Concordia University

ELEC372 Fundamentals of Control Systems

Supplementary Problem Set 1 (Not to be handed in)

The problems form the foundation of Quiz 1

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In these problems $u(t)$ denotes the unit step function.

1. The positioning system of a printer can be modeled as

$$Y(s) = \frac{10(s+50)}{s^2+60s+500} R(s)$$

where the input $R(s)$ represents the desired position and $Y(s)$ is the output position. If the input is a unit step, calculate the final value of the output.

2. Consider a system with the closed-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{20(s+4)}{s^2+8s+15}$$

with input $R(s)$ and output $Y(s)$. When all initial conditions are zero and the input is a unit impulse, calculate the output $y(t)$.

3. Consider a system with transfer function

$$H(s) = \frac{20}{s^2+10s+25}$$

Calculate the system step response.

4. Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{du(t)}{dt} - 5u(t)$$

Solve the equation when $u(t)$ is a unit step and $y(0) = -3$ and $\left.\frac{dy(t)}{dt}\right|_{t=0} = 4$.

5. Find the response $y(t)$ of a system with transfer function

$$H(s) = \frac{s+2}{s(s+1)}$$

to an input $x(t) = e^{-t}u(t)$.

6. Find the response $y(t)$ of a system with impulse response

$$h(t) = (e^{-t} + e^{-2t})u(t)$$

to an input $x(t) = 5u(t)$.

7. Consider four LTI systems *A*, *B*, *C*, and *D*. The following information is known about each system.

   (a) System *A* is described by a transfer function $H_A(s) = 10000$.

   (b) System *B* produces an output $y(t) = u(t)$ to an input $u(t) = e^{-t}u(t)$.

   (c) System *C* has an impulse response $h(t) = t^{10}e^{-0.1t}u(t)$.

   Discuss BIBO stability of each system.