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A disturbance-decoupled adaptive observer and its application to faulty parameters estimation of a hydraulically driven elevator

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SUMMARY

In this paper, a disturbance-decoupled adaptive observer is designed for the joint state-parameter estimation of a system with unknown disturbance inputs. The proposed Robust Adaptive Observer (RAO) integrates an Unknown Input Observer (UIO) to the parameter estimation process, where the unknown parameters are estimated as extended states of the system. An auxiliary input is added to the UIO in coping with the estimation errors so that the exponential stability and convergence of the observer are guaranteed. The proposed observer is applied to a hydraulically driven elevator for the faulty parameter estimation. The simulation results show the accuracy of the observer and its robustness to both disturbances and measurement noises. Copyright © 2010 John Wiley & Sons, Ltd.

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KEY WORDS: robust adaptive observer; unknown input observer; joint state-parameter estimation

1. INTRODUCTION

The study of adaptive observers is traced back to the joint state-parameter estimation in adaptive control systems [1, 2]. On one hand, the unmeasured states are evaluated for the purpose of state feedback control; on the other hand, the unknown parameters are estimated online so that the controller can be updated accordingly. When certain persistent excitation requirements are satisfied, the estimated states and parameters converge to their real values simultaneously. With the increasing reliability requirements on control systems, adaptive observers have also been applied to the fault diagnosis and evaluation, where faults are modeled and evaluated as the deviation of parameters from their nominal values [3–6]. In the framework of stochastic problem formulation, a joint state-parameter estimation algorithm based on a modified Extended Kalman filter (EKF)—a two-stage Kalman filter—has been developed for fault diagnosis [7] and fault tolerant control [8] applications.

The approaches of adaptive observers normally fall into two categories. (1) The unknown parameters are taken as extra states of the system; a state observer is then constructed to estimate all extended states including the unknown parameters [9, 10]. (2) Parameters estimation techniques are applied first using the input–output data; the states of the system are then estimated with the updated information of parameters [11–15]. A general comparison of different adaptive observers is shown in [16] with a unified approach.

One concern regarding adaptive observers is the robustness to disturbances such as measurement noises, modeling inaccuracies, and unknown external inputs while providing the desired accurate

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estimation for both states and parameters. In [10, 15], the performance of adaptive observers is discussed for the noise corrupted systems. It is stated in [10] that the expectation of the estimation errors is bounded if the magnitude of noises is bounded. Furthermore, the expectation of estimation errors converges to zero for systems with independent noises of zero means. Therefore, an adaptive observer is, as shown in [17], Bounded-Input Bounded-Output (BIBO) stable to unknown external signals, which means the estimation of both parameters and states are affected by disturbance inputs such that persistent estimation errors exist as functions of the disturbance. Since the disturbance is unknown and consequently the size of these errors is also unknown to the system, the accuracy and the reliability of the estimation are impaired.

For a Multiple-Input Multiple-Output (MIMO) system, the robustness to structured disturbances can be further enhanced for an adaptive observer by utilizing the measurement redundancy of the system. With proper measurements, the influence of disturbances can be decoupled from the estimation errors. One technique is to incorporate with an Unknown Input Observer (UIO). A UIO [18, 19] is a disturbance-decoupled observer for the accurate state estimation for a system with unknown disturbance inputs. In [20], an adaptive UIO has been designed to estimate the faulty parameters of an aircraft actuator. The approach, however, needs full states (n independent) measurements and hence reduces the necessity of states estimation.

This paper developed a robust adaptive observer (RAO) which integrates a UIO to the parameters estimation process. A UIO is first constructed for disturbance decoupling; an auxiliary input is added to the UIO so that the stability of the observer and the exponential convergence of all estimations to their real values are guaranteed if the given requirements on the input signals are satisfied. One advantage of the proposed observer is that the estimations of both unknown states and unknown parameters are free of disturbance. The influence of disturbance is decoupled from all estimations so that the persistent estimation errors caused by the disturbance are eliminated. The other advantage is that, compared to [20], the observer requires less measurements for the same type of unknown parameters estimation. To test its validity, the method is then applied to evaluate sudden faulty parameter changes in a hydraulically driven elevator, which serves as a control actuator of an airplane.

The paper is organized as follows. The problem studied in this paper is formulated in Section 2. The proposed RAO is presented in Section 3 with the proof of stability and convergence. The mathematical model of a hydraulically driven elevator is introduced in Section 4. Section 5 shows the simulation results. The paper ends in Section 6 with conclusions.

2. PROBLEM STATEMENT

The problem addressed in this paper is the joint state-parameter estimation of a linear system with structural disturbances. The dynamics of the system is described by the following equation:

$$\begin{aligned} \dot{x} &= Ax+Bu+Ed+\phi\theta \\ y &= Cx \end{aligned} \tag{1}$$

where $A\in R^{n\times n}$, $B\in R^{n\times m}$, and $C\in R^{r\times n}$ are respectively the system, input, and output matrices; $E\in R^{n\times p}$ is a known distribution matrix of disturbances; $d\in R^{p}$ is a vector of unknown disturbances; $\phi\in R^{n\times k}$ is a matrix of known signals, which can be linear or nonlinear functions of the output y and input u; $\theta\in R^{k}$ is a vector of unknown parameters.

The objective of this research is to build an RAO to evaluate all unknown states x and unknown parameters θ with the available inputs u, measurements y, and signal matrix ϕ. The stability of the observer and the convergence of all estimations to real values need to be guaranteed.

3. ROBUST ADAPTIVE OBSERVER

An RAO is proposed in this section for the system described by (1) with structured disturbance inputs. All estimates of the observer, including both states and parameters, converge exponentially

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to their real values free of disturbance. The structure of the observer is presented first and then followed by the proof of the stability and convergence.

In the proposed observer, a UIO is constructed to evaluate the states of the system using the estimated values of unknown parameters. The estimation of parameters is then updated as the extended states of the system. An auxiliary input is added to the UIO in coping with the transient estimation errors of parameters. The RAO has the following form:

$$
\begin{aligned}
\dot{z} &= Nz + TBu + Ky + T\phi\hat{\theta} + \Gamma\dot{\hat{\theta}} \\
\dot{\hat{\theta}} &= \Sigma\Gamma^{\mathrm{T}} C^{\mathrm{T}}(y - C\hat{x}) \\
\dot{\Gamma} &= N\Gamma + T\phi \\
\hat{x} &= z + Hy
\end{aligned}
\tag{2}
$$

where $\hat{x}$ and $\hat{\theta}$ are the estimation of x and θ; z is an intermediate states vector; the matrices H, T, K, N, and Σ in the observer are selected as:

$$
\begin{aligned}
N + N^{\mathrm{T}} &< 0 \\
(HC - I)E &= 0 \\
T &= I - HC \\
A - HCA - K_1 C &= N \\
NH &= K_2 \\
K &= K_1 + K_2 \\
\Sigma &= \Sigma^{\mathrm{T}} > 0
\end{aligned}
\tag{3}
$$

Since the update of $\hat{\theta}$ in (1) involves an evolving variable matrix of Γ and thus makes the observer a Linear Time-Varying (LTV) system, two lemmas regarding to the stability of an LTV system are introduced before the further discussion on the stability of the proposed observer.

Lemma 1

An LTV system

$$\dot{x} = A(t)x \tag{4}$$

with $A(t) = A(t)^{\mathrm{T}} \leqslant 0$ is uniformly exponentially stable if there exist positive constants T_o and α so that for any t_o

$$\int_{t_o}^{t_o+T_o} \underline{\sigma}(t)\,\mathrm{d}t \geqslant \alpha > 0 \tag{5}$$

where $\underline{\sigma}$ is the minimum singular value of $A(t)$.

Proof

According to [21] (Theorem 8.2, p. 132), the norm of the state vector in the autonomous system (4) satisfies the following inequality:

$$\|x(t)\| \leqslant \|x_o\| \mathrm{e}^{1/2\int_{t_o}^{t} \lambda_{\max}(\tau)\,\mathrm{d}\tau}, \quad t \geqslant t_o \tag{6}$$

where $\lambda_{\max}(t)$ is the maximum eigenvalue of $A(t) + A(t)^{\mathrm{T}}$. For a symmetric matrix $A(t) = A(t)^{\mathrm{T}} \leqslant 0$, its singular values satisfy

$$\sigma(t) = -\lambda[A(t)] = -\tfrac{1}{2}\lambda[A(t) + A(t)^{\mathrm{T}}] \tag{7}$$

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Then it is obvious that

$$\lambda_{\max}(t) = -2\underline{\sigma}(t) \tag{8}$$

and consequently

$$\|x(t)\| \leqslant \|x_o\| \mathrm{e}^{-\int_{t_o}^{t} \underline{\sigma}(\tau)\,\mathrm{d}\tau} \tag{9}$$

Since $\underline{\sigma}(t) \geqslant 0$ and for any t_o

$$\int_{t_o}^{t_o+T_o} \underline{\sigma}(t)\,\mathrm{d}t \geqslant \alpha > 0 \tag{10}$$

it can be seen that

$$\begin{aligned}\int_{t_o}^{t} \underline{\sigma}(\tau)\,\mathrm{d}\tau &= \int_{t_o}^{t_o+T_o} \underline{\sigma}(\tau)\,\mathrm{d}\tau + \int_{t_o+T_o}^{t_o+2T_o} \underline{\sigma}(\tau)\,\mathrm{d}\tau + \cdots + \int_{t_o+nT_o}^{t} \underline{\sigma}(\tau)\,\mathrm{d}\tau \\ &\geqslant n\alpha \geqslant \left(\frac{t-t_o}{T_o} - 1\right)\alpha > \left(\frac{t-t_o}{T_o}\right)\alpha > 0\end{aligned} \tag{11}$$

where

$$n = \text{floor}\left(\frac{t-t_o}{T_o}\right). \tag{12}$$

Therefore, one has

$$\|x(t)\| \leqslant \|x_o\| \mathrm{e}^{-\left(\frac{t-t_o}{T_o} - 1\right)\alpha} \leqslant \|x_o\| \mathrm{e}^{\alpha} \mathrm{e}^{-\frac{\alpha}{T_o}(t-t_o)} \tag{13}$$

which means that the system in (4) is uniformly exponentially stable (following the definition 6.5 in [21], p. 101). □

Remark 1
Lemma 1 claims that for an autonomous LTV system (4) with a semi-negative-definite system matrix, the exponential stability condition is that the integrated magnitude of the system matrix is lower bounded over some constant time period.

Lemma 2
If the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1(t) & 0 \\ 0 & A_2(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{14}$$

is uniformly exponentially stable, the following system

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} A_1(t) & A_{12}(t) \\ 0 & A_2(t) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{15}$$

is also uniformly exponentially stable if $\|A_{12}(t)\| \leqslant \bar{\sigma}$ is upper bounded, where $\|\bullet\|$ is the 2-norm of a matrix.

Proof
The solution to the differential equation (15) has the form of

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \Phi_1(t,t_o) & \Phi_{12}(t,t_o) \\ 0 & \Phi_2(t,t_o) \end{bmatrix} \begin{bmatrix} z_{10} \\ z_{20} \end{bmatrix} \tag{16}$$

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where $\Phi_1(t,t_o)$ and $\Phi_2(t,t_o)$ are the state transition matrices associated with $A_1(t)$ and $A_2(t)$; $[z_{10}\ \ z_{20}]^{\mathrm{T}}$ is the initial states at time t_o. From (16), it is obvious that the states of system (14) are

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \Phi_1(t,t_o) & 0 \\ 0 & \Phi_2(t,t_o) \end{bmatrix} \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix} \tag{17}$$

Since (14) is uniformly exponentially stable, there exist positive constants γ_1, λ_1, γ_2, and λ_2 such that for all $t \geqslant t_o$, the following inequalities hold:

$$\begin{aligned} \|\Phi_1(t,t_o)\| &\leqslant \gamma_1 \mathrm{e}^{-\lambda_1(t-t_o)} \\ \|\Phi_2(t,t_o)\| &\leqslant \gamma_2 \mathrm{e}^{-\lambda_2(t-t_o)} \end{aligned} \tag{18}$$

Taking z_2 as an external input to the dynamics of z_1 yields

$$\Phi_{12}(t,t_o) = \int_{t_0}^{t} \Phi_1(t,\tau) A_{12}(\tau) \Phi_2(\tau,t_o)\,\mathrm{d}\tau \tag{19}$$

Hence one has

$$z_1 = \Phi_1(t,t_o) z_{10} + \int_{t_0}^{t} \Phi_1(t,\tau) A_{12}(\tau) \Phi_2(\tau,t_o) z_{20}\,\mathrm{d}\tau \tag{20}$$

with

$$\|\Phi_{12}(t,t_o)\| \leqslant \int_{t_0}^{t} \|\Phi_1(t,\tau)\| \|A_{12}(\tau)\| \|\Phi_2(\tau,t_o)\|\,\mathrm{d}\tau \tag{21}$$

The above inequality becomes

$$\|\Phi_{12}(t,t_o)\| \leqslant \bar{\sigma}\gamma_1\gamma_2 \int_{t_0}^{t} \mathrm{e}^{-\lambda_1(t-\tau)} \mathrm{e}^{-\lambda_2(\tau-t_o)}\,\mathrm{d}\tau \tag{22}$$

and thus

$$\|\Phi_{12}(t,t_o)\| \leqslant \bar{\sigma}\gamma_1\gamma_2 \int_{t_0}^{t} \mathrm{e}^{-\lambda(t-t_0)}\,\mathrm{d}\tau = \sigma\gamma_1\gamma_2 \mathrm{e}^{-\lambda(t-t_0)} \tag{23}$$

where $\lambda = \min(\lambda_1, \lambda_2)$.

With the property of matrix norm

$$\left\| \begin{bmatrix} \Phi_1(t,t_o) & \Phi_{12}(t,t_o) \\ 0 & \Phi_2(t,t_o) \end{bmatrix} \right\| \leqslant \left\| \begin{bmatrix} \|\Phi_1(t,t_o)\| & \|\Phi_{12}(t,t_o)\| \\ 0 & \|\Phi_2(t,t_o)\| \end{bmatrix} \right\| \tag{24}$$

which means

$$\left\| \begin{bmatrix} \Phi_1(t,t_o) & \Phi_{12}(t,t_o) \\ 0 & \Phi_2(t,t_o) \end{bmatrix} \right\| \leqslant \left\| \begin{bmatrix} \gamma_1 \mathrm{e}^{-\lambda_1(t-t_o)} & \bar{\sigma}\gamma_1\gamma_2 \mathrm{e}^{-\lambda(t-t_0)} \\ 0 & \gamma_2 \mathrm{e}^{-\lambda_2(t-t_o)} \end{bmatrix} \right\| \tag{25}$$

the following inequality is obtained:

$$\left\| \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\| \leqslant \left\| \begin{bmatrix} \gamma_1 \mathrm{e}^{-\lambda_1(t-t_o)} & \bar{\sigma}\gamma_1\gamma_2 \mathrm{e}^{-\lambda(t-t_0)} \\ 0 & \gamma_2 \mathrm{e}^{-\lambda_2(t-t_o)} \end{bmatrix} \right\| \left\| \begin{bmatrix} z_{10} \\ z_{20} \end{bmatrix} \right\| \tag{26}$$

Since for any matrix $M \in R^{m\times n}$ one has

$$\|M\| \leqslant \sqrt{mn} |\overline{M}_{ij}| \tag{27}$$

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where $|\bar{M}_{ij}|$ is the maximum absolute value of all elements in M, the following inequality holds:

$$\left\| \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\| \leqslant \gamma e^{-\lambda_o(t-t_o)} \left\| \begin{bmatrix} z_{10} \\ z_{20} \end{bmatrix} \right\| \tag{28}$$

where $\gamma e^{-\lambda_o(t-t_o)}$ is the maximum element in the matrix. It thus follows that the system in (16) is uniformly exponentially stable. □

With the above two lemmas, the stability and convergence of the observer are summarized in the following theorem.

Theorem 1

For system (1) with unknown disturbances, an RAO can be designed in the form of (2) with its parameters H, T, K, N, and Σ selected as in (3). The adaptive observer is uniformly exponentially stable and the estimated states and parameters converge to the real values in an exponential rate if the following conditions are satisfied:

(1) $\mathrm{Rank}(CE) = \mathrm{Rank}(E)$;
(2) $(A - HCA, C)$ is an observable pair;
(3) $\int_{t_o}^{t_o+T_o} \underline{\sigma}(-\Sigma\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\Gamma)\,\mathrm{d}t$ is lower bounded;
(4) $\|\Sigma\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\|$ is upper bounded;
(5) ϕ is not a linear transformation of E, i.e. $\phi = EQ$ does not have solution Q for all time.

Proof

Define $\tilde{x} = x - \hat{x}$ and $\tilde{\theta} = \theta - \hat{\theta}$. One obtains

$$\dot{\tilde{x}} = Ax + Bu + Ed + \phi\theta - [Nz + TBu + KCx + T\phi\hat{\theta} + \Gamma\dot{\hat{\theta}}] - HC(Ax + Bu + Ed + \phi\theta) \tag{29}$$

After manipulations, Equation (29) can be written as

$$\dot{\tilde{x}} = (A - HCA - KC)x + (I - HC - T)Bu + (I - HC)Ed + (I - HC)\phi\theta - T\phi\hat{\theta} - Nz - \Gamma\dot{\hat{\theta}} \tag{30}$$

With $K = K_1 + K_2$, the above equation becomes

$$\dot{\tilde{x}} = (A - HCA - K_1C)x + (I - HC - T)Bu + (I - HC)Ed + (I - HC)\phi\theta - T\phi\hat{\theta} - (Nz + K_2Cx) - \Gamma\dot{\hat{\theta}} \tag{31}$$

With the parameters selected as in (3) and $\dot{\theta} = 0$, Equation (31) is written as

$$\dot{\tilde{x}} = Nx + T\phi\tilde{\theta} - (N\hat{x}) - \Gamma\dot{\hat{\theta}} = N\tilde{x} + T\phi\tilde{\theta} + \Gamma\dot{\tilde{\theta}} \tag{32}$$

One has

$$\dot{\Gamma} = N\Gamma + T\phi \tag{33}$$

and

$$\overset{\bullet}{(\tilde{x} - \Gamma\tilde{\theta})} = \dot{\tilde{x}} - \dot{\Gamma}\tilde{\theta} - \Gamma\dot{\tilde{\theta}} \tag{34}$$

Thus, the following equation is derived:

$$\overset{\bullet}{(\tilde{x} - \Gamma\tilde{\theta})} = N(\tilde{x} - \Gamma\tilde{\theta}) \tag{35}$$

With the updating law of the estimated parameters

$$\dot{\hat{\theta}} = \Sigma^{-1}\Gamma^{\mathrm{T}}C^{\mathrm{T}}(y - C\hat{x}) \tag{36}$$

the dynamics of parameter estimation errors has the form of

$$\dot{\tilde{\theta}} = -\Sigma^{-1}\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\tilde{x} \tag{37}$$

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or equivalently

$$\dot{\tilde{\theta}} = -\Sigma^{-1}\Gamma^{\mathrm{T}}C^{\mathrm{T}}C(\tilde{x}-\Gamma\tilde{\theta}) - \Sigma^{-1}\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\Gamma\tilde{\theta} \tag{38}$$

The dynamics of the adaptive observer is then changed to

$$\begin{bmatrix} \dot{\tilde{\theta}} \\ \overset{\bullet}{(\tilde{x}-\Gamma\tilde{\theta})} \end{bmatrix} = \begin{bmatrix} -\Sigma^{-1}\Gamma^{T}C^{T}C\Gamma & -\Sigma^{-1}\Gamma^{T}C^{T}C \\ 0 & N \end{bmatrix} \begin{bmatrix} \tilde{\theta} \\ (\tilde{x}-\Gamma\tilde{\theta}) \end{bmatrix} \tag{39}$$

Since $N+N^{\mathrm{T}}<0$ and condition 3 are satisfied, with Lemma 1, the following system

$$\begin{bmatrix} \dot{\tilde{\theta}} \\ \overset{\bullet}{(\tilde{x}-\Gamma\tilde{\theta})} \end{bmatrix} = \begin{bmatrix} -\Sigma^{-1}\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\Gamma & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} \tilde{\theta} \\ \left(\tilde{x}-\Gamma\tilde{\theta}\right) \end{bmatrix} \tag{40}$$

is uniformly exponentially stable.

With condition 4 that $\|\Sigma\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\|$ is upper bounded, system (39) is also uniformly exponentially stable by following Lemma 2. This implies that $\tilde{\theta}\to 0$ and $(\tilde{x}-\Gamma\tilde{\theta})\to 0$ in an exponential rate. □

Remark 2

The five conditions in the theorem above are further explained as follows:

(1) Condition 1 is required as the measurement redundancy. Disturbances can be decoupled from the observation only when there are enough independent measurements. Hence the equation $(HC-I)E=0$ is solvable.
(2) Condition 2 is required so that the poles of the observer, i.e. the eigenvalues of N, can be assigned freely with the feedback gain of K_1.
(3) Condition 3 is the persistent excitation requirement on the richness of signals matrix ϕ, which states that the filtered output Γ of ϕ must have enough energy in all channels.
(4) Condition 4 is the requirement on the size of ϕ. Since the output matrix C and the selected parameter $\sum$ are all known, the upper boundedness of $\|\Sigma\Gamma^{\mathrm{T}}C^{\mathrm{T}}C\|$ implies the norm of ϕ is bounded.
(5) Condition 5 is required so that the disturbance inputs and the unknown parameters are separable; otherwise, there will always exist a matrix Q such that $\phi=EQ$, which means the parameter variations are covered by the disturbance inputs $d=Q\theta$.

Remark 3

Besides the state estimation of an UIO, the proposed observer has the capability of disturbance-decoupled parameters estimation. Compared to the normal adaptive observer and EKF [22] which can also handle noise and estimate states and parameters, the salient property of the proposed observer is the disturbance-decoupled states and parameters estimation. The proposed observer decouples the influence of disturbance from all estimations and thus improves estimation accuracy and the robustness of estimation when the system is subject to unknown disturbances and measurement noises.

4. THE MODEL OF A HYDRAULICALLY DRIVEN ELEVATOR

To test its efficiency, the RAO proposed in Section 3 is applied to a hydraulically driven elevator with application to fault detection and identification. In the simulation, different faults of the elevator are modeled as abrupt parameters change and then estimated together with the states of the system. The mathematical model of the elevator is presented in this section.

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4.1. The model of the elevator

In a flight control system, an elevator is a part of the control actuator that adjusts the pitch angle of an airplane. A simple illustration of the elevator studied in this research is shown in Figure 1.

The elevator consists of a right and a left subsystem. Each subsystem has a panel, a hydraulic cylinder, and an Electro-Hydraulic Servo Valve (EHSV). The panel of each subsystem, which is fixed to a shaft, is driven by the EHSV-controlled hydraulic cylinder through a spring connection. The two shafts are connected through a joint so that the two panels will move synchronously. The joint is connected to the tail of the airplane through a hinge. The control command to the elevator is the current to the EHCVs. The mathematical model of the elevator (adopted from the Simulink model from THALES AVIONICS CANADA Inc.) is shown in the following equations:

$$
\begin{aligned}
\dot{x}_{1L} &= x_{2L}\\
\dot{x}_{2L} &= \frac{1}{m_L}[A_{pL}P_L - b_L x_{2L} - K_{sL}(x_{1L} - l_L x_{5L})]\\
\dot{x}_{3L} &= x_{4L}\\
\dot{x}_{4L} &= -\omega_{vL}^2 x_{3L} - 2\xi_{vL}\omega_{vL}x_{4L} + k_{vL}u_L\\
\dot{x}_{5L} &= x_{6L}\\
\dot{x}_{6L} &= \frac{K_{sL}(x_{1L} - l_L x_{5L}) - B_{sL}x_{6L} - K_{Ps}(x_{5L} - x_{5R}) - 0.5H_m(x_{5L} + x_{5R})}{J_{sL}}\\
\dot{x}_{1R} &= x_{2R}\\
\dot{x}_{2R} &= \frac{1}{m_R}[A_{pR}P_R - b_R x_{2R} - K_{sR}(x_{1R} - l_R x_{5R})]\\
\dot{x}_{3R} &= x_{4R}\\
\dot{x}_{4R} &= -\omega_{vR}^2 x_{3R} - 2\xi_{vR}\omega_{vR}x_{4R} + k_{vR}u_R\\
\dot{x}_{5R} &= x_{6R}\\
\dot{x}_{6R} &= \frac{K_{sR}(x_{1R} - l_R x_{5R}) - B_{sR}x_{6R} + K_{Ps}(x_{5L} - x_{5R}) - 0.5H_m(x_{5L} + x_{5R})}{J_{sR}}
\end{aligned}
\tag{41}
$$

The available measurements are

$$y = [x_{1L}\ \ x_{2L}\ \ x_{3L}\ \ x_{5L}\ \ x_{1R}\ \ x_{2R}\ \ x_{3R}\ \ x_{5R}]^{\mathrm{T}} \tag{42}$$

where the physical meanings of the states and parameters (and the nominal values of the parameters) are given in Table I.

The elevator has nonlinear components in the dynamics of hydraulic pressures. The pressure difference P between the active and passive chambers has the form of (same for P_L and P_R):

$$
\begin{aligned}
\dot{P}_1 &= \frac{\beta\left(C_V w x_3\sqrt{\dfrac{2(P_s - P_1)}{\rho}} - C_{12}A_{12}(P_1 - P_2) - A_1 P_2\right)}{V_1 + A_1 x_1}\\
\dot{P}_2 &= \frac{\beta\left(-C_V w x_3\sqrt{\dfrac{2(P_2 - P_r)}{\rho}} + C_{12}A_{12}(P_1 - P_2) + A_2 P_2\right)}{V_2 - A_2 x_1}\\
P &= P_1 - P_2
\end{aligned}
\tag{43}
$$

where P_1 and P_2 are the pressures in the active and passive chambers, respectively. Physical meanings and values of hydraulic parameters are shown in Table II.

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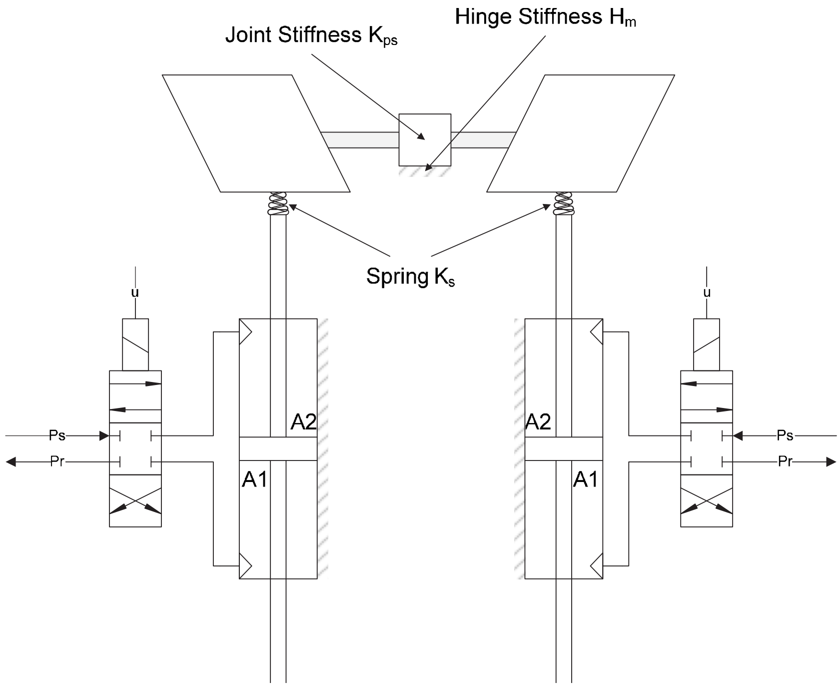

Figure 1. Structure of the elevator.

Table I. Nomenclatures.

Left system	Right system	Physical meanings	Values
x_{1L}	x_{1R}	Cylinder piston position (in.)	—
x_{2L}	x_{2R}	Piston velocity (in./s)	—
x_{3L}	x_{3R}	EHSV spool position (in.)	—
x_{4L}	x_{4R}	EHSV spool velocity (in./s)	—
x_{5L}	x_{5R}	Panel angel (°)	—
x_{6L}	x_{6R}	Panel velocity (°/s)	—
u_L	u_R	Control current to EHSV	—
P_L	P_R	Pressure difference between chambers (psi)	—
m_L	m_R	Piston mass	7.88×10^{-3}(lb s^2/in.)
A_{pL}	A_{pR}	Cylinder chamber area	3.624(in.^{2})
b_L	b_R	Cylinder damping	9.27(lb/in./s)
K_{sL}	K_{sR}	Spring stiffness	2.5×10^5(lbf/in.)
l_L	l_R	Arm length to the shaft	2.924(in.)
d_L	d_R	Pressure dynamics	—
ω_L	ω_R	EHSV natural frequency	817(rad/s)
ξ_L	ξ_R	EHSV damping ratio	0.8
k_{vL}	k_{vR}	EHSV actuation gain	0.00337(in./s^2/mA)
B_{sL}	B_{sR}	Shaft damping	82(lbf in. s/rad)
J_{sL}	J_{sR}	Equivalent inertia	7.5(lbf in. s^2)
K_{ps}		Joint stiffness	5×10^5(lbf in./rad)
H_m		Hinge stiffness	8.594×10^3(lbf in./rad)

In this research, the model with both the linear component (41) and the nonlinear component (43) is applied to simulate the dynamics of the elevator in Matlab/Simulink environment. For the proposed method in Section 3 to be applicable, the linear model in (41) is adopted in constructing the RAO, where the pressure difference P_L and P_R are taken as disturbance inputs that are

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Table II. Hydraulic parameters.

Variables	Physical meanings	Values
β	The oil bulk modulus	10^5(psi)
$C_V w\sqrt{\frac{2}{\rho}}$	The flow rate gains of the EHSVs	9.6571(in.^{2}/psi$^{1/2}$)
P_S	The oil supply pressure	3000 (psi)
P_r	The oil reservoir pressure	50 (psi)
$C_{12}A_{12}$	The leaking coefficients between chambers	3.208(in.^{3}/s/psi)
A_1	The piston area (active)	3.6264(in.^{2})
A_2	The piston area (passive)	3.6264(in.^{2})
V_1	Null volume of the active chamber	4.1(in.^{3})
V_2	Null volume of the passive chamber	4.1(in.^{3})

totally unknown to the system. Thus, the dynamics of the elevator in the state space form can be expressed as

$$\begin{aligned}\dot{x} &= Ax+Bu+Ed\\ y &= Cx\end{aligned} \tag{44}$$

with

$$\begin{aligned}x &= [x_{1L} \ \ \dots \ \ x_{6L} x_{1R} \ \ \dots \ \ x_{6R}]^{\mathrm{T}}\\ u &= [u_L \ \ u_R]^{\mathrm{T}}\\ d &= [P_L \ \ P_R]^{\mathrm{T}}\end{aligned}$$

where matrices A, B, C, and E can be derived from (41) and (42) as

$$A = \left[\begin{array}{cc} A_L & \begin{array}{cccccc} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&\dfrac{(K_{Ps}-0.5H_m)}{J_{sR}}&0 \end{array}\\ \begin{array}{cccccc} 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&\dfrac{(K_{Ps}-0.5H_m)}{J_{sL}}&0 \end{array} & A_R \end{array}\right],$$

$$B = \begin{bmatrix} B_L & 0_{6\times1}\\ 0_{6\times1} & B_R\end{bmatrix}, \quad E=\begin{bmatrix} E_L & 0_{1\times6}\\ 0_{1\times6} & E_R\end{bmatrix}, \quad C=\begin{bmatrix} C_L & 0_{4\times6}\\ 0_{4\times6} & C_R\end{bmatrix},$$

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with

$$
A_L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-K_{sL}}{m_L} & \frac{-b_L}{m_L} & 0 & 0 & \frac{K_{sL} l_L}{m_L} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_{vL}^2 & -2\xi_{vL}\omega_{vL} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{K_{sL}}{J_{sL}} & 0 & 0 & 0 & \frac{-(K_{sL} l_L + K_{Ps} + 0.5H_m)}{J_{sL}} & \frac{-B_{sL}}{J_{sL}} \end{bmatrix}
$$

$$
A_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-K_{sR}}{m_R} & \frac{-b_R}{m_R} & 0 & 0 & \frac{K_{sR} l_R}{m_R} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_{vR}^2 & -2\xi_{vR}\omega_{vR} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{K_{sR}}{J_{sR}} & 0 & 0 & 0 & \frac{-(K_{sR} l_R + K_{Ps} + 0.5H_m)}{J_{sR}} & \frac{-B_{sR}}{J_{sR}} \end{bmatrix}
$$

$$
B_L = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_{vL} \\ 0 \\ 0 \end{bmatrix}, \quad B_R = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_{vR} \\ 0 \\ 0 \end{bmatrix}, \quad E_L = \begin{bmatrix} 0 \\ 0 \\ \frac{A_{pL}}{m_L} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_R = \begin{bmatrix} 0 \\ 0 \\ \frac{A_{pR}}{m_R} \\ 0 \\ 0 \\ 0 \end{bmatrix},
$$

$$
C_L = C_R = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}
$$

4.2. Model of the elevator with faults

Faults in an elevator can result in the loss of pitch control, which consequently reduces the elevation capability and thus deteriorates the performance, maneuverability, and safety of an airplane. Two types of faults are considered in this paper including the unsynchronized movement of panels and the gain loss of the elevator.

For the purpose of pitch control, the two panels of the elevator need to move synchronously. Unsynchronized movement of panels will lead to unexpected roll and yaw of the airplane. For the two identical subsystems (Figure 1) that have the same structure and follow the same command (the current to the EHSVs), the fault of unsynchronized movement is due to unmatched change in parameters. In this research, this fault was modeled as the changes in EHSV gain k_v, from its nominal value. The EHSV gain k_v is defined as a ratio of valve opening to its current command.

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The other fault considered in this research is the gain loss of the elevator, which means that the elevator moves slower than it is expected. One reason for such a fault is the stiffness change in the whole system, which is modeled as the increase in the hinge stiffness H_m.

The faults of the elevator are therefore modeled as the changes in parameters k_{vL}, k_{vR}, and H_m. The model of the elevator with faults θ (the deviations of the aforementioned parameters from their nominal values θ_o; θ_o is modeled as the fault-free dynamics) has the following form:

$$\begin{aligned} \dot{x} &= Ax+Bu+Ed+\phi\theta \\ y &= Cx \end{aligned} \tag{45}$$

where

$$\theta=\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad \phi=[F_1 \quad F_2]\begin{bmatrix} \varphi_1 & 0_{2\times 1} \\ 0_{2\times 2} & \varphi_2 \end{bmatrix}$$

with

$$\theta_1=\begin{bmatrix} \Delta k_{vL} \\ \Delta k_{vR} \end{bmatrix}, \quad \theta_2=\Delta H_M, \quad \varphi_1=\begin{bmatrix} u_L & 0 \\ 0 & u_R \end{bmatrix}, \quad \varphi_2=\begin{bmatrix} x_{5L} \\ x_{5R} \end{bmatrix},$$

$$F_1=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad F_2=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\dfrac{0.5}{J_{sL}} & -\dfrac{0.5}{J_{sL}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\dfrac{0.5}{J_{sR}} & -\dfrac{0.5}{J_{sR}} \end{bmatrix}.$$

5. SIMULATION EVALUATION RESULTS

In the simulations, the actuator gain loss for each EHSV was modeled as the change in parameter k_v to 0.5 of its original value (0.00337 in./s^2/mA) for the left EHSV and to 0.4 for the right. The stiffness change of the hinge was modeled as the change in parameter H_m to 10 times of its original value (8.594×10^3 lbf in./rad). All faulty parameters changed simultaneously at 1 s. To simulate the noise-corrupted system, white noises were added to the measurements. The size of the noise added to each channel was 5 percent of that specific measurement.

The simulation results of parameter estimations are shown in Figures 2 and 3 as the deviation ratio of each parameter from its nominal value. In each figure, the actual change in each parameter is shown as the dash-dot curve; the estimated parameter change in the proposed RAO is shown as the solid curve (RAO). As a comparison, the parameter estimation result of an adaptive observer (in the form of [11]) with the same poles and updating rate as those of the RAO is also shown as the dashed curve (Adpt. Obsv.).

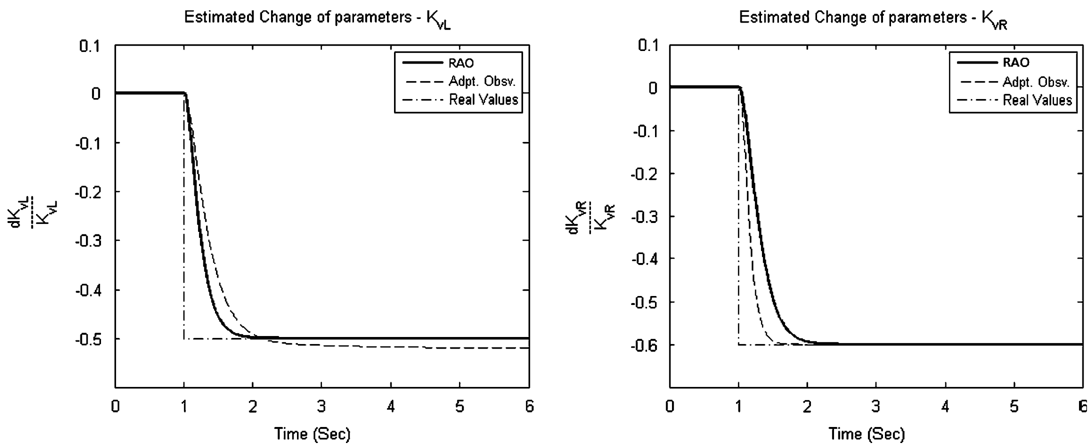

Figure 2. Estimation of k_v: RAO versus normal adaptive observer.

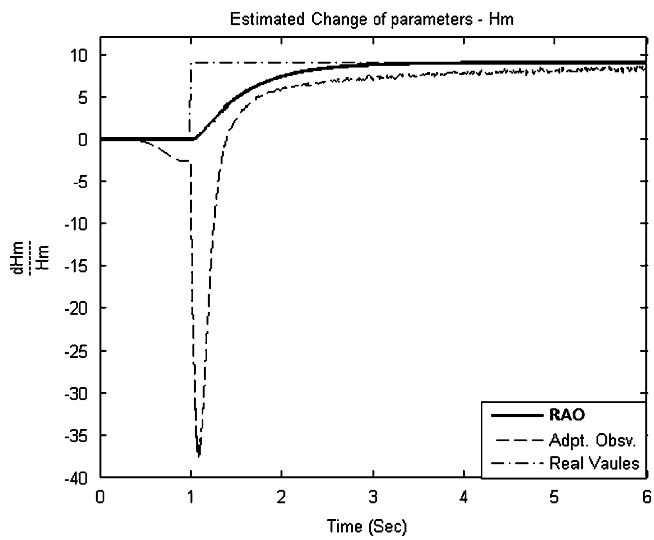

Figure 3. Estimation of H_m: RAO versus normal adaptive observer.

The state estimation errors of the proposed observer are shown in Figure 4, where in each sub-figure the solid curve is for the left subsystem and the dashed curve is for the right subsystem. The state estimation results of the compared observer are not shown because these errors are not in the same scale as those of the proposed observer.

The simulation results show that, for the proposed observer, the faulty parameters can be evaluated quickly and accurately even when the hydraulic pressure is unknown to the observer. For the compared adaptive observer, the lack of hydraulic pressure information leads to constant estimation errors.

To further illustrate the disturbance-decoupling capability of the proposed observer, we compare with an EKF [22], which also has the joint state and parameter estimation capability and the results of parameters estimation are shown in Figures 5 and 6. From these figures, one can see that the estimations of faulty parameters k_v are accurate since the states x_3 and x_4 of the servo valves, which are needed for the estimating of k_v, are not influenced by the disturbance (pressures in the cylinders). The estimated increase in H_m, however, converges to a false value of 16.2. This is because the states needed for the estimation of H_m are under the influence of the disturbance. The results show clearly the advantages of the proposed algorithm over the EKF in the convergence rate, accuracy of parameters estimation, and robustness to the disturbances.

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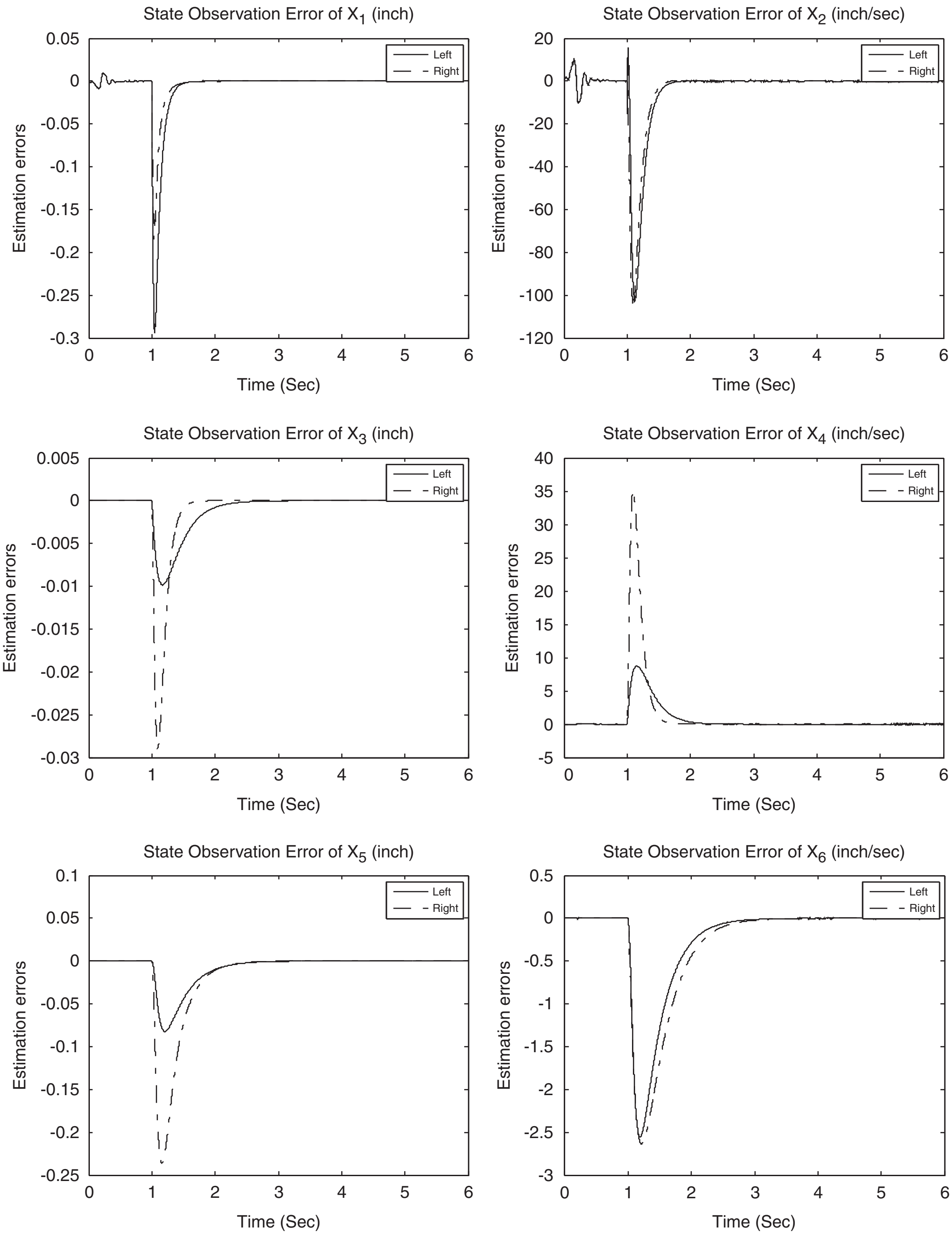

Figure 4. States observation errors of the RAO.

6. CONCLUSIONS

In this paper, an RAO was proposed for the linear systems with unknown disturbance inputs. The exponential stability of the observer and the exponential convergence of all estimations to the real values have been proved. The proposed observer enhances the robustness of estimation in the sense of disturbance rejection by integrating the estimation process with an unknown input

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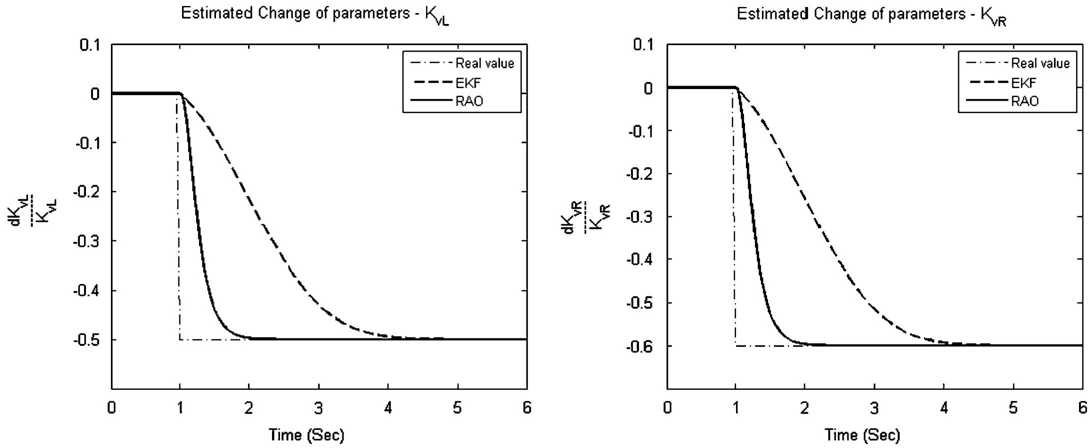

Figure 5. Estimation of k_v: EKF versus RAO.

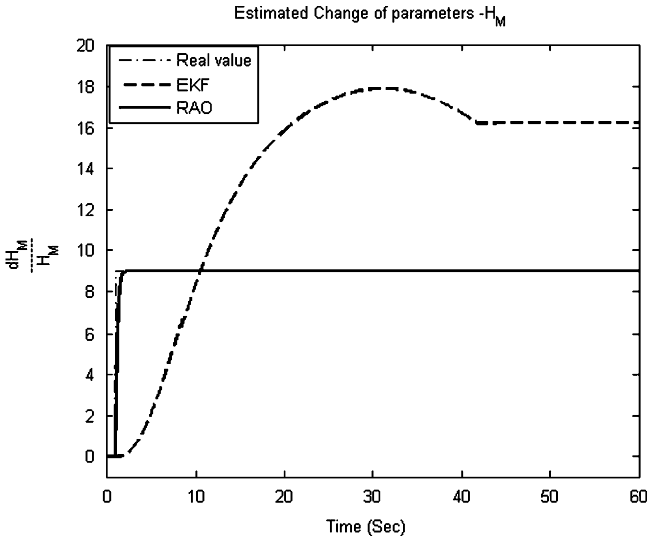

Figure 6. Estimation of H_m: EKF versus RAO.

observer. The results on comparison with the EKF demonstrate the improved estimation accuracy of the proposed observer for a system corrupted with unknown disturbances. Simulation on the faulty parameters estimation of an industrial hydraulically driven airplane elevator validates the effectiveness of the proposed observer.

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