JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 34, No. 5, September–October 2011

Engineering Notes

Fault-Tolerant Attitude Control for Flexible Spacecraft Without Angular Velocity Magnitude Measurement

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DOI: 10.2514/1.51148

I. Introduction

SINCE some catastrophic faults or failures may be induced due to the aging or damage of actuators and sensors during the mission of a spacecraft, those faults would lead to performance degradation of the spacecraft attitude control system or even result in the specified aerospace mission failure. Therefore, fault tolerance of the spacecraft attitude control system is one of the key issues that needs to be addressed. With a view to tackle such a challenge, fault-tolerant control (FTC) has received considerable attention in order to enhance the spacecraft reliability and to guarantee the attitude control performance [1–5]. In [5], an adaptive FTC is developed for the flexible spacecraft attitude tracking system where the persistent bounded disturbances, unknown inertia parameter, and even two types of reaction wheel faults are successfully accommodated.

Indeed, the aforementioned approaches offer many attractive conceptual features, but at the same time they are derived based on the availability of direct and exact measurements of both the angular velocity and the attitude orientation. It is important to note, however, that when it comes to practical implementation, the angular velocity measurements are not always available because of either cost limitations or implementation constraints. Motivated from such a practical consideration, it is therefore highly desirable to develop partial state feedback attitude control strategies with spacecraft angular velocity measurements eliminated. The issue has been addressed in the literature by using observer-based control [6,7], Lyapunov-based control [8,9], and variable structure control [10] under normal operation of spacecraft.

In this work, we provide solutions to two different problems of the flexible spacecraft attitude control system. The first problem consists of developing a control law to perform a attitude stabilization maneuver without angular velocity magnitude. In contrast with the velocity-free control schemes available in the literature, the presented approach can guarantee the attitude control performance be greatly robust to external disturbances and unknown inertia parameters. The second problem solved is the case where both loss of control effectiveness and additive fault occur in actuators simultaneously, but the attitude still requires stabilization with high resolution. To the best knowledge of the authors, this study is the first attempt to deal with fault-tolerant attitude stabilization control for flexible spacecraft with the angular velocity magnitude eliminated.

The Note is organized as follows. Section II presents the mathematical model and attitude control problems formation of a flexible spacecraft under normal and faulty actuator conditions. Section III presents the proposed fault-tolerant attitude stabilization controller without velocity magnitude in the presence of two types of actuator faults. Simulation results to demonstrate various features of the proposed scheme are given in Sec. IV followed by conclusions in Sec. V.

Received 13 June 2010; revision received 9 April 2011; accepted for publication 9 April 2011.

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II. Mathematical Model of Flexible Spacecraft

A. Kinematics Equation

The unit quaternion is adopted to describe the kinematics equation for its global rotation representation without singularity, and then the kinematics differential equation is given by [3]

$$\dot{\mathbf{q}} = \tfrac{1}{2}(\mathbf{q}^{\times} + q_0\mathbf{I}_3)\boldsymbol{\omega} \tag{1}$$

$$\dot{q}_0 = -\tfrac{1}{2}\mathbf{q}^T\boldsymbol{\omega} \tag{2}$$

where $\boldsymbol{\omega} \in \mathcal{R}^3$ is the angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $\mathbf{Q} = (q_0 \quad \mathbf{q}^T)^T \in \mathcal{R}^4$ denotes the unit quaternion vector representing the attitude orientation of the spacecraft in $\mathcal{B}$ with respect to $\mathcal{I}$, $\mathbf{I}_n$ is the identity matrix of the nth order, and the notation $\mathbf{q}^{\times}$ denotes the cross-product operator of $\mathbf{q}$.

B. Flexible Spacecraft Dynamics

When all the actuators run normally, the dynamic equations of a spacecraft with flexible appendages can be written under the hypothesis of small elastic deformations [10]:

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}(\mathbf{J}\boldsymbol{\omega} + \boldsymbol{\delta}\dot{\boldsymbol{\eta}}) + \boldsymbol{\delta}^T\ddot{\boldsymbol{\eta}} = \mathbf{u} + \mathbf{d} \tag{3}$$

$$\ddot{\boldsymbol{\eta}} + \mathbf{C}\dot{\boldsymbol{\eta}} + \mathbf{K}\boldsymbol{\eta} + \boldsymbol{\delta}\dot{\boldsymbol{\omega}} = 0 \tag{4}$$

where $\mathbf{J} \in \mathcal{R}^{3\times3}$ is the total inertia matrix of the spacecraft, $\mathbf{u} \in \mathcal{R}^3$ is the control torque input vector while $\mathbf{d} \in \mathcal{R}^3$ is the disturbance torque vector, and $\boldsymbol{\eta} \in \mathcal{R}^N$ is the modal coordinate vector relative to the main body. In addition, $\boldsymbol{\delta} \in \mathbb{R}^{N\times3}$ denotes the coupling matrix between flexible and rigid dynamics, and $\mathbf{K} = \mathrm{diag}(2\xi_1\Lambda_1, \ldots, 2\xi_N\Lambda_N)$ and $\mathbf{C} = \mathrm{diag}(\Lambda_1^2, \ldots, \Lambda_N^2)$ are the damping and stiffness matrices (with N the number of elastic modes considered; Λ_i, $i = 1, \ldots, N$, the natural frequencies; and ξ_i the associated damping), respectively.

Now, consider the situation in which the additive and partial loss of actuator effectiveness fault occurs. Then, the general nonlinear spacecraft attitude dynamics model in Eq. (3) can be rewritten as

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^{\times}(\mathbf{J}\boldsymbol{\omega} + \boldsymbol{\delta}\dot{\boldsymbol{\eta}}) + \boldsymbol{\delta}\ddot{\boldsymbol{\eta}} = \boldsymbol{\alpha}(t)\mathbf{u} + \mathbf{d} + \mathbf{f}(t, \boldsymbol{\omega}, \mathbf{q}) \tag{5}$$

where $\boldsymbol{\alpha}(t) = \mathrm{diag}[\alpha_{11}(t), \alpha_{22}(t), \alpha_{33}(t)]$ represents the partial loss of actuator effectiveness fault with $0 < \mu_0 \le \alpha_{ii}(t) \le 1$, $i = 1, 2, 3$; and $\mathbf{f}(t, \boldsymbol{\omega}, \mathbf{q})$ denotes the actuator fault entering the spacecraft dynamics in an additive way and is subject to $\|\mathbf{f}(t,\boldsymbol{\omega},\mathbf{q})\| \le \mu_1(t)$, where $\mu_1(t)$ is a positive continuous function.

Assumption 1: The disturbance $\mathbf{d}$ considered in Eq. (5) is bounded; that is, $\sup_{0\le\tau\le t}\|\mathbf{d}(\tau)\|$ exists and is bounded for $\forall\ t \ge 0$.

The control objective to be achieved in this Note can be stated as follows. Considering the faulty attitude control system given by Eq. (5), design a control law to guarantee the attitude to be converged to zero or an arbitrary small set containing the origin. Moreover, such a control objective is to be achieved under the following conditions: 1) without angular velocity magnitude measurement, 2) in the presence of uncertainties in spacecraft mass moment of inertia and unknown bounded external disturbances, and 3) in the presence of possible additive fault $\mathbf{f}(t,\boldsymbol{\omega},\mathbf{q})$ and partial loss of actuator effectiveness fault $\boldsymbol{\alpha}(t)$.

III. Control Law Design

Before going to the specific control law design, we first introduce the following variable:

$$\boldsymbol{\gamma} = \dot{\boldsymbol{\eta}} + \boldsymbol{\delta}\boldsymbol{\omega} \tag{6}$$

Define a new state variable $\boldsymbol{\psi} = (\boldsymbol{\eta}^T \quad \boldsymbol{\gamma}^T)^T$, then Eqs. (4) and (6) can be summarized as

$$\dot{\boldsymbol{\psi}} = \begin{pmatrix} 0 & \mathbf{I}_N \\ -\mathbf{K} & -\mathbf{C} \end{pmatrix}\boldsymbol{\psi} + \begin{pmatrix} -\boldsymbol{\delta} \\ \mathbf{C}\boldsymbol{\delta} \end{pmatrix}\boldsymbol{\omega} \tag{7}$$

In view of the faulty dynamics in Eq. (5), it follows that

$$\begin{aligned}\mathbf{J}_0\dot{\boldsymbol{\omega}} = &-\boldsymbol{\omega}^\times[\mathbf{J}_0\boldsymbol{\omega} + \boldsymbol{\delta}(\varphi - \boldsymbol{\delta}\boldsymbol{\omega})] + \boldsymbol{\delta}^T(\mathbf{K} \quad \mathbf{C})\boldsymbol{\psi} \\ &- (\boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta} + \boldsymbol{\omega}^\times\boldsymbol{\delta}^T\boldsymbol{\delta})\boldsymbol{\omega} + \boldsymbol{\alpha}\mathbf{u} + \mathbf{d} + \mathbf{f}\end{aligned} \tag{8}$$

where $\mathbf{J}_0 = \mathbf{J} - \boldsymbol{\delta}^T\boldsymbol{\delta}$ denotes the main body inertia matrix.

Assumption 2: For a flexible spacecraft, the main body inertia matrix $\mathbf{J}_0$ is positive definite symmetric and bounded but unknown during the entire orbiting operation.

Remark 1: For Assumption 2, the structural parameters are supposed to be poorly known, and they are constant or can vary during spacecraft operations. In both cases, since their variation is assumed to be slow with respect to the spacecraft dynamics, their derivatives are or can be considered as zero.

Since the measurement of angular velocity $\boldsymbol{\omega}$ is not exactly available, a filter is introduced to generate auxiliary signal from attitude quaternion measurement only. The auxiliary signal is the output of the following first-order dynamics [9]:

$$\dot{\boldsymbol{\chi}}(t) = -\boldsymbol{\chi}(t) + 2k_x\mathbf{q}(t) + k_i\int_0^t \mathbf{q}(\tau)\,\mathrm{d}\tau \tag{9}$$

To that end, we propose the following control law to perform the attitude stabilization maneuver

$$\mathbf{u} = \mathbf{u}_N + \mathbf{u}_F \tag{10}$$

where $\mathbf{u}_N$ is the normal controller, $\mathbf{u}_F$ is the fault-tolerant controller added in order to compensate for the possible actuator faults effect on the system, and the following two items are developed as

$$\begin{aligned}\mathbf{u}_N = &-[k_p + 2k_x(k_x - k_i)q_0]\mathbf{q} - \left(\frac{k_i}{2} - k_x\right)(\mathbf{q}^\times + q_0\mathbf{I}_3)^T\boldsymbol{\chi} \\ &- k_i\left(k_x - \frac{k_i}{2}\right)(\mathbf{q}^\times + q_0\mathbf{I}_3)^T\int_0^t \mathbf{q}(\tau)\,\mathrm{d}\tau\end{aligned} \tag{11}$$

$$\mathbf{u}_F = -\left[\frac{1-\mu_0}{\mu_0}(\|\mathbf{u}_N\| + \varepsilon_0) + \frac{1}{\mu_0}\mu_1(t)\right]\mathrm{sign}(\boldsymbol{\omega}) \tag{12}$$

where k_p, ε_0, k_x, and k_i are positive control gains; and $\mathrm{sign}(\boldsymbol{\omega})$ is a vector-valued sign function defined as

$$\mathrm{sign}(\boldsymbol{\omega}) = (\,\mathrm{sgn}(\omega_1) \quad \mathrm{sgn}(\omega_2) \quad \mathrm{sgn}(\omega_3)\,)^T \tag{13}$$

Now, we are ready to summarize the FTC solution to the underlying attitude stabilization problem without angular velocity magnitude measurement.

Theorem 1: Consider the faulty flexible spacecraft attitude control system given by Eqs. (1), (2), (4), and (5) under partial loss of actuator effectiveness and additive faults. With application of the control law equation (10), suppose that the control parameters are chosen such that $k_p - 0.5k_i^2 > 0$ and $2k_x \ne k_i$. Then, all of the signals in the resulting closed-loop attitude system are bounded and continuous, and global asymptotic stability is guaranteed in case of $\mathbf{d}(t) \equiv 0$. That is, the attitude and the angular velocity converge to zero; i.e.,

$$\lim_{t\to\infty}\mathbf{q} = 0$$

and

$$\lim_{t\to\infty}\boldsymbol{\omega} = 0$$

Proof: Consider the Lyapunov function candidate

$$V = \tfrac{1}{2}\boldsymbol{\omega}^T\mathbf{J}_0\boldsymbol{\omega} + k_p[(q_0-1)^2 + \mathbf{q}^T\mathbf{q}] + \tfrac{1}{2}\dot{\boldsymbol{\chi}}^T\dot{\boldsymbol{\chi}} - k_i\mathbf{q}^T\dot{\boldsymbol{\chi}} + \tfrac{1}{2}\boldsymbol{\psi}^T\mathbf{P}\boldsymbol{\psi} \tag{14}$$

where $\mathbf{P}$ is a positive definite matrix, and it is the solution of the following Lyapunov equation:

$$\mathbf{P}\begin{pmatrix} 0 & \mathbf{I}_N \\ -\mathbf{K} & -\mathbf{C} \end{pmatrix} + \begin{pmatrix} 0 & \mathbf{I}_N \\ -\mathbf{K} & -\mathbf{C} \end{pmatrix}\mathbf{P}^T = -2\mathbf{Q} \tag{15}$$

where $\mathbf{Q}$ is a positive definite symmetric matrix.

Defining

$$\mathbf{x} = [\,(1-q_0) \quad \mathbf{q}^T \quad \boldsymbol{\omega}^T \quad (\dot{\boldsymbol{\chi}} - k_i\mathbf{q})^T \quad \boldsymbol{\psi}^T\,]^T$$

then V could be bounded by

$$\lambda_{\min}(\mathbf{R})\|\mathbf{x}\|^2 \le V = \mathbf{x}^T\mathbf{R}\mathbf{x} \le \lambda_{\max}(\mathbf{R})\|\mathbf{x}\|^2 \tag{16}$$

where

$$\mathbf{R} = \mathrm{diag}(\,k_p, \quad (k_p - 0.5k_i^2), \quad 0.5\mathbf{J}_0, \quad 0.5, \quad 0.5\mathbf{P}\,)$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimal and maximal eigenvalues of a positive matrix, respectively. Since the inequality $k_p - 0.5k_i^2 > 0$ holds, $\mathbf{R}$ is positive definite that implies that V is globally positive definite from Eq. (16).

In the case of $\mathbf{d}(t) \equiv 0$, the time derivative of the Lyapunov function equation (14) along the faulty spacecraft dynamics equation (5) can be calculated as

$$\begin{aligned}\dot{V} = &\,\boldsymbol{\omega}^T\mathbf{J}_0\dot{\boldsymbol{\omega}} + k_p[2(q_0-1)\dot{q}_0 + 2\mathbf{q}^T\dot{\mathbf{q}}] + \dot{\boldsymbol{\chi}}^T\ddot{\boldsymbol{\chi}} - k_i\dot{\mathbf{q}}^T\dot{\boldsymbol{\chi}} - k_i\mathbf{q}^T\ddot{\boldsymbol{\chi}} \\ &+ \boldsymbol{\psi}^T\mathbf{P}\dot{\boldsymbol{\psi}} = \boldsymbol{\omega}^T\{\boldsymbol{\alpha}\mathbf{u} + \boldsymbol{\delta}^T(\mathbf{K} \quad \mathbf{C})\boldsymbol{\psi} + k_p\mathbf{q} - (\dot{\boldsymbol{\chi}} - k_i\mathbf{q})^T \\ &\times(\dot{\boldsymbol{\chi}} - k_i\mathbf{q}) + 2k_x(\mathbf{q}^\times + q_0\mathbf{I}_3)^T\Big[-\boldsymbol{\chi} + (2k_x - k_i)\mathbf{q} \\ &+ k_i\int_0^t \mathbf{q}(\tau)\,\mathrm{d}\tau\Big] - \frac{k_i}{2}(\mathbf{q}^\times + q_0\mathbf{I}_3)^T\Big[-\chi + 2k_x\mathbf{q} \\ &+ k_i\int_0^t \mathbf{q}(\tau)\,\mathrm{d}\tau\Big]\Big\} - \boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta}\boldsymbol{\omega} + \mathbf{f} + \boldsymbol{\psi}^T\mathbf{P}\begin{pmatrix} 0 & \mathbf{I}_N \\ -\mathbf{K} & -\mathbf{C} \end{pmatrix}\boldsymbol{\psi} \\ &+ \boldsymbol{\psi}^T\mathbf{P}\begin{pmatrix} -\boldsymbol{\delta} \\ \mathbf{C}\boldsymbol{\delta} \end{pmatrix}\boldsymbol{\omega}\end{aligned} \tag{17}$$

Substituting the controller equation (10) into Eq. (17) yields

$$
\begin{aligned}
\dot{V} &= \boldsymbol{\omega}^T\left\{-\boldsymbol{\alpha}\left[\frac{1-\mu_0}{\mu_0}(\|\mathbf{u}_N\|+\varepsilon_0)+\frac{1}{\mu_0}\mu_1(t)\right]\mathrm{sign}(\boldsymbol{\omega})\right.\\
&\quad \left.-(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})^T(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})+\mathbf{f}+\boldsymbol{\delta}^T(\mathbf{K}\quad \mathbf{C})\boldsymbol{\psi}-\boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta}\boldsymbol{\omega}\right\}\\
&\quad -(\mathbf{I}_3-\boldsymbol{\alpha})\mathbf{u}_N+\boldsymbol{\psi}^T\mathbf{P}\begin{pmatrix}0 & \mathbf{I}_N\\ -\mathbf{K} & -\mathbf{C}\end{pmatrix}\boldsymbol{\psi}+\boldsymbol{\psi}^T\mathbf{P}\begin{pmatrix}-\boldsymbol{\delta}\\ \mathbf{C}\boldsymbol{\delta}\end{pmatrix}\boldsymbol{\omega}\\
&\le (1-\mu_0)\|\boldsymbol{\omega}\|\|\mathbf{u}_N\|-(1-\mu_0)(\|\mathbf{u}_N\|+\varepsilon_0)\|\boldsymbol{\omega}\|\\
&\quad -(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})^T(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})-(\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T)\,\Xi\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{pmatrix}\\
&\le -(1-\mu_0)\varepsilon_0\|\boldsymbol{\omega}\|-\|\dot{\boldsymbol{\chi}}-k_i\mathbf{q}\|^2-(\boldsymbol{\omega}^T\quad \boldsymbol{\psi}^T)\,\Xi\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{\psi}\end{pmatrix} \qquad (18)
\end{aligned}
$$

where Ξ is given by

$$
\Xi=\begin{pmatrix}\boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta} & \frac{\boldsymbol{\delta}^T\left(\mathbf{K}-\mathbf{P}\quad \mathbf{C}+\mathbf{C}\mathbf{P}\right)}{2}\\ \frac{\left(\mathbf{K}-\mathbf{P}\quad \mathbf{C}+\mathbf{C}\mathbf{P}\right)^T\boldsymbol{\delta}}{2} & \mathbf{Q}\end{pmatrix} \qquad (19)
$$

By using the Schur complement lemma [11], and for the appropriate choice of matrix $\mathbf{Q}$, Ξ could be a positive definite matrix. With $\mu_0<1$, it is easily obtained from Eq. (18) that

$$
\dot{V}\le -\|\dot{\boldsymbol{\chi}}-k_i\mathbf{q}\|^2\le 0 \qquad (20)
$$

implying that

$$
\lim_{t\to\infty}V(t)=V(\infty)
$$

exists; it can be further concluded that $V\in\mathcal{L}_\infty$, and then q_0, $\mathbf{q}$, $\boldsymbol{\omega}\in\mathcal{L}_\infty$. Thus, we have $\dot{\mathbf{q}}\in\mathcal{L}_\infty$ from Eqs. (1) and (2). By integrating $\dot{V}$ from 0 to ∞, one has

$$
\lim_{t\to\infty}\|\dot{\boldsymbol{\chi}}-k_i\mathbf{q}\|^2\le V(0)-V(\infty) \qquad (21)
$$

Because the term on the right-hand side of inequality equation (21) is bounded, it follows that $\dot{\boldsymbol{\chi}}-k_i\mathbf{q}\in\mathcal{L}_2$. From Eq. (9), we have $\ddot{\boldsymbol{\chi}}=-\dot{\boldsymbol{\chi}}+2k_x\dot{\mathbf{q}}+k_i\mathbf{q}$, which together with $\dot{\mathbf{q}}\in\mathcal{L}_\infty$ implies that

$$
\ddot{\boldsymbol{\chi}}-k_i\dot{\mathbf{q}}=-(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})+(2k_x-k_i)\dot{\mathbf{q}}\mathcal{L}_\infty
$$

Hence, we have $\dot{\boldsymbol{\chi}}-k_i\mathbf{q}\in\mathcal{L}_2\cap\mathcal{L}_\infty$ and $\ddot{\boldsymbol{\chi}}-k_i\dot{\mathbf{q}}\in\mathcal{L}_\infty$. Using the Barbalat's lemma, it follows that

$$
\lim_{t\to\infty}(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})=0 \qquad (22)
$$

Now, consider another variable

$$
\mathbf{y}(t)\triangleq\boldsymbol{\chi}-k_i\int_o^t\mathbf{q}(\tau)\,\mathrm{d}\tau
$$

which is uniformly bounded, and then we conclude the uniform continuity of $\ddot{\mathbf{y}}$. Furthermore, due to $\dot{\mathbf{y}}\to 0$ as $t\to\infty$, it follows

$$
\lim_{t\to\infty}\int_0^t\ddot{\mathbf{y}}(\tau)\,\mathrm{d}\tau+\dot{\mathbf{y}}(0)=0 \qquad (23)
$$

By virtue of the alternate statement of Barbalat's lemma [12], together with the uniform continuity of $\ddot{\mathbf{y}}$, it leads to $\ddot{\mathbf{y}}\to 0$ as $t\to\infty$. Note that when $\ddot{\mathbf{y}}=-(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})+(2k_x-k_i)\dot{\mathbf{q}}$, we have

$$
\lim_{t\to\infty}\dot{\mathbf{q}}(t)=0
$$

whenever

$$
\lim_{t\to\infty}(\dot{\boldsymbol{\chi}}-k_i\mathbf{q})=0
$$

and $2k_x\neq k_i$.

With

$$
\lim_{t\to\infty}\dot{\mathbf{q}}(t)=0
$$

according to the result of [9], it has

$$
\lim_{t\to\infty}\mathbf{q}(t)=0,\qquad \lim_{t\to\infty}\boldsymbol{\omega}(t)=0 \qquad (24)
$$

for any initial attitude and angular velocity. Thereby, the globally asymptotic stability of the closed-loop system can be concluded. Here, the proof is completed. □

When we take external disturbances $\mathbf{d}$ into consideration, the stability analysis of the closed-loop system can be stated by the following corollary.

Corollary 1: Let the control parameters be chosen such that $k_p-0.5k_i^2>0$ and $2k_x\neq k_i$; the system in Eqs. (1), (2), (4), and (5) in the closed loop with the control law equation (10) is then ultimately uniformly bounded (UUB) in the presence of partial loss of actuator effectiveness and additive faults as well as external disturbances $\mathbf{d}(t)$.

Proof: In the case when the spacecraft attitude system is affected by external disturbances, calculating the time derivative of V in Eq. (14) gives

$$
\begin{aligned}
\dot{V}&=\boldsymbol{\omega}^T\{\boldsymbol{\alpha}\mathbf{u}+\boldsymbol{\delta}^T(\mathbf{K}\quad \mathbf{C})\boldsymbol{\psi}-\boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta}\boldsymbol{\omega}+\mathbf{f}+\mathbf{d}\}+\boldsymbol{\psi}^T\mathbf{P}\dot{\boldsymbol{\psi}}\\
&\quad +k_p[2(q_0-1)\dot{q}_0+2\mathbf{q}^T\dot{\mathbf{q}}]+\dot{\boldsymbol{\chi}}^T\ddot{\boldsymbol{\chi}}-k_i\dot{\mathbf{q}}^T\dot{\boldsymbol{\chi}}-k_i\mathbf{q}^T\ddot{\boldsymbol{\chi}} \qquad (25)
\end{aligned}
$$

Substituting Eq. (10) into Eq. (25) with the same derivation as in Theorem 1, it follows that

$$
\dot{V}\le\boldsymbol{\omega}^T\mathbf{d}-(\boldsymbol{\omega}^T\quad (\dot{\boldsymbol{\chi}}-k_i\mathbf{q})^T\quad \boldsymbol{\psi}^T)^T\Gamma\begin{pmatrix}\boldsymbol{\omega}\\ \dot{\boldsymbol{\chi}}-k_i\mathbf{q}\\ \boldsymbol{\psi}\end{pmatrix} \qquad (26)
$$

where Γ is defined by

$$
\Gamma=\begin{pmatrix}\boldsymbol{\delta}^T\mathbf{C}\boldsymbol{\delta} & 0 & \frac{\boldsymbol{\delta}^T(\mathbf{K}-\mathbf{P}\quad \mathbf{C}+\mathbf{C}\mathbf{P})}{2}\\ 0 & \mathbf{I}_3 & 0\\ \frac{(\mathbf{K}-\mathbf{P}\quad \mathbf{C}+\mathbf{C}\mathbf{P})^T\boldsymbol{\delta}}{2} & 0 & \mathbf{Q}\end{pmatrix} \qquad (27)
$$

Also, by using the Schur complement lemma [11], Γ can be guaranteed to be positive definite in the event of the appropriate choice of $\mathbf{Q}$. Since $\mathbf{q}^T\mathbf{q}+q_0^2=1$ results in $|1-q_0|^2\le(1-q_0)$ and $\|\mathbf{q}\|^2\le\|\mathbf{q}\|$, we can easily obtain that

$$
\begin{aligned}
\|\mathbf{x}\|^2&=\|((1-q_0)\quad \mathbf{q}^T\quad \boldsymbol{\omega}^T\quad (\dot{\boldsymbol{\chi}}-k_i\mathbf{q})^T\quad \boldsymbol{\psi}^T)^T\|^2\\
&\le(1-q_0)+\|\mathbf{q}\|+\|(\boldsymbol{\omega}^T\quad (\dot{\boldsymbol{\chi}}-k_i\mathbf{q})^T\quad \boldsymbol{\psi}^T)\|^2 \qquad (28)
\end{aligned}
$$

Then

$$
\begin{aligned}
\dot{V}&\le-\pi\|\mathbf{x}\|^2+\|\boldsymbol{\omega}\|\sup_{0\le\tau\le t}\|\mathbf{d}(\tau)\|+\pi((1-q_0)+\|\mathbf{q}\|)\\
&\le-\pi\|\mathbf{x}\|^2+\|\mathbf{x}\|(2\pi+\sup_{0\le\tau\le t}\|\mathbf{d}(\tau)\|) \qquad (29)
\end{aligned}
$$

where $\pi=\lambda_{\min}(\Gamma)$ has been introduced. Let $0<\theta<1$, and then Eq. (29) can be rewritten as

$$
\dot{V}\le-\pi\theta\|\mathbf{x}\|^2+\|\mathbf{x}\|[(2\pi+\sup_{0\le\tau\le t}\|\mathbf{d}(\tau)\|)-(1-\theta)\pi\|\mathbf{x}\|] \qquad (30)
$$

Clearly, if

$$
\|\mathbf{x}\|>\frac{\pi+\sup_{0\le\tau\le t}\|\mathbf{d}(\tau)\|}{(1-\theta)\pi}
$$

we obtain

$$
\dot{V}<-\pi\theta\|\mathbf{x}\|^2\le-\frac{\pi\theta}{\lambda_{\max}(\mathbf{R})}V \qquad (31)
$$

Therefore, the state is bounded ultimately by

$$\|\mathbf{x}(t)\| \leq \max \sqrt{\frac{\lambda_{\max}(\mathbf{R})}{\lambda_{\min}(\mathbf{R})}} \times \left\{ 2\|\mathbf{x}(0)\| e^{-[\pi\theta/\lambda_{\max}(\mathbf{R})]t}, 2\frac{(2\pi + \sup_{0\leq\tau\leq t}\|\mathbf{d}(\tau)\|)}{(1-\theta)\pi} \right\} \tag{32}$$

which is a small set containing the origin $\mathbf{x} = 0$. From Eqs. (10) and (32), using the boundedness theorem [13], it can be concluded that $(1 - q_0)$, $\mathbf{q}^T$, $\boldsymbol{\omega}^T$, $(\dot{\boldsymbol{\chi}} - k_i\mathbf{q})^T$, $\boldsymbol{\psi}^T$, and $\mathbf{u}$ are UUB. Thus, all signals in the closed-loop system are UUB. This completes the proof. □

Remark 2: It is worth mentioning that, although the proposed control law in Eq. (10) is independent on the magnitude measurements of spacecraft angular velocity, $\mathbf{u}_F$ in Eq. (10) requires the knowledge on the direction of the angular velocity. As a result, the spacecraft still needs low-cost gyroscopes to obtain the direction information of the angular velocity. In addition to this hardware-based scheme, there exists an alternative analytical methodology to determine the sign of $\boldsymbol{\omega}$ without any rate sensors. Actually, in practical aerospace engineering, the controller is implemented with a digital computer; hence, the value of $\dot{\mathbf{q}}$ in the time of $(k+1)T$ can be approximately estimated by using one-step previous information from attitude sensors as follows:

$$\dot{\mathbf{q}}[(k+1)T] = \frac{\mathbf{q}[(k+1)T] - \mathbf{q}(kT)}{T} \tag{33}$$

where T is the sampling time. From Eq. (1), one has

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \frac{2}{T} \begin{pmatrix} q_{0,k} & -q_{3,k} & q_{2,k} \\ q_{3,k} & q_{0,k} & -q_{1,k} \\ -q_{2,k} & q_{1,k} & q_{0,k} \end{pmatrix}^{-1} \begin{pmatrix} q_{1,k+1} - q_{1,k} \\ q_{2,k+1} - q_{2,k} \\ q_{3,k+1} - q_{3,k} \end{pmatrix} \tag{34}$$

where $q_{i,k}$ and $q_{i,k+1}$ $(i = 0, 1, 2, 3)$ are, respectively, the ith item of $\mathbf{Q}$ in the time of kT and $(k+1)T$. Based on Eq. (34), the sign of $\boldsymbol{\omega}$ can be derived.

Remark 3: It is worth mentioning that the chattering effect may be caused by the sign function of the controller equation (10). However, in practice, once the orbit and the target are determined, the direction of angular velocity would not vary and the positive direction of angular velocity is decided; in some sense, the sign of angular velocity will not change. Consequently, the proposed controller equation (10) is continuous; thus, vibration or instability will not be induced. This will be discussed further in the simulation study.

Remark 4: Note that when actuators are fault free, we have $\mu_0 \equiv 1$ and $\mu_1(t) \equiv 0$. Thus, $\mathbf{u} = \mathbf{u}_N$ can be obtained for the controller equation (10); that is to say, the system can be stabilized by controller equation (10).

Remark 5: The inequality equation (32) establishes the relationship between the control parameters and the attitude control accuracies $\mathbf{q}$ and $\boldsymbol{\omega}$. It is clear from Eq. (32) that the larger $\lambda_{\min}(\mathbf{R})$ and smaller θ are, the better attitude control accuracy can be obtained.

IV. Numerical Simulations

To verify the effectiveness and performance of the proposed control scheme in this Note, numerical simulations have been carried out using the flexible spacecraft system equations (1), (2), (4), and (5) with the developed control law equation (10). The same physical parameters as considered in [10] are used, which are given by

$$\mathbf{J} = \begin{pmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{pmatrix} \mathrm{kg \cdot m^2}$$

$$\boldsymbol{\delta} = \begin{pmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25819 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \end{pmatrix} \mathrm{kg^{1/2} \cdot m/s^2}$$

The natural frequencies are $\Lambda_1 = 0.7681$, $\Lambda_2 = 1.1038$, and $\Lambda_3 = 1.8733$ rad/s; and the damping ratios are $\xi_1 = 0.0056$, $\xi_2 = 0.0086$, and $\xi_3 = 0.013$. Moreover, bounded external disturbance $\mathbf{d} = [0.2 \quad 0.1 \quad -0.1]^T$ is also considered.

In the context of simulation, the following actuator fault scenario is considered. At $t = 40$ s, each actuator undergoes a partial loss of effectiveness, while at $t = 100$ s, these actuators experience also an additive fault induced by a stuck type of actuator fault, and the following nonlinearity summarized from [2,3,14] is used to generate the actuator faults scenario:

$$\alpha_i(t) = \begin{cases} 1, & t < 40 \text{ s} \\ 0.25 + 0.05\sin(2\pi t), & t \geq 40 \text{ s} \end{cases}$$
$$f_i(t) = \begin{cases} 0, & t < 100 \text{ s} \\ 0.35 + 0.15\sin(10t), & t \geq 100 \text{ s} \end{cases} \tag{35}$$

To implement the controller, the design parameters in Eq. (11) are chosen as $k_x = 6$, $k_i = 0.1$, $k_p = 3$, $\mu_0 = 0.15$, $\mu_1 = 1$, and $\varepsilon_0 = 0.25$ in Eq. (12). At time $t = 0$, the orientation of the spacecraft is set to be $\mathbf{q}(0) = [-0.5 \quad -0.26 \quad 0.79]^T$ with a zero initial body angular velocity and initial modal displacements $\eta_i(0) = 0.001$, as well as its time derivative $\dot{\eta}_i(0) = 0.0005$.

A. Response with Normal Controller

The fault scenario equation (35) was implemented in the case when the spacecraft attitude is governed by the normal controller equation (11). Because of the slowly time-varying additive fault and constant external disturbance, it is clear to see from Fig. 1 that the overall attitude system is stable. However, its attitude pointing accuracy is quite low, and it could not satisfy the requirement of the mission since such a controller does not have a mechanism to accommodate the actuator faults. This is due to the fact that once the actuators undergo partial loss of control effectiveness after 40 s, especially after the occurrence of additive fault in 100 s, the static value of control input, as shown in Fig. 2 (dashed line), is not big

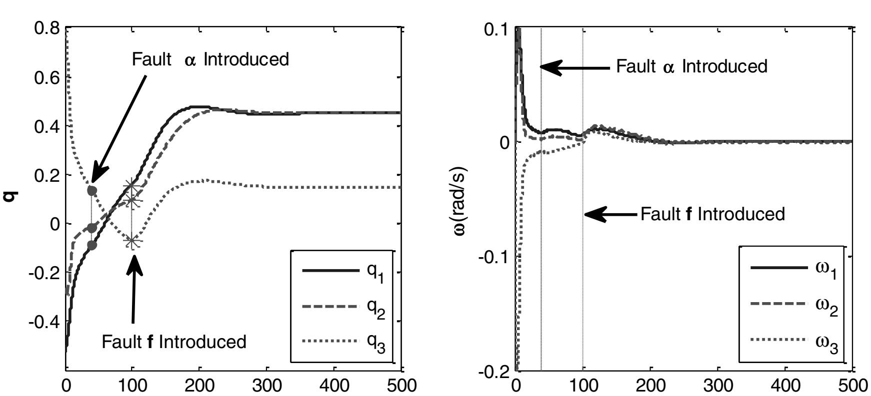

Fig. 1 Time response of attitude and angular velocity under normal controller equation (11) with the fault equation (35).

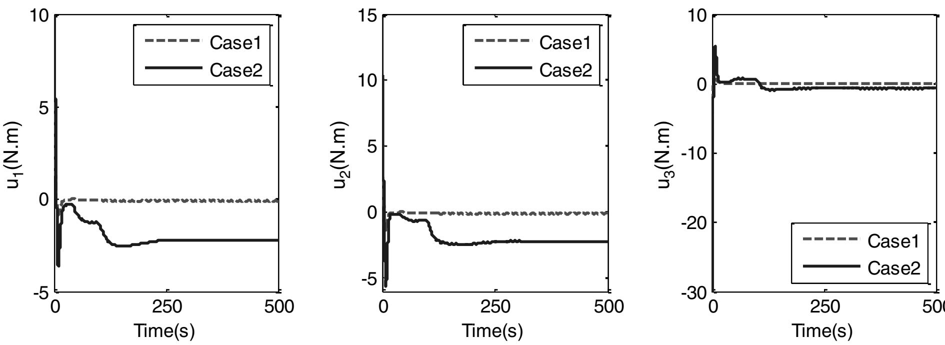

Fig. 2 Time response of control input. Case 1: normal controller equation (11); case 2: fault-tolerant controller equation (10).

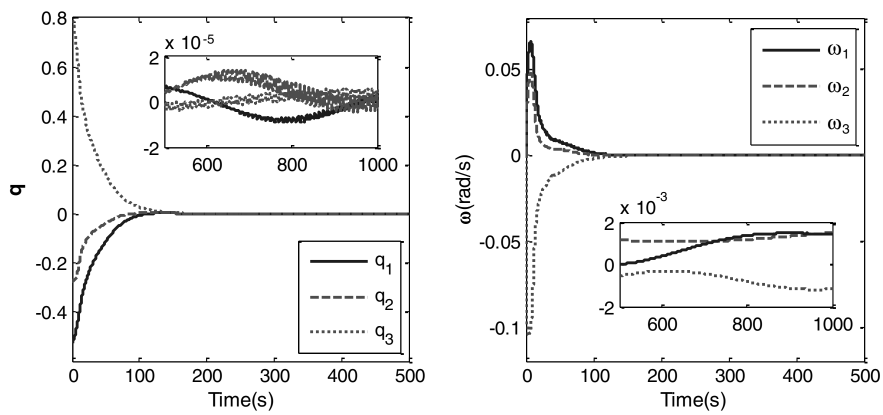

Fig. 3 Time response of attitude and angular velocity under fault-tolerant controller equation (10) with the fault equation (35).

enough to compensate the fault. Therefore, it can be concluded that a FTC design is greatly needed to accommodate the effect of actuator faults.

B. Response with Quaternion Feedback Fault-Tolerant Controller

In this case, we demonstrate the performance of the proposed strategy when the actuator faults equation (35) occurs in the system. When the designed controller equation (10) is implemented to the flexible spacecraft in the actuator faults case, the quaternion and angular velocity responses of the attitude system are presented in Fig. 3. As expected, we can see clearly that the control law equation (10) managed to compensate for the additive fault and partial loss of effectiveness, so that the closed-loop system is still stabilized within 150 s, and acceptable performance is also met despite of severe external disturbances. This is achieved by introducing the extra term $\mathbf{u}_F$ in Eq. (12). However, compared with the control input in case 1, a larger control effort is needed, as illustrated in Fig. 2 (solid line). Indeed, this is due to the fact that the fault-tolerant controller $\mathbf{u}_F$ is always active whenever the actuator undergoes faults or not. Moreover, as shown in Fig. 3b, the sign of angular velocity in each axis is invariable throughout the attitude maneuver. This further verified the analysis in Remark 3.

Summarizing the results from Figs. 1 and 3, the fault tolerance capability of Eq. (12) could be seen clearly. If the controller equation (10) is implemented without the $\mathbf{u}_F$ in Eq. (12), then the fault could not be accommodated, as shown in Fig. 1. Otherwise, the actuator faults and external disturbances can be successfully compensated, as shown in Fig. 3.

V. Conclusions

This Note proposed a novel FTC scheme for flexible spacecraft attitude stabilization in the presence of external disturbances, uncertain inertia matrix, and even two types of actuator faults. By using a first-order differentiation filter to account for the unmeasured magnitude of the angular velocity, the control law was derived with quaternion feedback and the knowledge on direction of the body angular velocity only. It is further shown that the developed control scheme does not involve any adaptive learning on system uncertainties or unknown bound; hence, the controller is more computationally favorable and practical for applications. Numerical simulations are also carried out to demonstrate the effectiveness of the proposed control structure. However, the drawback of the scheme remains its dependence on the availability of angular velocity direction. Extension of the proposed controller with consideration of control input constraints and elimination of the requirement of rate sensors will be carried out in the future work. Furthermore, extension to the fault-tolerant tracking control will also be carried out, in which the unknown inertia matrix is the main problem that needs to be addressed due to the extra item in the open-loop tracking error system in comparison with the attitude stabilization system.

Acknowledgments

This work was supported partially by the National Natural Science Foundation of China (project number 61004072), the Fundamental Research Funds for the Central Universities (project number HIT.NSRIF.2009003), the Research Fund for the Doctoral Program of Higher Education of China (project number 20070213061, 20102302110031), and the Scientific Research Foundation for the Returned Overseas Chinese Scholars of Harbin (project number 2010RFLXG001), as well as Natural Sciences and Engineering Research Council of Canada through a Strategic Project Grant and a Discovery Project Grant. The authors highly appreciate the preceding financial supports. The authors would also like to thank the reviewers and the Editor for their valuable comments and constructive suggestions that helped to improve the Note significantly.

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