On the role of phase in nonreciprocal vibration transmission in passive and active phononic lattices

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Abstract: We highlight the contribution of phase to nonreciprocal vibration transmission in nonlinear (passive) and spatiotemporally modulated (active) phononic lattices of finite length. We showcase nonreciprocal response regimes characterized by equal energies transmitted in opposite directions. A nonreciprocal phase shift is the only contributor to breaking of reciprocity in this scenario.

Nonreciprocity is an important problem from a fundamental perspective and in terms of additional functionalities that it enables in mechanical devices [1]. Common realizations of nonreciprocal response rely on implementation of nonlinearity or time-varying properties within the mechanical system. In most cases, the focus is on maximizing the nonreciprocal energy transfer: the difference in the transmitted energies or amplitudes when the locations of the source and receiver are interchanged. Here, we emphasize the role of phase in nonreciprocal vibration transmission in both nonlinear and time-varying phononic lattices. We focus on response regimes that are characterized by nonreciprocal phase shifts: the transmitted energies are equal for the left-to-right (LR) and right-to-left (RL) directions, but the transmitted phases are not.

Fig. 1 shows the systems we study to demonstrate nonreciprocal phase shifts. Panels (a) and (b) show, respectively the representative units used for the lattices with spatiotemporal modulation and nonlinear stiffness. In each case, the steady-state response of the system to a harmonic excitation is computed. Excitation is first applied to the left end of the system and the response at the right end is monitored (left-to-right transmission, LR). The locations of source and receiver are then interchanged and the procedure is repeated (right-to-left transmission, RL). The response is reciprocal if and only if the two outputs are identical.

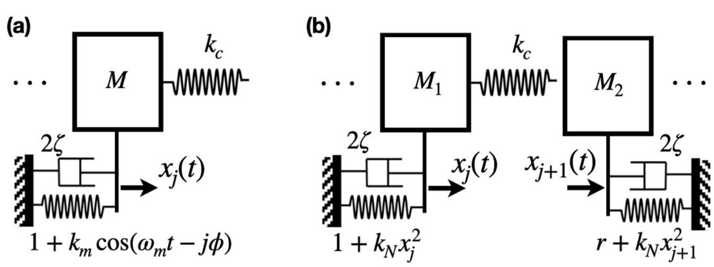

Figure 1: Representative units of the phononic lattice: (a) time-varying system, (b) nonlinear system.

In the nonlinear systems, nonreciprocal dynamics requires breaking the mirror symmetry of the system. This is enabled within the unit cell in Fig. 1(b) by the mass ratio (M_2/M_1) or stiffness ratio (r). Response regimes that exhibit nonreciprocal phase shifts ($\Delta\varphi$) can be computed for the unit cell or for the entire lattice using numerical continuation techniques [2,3]; see Fig. 2(a) for an example response. Computation of the nonreciprocal phase shifts also allows obtaining reciprocal dynamic response in a nonlinear system with broken mirror symmetry [4].

In systems subject to spatiotemporal modulations, the definition of phase is not as straightforward because the response of the system is quasi-periodic. This is due to the presence of two incommensurate frequencies: external and parametric drive. The envelope of the response, however, remains periodic. It is therefore possible to search for nonreciprocal phase shifts in the modulated system by considering the response envelope instead. If the response envelope exhibits nonreciprocal phase shift, so does the original signal. Fig. 2(b) shows an example of a response computed in this manner that exhibits nonreciprocal phase shift [5].

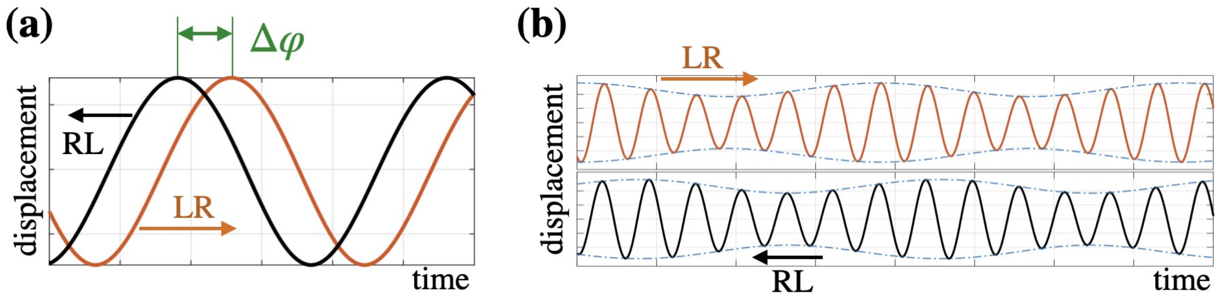

Figure 2: Response exhibiting nonreciprocal phase shift: (a) nonlinear system, (b) modulated system.

In summary, we showcased the contribution of nonreciprocal phase shifts in realizing nonreciprocal vibration transmission in short phononic lattices. To highlight the role of phase, we focused on nonreciprocal dynamics where the transmitted energies do not depend on the direction of travel but the transmitted phases do. For modulated systems, this became possible by formulating the response in terms of its response. We hope these findings contribute to enhancing the performance of devices that operate based on nonreciprocity.

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References

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