Two conjectures on lines in metric spaces

Vašek Chvátal
chvatal@cse.concordia.ca

Point v in a metric space with metric d is said to be *between* points u and w if $d(u,v) + d(v,w) = d(u,w)$. When x and y are distinct points in a metric space, the *line determined by x and y* is defined to consist of x, y, and all z such that one of x, y, z is between the other two. The line is called *universal* if it consists of all points of the space.

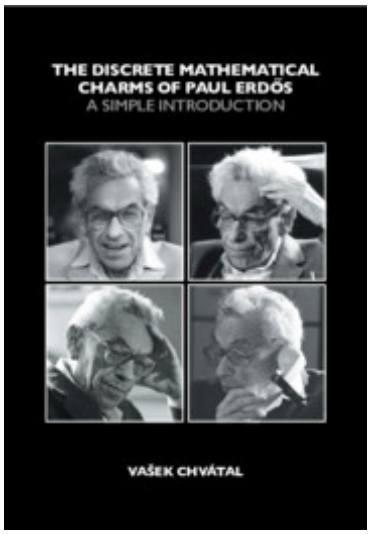
Conjecture 1 (Xiaomin Chen and V.C.)**.** *If a metric space on n points has no universal line, then it has at least n lines.*

Every connected graph induces a metric space on its vertex set.

Conjecture 2 (Yori Zwols and V.C.)**.** *The number of lines in a connected graph of order n without a universal line is minimized by a complete multipartite graph.*

References: A De Bruijn-Erdős theorem in graphs? In: *Graph Theory Favorite Conjectures and Open Problems–2* (Ralucca Gera, Teresa W. Haynes, and Stephen T. Hedetniemi, eds.), Springer (2018), pp. 149–176 [also in arXiv:1812.06288 [math.CO]]

and Chapter 2 of the forthcoming book



from Cambridge University Press.