Concordia University

ELEC372 Fundamentals of Control Systems

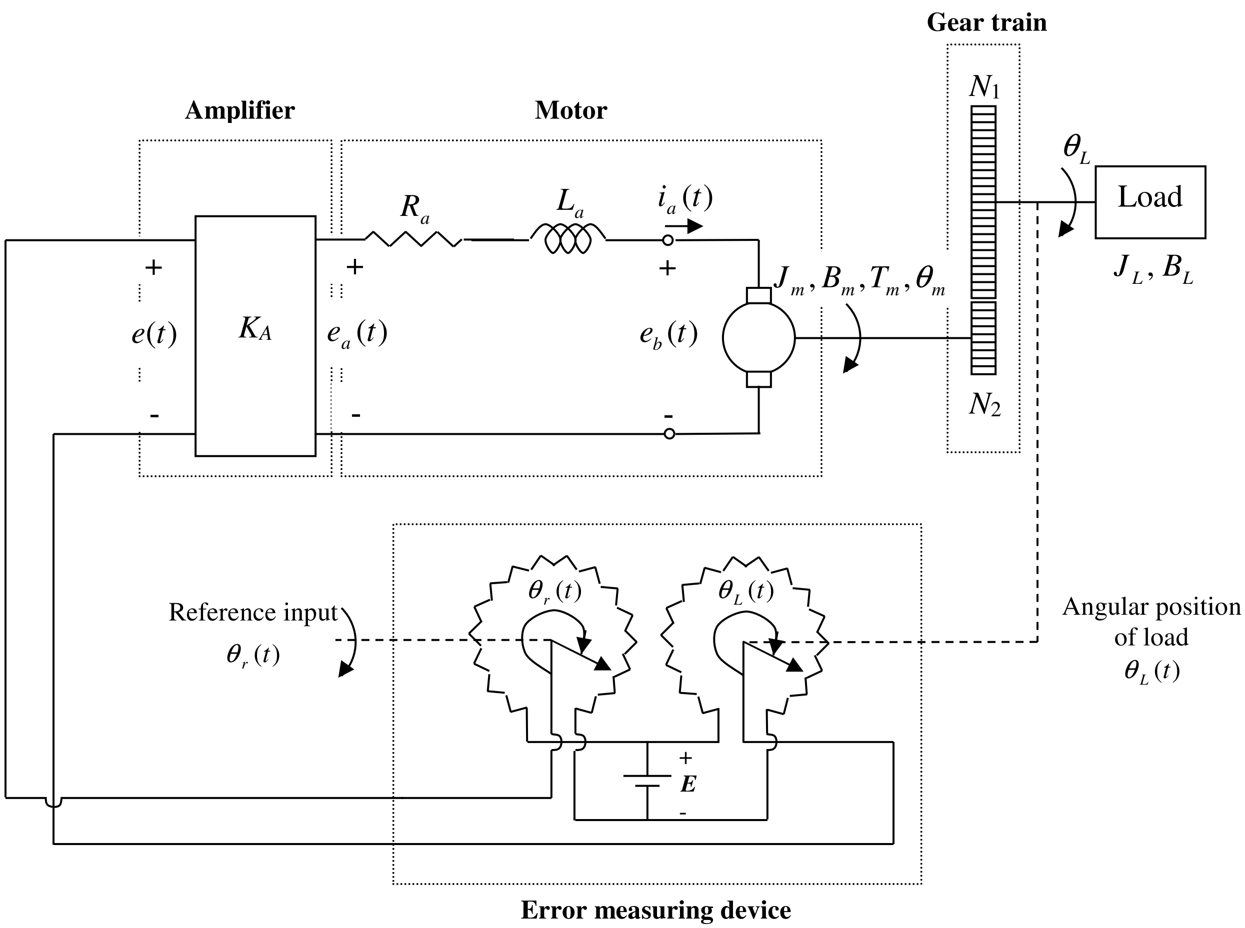
Homework #3

Amir G. Aghdam

1. Problem E2.15 from the 12th, 13th or 14th Edition of the main textbook (E2.14 from the 8th, 9th, 10th or 11th edition).

2. Problem P2.2 from the 8th, 9th, 10th, 11th, 12th, 13th or 14th edition of the main textbook.

3. (Katsuhiko Ogata, *Modern Control Engineering*, 4th Edition, Prentice Hall, 2002) "Consider the servo system shown in the following figure:



The motor shown is a servomotor, a DC motor designed specifically to be used in a control system. The operation of this system is as follows: A pair of potentiometers acts as an error-measuring device. They convert the input and output positions into proportional electric signals. The command input signal determines the angular position θ_r of the wiper arm of the input potentiometer. The angular position θ_r is the reference input to the system, and the electric potential of the arm (load) is proportional to the angular position of the arm. The output shaft position determines the angular position θ_L of the wiper arm of the output potentiometer. The difference between the input angular position θ_r and the output angular position θ_L is the error signal θ_e, or:

$$\theta_e(t) = \theta_r(t) - \theta_L(t).$$

The potential difference $e(t)$ is the error voltage, that is, $e(t) = K_0\theta_e(t)$, where K_0 is a proportionality constant. The error voltage that appears at the potentiometer terminals is amplified by the amplifier whose gain constant is K_A. The output voltage of this amplifier is applied to the armature circuit of the DC motor. A fixed voltage is applied to the field winding. If an error exists, the motor develops a torque to rotate the output load in asuch a way as to reduce the error to zero. Assume that the magnetic field is constant (armature-controlled DC motor) and the torque developed by the motor is:

$$T_m = K_m i_a$$

where K_m is the motor torque constant and i_a is the armature current.

When the armature is rotating, a voltage proportional to the product of the flux and angular velocity is induced in the armature. For a cosntant flux, the induced voltage e_b is directly proportional to the angular velocity or:

$$e_b(t) = K_b \frac{d\theta_m(t)}{dt}$$

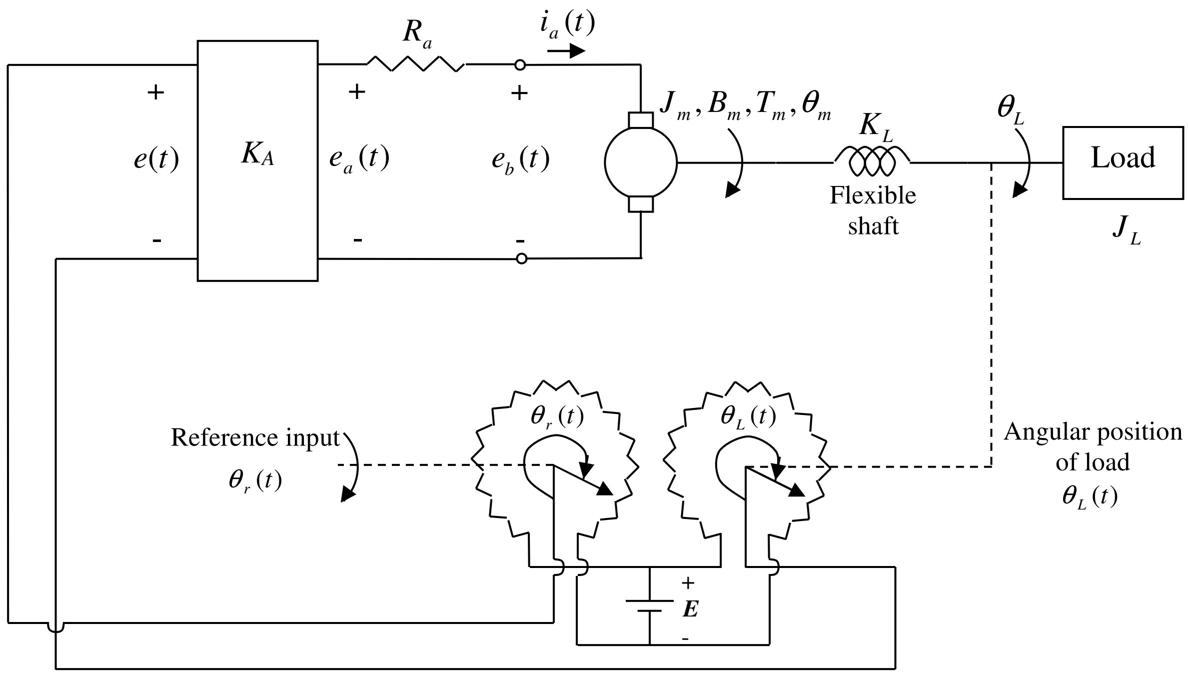
where $e_b(t)$ is the back emf, K_b is the back emf costant of the motor, and $\theta_m(t)$ is the angular displacement of the motor shaft, which is transformed to the angular displacement of the load through the gear train shown in the figure.

a) Obtain a block diagram for this system. The signals of the block diagram must be only θ_r, θ_e, e, θ_m , and θ_L.

b) Derive the transfer function from the error signal $\theta_e(t)$ to the output angular displacement $\theta_L(t)$.

c) Find a simplified transfer funcion from $\theta_e(t)$ to $\theta_L(t)$ when L_a is negligible."

4. (Farid Golnaraghi and Benjamin C. Kuo, *Automatic Control Systems*, 9h Edition, John Wiley & Sons, Inc., 2010). A dc-motor position-control system is shown in the following figure:



Assume that the magnetic field is constant (armature-controlled DC motor). "The following parameters and variables are defined: e is the error voltage; θ_r, the reference input; θ_L, the load position; K_A, the amplifier gain; e_a, the motor input voltage; e_b, the back emf; i_a, the motor current; T_m, the motor torque; J_m, the motor inertia = 0.03 oz-in.-sec^2; B_m, the motor viscous-friction coefficient = 10 oz-in.-sec; K_L, the torsional spring constant = 50,000 oz-in./rad; J_L, the load inertia = 0.05 oz-in.-sec^2; K_m, the motor torque constant ($T_m = K_m i_a$) = 21 oz-in./A; K_b, the back-emf constant = 15.5 V/1000 rpm; K_s, the error-detector gain = $E/2\pi$; E, the error-detector applied voltage = 2π V; R_a, the motor resistance = $1.15\,\Omega$; and $\theta_e = \theta_r - \theta_L$."

a) Draw a block diagram of the system with θ_r as the input and θ_L as the output, and show that $H_1(s)$ and $H_2(s)$ have the following form:

$$H_1(s) = \frac{\omega_m(s)}{T_m(s)} = \frac{J_L s^2 + K_L}{J_L J_m s^3 + B_m J_L s^2 + K_L(J_m + J_L)s + B_m K_L}$$

$$H_2(s) = \frac{\theta_L(s)}{\omega_m(s)} = \frac{K_L}{s(J_L s^2 + K_L)}$$

b) "Derive the closed-loop transfer function $M(s) = \dfrac{\theta_L(s)}{\theta_r(s)}$. Find the poles of $M(s)$ when $K_A = 1$, 2738, and 5476. Locate these poles in the *s*-plane, and comment on the significance of these values of K_A."