The discrete mathematical charms of Paul Erdős by Vašek Chvátal, pp 248, £22.99 (paper), ISBN 978-1-108-92740-6, Cambridge University Press (2021)

I was very happy to be asked to review this book. Like most readers of the *Gazette*, I knew something about Erdős —his love of collaboration, his eccentricity ('stateless by political conviction'), and his unending stream of conjectures, theorems and proofs. My one worry was that I might not be able to follow his genius-level mathematics. And so I was doubly happy when I unwrapped the book and saw the subtitle: 'a simple introduction'.

In the early seventies, Chvátal collaborated with Erdős and his book is clearly a labour of love. The background to each problem is meticulously researched and supported by a comprehensive bibliography, there are some interesting biographical details, and some amusing Erdős anecdotes, but all of these take second place to Chvátal's exposition of Erdős's mathematics. The arguments are so clever and so clearly explained that there is something to make you smile on almost every page. Here are some of the intriguing problems and theorems, each taken from a different chapter of the book:

From Chapter 1, A Glorious Beginning: Bertrand's Postulate

For every positive integer n there is at least one prime p such that $n < p \leqslant 2n$.

Erdős found a proof of this when he was just 18 years old. Bertrand's postulate says that the primes are not too infrequent. But "paradoxically Erdős's proof depends on the fact that they do not occur too often."

From Chapter 2, Discrete Geometry and Spinoffs

How many points (no three of them collinear) do you need to guarantee that five of them form a convex pentagon?

When Erdős took up this problem the upper bound was about 210 000. He reduced it by elementary arguments to 21. (If that makes you smile, I think you'll like this book).

From Chapter 5, Extremal Set Theory

Hypergraphs, finite projective planes and Steiner systems.

Whether or not there are projective planes (with orders other than a prime power) is an open question. It is fascinating to see how Erdős and others thought about things which might not even exist.

From Chapter 6, Van der Waerden's Theorem

If you colour all the positive integers either red or blue (any way you like!), then for every positive integer k can I find an arithmetic progression of k terms all of which are the same colour?

The answer is 'yes', but the problem is soon generalised. What is the smallest number n such that if the first n positive integers are coloured with r colours, there is a k-term arithmetic progression where every number is the same colour?

From Chapter 8. The Friendship Theorem

If in a group of people, every two people have precisely one common friend, then the group includes a 'politician' who is a friend of everybody.

Alongside such problems Chvátal contrasts 'reactionary' and 'revolutionary' mathematics. He says these adjectives were being used by Maoist Canadian category theorists in the 1970s. Reactionary mathematics was elitist and obscure (and presumably a bad thing), but revolutionary mathematics made mathematics accessible to the masses. That category theorists (of all people) should talk this way is a nice insider joke.

From Chapter 10. Thresholds of Graph Properties

> *What is the probability that a random graph with n vertices and m edges is connected?*

Roughly speaking there is a threshold size beneath which almost all random graphs are disconnected, and above which almost all of them are connected. Erdős was instrumental in developing the concept of 'threshold functions' to tackle such problems.

The chapters were derived from Chvátal's graduate course entitled 'Discrete mathematics of Paul Erdős', but he says that the ideal audience he had in mind while writing the book were contestants in a mathematical olympiad. I think he gets the level just right for such bright youngsters; for instance chapter one begins by defining the binomial coefficients, and even in the penultimate chapter the author takes time to explain the 'Inclusion-Exclusion principle' in elementary terms. While each chapter begins with the basics and there are few prerequisites beyond school mathematics, Chvátal clearly expects you to read with pencil in hand as he moves quickly to the frontiers of mathematical knowledge.

Chapter 11 is on Hamilton Cycles, which were not a major theme of Erdős work; however, they were the subject of the Erdős-Chvátal collaboration and the author has more than earned the right to include this final chapter.

This is a lovely book, beautifully written, beautifully designed and destined to become a familiar favourite, never left on the shelf for too long.

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