On Structurally Constrained Control Design with a Prespecified Form

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Abstract— **This paper deals with the decentralized overlapping control of interconnected systems. It is shown how the existing results aiming at designing a decentralized controller of a certain type such as static, dynamic, finite-dimensional linear time-varying, and sampled-data can be utilized to design a decentralized overlapping controller of a desired form, in order to achieve the design specifications. It is known that quotient fixed modes (QFM) of a decentralized control system are fixed with respect to any general (nonlinear and time-varying) decentralized control law. Generalization of this result to the decentralized overlapping control problem, i.e., the case when the control structure is only partially localized is not trivial at all. The notion of quotient overlapping fixed mode (QOFM) is introduced and it is shown that a mode of the interconnected system can be shifted by means of a general decentralized overlapping controller if and only if it is not a QOFM. It is then asserted that any interconnected system with no unstable QOFM can be stabilized by using an appropriate finite-dimensional linear time-varying controller. This work takes advantage of the new developments in analysis and design of decentralized control systems. The efficacy of the results is elucidated through a numerical example.**

I. INTRODUCTION

Control of interconnected systems has been of great interest in the literature in the past three decades, due to its enormous applications in important real-world problems. Such applications include for example power systems, communication networks, flexible space structures, etc. to name only a few. Due to the distributed nature of the problems of this type, the conventional control techniques are often not capable of handling them efficiently. More specifically, it is desired in the distributed interconnected systems to impose some constraints on the structure of the controller to be designed. These constraints specify the outputs of which subsystems can contribute to the construction of the input of any certain subsystem. To formulate the control problem, these constraints are usually represented by a matrix, which is often referred to as the information flow matrix [1].

A special case of structurally constrained controllers, is when the controller of each system operates independently of the other subsystems, i.e., there is no direct interaction between the control effort of each subsystem and the output signal of other subsystems. This case is of a particular interest in the control literature, and is usually referred to as the decentralized control problem [2], [3], [4], [5]. Each control component in a decentralized control system observes only the output of its corresponding subsystem to construct the input of that subsystem. The notion of decentralized fixed mode (DFM) was introduced in [2] to characterize the modes of an interconnected system which are fixed with respect to any decentralized linear time-invariant (LTI) controller. Since a DFM may not be fixed with respect to a nonlinear and time-varying controller, the notion of quotient fixed mode (QFM) was introduced in [6] to identify those modes that are fixed with respect to any type of decentralized control law (i.e., nonlinear and time-varying). Various properties of decentralized controllers are investigated thoroughly in the literature [3].

More recently, the case when the local controllers of an interconnected system can communicate with each other is studied intensively in the literature. This problem is referred to as decentralized overlapping control [7], [8], [9], and is motivated by the following practical issues:

1) The subsystems of many interconnected systems (referred to as overlapping subsystems) share some states [9], [10], [11]. In this case, it is often desired that the structure of the controller matches the overlapping structure of the system [9].
2) In some systems there are limitations on the availability of the states. In this case, only certain outputs of the system are available for constructing each control signal, and the controller need not be localized like the conventional decentralized control structures [12].

The control constraint in both cases discussed above is described by an information flow matrix which reflects the desired control structure. Decentralized overlapping control can, in fact, be envisaged as a general case of traditional decentralized control problem. Note that the terms *structurally constrained controller* and *decentralized overlapping controller* are interchangeably used in the literature [13], [14], [15]. Analogously to the definition of DFM, the notion of decentralized overlapping fixed mode (DOFM) was introduced in [12] to identify the modes of an interconnected system which are fixed with respect to LTI decentralized overlapping controllers. A procedure was then proposed to place the non-DOFMs freely in the complex plane. The question arises: Is there any non-LTI decentralized overlapping controller to shift a DOFM? It is also desired to employ the existing non-LTI decentralized control design techniques such as

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finite-dimensional linear time-varying (LTV), sampled-data, generalized sampled-data hold functions (GSHF), etc. to obtain a decentralized overlapping controller.

This paper aims to address the above-mentioned questions. A mapping is first introduced between the decentralized overlapping control and the decentralized control structures. This brings about advantages of using the existing results for traditional decentralized control design, in order to solve the decentralized overlapping control design problem. Some important properties of DOFMs are first investigated in detail. It is shown that the DOFMs of a system can be obtained in terms of the transmission zeros of a number of relevant systems. This is, in fact, generalization of the results given in [1]. Different types of decentralized overlapping control laws, namely static LTI, dynamic LTI, sampled-data, GSHF, and finite-dimensional LTV, are then studied. Finally, the important problem of stabilizability of an interconnected system by means of a general (nonlinear and time-varying) decentralized overlapping controller is addressed via the new notion of quotient overlapping fixed mode (QOFM). It is shown that any mode of the system is movable via a decentralized overlapping controller if and only if it is not a QOFM.

This paper is organized as follows. Some preliminary results, which are basically borrowed from [12], are presented in Section II. The main results of the paper are given in Section III, followed by a numerical example in Section IV. Finally, some concluding remarks are given in Section V.

II. Preliminaries

Consider a LTI interconnected system $\mathcal{S}$ consisting of ν subsystems with the following state-space representation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{\nu} B_i u_i(t) \\ y_i(t) &= C_i x(t) + \sum_{j=1}^{\nu} D_{ij} u_j(t), \quad i \in \bar{\nu} := \{1, 2, ..., \nu\} \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state, and $u_i(t) \in \Re^{m_i}$ and $y_i(t) \in \Re^{r_i}$, $i \in \bar{\nu}$, are the input and the output of the i^{th} subsystem S_i, respectively. Define the following vectors:

$$\begin{aligned} u(t) &:= \begin{bmatrix} u_1(t)^T & u_2(t)^T & \cdots & u_\nu(t)^T \end{bmatrix}^T \\ y(t) &:= \begin{bmatrix} y_1(t)^T & y_2(t)^T & \cdots & y_\nu(t)^T \end{bmatrix}^T \end{aligned} \tag{2}$$

Define also:

$$\begin{aligned} B &:= \begin{bmatrix} B_1 & B_2 & \cdots & B_\nu \end{bmatrix}, \\ C &:= \begin{bmatrix} C_1^T & C_2^T & \cdots & C_\nu^T \end{bmatrix}^T, \\ D &:= \begin{bmatrix} D_{11} & \cdots & D_{1\nu} \\ \vdots & \ddots & \vdots \\ D_{\nu 1} & \cdots & D_{\nu\nu} \end{bmatrix}, \\ m &:= \sum_{i=1}^{\nu} m_i, \quad r := \sum_{i=1}^{\nu} r_i \end{aligned} \tag{3}$$

It is desired to stabilize the system $\mathcal{S}$ by using a structurally constrained controller. The structure of this controller is often determined by means of a given matrix $\mathcal{K}$, introduced below.

Definition 1: Define the following matrices:

- Information flow matrix $\mathcal{K}$ is a matrix whose (i, j) entry, $i, j \in \bar{\nu}$, is a equal to 1 if the output $y_j(t)$ can contribute to the construction of the input $u_i(t)$, and is zero otherwise.
- Control *interaction* structure $\mathbf{K}$ is a matrix whose (i, j) block entry, $i, j \in \bar{\nu}$, is a $m_i \times r_j$ matrix denoted by k_{ij} if the output of the j^{th} subsystem can contribute to the construction of the input of the i^{th} subsystem, and is a $m_i \times r_j$ zero matrix otherwise.

The information flow matrix corresponding to any system is enclosed in parentheses throughout the paper, if necessary. For instance, $\mathcal{S}(\mathcal{K})$ indicates that the structure of the controller to be designed for the system $\mathcal{S}$ is to comply with the information flow matrix $\mathcal{K}$.

Note that k_{ij} in Definition 1 represents a component of the controller, which transforms the output of the j^{th} subsystem to the input of the i^{th} subsystem. Note also that the interaction structure matrix $\mathbf{K}$ not only conveys the information of the matrix $\mathcal{K}$, but also labels the control components. However, the matrices $\mathcal{K}$ and $\mathbf{K}$ will be utilized for different purposes hereafter.

In order to present the main results of this works, some concepts (partially borrowed from [12]) are to be introduced first.

Procedure 1: [12] Construct the graph $\mathcal{G}$ as follows:

1) Define two sets of ν vertices. Label the sets as set 1 and set 2, and the vertices in each set as vertex 1 to vertex ν.
2) For any $i, j \in \bar{\nu}$, connect the i^{th} vertex of set 1 to the j^{th} vertex of set 2 with an edge, if the (i, j) entry of $\mathcal{K}$ is equal to 1.

Procedure 2: Partition the graph $\mathcal{G}$ into a set of complete bipartite subgraphs such that each edge of the graph $\mathcal{G}$ appears in only one of the subgraphs. It is to be noted that this partition may require some of the vertices of the graph $\mathcal{G}$ to appear in several subgraphs.

It can be easily verified that Procedure 2 does not necessarily lead to a unique graph. Denote all the graphs which can be obtained through this procedure, with $\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_l$. Without loss of generality, assume that $\mathcal{G}_1$ and $\mathcal{G}_l$ are the ones with the following properties:

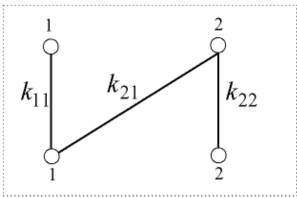
- $\mathcal{G}_1$ is obtained by considering any vertex in set 1 of the graph $\mathcal{G}$ along with all of the vertices in set 2 connected to that vertex as a complete bipartite graph.
- $\mathcal{G}_l$ is obtained by considering any edge in the graph $\mathcal{G}$ as a complete bipartite graph.

Procedure 3: Label the complete bipartite subgraphs of $\mathcal{G}_\mu$ ($\mu \in \bar{l} := \{1, 2, ..., l\}$) as subgraphs 1 to ν_μ. Consider subgraph number σ ($\forall \sigma \in \{1, 2, ..., \nu_\mu\}$). Label those vertices of this subgraph which belong to set 1 as vertex $1, ..., \eta_\sigma^\mu$. This group of vertices will be referred to as subset 1 (corresponding to subgraph number σ). Similarly, label those vertices which belong to set 2 of this subgraph as vertex $1, ..., \bar{\eta}^{\mu}_{\sigma}$, and define subset 2 accordingly. Define $\mathbf{K}_\mu$ as a block diagonal matrix, where its (σ, σ) block entry, $\sigma = 1, ..., \nu_\mu$, is a matrix itself, whose (i, j) block entry is equal to the gain of the edge connecting vertex i of subset 1 to vertex j of subset 2 in subgraph number σ of $\mathcal{G}_\mu$, for any $i \in \{1, ..., \eta^{\mu}_{\sigma}\}$, $j \in \{1, ..., \bar{\eta}^{\mu}_{\sigma}\}$. Denote the dimension of the (σ, σ) block entry of $\mathbf{K}_\mu$ with $m^{\mu}_{\sigma} \times r^{\mu}_{\sigma}$, for $\sigma = 1, 2, ..., \nu_\mu$, and the dimension of $\mathbf{K}_\mu$ with $m^\mu \times r^\mu$.

As an example, consider a system consisting of two subsystems with the following control interaction structure:

$$\mathbf{K} = \begin{bmatrix} k_{11} & 0 \\ k_{21} & k_{22} \end{bmatrix} \tag{4}$$

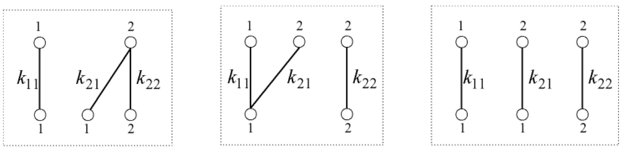
The graph $\mathcal{G}$ corresponding to the matrix $\mathbf{K}$, introduced in Procedure 1, is depicted in Figure 1.



$\mathcal{G}$

Fig. 1. The structural graph of the system $\mathcal{S}$ with the matrix $\mathbf{K}$ given by (4).

The graphs $\mathcal{G}_1, \mathcal{G}_2$ and $\mathcal{G}_3$ obtained based on Procedure 2 are sketched in Figure 2. It is obvious that some of the vertices are recurring.



$\mathcal{G}_1$ $\mathcal{G}_2$ $\mathcal{G}_3$

Fig. 2. The graphs $\mathcal{G}_1, \mathcal{G}_2$ and $\mathcal{G}_3$ corresponding to the system $\mathcal{S}$ with the matrix $\mathbf{K}$ given by (4).

Therefore, the matrices $\mathbf{K}_1, \mathbf{K}_2$ and $\mathbf{K}_3$ can be attained using Procedure 3 as follows:

$$\mathbf{K}_1 = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & k_{22} \end{bmatrix}, \ \mathbf{K}_2 = \begin{bmatrix} k_{11} & 0 \\ k_{21} & 0 \\ 0 & k_{22} \end{bmatrix}, \\ \mathbf{K}_3 = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{21} & 0 \\ 0 & 0 & k_{22} \end{bmatrix} \tag{5}$$

It is shown in [12] that there exist constant matrices Φ_μ and $\bar{\Phi}_\mu$ satisfying the following relation:

$$\mathbf{K} = \Phi_\mu \mathbf{K}_\mu \bar{\Phi}_\mu \tag{6}$$

for any $\mu \in \bar{l}$. Besides, a simple procedure is proposed to obtain these transformation matrices.

III. Main results

The following definitions will prove to be convenient in the development of main results.

Definition 2: Consider two arbitrary systems $\mathcal{S}_{d_1}$ and $\mathcal{S}_{d_2}$ associated with the information flow matrices $\mathcal{K}_{d_1}$ and $\mathcal{K}_{d_2}$, where $\mathcal{S}_{d_1}$ and $\mathcal{S}_{d_2}$ are of the same order and have the same initial state. Let $\mathbf{M}$ denote a given set of controllers. The systems $\mathcal{S}_{d_1}(\mathcal{K}_{d_1})$ and $\mathcal{S}_{d_2}(\mathcal{K}_{d_2})$ are called *analogous* with respect to $\mathbf{M}$ if for any controller K_{d_1} in $\mathbf{M}$ complying with the information flow matrix $\mathcal{K}_{d_1}$, there also exists a controller K_{d_2} in $\mathbf{M}$ complying with the information flow matrix $\mathcal{K}_{d_2}$ (and vice versa), such that the state of the system $\mathcal{S}_{d_1}$ under the controller K_{d_1} is equivalent to the state of $\mathcal{S}_{d_2}$ under K_{d_2}.

Definition 3: Define $\boldsymbol{\mathcal{S}}_\mu$, $\mu \in \bar{l}$, as an interconnected system with the following state-space representation:

$$\begin{aligned} \dot{\mathbf{x}}_\mu(t) &= A\mathbf{x}_\mu(t) + \mathbf{B}^\mu \mathbf{u}_\mu(t) \\ \mathbf{y}_\mu(t) &= \mathbf{C}^\mu \mathbf{x}_\mu(t) + \mathbf{D}^\mu \mathbf{u}_\mu(t) \end{aligned} \tag{7}$$

where the system parameters are related to the state-space matrices of the system $\mathcal{S}$ given by (1), as shown below:

$$\mathbf{B}^\mu = B\Phi_\mu, \quad \mathbf{C}^\mu = \bar{\Phi}_\mu C, \quad \mathbf{D}^\mu = \bar{\Phi}_\mu D \Phi_\mu \tag{8}$$

$\mathbf{u}_\mu(t) \in \Re^{m^\mu}$ and $\mathbf{y}_\mu(t) \in \Re^{r^\mu}$ are the input and the output of $\boldsymbol{\mathcal{S}}_\mu$, respectively, and $\mathbf{x}_\mu(0) = x(0)$. For any $\mu \in \bar{l}$, define the information flow matrix $\mathcal{K}_\mu$ for the system $\boldsymbol{\mathcal{S}}_\mu$ as a matrix obtained from $\mathbf{K}_\mu$ by replacing its nonzero block entry k_{ij}, with the scalar 1, for any $i, j \in \bar{\nu}$, and its zero block entries with the scalar 0.

One of the main objectives of this section is to prove that the systems $\mathcal{S}(\mathcal{K}), \boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), \boldsymbol{\mathcal{S}}_2(\mathcal{K}_2), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are *analogous* with respect to several classes of controllers, including LTI and LTV ones. The significance of this result will now be spelled out.

Consider a set of controllers, denoted by $\mathbf{M}$. Assume that $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, $\mu \in \bar{l}$, are *analogous* w.r.t. (with respect to) $\mathbf{M}$. In order to design a controller belonging to $\mathbf{M}$ for the system $\mathcal{S}$ w.r.t. the information flow structure $\mathcal{K}$ to achieve any design objectives, one can equivalently design a controller belonging to $\mathbf{M}$ for the system $\boldsymbol{\mathcal{S}}_\mu$, w.r.t. the information flow structure $\mathcal{K}_\mu$, to attain the same objective. The mapping between the components of $\mathbf{K}$ and $\mathbf{K}_\mu$ (derived from the equation (6)) can then be used to find the corresponding controller for the system $\mathcal{S}(\mathcal{K})$. The important advantage of this indirect design procedure is that the information flow structure $\mathcal{K}_\mu$ is block diagonal, and hence the problem is converted to the conventional decentralized control design framework.

A. LTI control law

Theorem 1: For any $\mu \in \bar{l}$, the systems $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ and $\mathcal{S}(\mathcal{K})$ are *analogous* w.r.t. the set of all LTI controllers.

Proof: The proof can be simply inferred from [12], and is omitted here. ∎

Corollary 1: For any $\mu \in \bar{l}$, the systems $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ and $\mathcal{S}(\mathcal{K})$ are *analogous* w.r.t. the set of all continuous-time *static* controllers.

Proof: The proof is similar to the proof of Theorem 1. ■

The work [12] states that the DOFMs of $\mathcal{S}(\mathcal{K})$ are those modes of the system $\mathcal{S}$ which are fixed w.r.t. any dynamic LTI controller with the information flow structure $\mathcal{K}$. Thus, it can be easily deduced from Theorem 1 that the DOFMs of the system $\mathcal{S}(\mathcal{K})$ are the same as the DFMs of the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ for any $\mu \in \bar{l}$, due to the property of analogousness.

Partition now the matrices $\mathbf{B}^\mu, \mathbf{C}^\mu$ and $\mathbf{D}^\mu$, $\mu \in \bar{l}$, as follows:

$$\mathbf{B}^\mu = \begin{bmatrix} \mathbf{B}_1^\mu & \mathbf{B}_2^\mu & \cdots & \mathbf{B}_{\nu_\mu}^\mu \end{bmatrix},$$

$$\mathbf{C}^\mu = \begin{bmatrix} \mathbf{C}_1^\mu \\ \mathbf{C}_2^\mu \\ \vdots \\ \mathbf{C}_{\nu_\mu}^\mu \end{bmatrix}, \quad \mathbf{D}^\mu = \begin{bmatrix} \mathbf{D}_{1,1}^\mu & \cdots & \mathbf{D}_{1,\nu_\mu}^\mu \\ \vdots & \ddots & \vdots \\ \mathbf{D}_{\nu_\mu,1}^\mu & \cdots & \mathbf{D}_{\nu_\mu,\nu_\mu}^\mu \end{bmatrix} \tag{9}$$

where:

$$\mathbf{B}_i^\mu \in \Re^{m_i^\mu}, \quad \mathbf{C}_i^\mu \in \Re^{r_i^\mu}, \quad \mathbf{D}_{ij}^\mu \in \Re^{r_i^\mu \times m_j^\mu} \tag{10}$$

for any $i, j \in \{1, 2, ..., \nu_\mu\}$. It is to be noted that m_i^μ and r_i^μ are defined in Procedure 3.

Theorem 2: $\lambda \in \mathrm{sp}(A)$ is a DOFM of the system $\mathcal{S}(\mathcal{K})$ if and only if it is a transmission zero of the following system:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + \begin{bmatrix} \mathbf{B}_{i_1}^l & \mathbf{B}_{i_2}^l & \cdots & \mathbf{B}_{i_q}^l \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) &= \begin{bmatrix} \mathbf{C}_{i_1}^l \\ \mathbf{C}_{i_2}^l \\ \vdots \\ \mathbf{C}_{i_q}^l \end{bmatrix} \mathbf{x}(t) \\ &+ \begin{bmatrix} 0 & \mathbf{D}_{i_1 i_2}^l & \cdots & \mathbf{D}_{i_1 i_q}^l \\ \mathbf{D}_{i_2 i_1}^l & 0 & \cdots & \mathbf{D}_{i_2 i_q}^l \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}_{i_q i_1}^l & \mathbf{D}_{i_q i_2}^l & \cdots & 0 \end{bmatrix} \mathbf{u}(t) \end{aligned} \tag{11}$$

for any $q \in \{1, ..., \nu_l\}$ and any arbitrary subset $\{i_1, i_2, ..., i_q\}$ of $\{1, 2, ..., \nu_l\}$.

Proof: It was shown earlier that the DOFMs of the system $\mathcal{S}(\mathcal{K})$ are the same as the DFMs of the system $\boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$. Furthermore, since the matrix $\mathcal{K}_l$ is diagonal (because the graph $\mathcal{G}_l$ is composed of some disjoint edges), it results from [1] that the DFMs of the system $\boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are the same as the common transmission zeros of the systems given by (11). This completes the proof. ■

Remark 1: The results of Theorem 2 are obtained in [1] for the particular case when the information flow matrix $\mathcal{K}$ is block diagonal. Furthermore, the system given by (11) is constructed by using Kronecker product in [1], while it is formed by means of graph theory in this paper. Therefore, Theorem 2 presents the results for the most general information flow structure compared to the ones given in [1].

B. Generalized sampled-data hold function

Periodic control design using generalized sampled-data hold function (GSHF) and its advantages have been studied intensively in the literature [16], [17], [13], [19]. Assume that it is desired to obtain a GSHF for the system $\mathcal{S}$, which complies with the information flow structure $\mathcal{K}$, and to achieve certain design objectives. Let this GSHF be denoted by $F(t)$. Hence, the hold controller will be as follows:

$$u(t) = F(t)y[\kappa], \quad \kappa h \leq t < (\kappa+1)h, \quad \kappa \geq 0 \tag{12}$$

where h represents the sampling period. Note that the discrete argument corresponding to the samples of any signal is enclosed in brackets (e.g., $y[\kappa] := y(\kappa h)$). In this subsection, assume that D is a zero matrix.

Theorem 3: The systems $\mathcal{S}(\mathcal{K}), \boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are *analogous* w.r.t. the set of all hold controllers (GSHFs).

Proof: To prove the theorem, it suffices to show that $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ are *analogous* w.r.t. all hold controllers, for any $\mu \in \bar{l}$. Consider a GSHF $F(t)$ which complies with the information flow structure $\mathcal{K}$. Utilize the proper transformation on $F(t)$ to obtain the equivalent hold function $F_\mu(t)$ for the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$. Note that $F_\mu(t)$ can be attained using the mapping between the components of $\mathbf{K}$ and $\mathbf{K}_\mu$. Since $F(t)$ and $F_\mu(t)$ comply with the information flow matrices $\mathcal{K}$ and $\mathcal{K}_\mu$, respectively, it is straightforward to show that $F(t) = \Phi_\mu F_\mu(t)\bar{\Phi}_\mu$. On the other hand, it follows from (12) that:

$$\dot{x}(t) = Ax(t) + BF(t)Cx[\kappa] \tag{13}$$

and consequently:

$$\begin{aligned} \dot{\mathbf{x}}_\mu(t) &= A\mathbf{x}_\mu(t) + \mathbf{B}^\mu F_\mu(t)\mathbf{C}^\mu \mathbf{x}_\mu[\kappa] \\ &= A\mathbf{x}_\mu(t) + B\Phi_\mu F_\mu(t)\bar{\Phi}_\mu C\mathbf{x}_\mu[\kappa] \\ &= A\mathbf{x}_\mu(t) + BF(t)C\mathbf{x}_\mu[\kappa] \end{aligned} \tag{14}$$

for all $t \in [\kappa h, (\kappa+1)h),\ \kappa \geq 0$. The equations (13) and (14), and the equality $x(0) = \mathbf{x}_\mu(0)$ result in the relation $x(t) = \mathbf{x}_\mu(t)$ for all $t \geq 0$. Conversely, for any GSHF $F_\mu(t)$ complying with the information flow matrix $\mathcal{K}_\mu$, it is straightforward to show that the state of the system $\mathcal{S}$ under the GSHF $F(t) = \Phi_\mu F_\mu(t)\bar{\Phi}_\mu$ is identical to that of the system $\boldsymbol{\mathcal{S}_\mu}$ under $F_\mu(t)$. ■

Theorem 3 states that the problem of designing a GSHF for the system $\mathcal{S}(\mathcal{K})$ can be formulated as the problem of designing a GSHF for the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ for any $\mu \in \bar{l}$. However, due to the decentralized structure of the control for $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, $\mu \in \bar{l}$, the corresponding GSHF design can be accomplished by using the existing methods [13], [18], [19].

C. Sampled-data controller

A typical sampled-data controller consists of a sampler, a zero-order hold (ZOH) and a discrete-time controller [20]. It is to be noted that a sampled-data controller acts as a time-varying control law for the continuous-time system. It is desired in this subsection to present a method for designing a sampled-data controller for the system $\mathcal{S}$, whose structure complies with a given information flow matrix $\mathcal{K}$. Throughout the remainder of this paper, the term *linear shift-invariant* (LSI) will be used instead of LTI, for discrete-time systems.

Theorem 4: The systems $\mathcal{S}(\mathcal{K}), \boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are all *analogous* w.r.t. the set of all LSI sampled-data controllers.

Proof: Denote the sampling period with h, and the discrete-time equivalent models of the systems $\mathcal{S}, \boldsymbol{\mathcal{S}}_1, ..., \boldsymbol{\mathcal{S}}_l$ with $\bar{\mathcal{S}}, \bar{\boldsymbol{\mathcal{S}}}_1, ..., \bar{\boldsymbol{\mathcal{S}}}_l$, respectively. Assume that the system $\bar{\mathcal{S}}$ is represented by:

$$\begin{aligned} x[\kappa+1] &= \bar{A}x[\kappa] + \bar{B}u[\kappa] \\ y[\kappa] &= Cx[\kappa] + Du[\kappa] \end{aligned} \tag{15}$$

Similarly, let the system $\bar{\boldsymbol{\mathcal{S}}}_\mu$ be represented by:

$$\begin{aligned} \mathbf{x}_\mu[\kappa+1] &= \bar{A}\mathbf{x}_\mu[\kappa] + \bar{\mathbf{B}}^\mu \mathbf{u}_\mu[\kappa] \\ \mathbf{y}_\mu[\kappa] &= \mathbf{C}^\mu \mathbf{x}_\mu[\kappa] + \mathbf{D}^\mu \mathbf{u}_\mu[\kappa], \qquad \mu \in \bar{l} \end{aligned} \tag{16}$$

It can be easily verified that:

$$\bar{\mathbf{B}}^\mu = \int_0^h e^{\tau A}\mathbf{B}^\mu \, d\tau = \int_0^h e^{\tau A} B \, d\tau \times \Phi_\mu = \bar{B}\Phi_\mu \tag{17}$$

It results from (15), (16), and (17) that the state-space matrices of $\bar{\mathcal{S}}$ are related to those of $\bar{\boldsymbol{\mathcal{S}}}_\mu$, exactly the same way the state-space matrices of $\mathcal{S}$ and $\boldsymbol{\mathcal{S}}_\mu$ are related. Hence, the systems $\bar{\mathcal{S}}$ and $\bar{\boldsymbol{\mathcal{S}}}_\mu$ are *analogous* w.r.t. the LSI sampled-data controllers. Consider now a discrete-time LSI controller with the transfer function matrix $\bar{K}(z)$ for the system $\bar{\mathcal{S}}(\mathcal{K})$. Construct a discrete-time LSI controller with the transfer function matrix $\bar{K}_\mu(z)$ for the system $\bar{\boldsymbol{\mathcal{S}}}_\mu(\mathcal{K}_\mu)$, such that it corresponds to the controller $\bar{K}(z)$ for $\bar{\mathcal{S}}(\mathcal{K})$. This controller can be obtained from the mapping between the components of $\mathbf{K}$ and $\mathbf{K}_\mu$. It is straightforward to show that $\bar{K}(z) = \Phi_\mu \bar{K}_\mu(z)\bar{\Phi}_\mu$. Applying the controller $\bar{K}(z)$ to the system $\bar{\mathcal{S}}$ and the controller $\bar{K}_\mu(z)$ to $\bar{\boldsymbol{\mathcal{S}}}_\mu$, one can conclude (using an approach similar to the proof of Theorem 1) that $x[\kappa] = \mathbf{x}_\mu[\kappa]$ and $u[\kappa] = \Phi_\mu \mathbf{u}_\mu[\kappa]$ for any $\kappa \geq 0$. Therefore:

$$\begin{aligned} x(t) &= e^{(t-\kappa h)A}x[\kappa] + \int_{\kappa h}^{t} e^{(\tau-\kappa h)A}Bu[\kappa] \, d\tau \\ &= e^{(t-\kappa h)A}\mathbf{x}_\mu[\kappa] + \int_{\kappa h}^{t} e^{(\tau-\kappa h)A}B\Phi_\mu \mathbf{u}_\mu[\kappa] \, d\tau \\ &= e^{(t-\kappa h)A}\mathbf{x}_\mu[\kappa] + \int_{\kappa h}^{t} e^{(\tau-\kappa h)A}\mathbf{B}^\mu \mathbf{u}_\mu[\kappa] \, d\tau \\ &= \mathbf{x}_\mu(t) \end{aligned} \tag{18}$$

for any $t \in [\kappa h, (\kappa+1)h)$, $k \geq 0$. Similarly, it can be easily verified that given any controller $\bar{K}_\mu(z)$ for the system $\bar{\boldsymbol{\mathcal{S}}}_\mu(\mathcal{K})$, the controller $\bar{K}(z) := \Phi_\mu \bar{K}_\mu(z)\bar{\Phi}_\mu$ corresponds to the information flow matrix $\mathcal{K}$. Moreover, the state of the system $\mathcal{S}$ under the controller $\bar{K}(z)$ is the same as that of $\boldsymbol{\mathcal{S}}_\mu$ under $\bar{K}_\mu(z)$. ■

Note that finding a sampled-data decentralized control law to achieve certain design objectives has been investigated in the literature, e.g, see [14].

D. Finite-dimensional linear time-varying controller

It is well-known that finite-dimensional linear time-varying (LTV) controllers are superior to their LTI counterparts in many control applications. It is desired in this subsection to present a procedure for designing a finite-dimensional LTV controller complying with the information flow matrix $\mathcal{K}$, for the system $\mathcal{S}$. Note that throughout this work, the term "finite-dimensional LTV controller" refers to a control law which can be represented by the following state-space model:

$$\begin{aligned} \dot{\tilde{z}}(t) &= \tilde{A}(t)\tilde{z}(t) + \tilde{B}(t)\tilde{u}(t) \\ \tilde{y}(t) &= \tilde{C}(t)\tilde{z}(t) + \tilde{D}(t)\tilde{u}(t) \end{aligned} \tag{19}$$

Theorem 5: The systems $\mathcal{S}(\mathcal{K}), \boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are *analogous* w.r.t. the set of all finite-dimensional LTV controllers.

Proof: This theorem extends the results of Theorem 1 to the case when the controllers are finite-dimensional LTV (as opposed to LTI). The proof is similar to that of Theorem 1 (but should be carried out in the time-domain). The details of the proof are omitted here due to space restrictions, and may be found in [21]. ■

Theorem 5 implies that to design a finite-dimensional LTV controller for the system $\mathcal{S}(\mathcal{K})$, one can first design a LTV controller for one of the systems $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$. This result will be exploited in the following section to present one of the main contributions of the present work.

E. General controller

The objective of this subsection is to find out under what conditions the system $\mathcal{S}(\mathcal{K})$ is stabilizable by means of a general control law (i.e. nonlinear and time-varying), when there exists no stabilizing constrained LTI controller.

Theorem 6: The systems $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ are *analogous* w.r.t. any type of controller (i.e. nonlinear or time-varying).

Proof: As pointed out in the discussion following Theorem 5 in [21], the configurations of the systems $\mathcal{S}$ and $\boldsymbol{\mathcal{S}}_1$ are essentially equivalent. In other words, the system $\boldsymbol{\mathcal{S}}_1$ is obtained from $\mathcal{S}$ by introducing some redundant outputs or control agents and reordering them, in such a way that the information flow structure $\mathcal{K}$ is converted to $\mathcal{K}_1$ (note that according to Lemma 1 in [12], $B = \mathbf{B}^1$). Hence, the state of the closed-loop system corresponding to the pair $(\mathcal{S}, \mathcal{K})$ is identical to that of the pair $(\boldsymbol{\mathcal{S}}_1, \mathcal{K}_1)$, regardless of the type of the control law. ■

It is to be noted that unlike $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$, the systems $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, $\mu \in \{2, 3, ..., l\}$, are not *analogous* w.r.t. any type of controller, in general. This results from the fact that the superposition principle presented in item 2 of the discussion following Theorem 5 in [21] does not apply here, as the controllers are nonlinear.

Remark 2: It follows immediately from Theorem 6 that the system $\mathcal{S}(\mathcal{K})$ is stabilizable if and only if the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ is stabilizable.

Henceforth, assume that the matrix D is equal to zero. It is shown in [6] that a system is stabilizable w.r.t. a block-diagonal information flow matrix (i.e. decentralized control structure) if and only if the system does not any unstable quotient fixed mode (QFM). However, QFM is only defined for decentralized control structures. In the following, this notion is extended to the general information flow structure and its property is investigated accordingly.

Definition 4: $\lambda \in \text{sp}(A)$ is a quotient overlapping fixed mode (QOFM) of the system $\mathcal{S}$ w.r.t. the information flow matrix $\mathcal{K}$, if λ cannot be eliminated by using any type of controller whose structure complies with $\mathcal{K}$.

Theorem 7: The QOFMs of the system $\mathcal{S}(\mathcal{K})$ are the same as the QFMs of the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, $\forall \mu \in \bar{l}$.

Proof: It follows from Theorem 6 that the QOFMs of the system $\mathcal{S}(\mathcal{K})$ are the same as the QFMs of the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$. To complete the proof, it suffices to show that the QFMs of the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ are the same as those of the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, for $\mu = 2, 3, ..., l$. This can be deduced from the following argument:

- The systems $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ all have the same A-matrix, and hence the same modes.
- It is shown in [6] and [22] that all of the non-QFMs of any system can be eliminated by using a proper finite-dimensional LTV controller.
- Theorem 5 states that the systems $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1), ..., \boldsymbol{\mathcal{S}}_l(\mathcal{K}_l)$ are *analogous* to each other w.r.t. finite-dimensional LTV controllers. ■

Corollary 2: The system $\mathcal{S}(\mathcal{K})$ is stabilizable if and only if it does not have any unstable QOFM.

Proof: The proof follows immediately from Remark 2 and Theorem 7. ■

Once it is determined that the system $\mathcal{S}(\mathcal{K})$ has no unstable QOFM, it can be stabilized by means of a time-varying controller, due to the fact that the system $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$, $\forall \mu \in \bar{l}$, will be stabilizable accordingly, using the finite-dimensional LTV controller proposed in [22]. Note that, as shown earlier, the systems $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_\mu(\mathcal{K}_\mu)$ are *analogous* w.r.t. finite-dimensional LTV controllers.

IV. NUMERICAL EXAMPLE

Consider the system $\mathcal{S}$ consisting of four SISO subsystems with the following state-space matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & -2 & 0 \\ 1 & 1 & 1 & 0 & 0 & -3 \end{bmatrix},$$
$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{20}$$
$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{bmatrix}$$

and $D = 0_{4\times 4}$. Assume that the information flow matrix for this system is given as follows:

$$\mathcal{K} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

which results in a control interaction structure as:

$$\mathbf{K} = \begin{bmatrix} 0 & 0 & k_{13} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ 0 & 0 & 0 & k_{34} \\ k_{41} & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

Using Procedures 1, 2 and 3, the block diagonal matrix $\mathbf{K}_1$ will be obtained as:

$$\mathbf{K}_1 = \begin{bmatrix} k_{13} & 0 & 0 & 0 & 0 \\ 0 & k_{22} & k_{24} & 0 & 0 \\ 0 & 0 & 0 & k_{34} & 0 \\ 0 & 0 & 0 & 0 & k_{41} \end{bmatrix} \tag{23}$$

As a result, the matrices Φ_1 and $\bar{\Phi}_1$ satisfying the relation $\mathbf{K} = \Phi_1 \mathbf{K}_1 \bar{\Phi}_1$ can be found by employing the method given in [12] which leads to the system $\boldsymbol{\mathcal{S}}_1$ with the parameters $\mathbf{B}^1 = B$, $\mathbf{D}^1 = 0$, and:

$$\mathbf{C}^1 = \begin{bmatrix} 0 & 1 & -4 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{24}$$

As pointed out earlier, the DOFMs of the system $\mathcal{S}(\mathcal{K})$ are identical to the DFMs of the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$. Note that $\mathcal{K}_1$ can be computed from the matrix $\mathbf{K}_1$ given by (23). It is straightforward to verify (using the method proposed in [1]) that $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ has two DFMs at $\lambda = +1$. Therefore, the system $\mathcal{S}$ has two identical DOFMs at $\lambda = +1$ w.r.t. the information flow matrix $\mathcal{K}$ given by (21). Consequently, this system cannot be stabilized by means of a structurally constrained LTI controller.

On the other hand, it follows from the characterization of QFM given in [6] that the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ has no QFM w.r.t. the information flow matrix $\mathcal{K}_1$. From the results of this work, it is known that the QOFMs of the system $\mathcal{S}(\mathcal{K})$ are the same as the QFMs of the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$. Thus, one can conclude that the system $\mathcal{S}$ has no QOFM w.r.t. the information flow matrix $\mathcal{K}$. Therefore, the system can be stabilized by utilizing a proper non-LTI controller.

It is desired now to find out a structurally constrained controller to stabilize the system $\mathcal{S}(\mathcal{K})$. Since it is shown that the systems $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ are *analogous* to each other w.r.t. any type of control law, the control design problem can be equivalently solved for the system $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$, instead. Due to the fact that $\boldsymbol{\mathcal{S}}_1$ has two DFMs w.r.t. $\mathcal{K}_1$, which are not QFMs, they can be eliminated by means of sampling, as pointed out in [14]. Consider the sampling period as $h = 3$, and find the discrete-time equivalent systems of $\mathcal{S}$ and $\boldsymbol{\mathcal{S}}_1$, denoted by $\bar{\mathcal{S}}$ and $\bar{\boldsymbol{\mathcal{S}}}_1$, respectively. It can be easily verified that $\bar{\boldsymbol{\mathcal{S}}}_1(\mathcal{K}_1)$ has no DFM, as expected. Moreover, the gain scheduling technique in [22] can be utilized to control the system from only one subsystem. More precisely, since the structural graph of the system $\bar{\boldsymbol{\mathcal{S}}}_1$ is strongly connected, the transfer functions $\bar{K}_{22}(z), \bar{K}_{24}(z), \bar{K}_{34}(z)$ and $\bar{K}_{41}(z)$ can be generically chosen as:

$$\bar{K}_{22}(z) = 1, \quad \bar{K}_{24}(z) = 2, \quad \bar{K}_{34}(z) = 3, \quad \bar{K}_{41}(z) = 4 \tag{25}$$

where $\bar{K}_{ij}(z)$ represents the discrete-time compensator for the system $\bar{\boldsymbol{\mathcal{S}}}_1$, which along with a ZOH builds the control component k_{ij}. Let the closed-loop system consisting of $\bar{\boldsymbol{\mathcal{S}}}_1$ and the controllers $\bar{K}_{22}(z), \bar{K}_{24}(z), \bar{K}_{34}(z)$ and $\bar{K}_{41}(z)$ given above be denoted by $\bar{\boldsymbol{\mathcal{S}}}_{1_{cl}}$. This closed-loop system is controllable from its first input, and is observable from its first output. Therefore, the controller $\bar{K}_{13}(z)$ can be designed for the system $\bar{\boldsymbol{\mathcal{S}}}_{1_{cl}}$ to not only stabilize the system but also place the modes at desired locations, by using the conventional pole-assignment techniques. Since $\mathcal{S}(\mathcal{K})$ and $\boldsymbol{\mathcal{S}}_1(\mathcal{K}_1)$ are *analogous* to each other w.r.t. the sampled-data controllers, the designed components for controlling the system $\boldsymbol{\mathcal{S}}_1$ can be used for controlling the system $\mathcal{S}$ as well, with identical properties.

V. Conclusions

This work tackles the structurally constrained control design problem for interconnected systems. The convectional techniques for designing a decentralized controller with a ceratin type, such as linear time-invariant (LTI), finite dimensional linear time-varying (LTV), generalized sampled-data hold functions (GSHF), are developed to handle the problem of designing a decentralized overlapping controller for an interconnected system. The notion of quotient overlapping fixed mode (QOFM) is then defined to investigate the stabilizability of the system via a general (nonlinear and time-varying) decentralized overlapping control law. It is shown that a mode of the system can be placed freely in the complex plane by means of a structurally constrained controller if and only if it is not a QFM. Moreover, it is asserted that once the system has no QOFMs in the closed right-half plane, there exists a finite dimensional LTV constrained controller to stabilize the system. The importance of the work is clarified and illustrated in a numerical example.

References

[1] E. J. Davison and T. N. Chang, "Decentralized stabilization and pole assignment for general proper systems," *IEEE Transactions on Automatic Control*, vol. 35, no. 6, pp. 652-664, 1990.

[2] S. H. Wang and E. J. Davison, "On the stabilization of decentralized control systems," *IEEE Transactions on Automatic Control*, vol. 18, no. 5, pp. 473-478, 1973.

[3] D. D. Šiljak, Decentralized control of complex systems, Cambridge: Academic Press, 1991.

[4] J. Lavaei, A. Momeni and A. G. Aghdam, "High-performance decentralized control for formation flying with leader-follower structure," *to appear in Proc. of 45th IEEE Conference on Decision and Control*, San Diego, CA, 2006.

[5] J. Lavaei and A. G. Aghdam, "Decentralized control design for interconnected systems based on a centralized reference controller," *to appear in Proc. of 45th IEEE Conference on Decision and Control*, San Diego, CA, 2006.

[6] Z. Gong and M. Aldeen, "Stabilization of decentralized control systems," *Journal of Mathematical Systems, Estimation, and Control*, vol. 7, no. 1, pp. 1-16, 1997.

[7] A. I. Zecevic and D. D. Šiljak, "A new approach to control design with overlapping information tructure constraints," *Automatica*, vol. 41, no. 2, pp. 265-272, 2005.

[8] S. S. Stankovic, M. J. Stanojevic, and D. D. Šiljak, "Decentralized overlapping control of a platoon of vehicles," *IEEE Transactions on Control Systems Technology*, vol. 8, no. 5, pp. 816-832, 2000.

[9] D. D. Šiljak and A. I. Zecevic, "Control of large-scale systems: Beyond decentralized feedback," *Annual Reviews in Control*, vol. 29, no. 2, pp. 169-179, 2005.

[10] A. Iftar, "Overlapping decentralized dynamic optimal control," *International Journal of Control*, vol. 58, no. 1, pp. 187-209, 1993.

[11] A. Iftar, "Decentralized optimal control with overlapping decompositions," *IEEE International Conference on Systems Engineering*, pp. 299-302, 1991.

[12] J. Lavaei and A. G. Aghdam, "A necessary and sufficient condition for the existence of a LTI stabilizing decentralized overlapping controller," *to appear in Proc. of 45th IEEE Conference on Decision and Control*, San Diego, CA, 2006 (available online at http://users.encs.concordia.ca/∼aghdam/TechnicalReports/CDC2006.pdf).

[13] J. Lavaei and A. G. Aghdam, "Optimal periodic feedback design for continuous-time LTI systems with constrained control structure," *International Journal of Control*, to appear.

[14] J. Lavaei and A. G. Aghdam, "Elimination of fixed modes by means of high-performance constrained periodic control," *to appear in Proc. of 45th IEEE Conference on Decision and Control*, San Diego, CA, 2006.

[15] Y. Ebihara and T. Hagiwara, "Structured controller synthesis using LMI and alternating projection method," *Proceedings of the 42nd IEEE Conference on Decision and Control*, pp. 5632-5637, 2003.

[16] P. T. Kabamba, "Control of linear systems using generalized sampled-data hold functions," *IEEE Transactions on Automatic Control*, vol. 32, no. 9, pp. 772-783, 1987.

[17] M. Rossi and D. E. Miller, "Gain/phase margin improvement using static generalized sampled-data hold functions," *Systems & Control Letters*, vol. 37, no. 3, pp. 163-172, 1999.

[18] A. G. Aghdam, "Decentralized control design using piecewise constant hold functions," *Proceedings of the 2006 American Control Conference*, Minneapolis, Minnesota, 2006.

[19] J. Lavaei and A. G. Aghdam, "Simultaneous LQ control of a set of LTI systems using constrained generalized sampled-data hold functions," *Automatica*, to appear.

[20] T. Chen and B. Francis, Optimal Sampled-Data Control Systems, Springer, 1995.

[21] J. Lavaei and A. G. Aghdam, "On structurally constrained control design with a prespecified form," *Technical Report*, Concordia University, 2006 (available online at http://users.encs.concordia.ca/∼aghdam/TechnicalReports/techrep4.pdf).

[22] B. Anderson and J. Moore, "Time-varying feedback laws for decentralized control," *IEEE Transactions on Automatic Control*, vol. 26, no. 5, pp. 1133-1139, 1981.