Flocking of Multi-Agent Systems with Limited Rotating Field of Views

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Abstract

In this paper, the flocking problem in a network of double-integrator agents with limited angular fields of view (FOV) is investigated. The angular FOV represents the limitation imposed on the velocity sensor mounted on every agent in the network, while the position sensors are assumed to be omnidirectional. To increase the sensing capabilities of agents and preserve network connectivity, it is assumed that the FOV of all agents rotate with constant angular velocity. The problem is formulated in the framework of switched nonlinear systems to address the switching topology of the network. The control inputs are subsequently designed as a combination of alignment and attractive/repulsive forces, such that the velocity vectors reach consensus and the inter-agent collision is avoided. The asymptotic convergence of the network configuration is guaranteed under the proposed controller. Simulations confirm the efficacy of the controller.

I. INTRODUCTION

Cooperative control of multi-agent systems has received an increasing attention in the past decade due to its applications in different fields of science and technology. Such applications include air traffic control, forest fire detection and reconnaissance missions, to name only a few [1], [2], [3], [4]. In this type of system, a local control law is designed for each particular agent in such a way that a global objective is achieved over the entire network. The information exchange among the agents is governed by a graph called information flow graph. A proper global objective is chosen in accordance with the particular applications such as consensus, rendezvous, containment, flocking, and formation control [5], [6], [7], [8].

As a problem of special interest in multi-agent systems, the flocking problem has been investigated by researchers in various fields such as biology, physics, robotics, and control engineering. A flocking model is proposed in [9] to mimic the collective behavior of birds, which contains flocking cohesion, collision avoidance, and velocity matching as three basic flocking rules. Motivated by [9], a theoretical framework for design and analysis of the flocking algorithms with and without obstacles is developed in [6]. In [10], the flocking and coordination problem for a network of kinematic agents is investigated, where the agents can only use the visual information received by their onboard sensors. Tracking the trajectories of a virtual leader and achieving flocking is addressed in [11], while the problem is formulated in switching framework due to the network’s time-varying topology. In [12], a set of coordination control laws are introduced to generate the desired flocking motion, resulting in convergence to the velocity vector of a virtual leader using asymmetric interactions.

On the other hand, the limitation of sensing devices can be a major hurdle in the control of multi-agent systems. The radial limitation in the visual sensing devices is discussed in [13] and [14]. In [15], a switching control law is proposed to solve the consensus and containment problems in a network of single integrators, where the sensing area of every position sensor is a half-plane. A distributed controller is proposed for a team of nonholonomic robots in [16] to achieve consensus on the heading angle of agents using visual sensors with constrained angular FOVs.

In this paper, the results of [12] and [15] are extended to the design of a distributed controller for convergent flocking in a network of double integrators with limited angular FOVs. It is assumed that the position sensors are omnidirectional, while the velocity sensors have sensing areas with angular constraints. Applying the proposed controller to a network with the above limitations results in an information flow graph with switching topology. Under the proposed control law, the convergence of flocking and uniformly preservation of the network connectivity are guaranteed. At the same time, the collision is avoided and the velocity vectors of all agents converge to steady-state velocity of network’s center of mass, asymptotically.

The remainder of the paper is organized as follows. Section II is devoted to definitions and formulation of the problem. In Section III, a control law is designed to address the flocking objective, and the main result of the work is subsequently provided in Section IV. Simulation results are presented in Section V, and finally the concluding remarks are drawn in Section VI.

II. PROBLEM FORMULATION

Notation 1: Let \( \mathbb{Z}_{\geq 0} \) and \( \mathbb{R}_{\geq 0} \) denote the set of nonnegative integer and nonnegative real numbers, respectively. Also, define \( \mathbb{N}_n := \{1, 2, ..., n\} \) for any integer \( n \). Given a real matrix \( \mathcal{A} \in \mathbb{R}^{n \times n} \), kernel \( \mathcal{A} \) is the null space of \( \mathcal{A} \) in the \( n \)-dimensional space \( \mathbb{R}^n \). Also, \( \| \cdot \| \) is the Euclidean norm in \( \mathbb{R}^n \), and the cardinality of an arbitrary set \( \Gamma \) is denoted by \( \text{Card}(\Gamma) \).

Consider a group of \( n \) double integrator agents moving in a 2D plane. The dynamics of each agent can be described by:

\[
\begin{align*}
q_i(t) &= v_i(t) \\
v_i(t) &= u_i(t)
\end{align*}
\]
for any $i \in N_n$, where $q_i(t), v_i(t) \in \mathbb{R}^2$ are the position and velocity vectors of agent $i$, respectively, and $u_i(t) \in \mathbb{R}^2$ is its acceleration vector, which is the control input driving the agent. Assume agent $i, i \in N_n$, can only detect those agents within a certain FOV, denoted by $\Omega_i$. It is also assumed that $\Omega_i$ is a conic area centered at $q_i(t)$ with apex angle $\alpha_i$, and that the angle $\theta_i(t)$ measures the direction of the bisector of $\Omega_i$ w.r.t. a fixed inertial frame. Fig. 1 depicts an example of an agent and its angularly limited FOV along with the corresponding variables. To achieve the design objectives, it is realistic to assume that every agent is capable of rotating its FOV with a constant angular velocity. The rotational dynamics of the $i$-th agent’s FOV is formulated as:

$$\dot{\theta}_i(t) = \omega_i$$  \hspace{1cm} (2)

where $\theta_i(t) \in (-\pi, \pi]$, and $\omega_i \in \mathbb{R}$ is a constant scalar applied as a rotational control input. One can represent the dynamic equations of all agents in following form:

$$q(t) = v(t)$$  \hspace{1cm} (3a)
$$v(t) = u(t)$$  \hspace{1cm} (3b)
$$\dot{\theta}_i(t) = \omega_i$$  \hspace{1cm} (3c)

where $q(t), v(t), u(t) \in \mathbb{R}^{2n}$, $\theta_i(t) \in (-\pi, \pi]^{n}$, and $\omega \in \mathbb{R}^{n}$. Throughout this paper, it is assumed that each agent is equipped with two types of sensing devices: (i) the position sensor which is omnidirectional and is used to sense the relative distance between every agent and its flockmates, (ii) the velocity sensor with limitation in its angular FOV which is used to measure the relative velocity of an agent’s neighbors.

The next definitions address some important concepts related to the connectivity of the network.

**Definition 1:** Given a directed graph (digraph) $G = (V, E)$ where $V$ is the set of nodes and $E$ is the set of edges, the node $j$ is said to belong to the neighbor set of agent $i$, denoted by $N_i$, if the directed edge $(j, i)$ pointing from $j$ to $i$ belongs to the edge set of the graph, i.e. $(j, i) \in E$.

**Definition 2:** A digraph $G = (V, E)$ is said to be uniformly strongly connected (USC) if there exists a finite $T > 0$ such that for any ordered pair of nodes $(i, j)$ belonging to the union edge set $\bigcup_{\tau \in [t, t+T]} E(\tau)$ and for any time $t \geq 0$, there exists a directed path from node $i$ to node $j$.

Note that the angular limitation on the FOVs is only for velocity measuring devices. In the sequel, the directed sensing graph $\overline{G}$ which is formed based on the relative velocity measurements is introduced.

**Definition 3:** The information flow of the velocity sensors in a network consisting of $n$ nodes with angular FOV limitation is represented by a so-called sensing digraph $\overline{G} = (\overline{V}, \overline{E})$, whose node set $\overline{V}$ and the edge set $\overline{E}$ are defined as:

$$\overline{V} = \{1, 2, ..., n\}$$  \hspace{1cm} (4a)
$$\overline{E} = \{(i, j) \mid i, j \in V, j \in \Omega_i\}$$  \hspace{1cm} (4b)

In the problem at hand, it is desired to develop a flocking control law to address the following four objectives:

- Avoiding the inter-agent collision.
- Convergence of the network configuration to the locally optimal configuration (corresponding to the local minimum of the collective potential function).
- Reaching agreement on velocity vectors.
- Preserving the connectivity of sensing digraph $\overline{G}$ uniformly over sufficient large time intervals.

For the purpose of collision avoidance and convergence of the network to an optimal configuration (as the first and second global objectives), the control law for the flocking problem is designed based on a suitable inter-agent potential function.

![Fig. 1. An example of a double-integrator agent $i$ with limited angular FOV $\Omega_i$.](image-url)
Thus, the previous problem is converted into designing \( \tilde{x} \). Moreover, the motion of the CoM is governed by the following equations:

\begin{align}
L \tilde{x} & = F(x) + \text{dist}(x) \xi + u(t) \\
\text{dist}(x) & = \begin{cases} 0 & \text{if } x \in D_i \cap D_j \\
1 & \text{otherwise}
\end{cases}
\end{align}

where \( L \) is a skew-symmetric matrix, \( F(x) \) is the force due to the interaction of agents, \( \text{dist}(x) \) is the distance function, and \( u(t) \) is the control input.

Remark 1: The digraph \( G \) is strongly connected, then its Laplacian matrix \( L \) is irreducible. This means that the real parts of all eigenvalues of \( L \) are positive except for a single eigenvalue 0. Also, \( [1, 1, \ldots, 1]^T \) is a right eigenvector corresponding to the single eigenvalue 0.

Remark 2: As a result of the FOV limitations described earlier, the underlying sensing digraph \( G_s \) has a switching topology. The switching is governed by a piecewise continuous switching signal \( \sigma(t) : \mathbb{R}_+ \rightarrow \Gamma \), where the set \( \Gamma \) contains the indices of all possible digraphs formed by \( n \) nodes. It is straightforward to show that the cardinality of the set \( \Gamma \) is \( 2^{n(n-1)} \). For simplicity, it is assumed that the switching of all agents is synchronized and occurs at time instants \( t_k \in \mathcal{S} \), where \( \mathcal{S} = \{ t_k < t_{k+1}, k \in \mathbb{Z}_{\geq 0} \} \) contains all switching time instants. Also, \( \tau_0 \) is the dwell-time of the switching signal \( \sigma(t) \), i.e., \( t_{k+1} - t_k \geq \tau_0 \) for all \( t_k \in \mathcal{S} \) and \( k \in \mathbb{Z}_{\geq 0} \).

Remark 3: Since the switchings of the sensing digraph \( G_s \) occur at the time instants \( t_k \in \mathcal{S} \), it can be inferred that during the time intervals \( [t_k, t_{k+1}) \), \( k \in \mathbb{Z}_{\geq 0} \), the sensing digraph \( G_s \) remains static for some \( \gamma \in \Gamma \). Also, the neighbor set of an arbitrary agent \( i \in V \) in a switching sensing digraph \( G_s \) is defined as follows:

\[ N_s(i) = \{ j \mid j \in V, \sigma(t) = \gamma, (j, i) \in \mathcal{E}_\gamma \} \]

for all \( t \in [t_k, t_{k+1}) \) and an index \( \gamma \in \Gamma \).

III. CONTROL DESIGN

In order to design the desired control law, the dynamic equations of the network are described in a frame attached to the center of mass (CoM) of the network. For defining this frame, the position and velocity vectors of the CoM are expressed as follows:

\begin{align}
q_c &= \frac{1}{n} \sum_{i=1}^{n} q_i \\
v_c &= \frac{1}{n} \sum_{i=1}^{n} v_i
\end{align}

Then, the position and velocity vectors of the agents w.r.t. the CoM frame are defined as:

\begin{align}
\tilde{q} &= q - I_n \otimes q_c \\
\tilde{v} &= v - I_n \otimes v_c
\end{align}

The dynamics of the network in a moving non-rotating frame centered at the CoM of the network is given by:

\begin{align}
\ddot{q}(t) &= \tilde{v}(t) \\
\dot{v}(t) &= \tilde{a}(t) \\
\dot{\theta}(t) &= \omega(t)
\end{align}

Moreover, the motion of the CoM is governed by the following equations:

\begin{align}
\dot{q}_c(t) &= v_c(t) \\
\dot{v}_c(t) &= u_c(t)
\end{align}

Thus, the previous problem is converted into designing \( \tilde{u}(t) \) and \( \omega(t) \) as distributed control inputs in the CoM frame. Then, the kinetic energy and collective potential energy of the network w.r.t. the CoM frame, denoted respectively by \( K(v) \) and \( V(q) \).
$H(q)$, are defined by:

$$K(v) = \frac{1}{2} \sum_{i=1}^{n} v_i^2 = \frac{1}{2} \sum_{i=1}^{n} (v_i - v_c)^2$$  \hspace{1cm} (11)$$

$$H(v) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j \neq i} \mu(q_i - q_j)$$  \hspace{1cm} (12)$$

The Lyapunov function considered for the analysis of the network in this framework has the following form:

$$V(q,v) = K(v) + H(q)$$  \hspace{1cm} (13)$$

For the stability analysis and controller design, it is assumed that the initial network configuration is limited to a level-set $\Psi_c$ defined as:

$$\Psi_c = \{ (q,v) | V(q,v) \leq c, c > 0 \}$$  \hspace{1cm} (14)$$

Based on the definition of the level-set and on noting that $H(q) \geq 0$, one can write:

$$\frac{1}{2} \sum_{i=1}^{n} \dot{q}_i^2 \leq V(q,v) \leq c$$  \hspace{1cm} (15)$$

Then, an upper bound on the magnitude of the velocity vector of any agent $i \in V$ is obtained as follows:

$$\| \dot{q}_i \| \leq \dot{q}_{\text{max}} = \sqrt{2c}$$  \hspace{1cm} (16)$$

The following lemma provides sufficient conditions for the realization of collision avoidance property in the network.

**Lemma 1:** Let the initial level-set $\Psi_c$ under a suitable control law be positively invariant while $c < \mu(r_c)$. Let also Assumption 1 hold. Then, the inter-agent collision is avoided for all $t \geq 0$.

**Proof:** From the definition of the level-set, $\Psi_c$ is positively invariant if the Lyapunov function $V(q,v)$ satisfies the following property:

$$V(q(t),v(t)) \leq c, \forall t \geq 0$$  \hspace{1cm} (17)$$

Choose the initial configuration of the network such that $c < \mu(r_c)$, and recall that there is no initial inter-agent collision at $t = 0$ by assumption. To prove the lemma by contradiction, let two arbitrary distinct agents $m,n \in V$ collide at time instant $t' \neq 0$, meaning that $\|q_m(t') - q_n(t')\| = r_c$. Also, it is straightforward to show that for any $t \geq 0$ the collective potential function $H(q(t))$ can be written for two distinct colliding agents $m,n \in V$ as follows:

$$H(q(t)) = \mu(\|q_m(t') - q_n(t')\|)$$

$$+ \frac{1}{2} \sum_{i \in V \setminus \{m,n\}} \sum_{j \in V \setminus \{m,n\}} \mu(\|q_i(t) - q_j(t)\|)$$

$$+ \frac{1}{2} \sum_{j \in V \setminus \{n\}} \mu(\|q_m(t) - q_j(t)\|)$$

$$+ \frac{1}{2} \sum_{j \in V \setminus \{m\}} \mu(\|q_n(t) - q_j(t)\|)$$

Since the inter-agent potential function is nonnegative, it can be concluded that at the time instant $t'$:

$$H(q(t')) \geq \mu(r_c) > c$$  \hspace{1cm} (18)$$

On the other hand, using (17) and on noting that $K(v)$ is a nonnegative function, one arrives at:

$$H(q(t)) \leq V(q(t),v(t)) \leq c < \mu(r_c)$$  \hspace{1cm} (19)$$

which is in contradiction with (18). Therefore, all inter-agent distances are greater than $r_c$ for any $t \geq 0$ as long as the level-set $\Psi_c$ is positively invariant and $c < \mu(r_c)$. This proves the collision avoidance property. 

In the sequel, the constant angular velocity of the FOVs is obtained in such a way that the uniform connectivity of the underlying network is guaranteed. To this end, the following lemma is presented.

**Lemma 2:** Consider two arbitrary agents $i,j \in V$ in a network of double integrators (9) with limited angular FOVs $\Omega_i$ and $\Omega_j$. Assume that the FOV of every agent rotates with constant angular velocity $\omega$ whose magnitude is greater than $\frac{2e}{r_c \sqrt{2c}}$, and that the collision avoidance condition holds. Then, for any $t \geq 0$ there exists time instants $t',t'' \in [t,t + T]$ such that $j \in \Omega_i(t')$ and $i \in \Omega_j(t'')$ while $T = \frac{2e}{|\omega| - \omega}$, where $\omega = \frac{2}{r_c \sqrt{2c}}$. 

Proof: Define \( \mathcal{F}_i \) as a frame attached to agent \( i \) and non-rotating w.r.t. a fixed inertial frame. Then the position and velocity vectors of agent \( j \) w.r.t. this frame, denoted by \( \tilde{q}_{ij} \) and \( \tilde{v}_{ij} \), are described as follows:

\[
\begin{align*}
\dot{\tilde{q}}_{ij} &= \tilde{q}_{ij}^r e_i^r \\
\dot{\tilde{v}}_{ij} &= \tilde{v}_{ij}^r e_i^r + \epsilon_{ij}^\theta \theta_i^\theta
\end{align*}
\]

(20a) where \( e_i^r \) and \( e_i^\theta \) are unit vectors which denote the radial and angular directions of the polar coordinates used for frame \( \mathcal{F}_i \), respectively, and \( \tilde{q}_{ij}^r, \tilde{v}_{ij}^r, \) and \( \epsilon_{ij}^\theta \) are proper scalars representing the magnitudes of the corresponding vectors. Since the defined coordinates in \( \mathcal{F}_i \) are orthogonal, \( ||\tilde{q}_{ij}|| = \tilde{q}_{ij}^r \) and \( ||\tilde{v}_{ij}|| = \sqrt{(\tilde{v}_{ij}^r)^2 + (\epsilon_{ij}^\theta)^2} \). The angular velocity of agent \( j \) w.r.t. frame \( \mathcal{F}_i \), denoted by \( \omega_{ij} \), is as follows:

\[
\omega_{ij} = \frac{\tilde{v}_{ij}^r}{\tilde{q}_{ij}^r}
\]

(21)

It follows from the collision avoidance assumption that \( \tilde{q}_{ij}^r > r_c \). Moreover, \( ||\tilde{v}_{ij}|| < 2\sqrt{2}c \) as a result of the limitation of system’s configuration to invariant level-set \( \Psi_c \). Thus, the following relation holds:

\[
\omega_{ij} \leq \overline{\omega}
\]

(22)

By choosing \( |\omega_i| > \overline{\omega} \), it is guaranteed that the \( i \)-th agent can sense agent \( j \) through its rotating FOV \( \Omega_i \) over a finite time interval of length \( T \), where \( T = \frac{2\pi}{|\omega_i|} \). Also, the effect of the apex angle \( \omega_i \) and initial heading of \( \Omega_i \) on the length of the interval can be neglected due to the above choice of \( T \), which is proportional to the inverse of the difference between \( \overline{\omega} \) and the absolute value of \( \omega_i \). Therefore, for any \( t \geq 0 \) there exists \( t' \in [t, t + T] \) such that \( j \in \Omega_i(t') \). The same argument applies to agent \( j \). Define \( \mathcal{F}_j \) as a frame attached to agent \( j \) and non-rotating w.r.t. a fixed inertial frame such that \( |\epsilon_{ij}^\theta > \overline{\omega} \). Then, for any arbitrary time instant \( t \geq 0 \), the FOV \( \Omega_j \) can detect agent \( i \) over the interval \( [t, t + T] \), where \( T = \frac{2\pi}{|\epsilon_{ij}^\theta|} \). In other words, there exists \( t'' \in [t, t + T] \) such that \( i \in \Omega_j(t'') \). The obtained result holds for both agents \( i \) and \( j \) by choosing \( \omega = \min(|\omega_i|, |\omega_j|) \) as the common angular velocity of \( \Omega_i \) and \( \Omega_j \), and \( T = \frac{2\pi}{|\omega|} \). This completes the proof.

The uniform connectivity of the underlying network is addressed in the next lemma.

Lemma 3: The union sensing digraph \( \mathcal{G} \) over the time interval \([t, t + T]\) is complete for every \( t \geq 0 \) if the absolute value of the common angular velocity of all FOVs, denoted by \( \omega \), is lower bounded by \( \overline{\omega} = \frac{2}{r_c} \sqrt{2c} \), where \( T = \frac{2\pi}{|\omega|} \).

Proof: Using the result of Lemma 2, if \( |\omega_i| > \overline{\omega} \) for all \( i \in V \), then \( \bigcup_{\tau \in [t, t + T]} N_i(\tau) = V \setminus \{i\} \) for all \( t \geq 0 \). Define the common angular velocity of all FOVs as \( \omega = \min_{i \in V} |\omega_i| \), which guarantees that all angular velocities are lower bounded by \( \overline{\omega} \). Thus, the union digraph \( \bigcup_{\tau \in [t, t + T]} \mathcal{G}(\tau) \) is complete, independent of \( t \), where \( T = \frac{2\pi}{|\omega|} \). In other words, there is a directed edge between any ordered pair of distinct nodes in the union digraph \( \mathcal{G} \) formed over the interval \([t, t + T]\) for any arbitrary \( t \geq 0 \). This completes the proof.

IV. MAIN RESULT

The following theorem provides the control law to achieve the desired flocking in a network of double integrators, as the main result of this work.

Theorem 1: Consider a multi-agent system consisting of \( n \) double-integrator agents described by (3) with limited angular FOVs. Also, let Assumption 1 hold and choose the initial configuration of the network such that the system trajectories are confined to the level-set \( \Psi_c \), where \( c < \mu(r_c) \). Apply an angular velocity \( \omega_i \) with magnitude greater than \( \frac{2}{r_c} \sqrt{2c} \) to every agent \( i \in V \), as well as the following acceleration:

\[
\dot{u}_i = -\sum_{j \in N_i(\tau)} (v_i - v_j) - \sum_{j=1, j \neq i}^{n} \phi(||q_i - q_j||)n_{ij}
\]

(23)

Then, the velocity vectors reach consensus, which no collision between agents. Furthermore, the configuration of the network asymptotically converges to a local minimum associated with the collective potential function.

Proof: For stability analysis of the system (3) under the proposed control law (23), transform the system equations into the CoM frame using (8). In this frame, the network dynamics is represented by (9) and the problem is converted to designing \( \dot{u}_i \) and \( \omega_i \) as the control inputs. Consider (13) as the common Lyapunov function of the network. Taking the time derivative of the Lyapunov function along the state trajectories (9) yields:

\[
\dot{V}(q, v) = \sum_{i=1}^{n} \dot{v}_i \dot{a}_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} \nabla_{\tilde{q}_i} \mu(||\tilde{q}_i - \tilde{q}_j||)(\dot{v}_i - \dot{v}_j)
\]

(24)
By defining $\phi(x) = \frac{d\mu(x)}{ds}$ and $n_{ij} = \frac{\hat{q}_i - \hat{q}_j}{|\hat{q}_i - \hat{q}_j|}$, the above equation can be rewritten as:

$$V(q,v) = \sum_{i=1}^{n} \tilde{v}_i \hat{u}_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j \neq i}^{n} \phi(\|\hat{q}_i - \hat{q}_j\|) n_{ij} (\tilde{v}_i - \tilde{v}_j)$$

(25)

Since $q_i - q_j = \hat{q}_i - \hat{q}_j$ and $v_i - v_j = \tilde{v}_i - \tilde{v}_j$, the following acceleration is considered as the control input of agent $i \in V$ in the CoM frame:

$$\hat{a}_i = u_i = - \sum_{j \in \mathcal{N}(i)} (\tilde{v}_i - \tilde{v}_j) - \sum_{j \neq i} \phi(\|\hat{q}_i - \hat{q}_j\|) n_{ij}$$

(26)

From the symmetry of relative position measurements, and by substituting (26) in (25), one has:

$$V(q,v) = -\tilde{v}^T \mathcal{L}_\gamma(\tilde{v})$$

(27)

Since by definition the Laplacian matrix $\mathcal{L}_\gamma$ is positive semi-definite for all $\gamma \in \Gamma$, one can write:

$$V(q,v) \leq 0$$

(28)

Thus, the level-set $\Psi_c$ is positively invariant. Therefore, the collision avoidance property of the network follows from Lemma 1 and considering the fact that $c < \mu(r_c)$. Let the angular velocity of agents satisfy the inequality $|\omega_i| > \frac{2}{\nu_0} \sqrt{2r_0}$, $i \in V$. Since there is no collision, the uniform completeness of the sensing digraph $\mathcal{G}$ over any interval of length $T = \frac{2\pi}{|\omega_0| - |\omega|}$ is concluded from Lemma 3. Define the positive integer $\eta$ as the maximum number of different digraphs which represent the interconnection between agents over time intervals of length $T$. Let:

$$\eta = \left\lceil \frac{T}{\tau_0} \right\rceil$$

(29)

where $\lceil \cdot \rceil$ denotes the ceiling operator (the minimum integer no smaller than the argument, which is $T/\tau_0$ in this case). Let $\{\mathcal{G}_{\gamma_1}, \mathcal{G}_{\gamma_2}, \ldots, \mathcal{G}_{\gamma_\eta}\}$ be the set of digraphs representing network interconnection during the interval $[t, t + T]$, where $\gamma_1, \gamma_2, \ldots, \gamma_\eta \in \Gamma$. Then, using the results of [13] the following equality is concluded for the corresponding Laplacian matrices $\mathcal{L}_{\gamma_1}, \mathcal{L}_{\gamma_2}, \ldots, \mathcal{L}_{\gamma_\eta}$:

$$\bigcap_{i=1}^{\eta} \text{kernel} \mathcal{L}_{\gamma_i} = \text{span}\{1_n\}$$

(30)

This means that $\text{span}\{1_n\}$ is the intersection of the null spaces associated with this set of Laplacian matrices. Now, it results from (28) that the state vector of the network remains bounded and evolves in such a way that the system’s trajectories are piecewise continuous, while the proposed Lyapunov function is invariant. From the boundedness and invariance properties discussed above, the compactness of the network’s trajectories is implied. Then, according to LaSalle’s invariance principle the trajectories of the network, starting from $\Psi_c$, converge to the largest invariant set $M$, contained in following set:

$$E = \{(q,v)|V(q,v) = 0\}$$

(31)

Since $\mathcal{G}_{\gamma(t)}$ is a uniformly connected and complete digraph, it follows from (30) that $M$ has following form:

$$M = \{(q,v)|\hat{v}_1 = \hat{v}_2 = \ldots = \hat{v}_n\}$$

(32)

or,

$$M = \{(q,v)|v_1 = v_2 = \ldots = v_n = v_c(t)\}$$

(33)

Thus, the velocity vector reaches consensus asymptotically.

The motion of the CoM is investigated in the next step. Since the relative position sensors are omnidirectional, the acceleration input $u_c$ applied to the CoM is given by:

$$u_c(t) = -\frac{1}{n} \sum_{i=1}^{n} \sum_{j \in \mathcal{N}(i)} (\tilde{v}_i - \tilde{v}_j)$$

(34)

To proceed further, it is desired to demonstrate that $u_c(t)$ is an absolutely integrable function, i.e.:

$$\int_{0}^{\infty} \|u_c(t)\| dt < \infty$$

(35)
Using the Cauchy-Schwarz inequality and assuming that the sensing digraph \( \mathcal{G} \) is complete for all \( t \geq 0 \), the left side of (35) can be recast as:

\[
\int_0^\infty \left\| \sum_{i=1}^n \sum_{j \neq i}^n (\tilde{v}_i - \tilde{v}_j) \right\| dt \leq \int_0^\infty \sum_{i=1}^n \sum_{j \neq i}^n ||\tilde{v}_i - \tilde{v}_j|| dt
\]

By changing the order of summation and integration, one arrives at:

\[
\int_0^\infty \sum_{i=1}^n \sum_{j \neq i}^n ||\tilde{v}_i - \tilde{v}_j|| dt = \int_0^\infty \sum_{i=1}^n \sum_{j \neq i}^n ||\hat{q}_i(t) - \hat{q}_j(t)||_{t=0} \]

or,

\[
\frac{1}{n} \sum_{i=1}^n \sum_{j \neq i}^n ||\hat{q}_i(t) - \hat{q}_j(t)||_{t=0} \leq (n-1) \max_{i,j \in \mathcal{V}} (||\hat{q}_i(\infty) - \hat{q}_j(\infty)|| - ||\hat{q}_i(0) - \hat{q}_j(0)||)
\]

Since the initial configuration of the network is limited to the positively invariant level-set \( \Psi_c \) with \( c < \mu(r_c) \), one can conclude that for all \( t \geq 0 \):

\[
\max_{i,j \in \mathcal{V}} (||\hat{q}_i(t) - \hat{q}_j(t)||) \leq \mu^{-1}(2c) < \infty
\]

Now, from (36)-(38) the following upper bound is deduced:

\[
\int_0^\infty \|u_c(t)\| dt \leq (n-1)\mu^{-1}(2c) < \infty
\]

Therefore, \( u_c(t) \) is an absolutely integrable function. Since the acceleration vector of the CoM is absolutely integrable and also \( u_c(t) \rightarrow 0 \) as \( t \rightarrow \infty \) (due to the velocity consensus specified by (34)), it can be concluded that \( v_c(t) \) approaches \( v_c^\infty \) asymptotically, where \( v_c^\infty \) denotes the constant finite velocity of the CoM. The value of \( v_c^\infty \) depends on the initial configuration of the network as well as the initial heading of the FOVs and their corresponding angular velocities. Hence, the largest invariant set \( M \) would have the following final form:

\[
M = \{ (q, v) | v_1 = v_2 = \ldots = v_n = v_c^\infty \}
\]

Once the trajectories inside the set \( M \) converge, the following equality holds:

\[
\dot{v}_1 = \dot{v}_2 = \ldots = \dot{v}_n = 0
\]

After reaching the velocity consensus, the network dynamics w.r.t. the CoM frame is simplified as follows:

\[
\dot{q} = \ddot{v} \quad (42a)
\]

\[
\dot{v} = -\nabla H(q) \quad (42b)
\]

\[
\dot{\theta} = \omega \quad (42c)
\]

Since \( \dot{v} = 0 \) (according to (41)), hence from (42b) \( \nabla H(q) = 0 \), which means that the network configuration \( q \in \mathbb{R}^{2n} \) converges to a local minimum associated with the collective potential function \( H(q) \). This completes the proof of Theorem 1. ■

V. SIMULATION RESULTS

Consider the flocking problem for a network composed of 5 double-integrator agents with limited angular FOVs as described by (3). The general form of the inter-agent potential function which is used in this example is defined next.

**Definition 4:** Let the function \( \mu(x) \) have the following form:

\[
\mu(x) = A \ln(x - r_c)^2 + \frac{B}{(x - r_c)^2} - A \left( 1 + \ln \left( \frac{B}{A} \right) \right)
\]

which satisfies all four conditions required for potential functions as described earlier. The positive real numbers \( A \) and \( B \) are two parameters of the function and \( r_c \) is the critical collision distance. Also, the derivative of \( \mu(x) \), denoted by \( \phi(x) \), is given by:

\[
\phi(x) = \frac{2A}{x - r_c} - \frac{2B}{(x - r_c)^3}
\]

The function \( \mu(x) \) has a global minimum at \( x_{\mu}^{\min} = r_c + \sqrt{\frac{B}{A}} \) with \( \mu(x_{\mu}^{\min}) = 0 \). Also, the global maximum of \( \phi(x) \) occurs at \( x_{\phi}^{\max} = r_c + \sqrt{\frac{2B}{A}} \) while \( \phi(x_{\phi}^{\max}) = \frac{4A}{3B} \sqrt{\frac{4B}{A}} \).
Fig. 2. The initial configuration of the multi-agent system in the example.

Fig. 3. The trajectories of the controlled multi-agent system in the example.

For this example, consider $A = 4$, $B = 3$, $r_c = 1$, and $r_d = \frac{x^\text{min}}{\mu} = 1.866$. Also, the apex angles which characterize the FOVs of agents are chosen as follows:

$$\alpha_1 = \frac{\pi}{2}, \alpha_2 = \frac{\pi}{4}, \alpha_3 = \frac{\pi}{2}, \alpha_4 = \frac{2\pi}{3}, \alpha_5 = \frac{\pi}{3}$$  \hspace{1cm} (45)

Moreover, the initial configuration of the network is chosen properly to satisfy Assumption 1 with $c \leq 180$. Using the results of Lemma 3, the lower bound $\varpi$ on the FOVs’ angular velocities is obtained as follows:

$$\varpi = \frac{2}{r_c} \sqrt{2c} \approx 38$$  \hspace{1cm} (46)

Let the common dwell-time of the synchronized switching signals be $\tau_D = 0.1\text{sec}$, and apply the control law (23) to the multi-agent system (3). Fig. 2 shows the initial position of agents along with their limited FOVs depicted as conic areas. Also, the black arrows show the initial velocity vectors of the double-integrator agents. The evolution of the system trajectories and the way they reach the desired flocking is depicted in Fig. 3. In this Figure, the initial and final positions of agents are marked by circles and squares, respectively, while the black lines show the final velocity vectors of agents.

The magnitude of the velocity vector of every agent versus time is depicted in Fig. 4. This figure shows that the velocity vectors reach consensus and asymptotically converge to $\|v^s_c\|$, indicating the constant steady-state velocity magnitude of the network’s CoM. Finally, Fig. 5 shows the evolution of all inter-agent distances, which asymptotically converge to a configuration corresponding to a local minimum of the collective potential function $H(q)$. It also demonstrates that no inter-agent collision happens during the motion of agents.
VI. CONCLUSIONS

The flocking problem for a network of double-integrator agents with limited rotating field of view (FOV) for velocity sensors is considered in this paper. Each agent is equipped with an omnidirectional position sensor, and a velocity sensor with a conic shape sensing area. It is assumed that all FOVs rotate with constant and sufficiently large angular velocities to uniformly preserve the network’s connectivity. Moreover, the control inputs are designed in such a way that the velocity vectors reach consensus and the collision avoidance objective is satisfied, while the configuration of network asymptotically converges to a local minimum corresponding to a collective potential function. The effectiveness of the proposed control scheme is confirmed by simulation. As a suggestion for future work, the proposed controller for flocking of double integrators can be generalized to the case where the sensing areas of both the position and velocity sensors have conic shapes.

REFERENCES


