MR4282416 05-02 01A60 01A70 05C55 05D10 11B25 11N05

Chvátal, Vašek (3-CONC-NDM; Montreal, QC)

★**The discrete mathematical charms of Paul Erdős—a simple introduction.**
With a foreword by János Pach.

Cambridge University Press, Cambridge, 2021. *xix*+248 *pp. ISBN* 978-1-108-83183-3; 978-1-108-92740-6

This wonderfully written book is undoubtedly a significant contribution to the growing body of literature on the various developments in discrete mathematics over the last several decades. Still, to reduce it to only its mathematical dimension would be an act of injustice not only towards the book but also towards its author. The book is a powerful homage to Paul Erdős as one of the leading mathematicians of the twentieth century as well as a person who, with his unprecedented level of academic generosity and overall human kindness, was one of the pillars of the discrete mathematics community during his lifetime. The author demonstrates that it would be impossible to look at Erdős's mathematical opus in isolation, i.e., without celebrating Erdős's collaborators and all of those who established, or were just curious about, the mathematical results that stem from questions that Erdős asked, techniques that he introduced, or results that he obtained.

The book also reads as the author's mathematical and personal memoir. When presenting mathematical topics, the author is a knowledgeable and objective narrator. Through anecdotal vignettes about Erdős sprinkled throughout the book, the author personalizes the topic. Those vignettes are welcome reminders that learning and doing mathematics are both personal and cultural experiences that intertwine with an individual's everyday life and also include interactions with others who share similar mathematical and non-mathematical interests, knowledge, history, set of values, and so on. The author's wit and humor further distance this text from commonly impersonal mathematical books.

The book is based on the lecture notes of a graduate course that the author taught in the late 2000s at Concordia University in Montreal, Canada. Consequently, this is a book for a mature mathematical audience. In general, the book chapters are self-contained, so a reader interested in a particular topic will be able to easily find an appropriate point of entry and go back and forth, if necessary. Moreover, to make the presented material more approachable, the author provides an appendix (pp. 215–217) that contains the relevant definitions, terminology, and notation. Another appendix entitled "A few tricks of the trade" (pp. 192–214) will be greatly appreciated by all those readers who need a reminder—or are just learning—about common and not-so-common techniques used in modern discrete mathematics.

Each chapter is typically organized in chronological order, starting with the origin of the chapter topic, and then presenting its development over time. This includes the relevant results, often with their proofs; historical reflections; the names of people who have contributed to the topic; and an extensive bibliography. This approach makes the book under review an invaluable resource for any working mathematician.

The last part of the book (Appendix C, pp. 218–225) is devoted to Erdős's life.

Here is a brief description of the content of each chapter:

Chapter 1: A glorious beginning: Bertrand's postulate (pp. 3–17)

In addition to Erdős's proof from 1932 [Acta Litt. Sci. Szeged **5** (1932), 194–198], the chapter contains a few earlier proofs of Bertrand's postulate together with some further results and problems concerning primes.

Chapter 2: Discrete geometry and spinoffs (pp. 18–35)

The chapter starts with the Happy Ending Problem [P. Erdős and G. Szekeres, Compositio Math. **2** (1935), 463–470; MR1556929] and continues with the Sylvester-Gallai theorem and the de Bruijn–Erdős theorem [N. G. de Bruijn and P. Erdős, Indag. Math. **10** (1948), 421–423; MR0028289]. The chapter includes several proofs of the latter theorem.

Chapter 3: Ramsey's Theorem (pp. 36–50)

The chapter includes proof of Ramsey's theorem from a 1935 paper by Erdős and Szekeres [op. cit.] and discusses Ramsey numbers, including the lower bound for diagonal Ramsey numbers established by Erdős in [Bull. Amer. Math. Soc. **53** (1947), 292–294; MR0019911].

Chapter 4: Delta-systems (pp. 51–58)

The chapter is about delta-systems, a notion introduced by Erdős and R. Rado in [J. London Math. Soc. **35** (1960), 85–90; MR0111692]. The chapter includes several related results including one by the author.

Chapter 5: Extremal set theory (pp. 59–81)

The chapter begins with Sperner's lemma [E. Sperner, Math. Z. **27** (1928), no. 1, 544–548; MR1544925] and includes the Erdős-Ko-Rado theorem [P. Erdős, C. Ko and R. Rado, Quart. J. Math. Oxford Ser. (2) **12** (1961), 313–320; MR0140419] and a section on the chromatic number of a hypergraph.

Chapter 6: van der Waerden's theorem (pp. 82–96)

This chapter includes a proof of van der Waerden's theorem [B. L. van der Waerden, Nieuw Arch. Wiskd. (2) **15** (1927), 212–216] and a discussion about van der Waerden numbers.

Chapter 7: Extremal graph theory (pp. 97–113)

The chapter starts with Turán's theorem [P. Turán, Mat. Fiz. Lapok **48** (1941), 436–452; MR0018405] and explores various related questions.

Chapter 8: The Friendship Theorem (pp. 114–124)

This chapter is about the Friendship Theorem of Erdős, A. Rényi, and V. T. Sós [Studia Sci. Math. Hungar. **1** (1966), 215–235; MR0223262]. It also includes discussion about strongly regular graphs.

Chapter 9: Chromatic number (pp. 125–150)

This chapter is devoted to the chromatic number of a graph. Among other relevant topics, the chapter includes five different proofs of the existence of graphs with a large chromatic number and no triangles.

Chapter 10: Thresholds of graphs properties (151–174)

The starting point of this chapter is the notion of a random graph as it was introduced by Erdős and Rényi, in [Publ. Math. Debrecen **6** (1959), 290–297; MR0120167; Magyar Tud. Akad. Mat. Kutató Int. Közl. **5** (1960), 17–61; MR0125031]. This is followed by several results including a theorem about balanced graphs.

Chapter 11: Hamilton cycles (pp. 175–191)

This chapter, in the author's words, is "more of an appendix than a genuine chapter: its theme, Hamiltonian graphs, was far from central among Erdős's interests in discrete mathematics."

Veselin Jungić