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Adaptive Variable Structure Control of a Class of Nonlinear Systems With Unknown Prandtl–Ishlinskii Hysteresis

Chun-Yi Su, Qingqing Wang, Xinkai Chen, and Subhash Rakheja

***Abstract*—Control of nonlinear systems preceded by unknown hysteresis nonlinearities is a challenging task and has received increasing attention in recent years due to growing industrial demands involving varied applications. In the literature, many mathematical models have been proposed to describe the hysteresis nonlinearities. The challenge addressed here is how to fuse those hysteresis models with available robust control techniques to have the basic requirement of stability of the system. The purpose of the note is to show such a possibility by using the Prandtl–Ishlinskii (PI) hysteresis model. An adaptive variable structure control approach, serving as an illustration, is fused with the PI model without necessarily constructing a hysteresis inverse. The global stability of the system and tracking a desired trajectory to a certain precision are achieved. Simulation results attained for a nonlinear system are presented to illustrate and further validate the effectiveness of the proposed approach.**

***Index Terms*—Adaptive control, cascade systems, hysteresis, nonlinear systems, Prandtl–Ishlinskii (PI) hysteresis model, robust control.**

I. Introduction

The hysteresis phenomenon occurs in all the smart material-based actuators, such as piezoceramics and shape memory alloys [1]. When a nonlinear plant is preceded by the hysteresis nonlinearity, the system usually exhibits undesirable inaccuracies or oscillations and even instability [14] due to the nondifferentiable and nonmemoryless character of the hysteresis. The development of control techniques to mitigate the effects of hystereses has been studied for decades and has recently reattracted significant attention, as can be seen in [10] and the references therein. Much of this renewed interest is a direct consequence of the importance of hysteresis in numerous current applications. Interest in studying dynamic systems with actuator hysteresis is also motivated by the fact that they are nonlinear systems with *nonsmooth* nonlinearities for which traditional control methods are insufficient and thus require development of alternate effective approaches [15]. Development of a general frame for control of a system in the presence of unknown hysteresis nonlinearities is a quite challenging task.

To address such a challenge, the thorough characterization of these nonlinearities forms the foremost task. Appropriate hysteresis models may then be applied to describe the nonsmooth nonlinearities for their potential usage in formulating the control algorithms. Hysteresis models can be roughly classified into physics based models and purely phenomenological models. Physics-based models are built on first principles of physics. Phenomenological models, on the other hand, are used to produce behaviors similar to those of the physical systems without necessarily providing physical insight into the problems [19]. The basic idea consists of the modeling of the real complex hysteresis nonlinearities by the weighted aggregate effect of all possible so-called elementary hysteresis operators. Elementary hysteresis operators are

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noncomplex hysteretic nonlinearities with a simple mathematical structure. Models set up by the composition of operators of play and stop type are referred to as Prandtl–Ishlinskii (PI) models in the literature (see, e.g., [6] and [17]). The reader may refer to [9] for a recent review of the hysteresis models.

With the developments in various hysteresis models, it is by nature to seek means to fuse these hysteresis models with the available robust control techniques to mitigate the effects of hysteresis, especially when the hystereis is unknown, which is a typical case in many practical applications. However, the discussions on the fusion of the available hysteresis models with the available control techniques is surprisingly spare [13], [18] in the literature. The most common approach is to construct an inverse operator, which was pioneered by Tao and Kokotovic [14], and the reader may refer to, for instance, [3], [4], [7], and the references therein.

The challenge addressed here is to fuse those hysteresis models with the available control techniques to have the basic stability requirements for the concerned system. As an illustration, this note presents such a possibility by fusing the PI models with the adaptive variable structure control approach [18] to mitigate the effects of the hysteresis without constructing the inverse hysteresis nonlinearity. The proposed control law ensures the global stability of the adaptive system and achieves both stabilization and strict tracking precision. Simulations performed on a nonlinear system illustrate and further validate the effectiveness of the proposed approach. The proposed method can be observed as an initial step to fuse the available hysteresis models with the available control techniques.

II. Problem Statement

Consider a controlled system consisting of a nonlinear plant preceded by an actuator with hysteresis nonlinearity, that is, the hysteresis is presented as an input to the nonlinear plant. The hysteresis is denoted as an operator, such that

$$w(t) = P[v](t) \tag{1}$$

with $v(t)$ as the input and $w(t)$ as the output. The operator $P[v]$ will be discussed in detail in the forthcoming section. The nonlinear dynamic system being preceded by the previous hysteresis is described in the canonical form as

$$x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i\left(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)\right) = bw(t) \tag{2}$$

where Y_i are known continuous, linear or nonlinear functions. Parameters a_i and control gain b are unknown constants. It is a common assumption that the sign of b is known. Without losing generality, we assume $b > 0$. It should be noted that more general classes of nonlinear systems can be transformed into this structure [5].

The control objective is to design a control law for $v(t)$ in (1), to force the plant state $x(t)$ to follow a specified desired trajectory, $x_d(t)$, i.e., $x(t) \to x_d(t)$ as $t \to \infty$.

Through the note the following assumption is made.

Assumption: The desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^T$ is continuous. Furthermore, $[\mathbf{x}_d^T, x_d^{(n)}]^T \in \Omega_d \subset R^{n+1}$ with Ω_d being a compact set.

III. Hysteresis Models

Although a large number of hysteresis models have been reported, this note focuses on the PI model to illustrate its fusion with the adaptive variable structure control approach to mitigate the effects of the hysteresis.

A. Stop and Play Operators

The PI model involves some basic well-known hysteresis operators. A detailed discussion on this subject can be found in the monographs [2], [6], and [17]. One of the basic elements of the theory of hysteresis operators is expressed by a stop operator $E_r[v]$ with threshold r.

Analytically, suppose that $C_m[0, t_E]$ is the space of piecewise monotone continuous functions. For any input $v(t) \in C_m[0, t_E]$, let $e_r : R \mapsto R$ be defined by

$$e_r(v) = \min\left(r, \max(-r, v)\right). \tag{3}$$

Then, for any initial value[1] $w_{-1} \in R$ and $r \geq 0$, the stop operator $E_r[\cdot; w_{-1}]$ is defined as [2]

$$\begin{aligned} E_r\,[v; w_{-1}](0) &= e_r(v(0) - w_{-1}) \\ E_r\,[v; w_{-1}](t) &= e_r(v(t) - v(t_i) + E_r[v; w_{-1}](t_i)) \\ &\quad \text{for } t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1 \end{aligned} \tag{4}$$

where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$ such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$. The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to a function. The stop operator however is mainly characterized by its threshold parameter r which determines the height of the hysteresis region in the (v, w) plane.

There is another basic hysteresis nonlinearity operator, called the play operator [2]. For $r \geq 0$, the play operator $F_r[\cdot; w_{-1}] : C_m[0, t_E] \times w_{-1} \mapsto C_m[0, t_E]$ for a general initial value[2] $w_{-1} \in R$, is defined by

$$\begin{aligned} F_r\,[v; w_{-1}](0) &= f_r(v(0), w_{-1}) \\ F_r\,[v; w_{-1}](t) &= f_r(v(t), F_r[v; w_{-1}](t_i)) \\ &\quad \text{for } t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1 \end{aligned} \tag{5}$$

with

$$f_r(v, w) = \max\left(v - r, \min(v + r, w)\right) \tag{6}$$

where the partition $0 = t_0 < t_1 < \cdots < t_N = t_E$ is the same as defined for the stop operator.

From the definitions given in (4) and (5), it can be proved [2] that the operator F_r is the complement of E_r, i.e., they are closely related through the following equation:

$$E_r[v; w_{-1}](t) + F_r[v; w_{-1}](t) = v(t) \tag{7}$$

for any piecewise monotone input function v and $r \geq 0$.

In the following sections, both the stop and play operators are denoted by $E_r[v]$ or $F_r[v]$ instead of $E_r[v; w_{-1}]$ or $F_r[v; w_{-1}]$ so long as it does not affect the proof. Owing to the nature of the play and stop operators, above discussions are defined on the space $C_m[0, t_E]$ of continuous and piecewise monotone functions, although they can also be extended to the space $C[0, t_E]$ of continuous functions.

B. PI Model

We are ready to introduce the PI model defined by the stop or play hysteresis operators. The PI model [11] was formulated to describe the elastic-plastic behavior through a weighted superposition of basic elastic-plastic elements $E_r[v]$, or stop operator, as follows:

$$w(t) = \int_0^R p(r) E_r[v](t)\, dr \tag{8}$$

[1] w_{-1} represent the value of $v - w$ before $v(0)$ is applied at time $\mathrm{t} = 0$.

[2] w_{-1} represent the initial state before $v(0)$ is applied at time $\mathrm{t} = 0$.

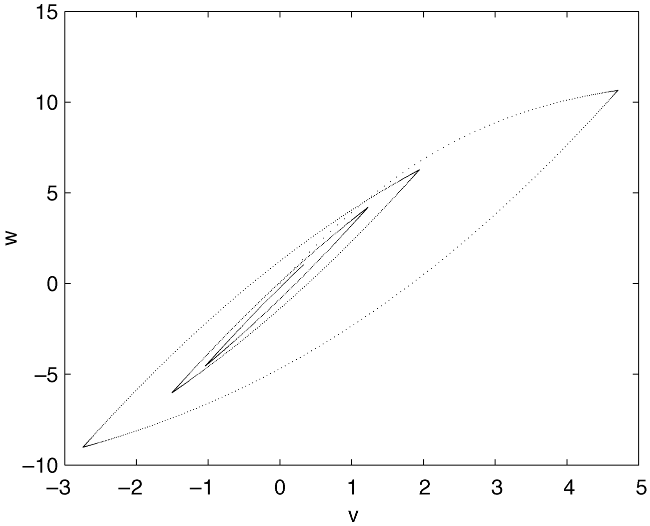

Fig. 1. Hysteresis curves given by (8).

where $p(r)$ is a given continuous density function, satisfying $p(r) \geq 0$ with $\int^{\infty} rp(r)dr < \infty$, and is expected to be identified from experimental data. With the defined density function, this operator maps $C[t_0, \infty)$ into $C[t_0, \infty)$, i.e., Lipschitz continuous inputs will yield Lipschitz continuous outputs [6]. Since the density function $p(r)$ vanishes for large values of r, the choice of $R = \infty$ as the upper limit of integration in the literature is just a matter of convenience [2].

It can be seen that the stop operator E_r serves as the building element in the PI model (8). Moreover, it needs to be mentioned that the stop and play operators are rate-independent, and thus the PI model is also rate-independent. As an illustration, Fig. 1 shows $w(t)$ derived from the model given in (8), with $p(r) = e^{-0.067(r-1)^2}$, $r \in [0, 10]$, and input $v(t) = 7sin(3t)/(1+t)$, $t \in [0, 2\pi]$ with $w_{-1} = 0$. This numerical result shows that the PI model (8) indeed generates the hysteresis curves and can be considered to be well-suited to describe the rate-independent hysteretic behavior.

Since the operator F_r is the complement of E_r, the PI model can also be expressed through the play operator. Using (7) and substituting for E_r in (8) by F_r, the PI model defined by the play hysteresis operator is expressed as follows:

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) dr \tag{9}$$

where $p_0 = \int_0^R p(r)dr$ is a constant which depends on the density function $p(r)$. It should be noted that (9) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part, while the second term describes the nonlinear hysteretic behavior. This decomposition is crucial since it facilitates the utilization of the currently available control techniques for the controller design.

IV. Controller Design

In this section, instead of constructing the inverse of the hysteresis model to mimic the hysteresis effects as frequently done in the literature [3], [4], [7], [14], we shall propose, as an illustration, an adaptive variable structure controller for plants of the form described by (2) preceded by the hysteresis that is described by the PI model. The proposed controller will lead to global stability and yield tracking within a desired precision.

Consider the PI model expressed by the play operator given in (9), the hysteresis output $w(t)$ can be expressed as

$$w(t) = p_0 v(t) - d[v](t) \tag{10}$$

where

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr \tag{11}$$

with $p_0 = \int_0^R p(r)dr$.

Using the hysteresis model of (10), the nonlinear system dynamics described in (2) can be expressed as

$$x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i \left(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t) \right) = b \left\{ p_0 v(t) - d[v](t) \right\} \tag{12}$$

which yields a linear function of the input signal $v(t)$ together with a shifting term $bd[v]$.

Remark: It is clear that the first term on the right-hand side of (12) is expressed as a linear function of the control signal $v(t)$. In this case, it is possible to fuse the currently available controller design techniques with the hysteresis model for the controller design. Such a structure would thus permit for the design of the adaptive variable structure control algorithm. This particular aspect of the fusion would become clear with the formulations presented later. Furthermore, the integrated model in (12) was also our primary motivation behind using the PI model.

In the following development, we will propose an adaptive variable structure controller for (12).

Equation (12) can be re-expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= -\sum_{i=1}^{k} a_i Y_i \left(x_1(t), x_2(t), \ldots, x_{n-1}(t) \right) \\ &\quad + b \left\{ p_0 v(t) - d[v](t) \right\} \\ &= \mathbf{a}^T Y + b_p v(t) - d_b[v](t) \end{aligned} \tag{13}$$

where $x_1(t) = x(t), x_2(t) = \dot{x}(t), \ldots, x_n(t) = x^{(n-1)}(t)$, $\mathbf{a} = [-a_1, -a_2, \ldots, -a_k]^T$, $Y = [Y_1, Y_2, \ldots, Y_k]^T$, $b_p = bp_0$, and $d_b[v](t) = \int_0^R p_b(r) F_r[v](t) dr$, with $p_b(r) = bp(r)$.

In presenting the developed adaptive variable structure control law, the following definitions are required:

$$\tilde{\mathbf{a}}(t) = \mathbf{a} - \hat{\mathbf{a}}(t) \tag{14}$$

$$\tilde{\phi}(t) = \phi - \hat{\phi}(t) \tag{15}$$

$$\tilde{p}_b(t, r) = p_b(r) - \hat{p}_b(t, r), \quad \text{for all } r \in [0, R] \tag{16}$$

$\hat{\mathbf{a}}$ is an estimate of $\mathbf{a}$, $\hat{\phi}$ is an estimate of ϕ, which is defined as $\phi \triangleq (b_p)^{-1}$, $\hat{p}_b(t, r)$ is an estimate of the density function $p_b(r)$. Let

$$B\left(v(t)\right) \triangleq \int_0^R p_b(r) \left| F_r[v](t) \right| dr \tag{17}$$

and the estimation $\hat{B}(t)$ is given by

$$\hat{B}\left(v(t)\right) \triangleq \int_0^R \hat{p}_b(t, r) \left| F_r[v](t) \right| dr \tag{18}$$

which leads to

$$\tilde{B}(t) = \int_0^R \left(\hat{p}_b(t, r) - p_b(r) \right) \left| F_r[v](t) \right| dr. \tag{19}$$

Given the plant and hysteresis model subject to the assumptions described above, we propose the following control law:

$$v(t) = \hat{\phi}(t)v_1(t) \tag{20}$$

with

$$v_1(t) = -c_n z_n - z_{n-1} - \hat{\mathbf{a}}^T Y + u_N + x_d^{(n)} + \dot{\alpha}_{n-1} \tag{21}$$

where

$$\begin{aligned} z_1(t) &= x_1(t) - x_d(t) \\ z_i &= x_i(t) - x_d^{(i-1)} - \alpha_{i-1}, \qquad i = 2,3,\ldots,n \qquad (22) \\ \alpha_1(t) &= -c_1 z_1(t) \\ \alpha_i(t) &= -c_i z_i(t) - z_{i-1}(t) \\ &\quad + \dot{\alpha}_{i-1}\left(x_1,\ldots,x_{i-1},x_d,\ldots,x_d^{i-1}\right), \\ &\quad i = 2,3,\ldots,n-1 \qquad (23) \end{aligned}$$

where $u_N = \mathrm{sign}(z_n)\hat{B}$ and c_i, $i = 1,2,\ldots,n-1$, are positive design parameters. The parameters $\hat{\phi}$, $\hat{\mathbf{a}}$, and function $\hat{p}_b(t,r)$ will be updated by the following adaptation laws:

$$\dot{\hat{\mathbf{a}}} = \gamma Y z_n \tag{24}$$

$$\dot{\hat{\phi}} = -\eta v_1 z_n \tag{25}$$

$$\frac{\partial}{\partial t}\hat{p}_b(t,r) = q\,|F_r[v](t)|\,|z_n|, \quad \text{for } r \in [0,R] \tag{26}$$

where parameters γ, η and q are positive constants determining the rates of the adaptations.

Remarks:

1) The term $u_N(t)$ represents the compensation component for the function $d[v](t)$. Unlike the traditional adaptive variable structure controller designs, where $d[v](t)$ is either assumed to be bounded by a constant or a known function [16], $d[v](t)$ is presented as an integral equation, and there is no assumption on its boundedness. Considering that the density function $p(r)$ is not a function in time, it can be treated as a parameter of the hysteresis model and adaptive law can be developed to obtain an estimate of it. This is crucial for the success of the adaptive law design.
2) The function $\hat{B}(t) = \int_0^R \hat{p}_b(r,t)|F_r[v](t)|dr$ in the implementation can be computed using numerical techniques by replacing the integration with the sum, $\hat{B}(t) = \sum_{l=0}^{N-1}\hat{p}_b(l\Delta r,t)|F_{l\Delta r}[v](t)|\Delta r$, where N determines the size of the intervals of R such that $\Delta r = R/N$. The selection of the size of the intervals depends on the accuracy requirement. As will be shown in the simulation example, the size of the intervals may not necessarily be very small.

The stability of the closed-loop system described in (12), (20) and (24)–(26) is established in the following theorem.

Theorem: For the plant given in (2) with the hysteresis (9), subject to Assumption 1, the adaptive variable structure controller specified by (20) and (24)–(26) ensures that all the closed-loop signals are bounded and $x(t) \to x_d(t)$ as $t \to \infty$.

Proof: Using the expression (13) and the definition of z_n in (22), noticing that $b_p v(t) = b_p\hat{\phi}v_1(t) = v_1(t) - b_p\tilde{\phi}v_1(t)$, one can obtain

$$\begin{aligned} z_1\dot{z}_1 &= -c_1 z_1^2 + z_1 z_2 \\ z_i\dot{z}_i &= -z_{i-1}z_i - c_i z_i^2 + z_i z_{i+1}, \qquad i = 2,3,\ldots,n-1 \\ \dot{z}_n &= -c_n z_n - z_{n-1} + \tilde{\mathbf{a}}^T Y \\ &\quad - \mathrm{sign}(z_n)\hat{B} - d_b[v](t) - b_p\tilde{\phi}v_1(t). \end{aligned} \tag{27}$$

To establish global boundedness, we define the following Lyapunov function candidate:

$$V(t) = \sum_{i=1}^n \frac{1}{2}z_i^2 + \frac{1}{2\gamma}\tilde{\mathbf{a}}^T\tilde{\mathbf{a}} + \frac{b_p}{2\eta}\tilde{\phi}^2 + \frac{1}{2q}\int_0^R \tilde{p}_b^2(t,r)dr. \tag{28}$$

The derivative $\dot{V}$ is given by

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^n z_i\dot{z}_i + \frac{1}{\gamma}\tilde{\mathbf{a}}^T\dot{\tilde{\mathbf{a}}} + \frac{b_p}{\eta}\tilde{\phi}\dot{\tilde{\phi}} + \frac{1}{q}\int_0^R \tilde{p}_b(t,r)\frac{\partial}{\partial t}\tilde{p}_b(t,r)dr \\ &\le -\sum_{i=1}^n c_i z_i^2 + \frac{1}{\gamma}\tilde{\mathbf{a}}^T(\dot{\tilde{\mathbf{a}}} + \gamma Y z_n) + \frac{b_p}{\eta}\tilde{\phi}(\dot{\tilde{\phi}} - \eta v_1 z_n) \\ &\quad - |z_n|\hat{B} + |d_b[v](t)|\,|z_n| + \int_0^R \tilde{p}_b(t,r)\,|F_r[v](t)|\,|z_n|dr \\ &\le -\sum_{i=1}^n c_i z_i^2 + \frac{1}{\gamma}\tilde{\mathbf{a}}^T(\dot{\tilde{\mathbf{a}}} + \gamma Y z_n) + \frac{b_p}{\eta}\tilde{\phi}(\dot{\tilde{\phi}} - \eta v_1 z_n) \\ &\quad - |z_n|\tilde{B} + \int_0^R \tilde{p}_b(t,r)\,|F_r[v](t)|\,|z_n|dr \\ &= -\sum_{i=1}^n c_i z_i^2. \end{aligned} \tag{29}$$

Equations (28) and (29) imply that V is nonincreasing. Hence, z_i (i = 1,...,n), $\hat{\mathbf{a}}$, $\hat{\phi}$, and $\hat{p}_b(t,r)$ are bounded. By applying the Lasalle–Yoshizawa theorem in [8] to (29), it further follows that $z_i \to 0$ (i = 1,...,n) as $t \to \infty$, which implies that $\lim_{t\to\infty}[x(t) - x_d(t)] = 0$.

Remark: It is now clear that the proposed control strategy to deal with the hysteresis nonlinearities can be applied to many systems and may not necessarily be limited to the system described by (2). However, we should emphasize that our goal in this note is to illustrate the fusion of the hysteresis models with available control techniques in a simpler setting that reveals its essential features.

V. Simulation Studies

In this section, we illustrate the methodology presented in the previous sections using a simple nonlinear system described by

$$\dot{x} = a\frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + bw(t) \tag{30}$$

where $w(t)$ represents the output of the hysteresis. The actual parameter values are $b = 1$ and $a = 1$. The objective is to control the system state x to follow the desired trajectory $x_d = 5sin(2t) + cos(3.2t)$. The hysteresis is described by

$$w(t) = p_0 v(t) - \int_0^R p(r)F_r[v](t)dr \tag{31}$$

where $p(r) = \alpha e^{-\beta(r-\sigma)^2}$ for $r \in [0, 100]$, with parameters $\alpha = 0.5$, $\beta = 0.0014$, and $\sigma = 1$.

As yet, no analytical approach has been developed for the selection of the control constants. The approach to select their values was through iterative simulation. In this simulation, the adaptive variable structure control law (12) and (24)–(26) were used, taking $c_1 = 0.9368$. In the adaptation laws, we choose $\gamma = 0.13$, $\eta = 0.05$, $q = 0.437$, and the initial parameters $\hat{a}(0) = 0.13$, $\hat{\phi}(0) = 0.431$, and $\hat{p}_b(0,r) = 0$. The initial state is chosen as $x(0) = 2.05$, sample time is 0.002. We also assume that the hysteresis internal state was $w_{-1} = 0.07$ for $r \in [0,R]$ before $v(0)$ was applied. For the calculation of $\hat{B}(t)$, we replace the integration by the sum $\sum_0^N$, while N is chosen as $N = 6000$.

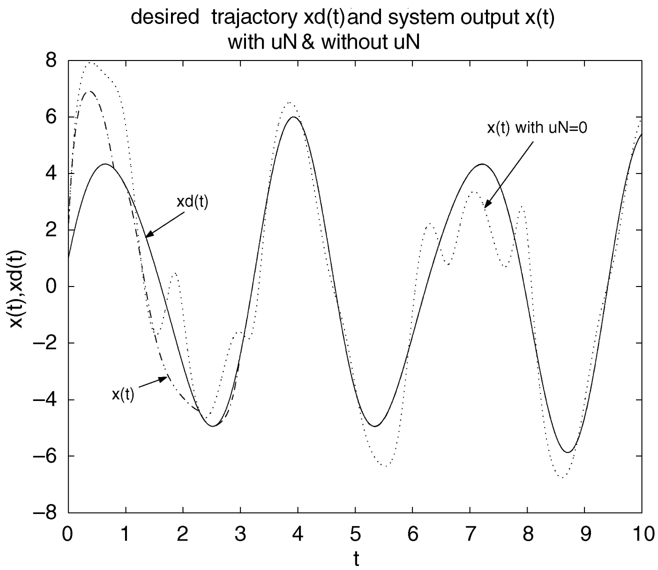

Fig. 2. Desired trajectory $x_d(t) = 5sin(2t) + cos(3.2t)$, system outputs $x(t)$ with control term u_N (-.) and $u_N = 0$ (dotted line).

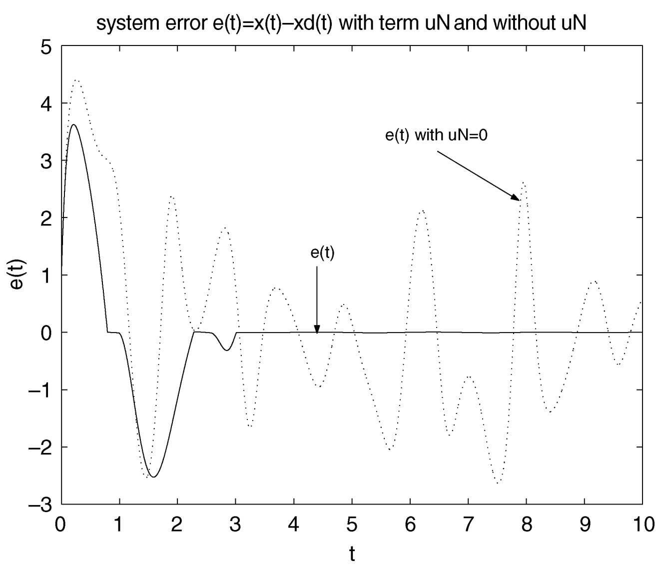

Fig. 3. Tracking errors of the state with control term u_N and $u_N = 0$ (dotted line).

Also, in the simulation, the function $\mathrm{sign}(z_n)$ in u_N is replaced by the saturation function $\mathrm{sat}(z_n/\varepsilon$ with $\varepsilon = 0.01$ to avoid the control chatter.

To illustrate the effectiveness of the proposed control scheme, the simulations were performed with and without controlling the effects of hysteresis. The analysis without consideration of the effects of hysteresis is implemented by setting $u_N(t) = 0$ in the controller $v(t)$, which implies that the control compensation for the hysteresis nonlinearity is ignored. Simulation results obtained for both cases ($u_N(t) = 0$ and $u_N(t) \neq 0$) are shown in Figs. 2–4 for the system (12) to track the desired trajectory $x_d(t) = 5sin(2t) + cos(3.2t)$. Figs. 2 and 3 show the state trajectories and tracking errors for the desired trajectory with and without considering the effects of hysteresis, where the solid line is the results with $u_N(t) \neq 0$ and the dotted line is with $u_N(t) = 0$. Fig. 4 shows the input control signal $v(t)$. The proposed robust controller clearly demonstrates excellent tracking performance as evident from the results presented in Figs. 2 and 3. The developed control algorithm can thus effectively overcome the effects of the hysteresis.

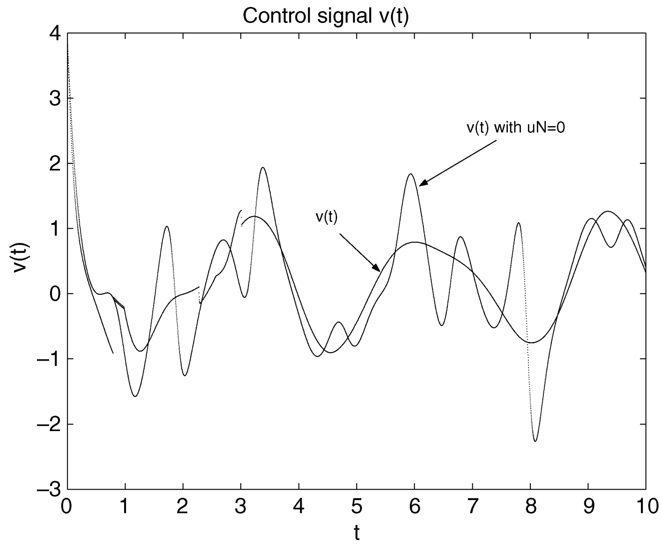

Fig. 4. Control signals $v(t)$ with control term u_N and without u_N (showing jumps).

VI. Conclusion

In practical control systems, hysteresis nonlinearity with unknown parameters in physical components may severely limit the performance of control. In this note, an adaptive variable structure control architecture is proposed for a class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity with the Prandtl–Ishlinskii model representation. The control law ensures global stability of the entire system and achieves both stabilization and tracking within a desired precision. Simulations performed on an unstable nonlinear system illustrate and further validate the effectiveness of the proposed approach. The primary purpose of exploring new avenues to fuse the model of hysteresis nonlinearities with the available adaptive controller design methodology without constructing a hysteresis inverse is achieved with highly promising results. The results presented in this note can be considered as a stepping stone to be used toward the development of a general control framework for the systems with hysteretic behavior.

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A New Solution to the Problem of Range Identification in Perspective Vision Systems

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***Abstract*—A new solution to the problem of range identification for perspective vision systems is proposed. These systems arise in machine vision problems, where the position of an object moving in the three-dimensional space has to be identified through two-dimensional images obtained from a single camera. The proposed identifier yields asymptotic estimates of the object coordinates and is significantly simpler than existing designs. Moreover, it can be easily tuned to achieve the desired convergence rate. Simulations are provided demonstrating the enhanced performance of the proposed scheme and its robustness to measurement noise.**

***Index Terms*—Machine vision, nonlinear observer, perspective system.**

I. Introduction

A classical problem in machine vision is to determine the position of an object moving in the three-dimensional space by observing the motion of its projected feature on the two-dimensional image space of a charge-coupled device (CCD) camera. The case where the motion of the object is described by linear (time-varying) dynamics with known parameters has received particular attention, see e.g. [1]–[4].

The systems that arise in this case are known as perspective dynamical systems and the problem of determining the object space coordinates reduces to the problem of estimating the depth (or range) of the object. Higher-dimensional perspective systems, but with constant motion parameters, have also been considered, see e.g. [5]. Alternatively, the (dual) problem of estimating the motion parameters when the three-dimensional coordinates are available for measurement has been studied in [6]–[9].

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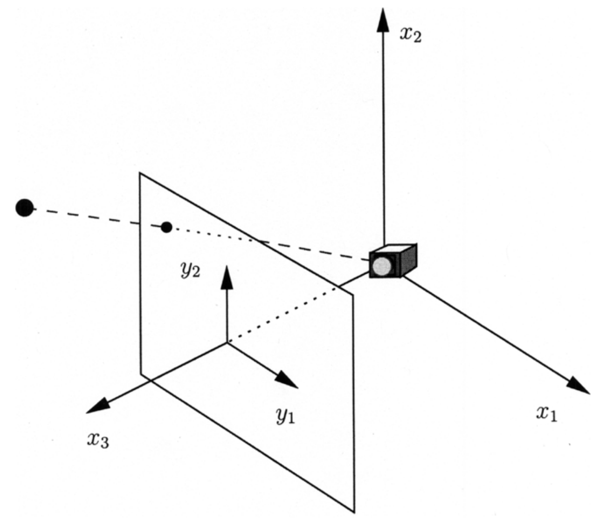

Fig. 1. Diagram of the perspective vision system.

In this note, a solution to the range identification problem is proposed based on a new nonlinear observer design which is inspired by the recently developed immersion and invariance methodology [10] and the reduced-order observer in [11].

The proposed scheme achieves asymptotic convergence of the observation error to zero and is considerably simpler than the fourth-order asymptotic observer proposed in [3], as well as the fifth-order approximate observer in [2], and the high-gain observer proposed in [1]. Moreover, it can be easily tuned to achieve the desired convergence rate.

II. Problem Formulation

The motion of an object undergoing rotation, translation and linear deformation can be described by the affine system [3]

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix} \tag{1}$$

where $(x_1, x_2, x_3) \in \mathbb{R}^3$ are the unmeasurable coordinates of the object in an inertial reference frame with x_3 being perpendicular to the camera image space, as shown in Fig. 1. The motion parameters $a_{ij} = a_{ij}(t)$, $b_i = b_i(t)$ are possibly time-varying and are assumed known.

Using the perspective (or "pinhole") model for the camera, the measurable coordinates on the image space are given by

$$y = [y_1,\ y_2]^T = \epsilon\left[\frac{x_1}{x_3}, \frac{x_2}{x_3}\right]^T \tag{2}$$

where ϵ is the focal length of the camera, i.e. the distance between the camera and the origin of the image-space axes. Without loss of generality, we assume that $\epsilon = 1$.

The perspective system (1) must satisfy the following assumption.

Assumption 1: The parameters a_{ij}, b_i in (1) and the coordinates y_1, y_2 in (2) are bounded functions of time, i.e. $a_{ij}(t), b_i(t) \in \mathcal{L}_\infty$, for all i, j = 1,2,3 and $y(t) \in \mathcal{L}_\infty$. Moreover, $a_{ij}(t)$ and $b_i(t)$ are first-order differentiable and $x_3(t) < \epsilon = 1$, where ϵ is as in (2).

Remark 1: Assumption 1 is motivated by the physical properties of the perspective system, see [1] and [3]. Note that in [3] it is further assumed that the functions $b_i(t)$ are twice differentiable and that $x_3(t) \in \mathcal{L}_\infty$.

The design objective is to reconstruct the coordinates x_1, x_2, x_3 from measurements of the image-space coordinates y_1, y_2.