Concordia University

ELEC372 Fundamentals of Control Systems

Homework #2

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1. The flow of traffic in a single lane can be described by the following equation:

$$\frac{dy}{dt} = V - Ae^{\frac{-\alpha}{y(t)}}$$

where

$y(t)$: relative distance between two cars,

V : constant velocity of the lead car $(V > 0)$,

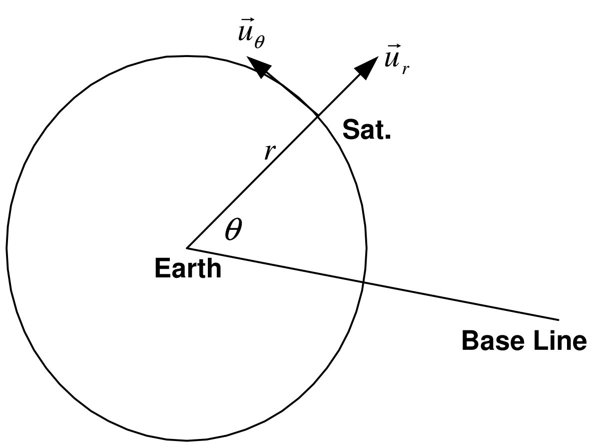
A, α : positive real constants.

a) Obtain the equilibrium value Y that results in $\dot{y}(t) = 0$.

b) Obtain the range of V / A for which $Y > 0$.

c) Linearize the equation around Y, and write the linearized equation.

2. Consider the following satellite orbit around the earth



The satellite motion is represented by the following set of differential equations:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{k}{r^2} + \frac{u_r}{m}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{u_\theta}{m}$$

where m is the satellite mass and k is a constant.

a) Assuming constant radius and angular speed ($r = r_0$ and $\dot{\theta} = \omega_0$), and $u_r = u_\theta = 0$, show that an equilibrium point can be found as $r_0^3\omega_0^2 = k$.

b) Show that the linearized satellite motion around the above equilibrium point will have the following form:

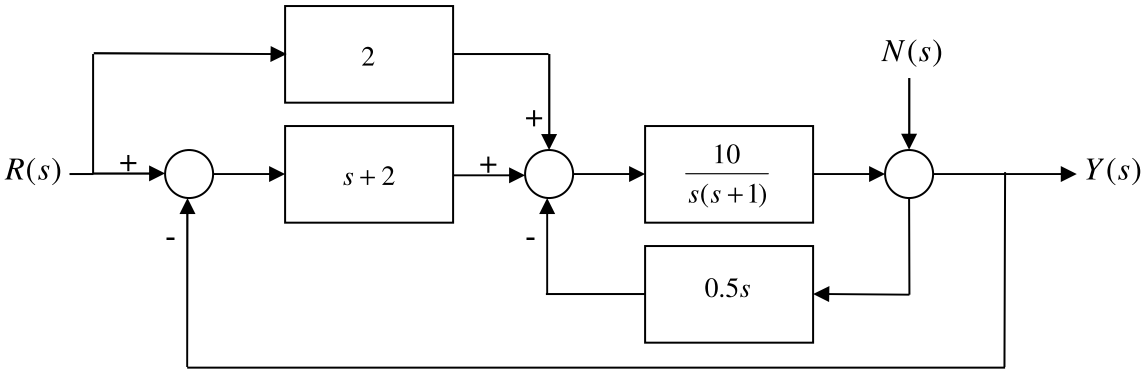
$$\delta\ddot{r} - 3\omega_0^2\delta r - 2r_0\,\omega_0\,\delta\omega = \frac{u_r}{m}$$

$$r_0\,\delta\dot{\omega} + 2\delta\dot{r}\,\omega_0 = \frac{u_\theta}{m}$$

c) Use Laplace transform to express the above linearized system in the following transfer function matrix form:

$$\begin{bmatrix}\delta r(s)\\ \delta\omega(s)\end{bmatrix} = \begin{bmatrix}\dfrac{1}{s^2+\omega_0^2} & \dfrac{2\omega_0}{s(s^2+\omega_0^2)}\\ \dfrac{-2\omega_0}{r_0(s^2+\omega_0^2)} & \dfrac{s^2-3\omega_0^2}{r_0 s(s^2+\omega_0^2)}\end{bmatrix}\begin{bmatrix}\dfrac{u_r(s)}{m}\\ \dfrac{u_\theta(s)}{m}\end{bmatrix}$$

3. The block diagram of a feedback control system is shown in the following figure.



Find the following transfer functions:

a) $\left.\dfrac{Y(s)}{R(s)}\right|_{N(s)=0}$

b) $\left.\dfrac{Y(s)}{N(s)}\right|_{R(s)=0}$

4. In the following block diagram find the following four transfer functions.

$$\left.\frac{C_1(s)}{R_1(s)}\right|_{R_2(s)=0},\ \left.\frac{C_1(s)}{R_2(s)}\right|_{R_1(s)=0},\ \left.\frac{C_2(s)}{R_1(s)}\right|_{R_2(s)=0},\ \left.\frac{C_2(s)}{R_2(s)}\right|_{R_1(s)=0}$$