 0 & 0 & R \end{bmatrix} \delta_{kj} \tag{8}$$

where $Q^x > 0$, $Q^{\gamma} > 0$, $R > 0$ and δ_{kj} is the Kronecker delta. The initial states $\mathbf{x}_0$ and $\boldsymbol{\gamma}_0$ are assumed to be uncorrelated with the white noise processes $\mathbf{w}_k^x$, w_k^{γ}, and $\mathbf{v}_k$.

The minimum variance solution is obtained by a direct application of the two-stage Kalman filter algorithm [15], with constant coefficient matrices in Reference [15] replaced by time-varying matrices.

Optimal bias estimator

$$\hat{\boldsymbol{\gamma}}_{k+1|k} = \hat{\boldsymbol{\gamma}}_{k|k} \tag{9}$$

$$P^{\gamma}_{k+1|k} = P^{\gamma}_{k|k} + Q^{\gamma}_{k} \tag{10}$$

$$\hat{\boldsymbol{\gamma}}_{k+1|k+1} = \hat{\boldsymbol{\gamma}}_{k+1|k} + K^{\gamma}_{k+1}(\tilde{\mathbf{r}}_{k+1} - H_{k+1|k}\hat{\boldsymbol{\gamma}}_{k|k}) \tag{11}$$

$$K^{\gamma}_{k+1} = P^{\gamma}_{k+1|k}H^{\mathrm{T}}_{k+1|k}(H_{k+1|k}P^{\gamma}_{k+1|k}H^{\mathrm{T}}_{k+1|k} + \tilde{S}_{k+1})^{-1} \tag{12}$$

$$P^{\gamma}_{k+1|k+1} = (I\text{-}K^{\gamma}_{k+1}H_{k+1|k})P^{\gamma}_{k+1|k} \tag{13}$$

Bias-free state estimator

$$\tilde{\mathbf{x}}_{k+1|k} = A_k\tilde{\mathbf{x}}_{k|k} + B_k\mathbf{u}_k + W_k\hat{\boldsymbol{\gamma}}_{k|k} - V_{k+1|k}\hat{\boldsymbol{\gamma}}_{k|k} \tag{14}$$

$$\tilde{P}^{x}_{k+1|k} = A_k\tilde{P}^{x}_{k|k}A^{\mathrm{T}}_{k} + Q^{x}_{k} + W_kP^{\gamma}_{k|k}W^{\mathrm{T}}_{x} - V_{k+1|k}P^{\gamma}_{k+1|k}V^{\mathrm{T}}_{k+1|k},$$

$$\tilde{\mathbf{x}}_{k+1|k+1} = \tilde{\mathbf{x}}_{k+1|k} + \tilde{K}^{x}_{k+1}(\mathbf{y}_{k+1} - C_{k+1}\tilde{\mathbf{x}}_{k+1|k}) \tag{15}$$

$$\tilde{K}^{x}_{k+1} = \tilde{P}^{x}_{k+1|k}C^{\mathrm{T}}_{k+1}(C_{k+1}\tilde{P}^{x}_{k+1|k}C^{\mathrm{T}}_{k+1} + R_{k+1})^{-1} \tag{16}$$

$$\tilde{P}^{x}_{k+1|k+1} = (I - \tilde{K}^{x}_{k+1}C_{k+1})\tilde{P}^{x}_{k+1|k} \tag{17}$$

where the filter residual and its covariance are given as

$$\tilde{\mathbf{r}}_{k+1} = \mathbf{y}_{k+1} - C_{k+1}\tilde{\mathbf{x}}_{k+1|k} \tag{18}$$

$$\tilde{S}_{k+1} = C_{k+1}\tilde{P}^{x}_{k+1|k}C^{\mathrm{T}}_{k+1} + R_{k+1} \tag{19}$$

Coupling equations

$$W_k = A_kV_{k|k} + E_k \tag{20}$$

$$V_{k+1|k} = W_kP^{\gamma}_{k|k}(P^{\gamma}_{k+1|k})^{-1} \tag{21}$$

$$H_{k+1|k} = C_{k+1}V_{k+1|k} \tag{22}$$

$$V_{k+1|k+1} = V_{k+1|k} - \tilde{K}^{x}_{k+1}H_{k+1|k} \tag{23}$$

And finally the compensated state and error covariance estimates

$$\hat{\mathbf{x}}_{k+1|k+1} = \tilde{\mathbf{x}}_{k+1|k+1} + V_{k+1|k+1}\hat{\boldsymbol{\gamma}}_{k+1|k+1} \tag{24}$$

$$P_{k+1|k+1} = \tilde{P}^{x}_{k+1|k+1} + V_{k+1|k+1}P^{\gamma}_{k+1|k+1} + V^{\mathrm{T}}_{k+1|k+1} \tag{25}$$

This last state estimate, instead of that given in (15), is **the** correct state estimate. It is used as the feedback variable to regulate the output of the plant described by (7). One may recall the separation principle and consider the control law of the form

$$\mathbf{u}_k = -F^{x}_{k}\hat{\mathbf{x}}_{k|k} - F^{\gamma}_{k}\hat{\boldsymbol{\gamma}}_{k|k} \tag{26}$$

The analysis of the closed-loop stability with the above control law involves non-linearity because E_k in (7) is dependent on the control input. Section 4 considers feeding back only the state estimate ($F_k^\gamma = 0$), i.e.,

$$\mathbf{u}_k = -F_k^x \hat{\mathbf{x}}_{k|k} \tag{27}$$

In this case the regulator stability analysis becomes tractable with the aid of some existing results on robust stability of interval systems. The details are given in Section 4.

3. ADAPTIVE ESTIMATION OF CONTROL EFFECTIVENESS FACTORS

In this section, a further measure is taken to modify the above filtering algorithm so that the estimates become more responsive to abrupt changes in the control effectiveness factors.

A well-known technique for estimating time-varying parameters is the use of forgetting factors. The basic idea is to enable a recursive algorithm to discount the past information so that the filter is more apt to recognize the changes in the system. Since the time update of the bias estimate governed by (9) is the dominant opposing force to acknowledge the abrupt changes in the biases, forgetting factors introduced into the time-propagation equation (10) of the bias covariance is likely to function most effectively.

In possibly the simplest consideration, a single constant forgetting factor is used, i.e.

$$P_{k+1|k}^\gamma = P_{k|k}^\gamma/\lambda + Q_k^\gamma, \quad 0 < \lambda \leqslant 1 \tag{28}$$

In this case the old information is discounted uniformly in time and in space. A main concern with this technique is the possible 'blow-up' of the estimation error covariance matrix. Some alternatives for improvement have been proposed, and used in recursive least-squares-based parameter identification schemes, where the forgetting factor is non-uniform in time or in space [18, 14].

Assume that covariance $P_{k|k}^\gamma$ adequately describes the bias estimation error along both temporal and spacial directions under the normal system operation condition. Then this covariance provides a basis for the selection of forgetting factors. The bias estimates should be prevented from being impetuous ($P_{k|k}^\gamma$ too large), as well as from being indifferent ($P_{k|k}^\gamma$ to small) to the changes shown in the measurements. A technique suggested in Reference [14] amounts to select forgetting factors that would force the adjusted covariance in (10) to stay within some prescribed bounds

$$\sigma_{\min} I \leqslant P_{k+1|k}^\gamma \leqslant \sigma_{\max} I \tag{29}$$

where $\sigma_{\min}, \sigma_{\max}$ are positive constants with $0 < \sigma_{\min} < \sigma_{\max} < \infty$, and I is the identity matrix. Let the dyadic expansion of $P_{k|k}^\gamma$ be given by

$$P_{k|k}^\gamma = \sum_{i=1}^{l} \alpha_{k|k}^i e_k^i (e_k^i)^{\mathrm{T}} \tag{30}$$

where $\alpha^1_{k|k}, \ldots, \alpha^l_{k|k}$ are the eigenvalues of $P^\gamma_{k|k}$ with $\alpha^1_{k|k} \geqslant \cdots \geqslant \alpha^l_{k|k}$, and $e^1_k, \ldots, e^l_k$ are the corresponding eigenvectors with $\|e^1_k\| = \cdots = \|e^l_k\| = 1$. Equation (10) can then be expressed as

$$P^\gamma_{k+1|k} = \sum_{i=1}^{l} \frac{\alpha^i_{k|k}}{\lambda^i_k} e^i_k (e^i_k)^{\mathrm{T}} + Q^\gamma_k, \quad 0 < \lambda^i_k \leqslant 1 \tag{31}$$

Following the argument in Reference [14], the forgetting factor λ^i_k can be chosen as a decreasing function of the amount of information received in the direction e^i_k. Since eigenvalue $\alpha^i_{k|k}$ of $P^\gamma_{k|k}$ is a measure of the uncertainty in the direction of e^i_k, a choice of forgetting factor λ^i_k based on the above constraints can be

$$\lambda^i_k = \begin{cases} 1, & \alpha^i_{k|k} > \alpha_{\max} \\ \alpha^i_{k|k}\left[\alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{\alpha_{\max}} \alpha^i_{k|k}\right]^{-1}, & \alpha^i_{k|k} \leqslant \alpha_{\max} \end{cases} \tag{32}$$

This selection of forgetting factors guarantees that the lower bound in (29) is satisfied. Under the condition that $Q^\gamma_k = 0$, $\forall k$, the upper bound in (29) is also satisfied. This condition however, contradicts our conclusion drawn from the previous section that Q^γ_k ought to be sufficiently large in order to weaken the bias evolution constraint described in (5). In fact, the forgetting factors introduced in (32) can be thought of as a way of adding a noise term with a varying covariance. Let $\bar{Q}^\gamma_k$ be the equivalent noise covariance. $\bar{Q}^\gamma_k$ relates to the forgetting factors through the following relation:

$$\bar{Q}^\gamma_k = \sum_{i=1}^{l} \left(\frac{1}{\lambda^i_k} - 1\right) \alpha^i_{k|k} e^i_k (e^i_k)^{\mathrm{T}} \geqslant 0$$

In comparison with the filtering algorithm described in the previous section, the modified algorithm is given by (9)–(23) with (10) replaced by (31) and (32). With the insertion of forgetting factors, the filter algorithm described in the previous section loses its optimality in the steady-state-bias accuracy. But our goal to render the estimates more responsive to changes in control effectiveness is achieved. It was observed in Reference [19] that in general a reinitialization is necessary in order for a filtering algorithm to correctly estimate parameters that characterize a subsequent fault. By using the above forgetting factor technique, this reinitialization issue can be avoided.

The rest of the section is devoted to the discussion of handling faulty situations that necessitate dramatic measures, such as the reconfiguration of the control law. Such situations need to be evaluated with great care. Our discussion is confined to the case of possible loss of control effectiveness in a system. Based on the bias estimates, statistical variables can be constructed for hypothesis tests. Under the normal condition, the ith component $\hat{\gamma}^i_{k|k}$ of the bias estimate $\hat{\boldsymbol{\gamma}}_{k|k}$ is a zero mean Gaussian variable. Define the weighted sum-squared bias estimate (WSSBE) as

$$d^i_k = \frac{1}{L} \sum_{j=k-L+1}^{k} (\hat{\gamma}^i_{j|j})^2 / P^{\gamma^i}_{j|j} \tag{33}$$

where $P^{\gamma^i}_{k|k}$ is the ith diagonal element of $P^{\gamma}_{k|k}$. d^i_k is small until there is a reduction of effectiveness in the ith control input channel. Therefore the following hypothesis test can be used:

$$d^i_k \underset{H_0}{\overset{H_i}{\gtreqless}} \varepsilon_i, \quad i = 1, \ldots, l \tag{34}$$

where $H_0 = \{$no significant reduction of effectiveness in ith control input$\}$, $H_i = \{i$th control input has significant reduction of effectiveness$\}$. What the designer means by significant reduction should be captured in the selection of thresholds ε_i, $i = 1, \ldots, l$. The threshold determination is beyond the scope of discussion in this paper.

When $P^{\gamma^i}_{k|k}$ adequately represents the variance of $\hat{\gamma}^i_{k|k}$, the WSSBE variable d^i_k is a chi-square random variable with L degrees of freedom, where L denotes the data window length. With the aid of a chi-square distribution table, it is possible to determine the probability that a fault of a certain severity has occurred, as a function of the window length L and the decision threshold ε_i. The selection of window length and decision threshold is a trade-off between the probability P_F of false alarm (declares H_1 and H_0) and the probability P_M of a missed detection (declares H_0 when H_1).

4. ADAPTIVE REGULATION AND ROBUSTNESS ANALYSIS

Following up the discussion at the end of Section 2, the control law of the form

$$\mathbf{u}_u = -F_k \hat{\mathbf{x}}_{k|k} \tag{35}$$

is attempted in this section to regulate the plant output $\mathbf{y}_k$, where $\hat{\mathbf{x}}_{k|k}$ is given in (24). In the subsequent development, A_k, B_k, and C_k in (7) are assumed to be constant matrices. Rearrange (2) and close the feedback loop to obtain

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B(I + \mathbf{\Gamma}_k)\mathbf{u}_k + \mathbf{w}^x_k, \; \mathbf{\Gamma}_k = \mathrm{diag}\{\gamma^1_k, \ldots, \gamma^l_k\} \tag{36}$$

$$= A\mathbf{x}_k - B(I + \mathbf{\Gamma}_k)F_k\hat{\mathbf{x}}_{k|k} + \mathbf{w}^x_k$$

$$= [A - B(I + \mathbf{\Gamma}_k)F_k]\mathbf{x}_k + B(I + \mathbf{\Gamma}_k)F_k(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}) + \mathbf{w}^x_k \tag{37}$$

The stability of the system described in (37) is defined by the boundedness of its response to any bounded inputs. $\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$ can be regarded as a part of the input, provided that it can be shown to be a bounded signal for all k regardless of the choice of F_x. We appeal to the separation principle in dealing with a feedback structure that involves the combination of an estimate feedback control law and a state estimator, so that the regulator stability analysis can be divided into two steps.

Step 1: The conditions that ensure the boundedness of $\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$ $\forall k$ are first stated. The algorithm presented in Section 2 gives the optimal estimates for both $\hat{\mathbf{x}}_{k|k}$ and $\hat{\boldsymbol{\gamma}}_{k|k}$ when (7) accurately describes the process under consideration. In this case, the expectations of the estimation errors are $\mathrm{tr}(P^{\gamma})$ for $\boldsymbol{\gamma}_k - \hat{\boldsymbol{\gamma}}_{k|k}$ and to $\mathrm{tr}(P)$ for $\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$, where P^{γ} and P are given by (13) and (25), respectively. The boundedness of the traces requires that the system described in (7) is uniformly completely controllable from $\mathbf{w}_k^x$ and $\mathbf{w}_k^\gamma$, and uniformly completely reconstructible [20]. These conditions for the special problem considered in this paper can be translated into the existence of an integer $k \geqslant 1$ and positive constants α_{0c}, α_{1c}, β_{0c}, β_{0c}, α_{0o}, α_{1o}, β_{0o}, and β_{1o}, such that for all i_0 and for all i_1,

$$\sum_{i=i_0}^{i_0+k-1} \mathbf{\Phi}(i_0+k, i+1)\mathbf{\Phi}^{\mathrm{T}}(i_0+k, i+1) > 0$$

$$\alpha_{0c} I \leqslant \left[\sum_{i=i_0}^{i_0+k-1} \mathbf{\Phi}(i_0+k, i+1)\mathbf{\Phi}^{\mathrm{T}}(i_0+k, i+1)\right]^{-1} \leqslant \alpha_{1c} I$$

$$\beta_{0c} I \leqslant \mathbf{\Phi}^{\mathrm{T}}(i_0+k, i_0)\left[\sum_{i=i_0}^{i_0+k-1} \mathbf{\Phi}(i_0+k, i+1)\mathbf{\Phi}^{\mathrm{T}}(i_0+k, i+1)\right]^{-1} \mathbf{\Phi}(i_0+k, i_0) \leqslant \beta_{1c} I$$

$$\sum_{i=i_1-k+1}^{i_i} \mathbf{\Phi}^{\mathrm{T}}(i, i_1-k+1)\begin{bmatrix} C^{\mathrm{T}}C & 0 \\ 0 & 0 \end{bmatrix}\mathbf{\Phi}(i, i_1-k+1) > 0$$

$$\alpha_{0o} I \leqslant \left[\sum_{i=i_1-k+1}^{i_1} \mathbf{\Phi}^{\mathrm{T}}(i, i_1-k+1)\begin{bmatrix} C^{\mathrm{T}}C & 0 \\ 0 & 0 \end{bmatrix}\mathbf{\Phi}(i, i_1-k+1)\right]^{-1} \leqslant \alpha_{1o} I$$

$$\beta_{0o} I \leqslant \mathbf{\Phi}(i_1, i_1-k)\left[\sum_{i=i_1-k+1}^{i_1} \mathbf{\Phi}^{\mathrm{T}}(i, i_1-k+1)\begin{bmatrix} C^{\mathrm{T}}C & 0 \\ 0 & 0 \end{bmatrix}\mathbf{\Phi}(i, i_1-k+1)\right]^{-1} \mathbf{\Phi}^{\mathrm{T}}(i_1, i_1-k) \leqslant \beta_{1o} I$$

where

$$\mathbf{\Phi}(i_1, i_0) = \begin{bmatrix} A^{i_1-i_0} & \sum_{j=0}^{i_1-i_0-1} A^{i_1-i_0-1-j} B U_{i_0+j} \\ 0 & I \end{bmatrix}, \quad i_1 > i_0 \tag{38}$$

is the transition matrix of the system described in (7) with U_k given in (5), and $\mathbf{\Phi}(i, i) = I$. From a different perspective it is seen that the presence of the noise term in the bias equation is necessary in order to satisfy the condition of uniformly completely controllable from $\mathbf{w}_k^\gamma$. Note that the expression of $\mathbf{\Phi}(i_1, i_0)$ in (38) contains a matrix of control signals. As a consequence, the boundedness and non-singularity of several of the above matrices depend on the control signals being persistently exciting. The feedback structure guarantees that whenever an observable fault occurs, the control signal would attempt to counteract its effect, and therefore becomes exciting.

Step 2: The selection of F_k to guarantee the stability of (37) is now discussed with the focus on the effect of replacing $\mathbf{\Gamma}_k$ by $\hat{\mathbf{\Gamma}}_k = \mathrm{diag}\{\hat{\gamma}_1, \ldots, \hat{\gamma}_l\}_{k|k}$. From (37) it can be seen that the selection of a stabilizing F_k is dictated by the values of A and $B(I + \mathbf{\Gamma}_k)$. It is noted that the change in F_k becomes necessary only if $\mathbf{\Gamma}_k$ changes. Since the diagonal elements in $\mathbf{\Gamma}_k$ characterize the reduction of control effectiveness, they can be reasonably assumed to change only a finite number of times during the course of a single system operation. If in addition, the duration between any consecutive changes in control effectiveness is longer than the time required for a transient to settle, the design of F_k can be much simplified. Let us name the above assumptions as the infrequent control impairment assumptions. Under these assumptions, F_k can be made adaptive to the change in $\mathbf{\Gamma}_k$ by ensuring that $A - B(I + \mathbf{\Gamma}_k)F_k$ a stability matrix for all k. As a side remark, it can be seen that the recovery of the system performance is impossible when $(A, B + B\mathbf{\Gamma}_k)$ becomes uncontrollable. Therefore, failures can be tolerated at most to the extent manageable by the redundant control authority.

On the other hand, though A and B are known, $\mathbf{\Gamma}_k$ is not. Only an estimate $\hat{\mathbf{\Gamma}}_k$ of $\mathbf{\Gamma}_k$ is available at any given time. Therefore, the design of F_k can only be based on the knowledge of $\hat{\mathbf{\Gamma}}_k$, besides that of A and B. To see the effect of replacing $\mathbf{\Gamma}_k$ by $\hat{\mathbf{\Gamma}}_k$, (37) is rewritten as

$$\mathbf{x}_{k+1} = [A - B(I + \hat{\mathbf{\Gamma}}_k)F_k + B(\hat{\mathbf{\Gamma}}_k - \mathbf{\Gamma}_k)F_k]\mathbf{x}_k + B(I + \mathbf{\Gamma}_k)F_k(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}) + \mathbf{w}_k^x \tag{39}$$

Let

$$\mu_{\gamma_k^i} = \hat{\gamma}_{k|k}^i - \gamma_k^i$$

for the ith control effectiveness factor at instant k, and write B and F_k in terms of their columns and rows, respectively, as

$$B = [\mathbf{b}_1 \;\; \mathbf{b}_2 \cdots \mathbf{b}_l], \quad F_k = \begin{bmatrix} \mathbf{f}_k^1 \\ \mathbf{f}_k^2 \\ \vdots \\ \mathbf{f}_k^l \end{bmatrix}$$

Define

$$E_k^i = \mathbf{b}_i \mathbf{f}_k^i, \quad i = 1, \ldots, 1 \tag{40}$$

$$M(\mathbf{\Gamma}_k) = A - B(I + \mathbf{\Gamma}_k)F_k \tag{41}$$

$$M(\hat{\mathbf{\Gamma}}_k) = A - B(I + \hat{\mathbf{\Gamma}}_k)F_k \tag{42}$$

$$\Delta M(\hat{\mathbf{\Gamma}}_k, \Delta\mathbf{\Gamma}_k) = B(\hat{\mathbf{\Gamma}}_k - \mathbf{\Gamma})F_k \tag{43}$$

Then ΔM can be expressed as

$$\Delta M(\hat{\mathbf{\Gamma}}_k, \Delta\mathbf{\Gamma}_k) = \sum_{i=1}^{l} \mu_{\gamma_k^i} E_k^i \tag{44}$$

Suppose the FDI scheme has a decision mechanism, such as that given in (34), that sends a current value of $\hat{\mathbf{\Gamma}}_k$ to the controller whenever the hypothesis test has confirmed a significant change in the value of γ_k^i for some i at some k. F_k is then re-calculated based on this $\hat{\mathbf{\Gamma}}_k$.

It has been shown [17] that if

$$\operatorname{rank}(E_k^i) = 1, \quad \forall i, k \tag{45}$$

which holds by definition (40), the coefficients of the characteristic polynomial of $M(\mathbf{\Gamma}_k)$ are multilinear functions of $\gamma_k^1, \ldots, \gamma_k^l$. In this case, the Mapping Theorem can be applied to derive a procedure [17 (Chapter 12, p. 512)] that determines the maximum ε_k so that matrix $M(\mathbf{\Gamma}_k)$ remains stable under all perturbations ranging over

$$\hat{\gamma}^i_{k|k} - \omega^i_k \varepsilon_k \leqslant \gamma^i_k \leqslant \hat{\gamma}^i_{k|k} + \omega^i_k \varepsilon_k, \quad i = 1, \ldots, l \tag{46}$$

for predetermined weights ω^i_k.

Although the determination of such perturbation boundaries does not result in a robust stabilization because our estimation algorithm is probabilistic rather than deterministic, it can assist through simulations to determine what level of accuracy in the control effectiveness estimates can be expected, and how long it takes for the estimates to converge to values sufficiently close to the true values, so that a re-design of F_k can allow the recovery of the closed-loop performance.

In the following development, the infrequent control impairment assumptions hold. F_k is obtained and updated as the steady-state LQ solution of a fictitious LQG problem [21] which minimizes

$$J = \lim_{N \to \infty} \frac{1}{N} \left\{ \sum_{k=0}^{N-1} (\mathbf{y}_k^{\mathrm{T}} Q_c \mathbf{y}_k + \mathbf{u}_k^{\mathrm{T}} R_c \mathbf{u}_k) \right\} \tag{47}$$

subject to the constraint

$$\begin{aligned} \mathbf{x}_{k+1} &= A\mathbf{x}_k + B(I + \hat{\mathbf{\Gamma}})\mathbf{u}_k + \mathbf{w}^x_k \\ \mathbf{y}_k &= C\mathbf{x}_k + \mathbf{v}_k \end{aligned} \tag{48}$$

where Q_c is a positive-semi-definite weighting matrix, and R_c is a positive-definite weighting matrix. The FDI unit is responsible for updating $\hat{\mathbf{\Gamma}}$ and activating the re-design of F_k when necessary. The current control law, given the validated control effectiveness estimate $\hat{\mathbf{\Gamma}}$, is

$$\mathbf{u}_k = -F_k \hat{\mathbf{x}}_k$$

where $\hat{\mathbf{x}}_k$ is estimated using the two-stage adaptive Kalman estimator given in Section 2, and

$$F_k = (R_c + (I + \hat{\mathbf{\Gamma}})B^{\mathrm{T}} P_c B(I + \hat{\mathbf{\Gamma}}))^{-1}(I + \hat{\mathbf{\Gamma}})B^{\mathrm{T}} P_c A \tag{49}$$

where P_c satisfies the algebraic Riccati equation

$$P_c = C^{\mathrm{T}} Q_c C + (A - B(I + \hat{\mathbf{\Gamma}})F_k)^{\mathrm{T}} P_c (A - B(I + \hat{\mathbf{\Gamma}})F_k) + F_k^{\mathrm{T}} R_c F_k$$

Note that the fictitious system in (48) is different from the bias augmented system (7) upon which the estimator is built. Therefore, the control gain F_k is not optimal in the LQG sense in either situation. Nevertheless, the analysis in this section has shown that as long as the estimates are sufficiently accurate, so that the true values of the control effectiveness factors fall within the bounds given by (46), the regulator stability is guaranteed.

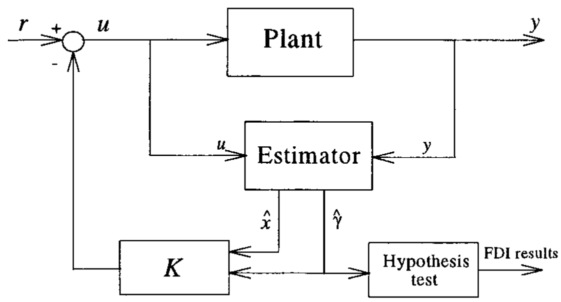


Figure 1. Regulator configuration.

A further consideration on how to maintain the closed-loop performance in the face of the reduction of control effectiveness can be to adjust the values of Q_c and R_c in the process of updating F_k. As suggested in Reference [22], the automatic redesign can be done by fixing the state weighting matrix Q_c and choosing a new value for R_c in an appropriate manner to offset the effect of control effectiveness reduction whenever it occurs. Figure 1 depicts the configuration of the proposed scheme.

5. EXAMPLES AND SIMULATION RESULTS

The effectiveness of the filtering algorithm and the correctness of the regulator design approach presented in earlier sections are demonstrated in this section through two examples. In the first example, a longitudinal aircraft model [23] is used mainly to demonstrate that the control effectiveness estimator can work effectively in the closed-loop setting. The reader is referred to Reference [16] for open-loop simulation results with the same model and the same test scenarios. The second example is taken from [24] where it was used to evaluate a reliable linear-quadratic state-feedback control method. The intention here is to show that the control effectiveness accommodation mechanism developed in this paper can work successfully.

5.1. Example 1

In this example, five scenarios involving reduction of control effectiveness are simulated. The example considers longitudinal dynamics only, which is taken to be decoupled from the lateral-directional dynamics. The aircraft model has four states: forward velocity, u, angle of attack, α, pitch rate, q, and pitch angle, θ; two control inputs: elevon, δ_e and canard δ_c; two outputs: angle of attack and pitch angle. The system, control and measurement matrices, A, B and C, in the state-space description at a given flight condition are given as follows:

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k + \mathbf{w}_k^x$$

$$\mathbf{y}_k = C\mathbf{x}_k + \mathbf{v}_k$$

Table I. Test scenarios

Scenarios	% of Control effectiveness	Time of effectiveness change				
		0	5.1	10.1	12.6	20
1	Elevon	100	25	25	25	25
	Canard	100	100	100	100	100
2	Elevon	100	50	0	0	0
	Canard	100	100	100	100	100
3	Elevon	100	50	50	50	50
	Canard	100	100	100	50	50
4	Elevon	100	50	10	10	10
	Canard	100	100	100	50	50
5	Elevon	100	50^{ramp}	10	10	10
	Canard	100	100	100	50	50

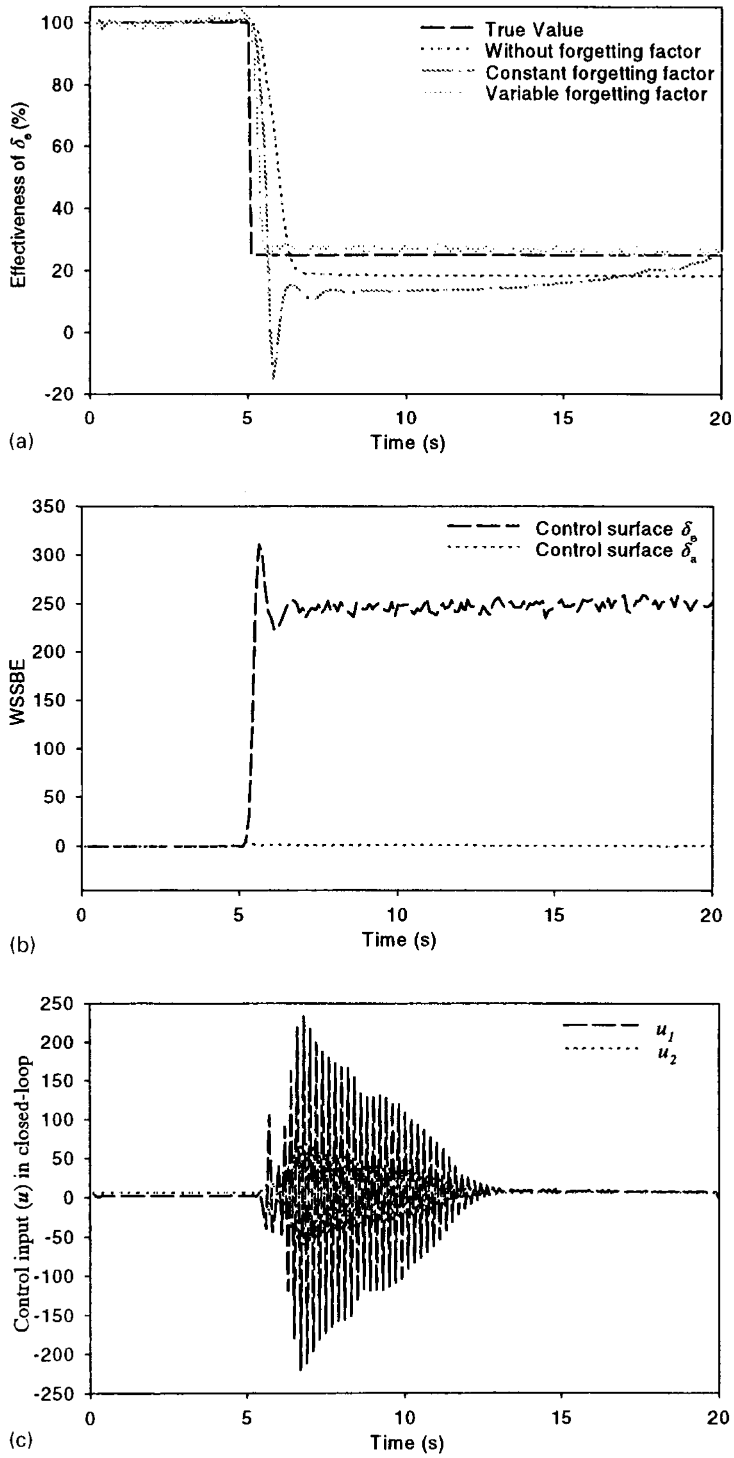
where

$$\mathbf{x} = [u \;\; \alpha \;\; q \;\; \theta]^{\mathrm{T}}, \quad \mathbf{u} = [\delta_{\mathrm{e}} \;\; \delta_{\mathrm{c}}]^{\mathrm{T}}, \quad \mathbf{y} = [\alpha \;\; \theta]^{\mathrm{T}}$$

$$A = \begin{bmatrix} -0.0226 & -36.6 & -18.9 & -32.1 \\ 0.0 & -1.9 & 0.983 & 0.0 \\ 0.0123 & -11.7 & -2.63 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0 & 0.0 \\ -0.414 & 0.0 \\ -77.8 & 22.4 \\ 0.0 & 0.0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Other parameters used in the simulation are given as follows. $Q^x = \mathrm{diag}\{0.01^2, 0.01^2, 0.01^2, 0.01^2\}$, $R = \mathrm{diag}\{0.1^2, 0.1^2\}$, $Q^\gamma = \mathrm{diag}\{0.001^2, 0.001^2\}$, $\mathbf{x}_0 = [50\; 0.05\; 5\; 0.06]^{\mathrm{T}}$ $\boldsymbol{\gamma}_0 = [0\; 0]^{\mathrm{T}}$. Initial parameters of filter are $\tilde{\mathbf{x}}_0 = \mathbf{x}_0$, $\hat{\boldsymbol{\gamma}}_0 = \boldsymbol{\gamma}_0$, $\tilde{P}_0 = 10I$, $P_0^\gamma = 10I$. For the output and control weighting matrices are chosen as $Q_c = \mathrm{diag}\{10, 50\}$ and $R_c = \mathrm{diag}\{10, 10\}$ for all k. The window length in (33) is $L = 3$ and the threshold in (34) is chosen as $\boldsymbol{\varepsilon} = [5\;\; 5]^{\mathrm{T}}$.

The reduction of control effectiveness is due to the actuator malfunction or the control surface impairment, such as stuck, floating, or partially missing elevons and/or canards. The five simulated fault scenarios are summarized in Table I. These scenarios include a reduction of control effectiveness in a single surface, two consecutive reductions of control effectiveness in a single surface, asynchronous reductions of control effectiveness in both surfaces, two consecutive reductions of control effectiveness in one surface followed by a reduction in the second surface, and an abrupt reduction and sequentially a gradual reduction in one surface, and followed by a reduction of control effectiveness in the second surface. In each case, the two-stage Kalman filter presented in Section 2 is used to obtain estimates of the control effectiveness, but first without forgetting factors (WFF), and then with the filter modified by inserting a constant forgetting factor (CFF), and finally with the filter modified by a set of time-varying forgetting factors (VFF) discussed in the previous section. The simulation results for all scenarios


True Value
Without forgetting factor
Constant forgetting factor
Variable forgetting factor
Effectiveness of δ_e (%)
Time (s)
(a)
Control surface δ_e
Control surface δ_a
WSSBE
Time (s)
(b)
u_1
u_2
Control input (u) in closed-loop
Time (s)
(c)

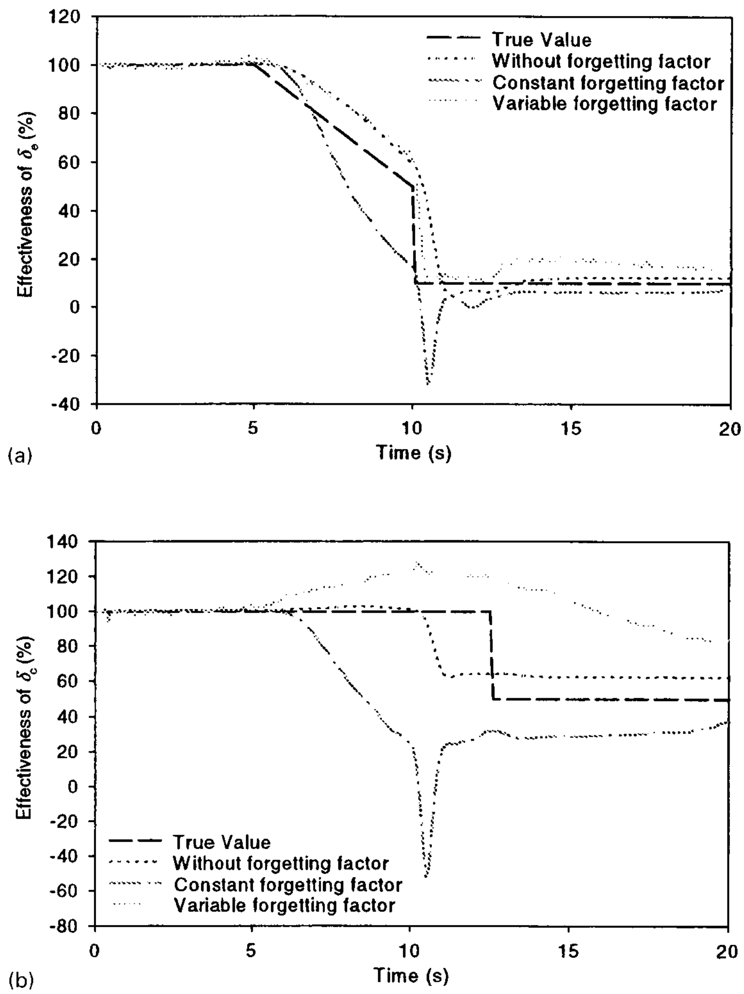


Figure 3. Effectiveness estimates for incipient and multiple channel occurrences: (a) effectiveness estimate for scenario 5 (closed-loop), (b) effectiveness estimate for scenario 5 (closed-loop).

demonstrate favourably on the optimal two-stage Kalman filter with variable forgetting factors. Figures 2–4 display the simulation results for selected test scenarios (1 and 5).

Figure 2. Effectiveness estimates and WSSBE for a single fault occurrence: (a) effectiveness estimate for scenario 1 (closed-loop), (b) WSSBE for scenario 1 (closed-loop), (c) control inputs in closed-loop with constant external input for scenario 1.

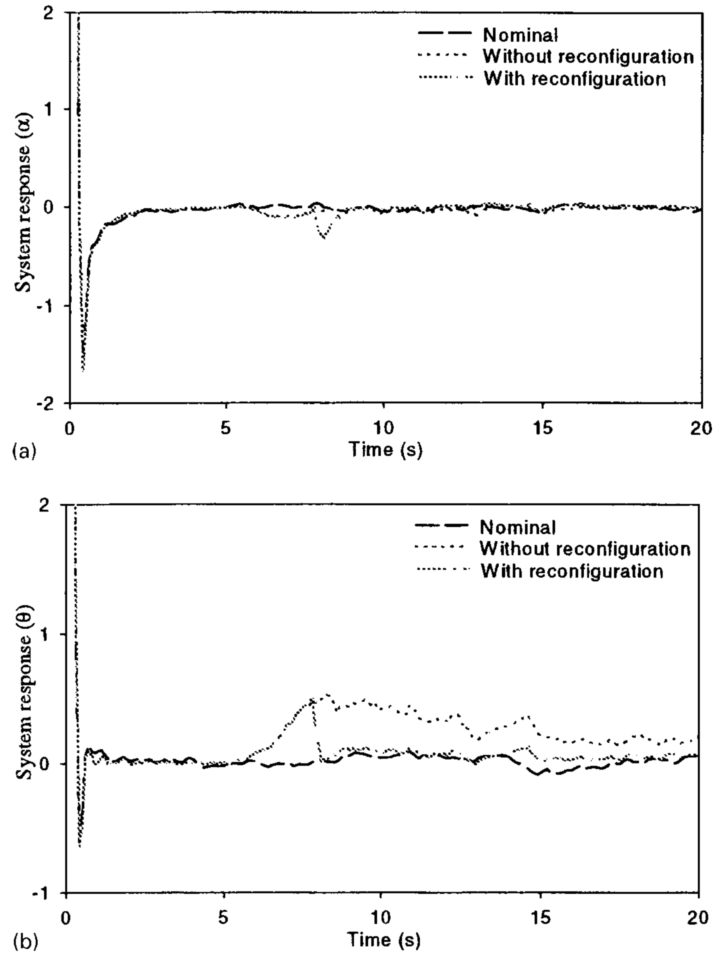


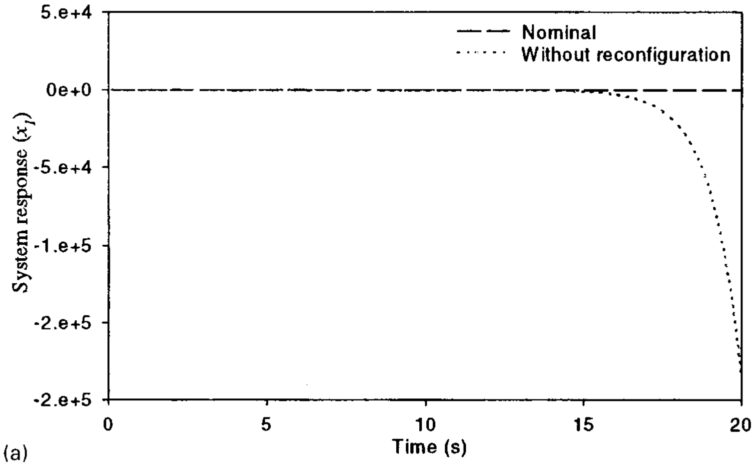
Figure 4. Regulated system response with elevon effectiveness reduction at $t = 5$ s: (a) closed-loop response of angle of attack with and without reconfiguration, (b) closed-loop response of pitch angle with and without reconfiguration.

5.2. Example 2

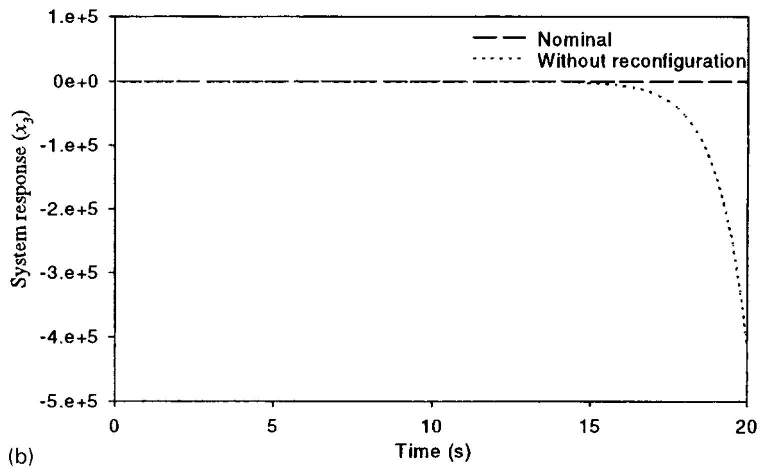
The state-space parameters of the model [24] are as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ -1 & -1 & 1 & 0 \\ 2 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that the plate is open-loop unstable and there is a lightly damped oscillatory mode. The control weighting matrices and the initial state for this example are chosen according to Reference



(a)



(b)

Figure 5. Regulator response with effectiveness loss in the first control channel at $t = 5$ s: (a) closed-loop response of the first state variable without reconfiguration, (b) closed-loop response of the third state variable without reconfiguration.

[24] as $Q_c = \mathrm{diag}\{1, 0, 1, 0\}$, $R_c = \mathrm{diag}\{1, 1\}$, $\mathbf{x}_0 = [1\ 0\ 1\ 0]^{\mathrm{T}}$. Process and measurement noises are added with $Q^x = \mathrm{diag}\{0.01^2, 0.01^2, 0.01^2, 0.01^2\}$, $R = \mathrm{diag}\{0.1^2, 0.1^2\}$, $Q^\gamma = \mathrm{diag}\{0.001^2, 0.001^2\}$, $\gamma_0 = [0\ 0]^{\mathrm{T}}$.

Figure 5 shows the first and third state variables of the system with a loss of first control effectiveness at $t = 5$ s. Without the control law adaptation after the loss of control effectiveness, the system becomes unstable. With an on-line re-design however, the regulator eventually settles down after a transient period as shown in Figure 6. Figure 7 shows the time history of WSSBE. The WSSBE of the system with an adaptive control law is more indicative of the change in the effectiveness of the first control. Figure 8 shows the effectiveness estimates in both control channels. More accurate estimates are obtained when the control law is adaptive.

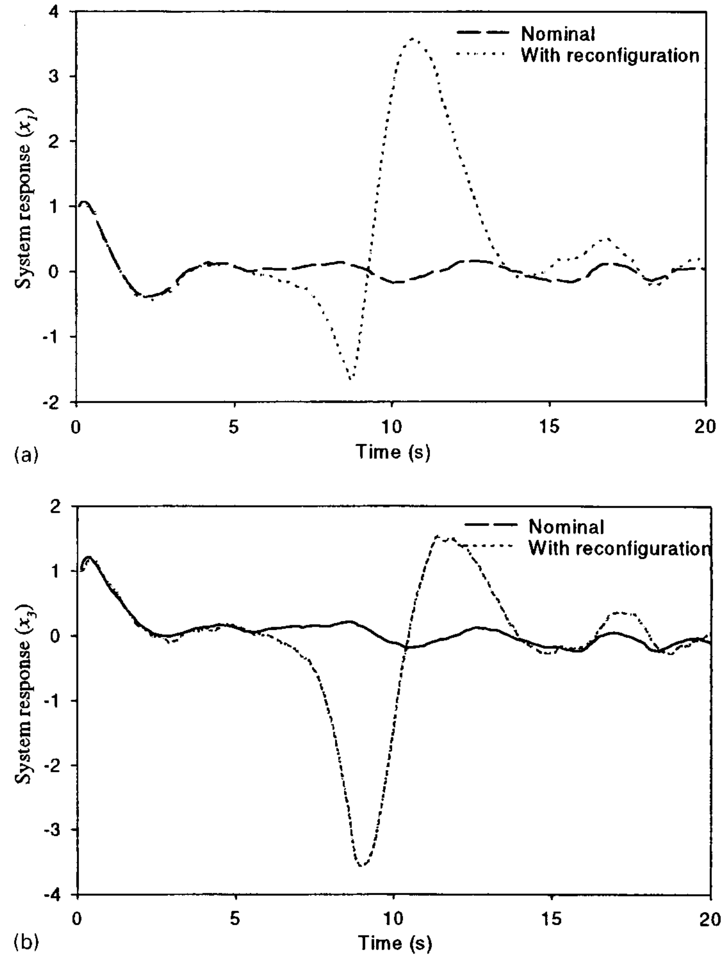


Figure 6. Regulator response with effectiveness loss in the first control channel at $t = 5$ s: (a) closed-loop response of first state variable with reconfiguration, (b) closed-loop response of third state variable with reconfiguration.

6. DISCUSSION AND CONCLUSIONS

An adaptive two-stage Kalman filter for the estimation and change detection of the control effectiveness has been developed for closed-loop feedback control systems. The algorithm is capable of simultaneously estimating the state and an unknown constant entering the state equation additively. Since it is the changes in the control effectiveness factors that are to be estimated, our development has been focused on sensitizing the filter estimates specifically to these changes. The objective has been achieved by introducing selective forgetting factors into the decoupled effectiveness factor estimator. Both abrupt and incipient reduction of control effectiveness in dynamic systems have been considered. Simulation results from Example 1 have

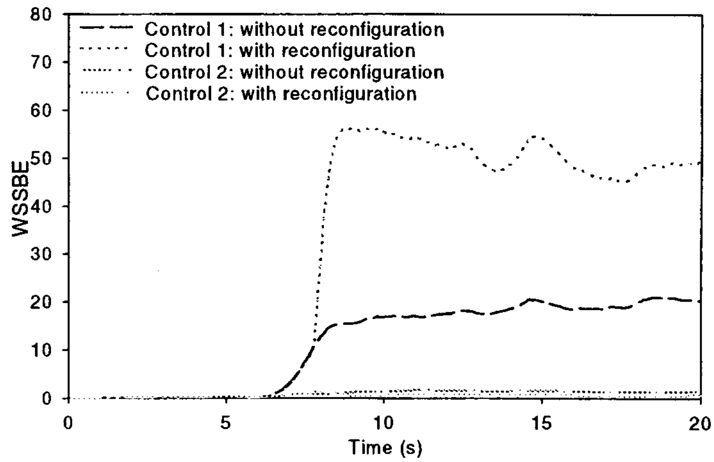


Figure 7. WSSBE with effectiveness loss in the first control channel at $t = 5$ s.

illustrated the effectiveness and superiority of our algorithm over the optimal two-stage Kalman filters without the forgetting factors. The feedback control law is solved in a fictitious LQG setting. The state estimate is fed back to regulate the system output, while the control effectiveness estimate is used to tune the gain in the control law. Tolerance of the control law to bias estimation error is analyzed. Simulation results from Example 2 have demonstrated the fault tolerance of the adaptive control law. Example 2 has also shown that, due to the more responsive control signal generated through the use of an adaptive control law, more pronounced detection results and more accurate estimation results have been produced.

One may wonder why is not the dual problem of sensor effectiveness factor estimation considered in this paper. It turns out that modeling the sensor effectiveness generates a bilinear term between the state and the bias in the output equation. One may start with one of the existing adaptive estimation schemes [25], and seek to improve the speed and accuracy of the estimates by exploiting the special structure of the non-linear model.

One of the most widely used fault detection and diagnosis approach for stochastic systems is the generalized likelihood ratio (GLR) approach originated in Reference [7] and modified in Reference [1]. The proposed two-stage adaptive Kalman filter is similar to the GLR approach in that it uses one single Kalman filter to not only detect a fault, it estimates the fault magnitude and identifies the fault occurrence time as well. In this paper, faults are detected and identified using a variable built upon the estimated fault parameters (control effectiveness factors), not on the Kalman innovations. The major advantage of our approach is that it gives accurate fault severity estimates, including sequential faults, without requiring any filter reinitialization.

A side issue which requires an equal attention is the issue of input scaling. Since the control effectiveness factors are directly attached to the control input vector, the signal (change in control effectiveness) to noise ratios are affected by both the magnitude and the direction of the input vector. As a result, estimates along some directions of input space at some time can have much poorer quality than estimates along other directions at the same time. Similarly, estimates along some directions of input space at some time can have much poorer quality than estimates along the same input direction but at a different time. Therefore, a focused effort is necessary in

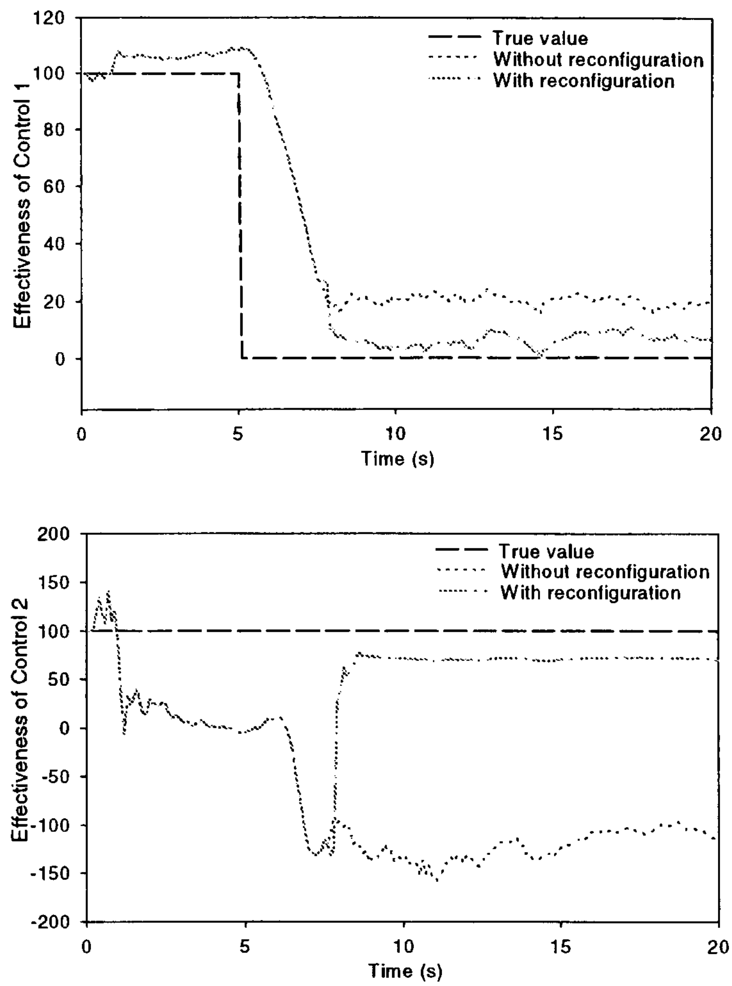


Figure 8. Control effectiveness estimates with effectiveness loss in the first control channel at $t = 5$ s.

developing an input scaling scheme that equalizes the quality of bias estimates along temporal and input-spacial directions whenever there are input excitations.

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