methods because the repulsion between the object and obstacle is available in closed-form. According to the simulation results, not only can an object configuration obtained with the proposed approach avoid obstacles with satisfactory (optimal) margins, a sequence of object configurations thus obtained also connect naturally into a spatially smooth object path. Preliminary results of connecting local paths obtained with the proposed local planner into a global one are also included.

Despite the aforementioned success in applying the proposed algorithm in 3-D path planning, several related issues are yet to be addressed. For example, the sampling of the object surface is not a trivial problem. There is certainly a tradeoff between the computation efficiency and the correctness in the resultant object path. Other issues include the developments of a systematic way of identifying free space bottlenecks of more complex geometry, suitable global planning strategies to connect the local paths, and other local planning algorithms. On the other hand, it is possible to combine the proposed algorithm with some other global path planning approaches, e.g., a probabilistic roadmaps method presented in [21]. Extensions of the proposed approach to more general problems, other than the one involvinga single rigid object among stationary obstacles, are also under investigation.

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Adaptive Control of a Class of Nonlinear Systems With a First-Order Parameterized Sugeno Fuzzy Approximator

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***Abstract*—In this paper, an adaptive fuzzy control scheme for tracking of a class of continuous-time plants is presented. A parameterized Sugeno fuzzy approximator is used to adaptively compensate for the plant nonlinearities. All parameters in the fuzzy approximator are tuned using a Luapunov-based design. In the fuzzy approximator, first-order Sugeno consequents are used in the IF–THEN rules of the fuzzy system, which has a better approximation capability than those using constant consequents. Global boundedness of the adaptive system is established. Finally, a simulation is used to demonstrate the effectiveness of the proposed controller.**

***Index Terms*—Adaptive control, fuzzy approximator, nonlinear systems, robustness, stability, Sugeno fuzzy systems.**

I. INTRODUCTION

The weakness of traditional quantitative techniques to adequately describe and control complex and ill-defined phenomena was summarized in the well known principle of incompatibility formulated by Zadeh [1]. This principle states that "as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviors diminishes." The idea of fuzzy modeling first emerged in Zadeh [1], and has subsequently been pursued by many others. Although fuzzy modeling and control is thought of as an alternative approach compared with traditional control methods, its effectiveness is now well proven. Over the past two decades, engineers have applied fuzzy modeling and control methods very successfully [2]–[7].

Recently, in [11], [12], and [18]–[20] fuzzy controllers have been justified by universal approximation theorems. In other words, these fuzzy controllers are general enough to perform any nonlinear control action. Therefore, by carefully choosing the parameters of the fuzzy controller, it is always possible to design a fuzzy controller that is suitable for the nonlinear system under consideration. Based on this fact, a global stable adaptive fuzzy controller is firstly synthesized from a collection of fuzzy IF–THEN rules [10]. The fuzzy system, used to approximate an optimal controller, is adjusted by an adaptive law based on Luapunov synthesis approach. An adaptive tracking control architecture is proposed in [8] for a class of continuous time nonlinear dynamic systems, where an explicit linear parameterization of the uncertainty in the dynamics is not possible. The architecture employs fuzzy

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systems, which are expressed as a series expansion of fuzzy basis functions (FBFs), to adaptively compensate for the plant nonlinearities. It is shown in [11] that Gaussian basis functions (GBFs) have the best approximation property. This is the main reason in choosing the Gaussian functions as the membership functions in this study.

In the GBF expansion, three parameter vectors are used; connection weights (constant consequents), variances and centers. It is obvious that as these parameters change, the shape of the GBF vary accordingly. However, in the fuzzy schemes presented in [8], [10], [13], [14] only connection weights are updated in the GBF expansion. To overcome this drawback, in the recently developed adaptive fuzzy controller [9] all three parameters are updated, which results in a better tracking performance. In [22], an adaptive controller using a similar approach to the one used in [10] is introduced, where a Sugeno fuzzy system is used to approximate the controller. In this paper, we introduce a controller along the lines of [9]. The principal difference is that our controller is designed based on the well known Sugeno first-order fuzzy system. The consequent part of IF–THEN rules is a linear combination of input variables and a constant term, and the final output is the weighted average of each rule's output. This introduces additional parameter vectors to be updated, but improves the tracking performance due to the better approximation ability of the higher order Sugeno consequents model [23]. The results of [9] can, therefore, be thought of as a special case of this extension. It is also shown in [15] that a model based on higher order Sugeno consequents could identify a system with less error for the same number of rules or could achieve the required performance with less rules than a model using lower order consequents.

II. Fuzzy Approximators

A. Problem Statement

In this paper, an adaptive control algorithm for a class of dynamic systems is to be developed. The considered systems have the following equation of motion:

$$\begin{aligned} x^{(n)}(t) + f\left(x(t),\, \dot{x}(t),\, \ldots,\, x^{(n-1)}(t)\right) \\ = b\left(x(t),\, \dot{x}(t),\, \ldots,\, x^{(n-2)}(t)\right) u(t) \end{aligned} \tag{1}$$

where

- $u(t)$ control input;
- f unknown linear or nonlinear function;
- b control gain.

The control objective is to force the state $X = [x, \dot{x}, \ldots, x^{(n-1)}]^T$ to follow a specified desired trajectory $X_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^T$. In the case considered, an explicit linear parameterization of the function $f(X)$ is unknown or not possible, i.e., $f(X)$ cannot be expressed as $f(X) = \sum_{j=1}^{N} \theta_j Y_j(X)$, where θ_j is a set of unknown parameters which appear linearly, and $Y_j(X)$ is a set of known regressors or basis functions. Therefore, the unknown function $f(X)$ will be approximated by a parameterized fuzzy approximator. The required tracking control is achieved by tuning the parameters of the fuzzy approximator.

B. Fuzzy Model

In the Sugeno model, a multi-input and single-output (MISO) system with n antecedents can be represented as a set of N rules of the following format:

$$\begin{aligned} R_j:\quad & \text{IF } x_1 \text{ is } A_1^j \text{ AND } x_2 \text{ is } A_2^j, \ldots, x_n \text{ is } A_n^j \\ & \text{THEN } w_j = b_j + a_j^1 x_1 + \cdots + a_j^n x_n \\ & \quad \text{for } j = 1, \ldots, N \end{aligned}$$

where R_j denotes the jth fuzzy rule, x_i $(i = 1, 2, \ldots, n)$ is the input, w_j is functional consequent of the fuzzy rule R_j and $(b_j, a_j^1, \ldots, a_j^n;\ j = 1, 2, \ldots, N)$ are adjustable design parameters. In this study, for simplicity, a MISO system is assumed. In case of a multi-output system, several output variables such as w_j^k $(k = 1, 2, \ldots, m)$ are used, where m represents the number of output variables. $A_1^j, A_2^j, \ldots, A_n^j$ are fuzzy labels of the membership functions. To combine the membership values of the input fuzzy sets in the rule antecedent, any type of fuzzy conjunction operator (AND operator) may be used.

The output of a fuzzy system with Gaussian membership function, product conjunction operator and functional consequent can be written as

$$C(X) = \sum_{j=1}^{N} w_j \left(\prod_{i=1}^{n} \mu_{A_j^i}(x_i) \right) \tag{2}$$

where $C\colon R^n \to R$, $X = (x_1, x_2, \ldots, x_n) \in R^n$, w_j is a functional consequent and $\mu_{A_j^i}(x_i)$ is the membership value when Gaussian membership function is evaluated at x_i. That is

$$\mu_{A_j^i}(x_i) = \exp\left[-\left(\sigma_j^i \left(x_i - \xi_j^i \right) \right)^2 \right]$$

where σ_j^i, ξ_j^i are real-valued parameters. The FBF is defined as

$$g_j(\sigma_j \|X - \xi_j\|) = \prod_{i=1}^{n} \mu_{A_j^i}(x_i) \qquad j = 1, 2, \ldots, N$$

where $\sigma_j = (\sigma_j^1, \sigma_j^2, \ldots, \sigma_j^n) \in U$ and $\xi_j = (\xi_j^1, \xi_j^2, \ldots, \xi_j^n) \in U$ and w_j is defined as

$$w_j = \sum_{i=1}^{n} a_j^i x_i + b_j. \tag{3}$$

Then, the fuzzy system is equivalent to an FBF expansion

$$\begin{aligned} C(X) &= \sum_{j=1}^{N} \sum_{i=1}^{n} (a_j^i x_i + b_j) g_j(\sigma_j \|X - \xi_j\|) \\ &= \sum_{j=1}^{N} \sum_{i=1}^{n} (a_j^i x_i) g_j(\sigma_j \|X - \xi_j\|) \\ &\quad + \sum_{j=1}^{N} b_j g_j(\sigma_j \|X - \xi_j\|). \end{aligned}$$

Define

$$\begin{aligned} C(X) &= A^T X G(X, \xi, \sigma) + B^T G(X, \xi, \sigma) \\ &= A^T L(X, \xi, \sigma) + B^T G(X, \xi, \sigma) \end{aligned}$$

where

$$\begin{aligned} A &= [A_1, A_2, \ldots, A_N] \\ A_j &= [a_j^1, a_j^2, \ldots, a_j^n]^T \\ X &= [x_1, x_2, \ldots, x_n]^T \\ B &= [b_1, b_2, \ldots, b_N]^T \\ \sigma_j &= [\sigma_1, \sigma_2, \ldots, \sigma_N]^T \\ \xi_j &= [\xi_1, \xi_2, \ldots, \xi_N]^T \\ G(X, \xi, \sigma) &= [g_1(\sigma_1 \|X - \xi_1\|), g_2(\sigma_2 \|X - \xi_2\|) \\ &\quad \ldots, g_N(\sigma_N \|X - \xi_N\|)]^T \end{aligned}$$

and

$$L(X, \xi, \sigma) = [Xg_1(\sigma_1\|X-\xi_1\|), Xg_2(\sigma_2\|X-\xi_2\|) \ldots, Xg_N(\sigma_N\|X-\xi_N\|)]^T.$$

Remark: A Sugeno first-order consequent model is expected to result in at least the same system performance with fewer rules when compared with Sugeno constant consequent model. This is due to a better approximation capability of higher order Sugeno consequents [15]. From (3) we can see that Sugeno first-order consequent model is reduced to constant consequent model when $a_j^i = 0$. Therefore, Sugeno constant consequent model is a special case of the Sugeno first-order consequent model, which means that the approximation ability of first-order consequent rule is at least as good as that of constant consequent one.

C. Fuzzy Systems as Universal Approximators

An important property to look for in the Sugeno fuzzy systems, when used as controllers, is the universal approximation property. That is, can a Sugeno model always be constructed to approximate any continuous and nonlinear control solution with any arbitrary accuracy? The issue of fuzzy systems as universal approximators is very important and much significant work has been done in this area. Many studies in literature consider Mamdani fuzzy systems [11], [12], [18], [19]. Recently, some researchers have studied the universal approximation property of Sugeno systems. It is proven in [20] that fuzzy systems with nonfuzzy consequents are universal approximators. Also, it has been constructively proven in [21], in a two step approach using polynomials as the bridge, that Sugeno first-order fuzzy systems are universal approximators. The Sugeno systems in [21] are general because they use any type of continuous fuzzy sets, any type of fuzzy conjunction operator, and fuzzy rules with linear consequent.

It is important to note that in [21] the weighted average *centroid defuzzifier* is adopted. However, in this paper the linguistic fuzzy IF–THEN rules are only used for the purpose of approximating the required functions, we, therefore, define the defuzzifier in (2) as a weighted sum of each rule's output, similar to [8]. This definition will not change the universal approximation property of the Sugeno model. The following theorem states that the above Sugeno FBF expansion is a universal approximator.

Theorem 1: For any real continuous function f on a compact set and arbitrary $\varepsilon > 0$, there exist $\overset{*}{C}(X) = \overset{*}{A}{}^T L(X, \overset{*}{\xi}, \overset{*}{\sigma}) + \overset{*}{B}{}^T G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ such that

$$\sup\left|f(X) - \overset{*}{C}(X)\right| < \varepsilon. \tag{4}$$

A proof of this theorem is in the same spirit as [21]. We omit the proof here for brevity.

This theorem states that $\overset{*}{C}(X)$ is universal approximator, i.e., $\overset{*}{C}(X)$ can approximate the unknown function $f(X)$ with the required accuracy. The universal approximation property of $\overset{*}{C}(X)$ is characterized by the parameters of the fuzzy sets (ξ, σ) and the parameters of the linear consequents (b_j, $a_j^1, \ldots, a_j^n$). $\overset{*}{C}(X)$ can be called as a nonlinearly parameterized fuzzy approximator since ξ and σ appear nonlinearly in the fuzzy system as shown in (2).

III. Adaptive Control Using Nonlinearly Parameterized Fuzzy Approximators

A. Approximation Error

Based on Theorem 1, there is a fuzzy approximator

$$\overset{*}{f}(X) = \overset{*}{A}{}^T L\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right) + \overset{*}{B}{}^T G\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)$$

that approximates f. The approximation error on the entire state space can be expressed as

$$f(X) - \overset{*}{f}(X) = \varepsilon_f(X).$$

Due to Theorem 1, it can be assumed that there exists a constant $\overset{*}{\varepsilon} \geq 0$ such that

$$|\varepsilon_f(X)| \leq \overset{*}{\varepsilon}.$$

To construct $\overset{*}{f}(X)$ the values of the parameters $\overset{*}{A}$, $\overset{*}{B}$, $\overset{*}{\xi}$, and $\overset{*}{\sigma}$ are required. These parameters are replaced with their estimates $\hat{A}$, $\hat{B}$, $\hat{\xi}$, and $\hat{\sigma}$, respectively, so $\hat{f}(X) = \hat{A}^T L(X, \hat{\xi}, \hat{\sigma}) + \hat{B}^T G(X, \hat{\xi}, \hat{\sigma})$ is used to approximate the unknown function f. In this paper, all parameters in the estimate $\hat{f}(X)$ are tuned. This should provide a better performance than tuning just the consequent parameters A and B and fix the other parameters before controller design. The consequent parameters are easy to tune because they appear linearly in the fuzzy approximator or in the approximation error. It will be possible to update the parameters that appear nonlinearly in the approximation error if it is possible to express the approximation error in a linear parameterized form with respect to each parameter. The approximation capability of the fuzzy approximator can be improved by using a first-order Sugeno consequent in the IF–THEN rules of the fuzzy approximator instead of a constant consequent used in [9]. Also higher order consequents will usually minimize the number of rules to be constructed to describe and control the system under consideration [15]. Because $\overset{*}{A}$, $\overset{*}{B}$, $\overset{*}{\xi}$, and $\overset{*}{\sigma}$ are unknown, the approximation function $\overset{*}{f}$ can not be used to directly construct the control law. Using the estimation function $\hat{f}(X)$ of $\overset{*}{f}$, the estimation error $\varepsilon(X)$ needs to be formulated.

Theorem 2: The function estimation error between f and $\hat{f}(X)$, written as $\varepsilon(X) = f(X) - \hat{f}(X)$ is equivalent to

$$\varepsilon(X) = \tilde{A}^T\left(\hat{L} - L'_\xi\hat{\xi} - L'_\sigma\hat{\sigma}\right) + \hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) + \tilde{B}^T\left(\hat{G} - G'_\xi\hat{\xi} - G'_\sigma\hat{\sigma}\right) + \hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) + d_f \tag{5}$$

where the estimation errors of the parameter vectors are defined as $\tilde{A} = \overset{*}{A} - \hat{A}$, $\tilde{B} = \overset{*}{B} - \hat{B}$, $\tilde{\xi} = \overset{*}{\xi} - \hat{\xi}$, $\tilde{\sigma} = \overset{*}{\sigma} - \hat{\sigma}$, G'_ξ and L'_ξ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\xi}$ at $\hat{\xi}$, respectively, also G'_σ and L'_σ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\sigma}$ at $\hat{\sigma}$, respectively; d_f is a residual term that satisfies

$$|d_f| \leq \overset{*}{\theta}{}_f^T \cdot Y_f$$

where $\overset{*}{\theta}_f$ is an unknown constant vector of optimal weights and bounded constants and

$$Y_f = \left[1, \left\|\hat{A}\right\|, \left\|\hat{B}\right\|, \left\|\hat{\xi}\right\|, \left\|\hat{\sigma}\right\|\right]^T$$

is a known function vector.

Proof: The approximation error between f and $\overset{*}{f}$ is denoted by $\varepsilon_f(X) = f - \overset{*}{f}$. The estimation error $\varepsilon(X) = f(X) - \hat{f}(X)$ can be written as

$$\begin{aligned}\varepsilon(X) &= f(X) - \hat{A}^T\hat{L} - \hat{B}^T\hat{G}\\ &= \overset{*}{f}(X) - \hat{A}^T\hat{L} - \hat{B}^T\hat{G} + \varepsilon_f(X)\\ &= \overset{*}{A}{}^T\overset{*}{L} + \overset{*}{B}{}^T\overset{*}{G} - \hat{A}^T\hat{L} - \hat{B}^T\hat{G} + \varepsilon_f(X)\\ &= \overset{*}{A}{}^T\overset{*}{L} - \overset{*}{A}{}^T\hat{L} + \overset{*}{A}{}^T\hat{L} + \overset{*}{B}{}^T\overset{*}{G} - \overset{*}{B}{}^T\hat{G}\end{aligned}$$

$$+ \overset{*}{B}^T \hat{G} - \hat{A}^T \hat{L} - \hat{B}^T \hat{G} + \varepsilon_f(X)$$
$$= \tilde{A}^T \tilde{L} + \hat{A}^T \tilde{L} + \tilde{B}^T \tilde{G} + \hat{B}^T \tilde{G} + \tilde{A}^T \hat{L} + \tilde{B}^T \hat{G} + \varepsilon_f(X).$$

Taking the Taylor's series expansion of $\overset{*}{G}$ and $\overset{*}{L}$ at $\overset{*}{\xi} = \hat{\xi}$ and $\overset{*}{\sigma} = \hat{\sigma}$, $\tilde{G}$ and $\tilde{L}$ can be expressed as

$$\tilde{G} = G'_\xi \tilde{\xi} + G'_\sigma \tilde{\sigma} + o\left(X, \tilde{\xi}, \tilde{\sigma}\right)$$
$$\tilde{L} = L'_\xi \tilde{\xi} + L'_\sigma \tilde{\sigma} + h\left(X, \tilde{\xi}, \tilde{\sigma}\right)$$

where $o(.)$ is the sum of the high order terms in Taylor's series expansion, G'_ξ and L'_ξ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\xi}$ at $\hat{\xi}$ and expressed as

$$G'_\xi = G'_\xi\left(x, \hat{\xi}\ \hat{\sigma}\right) = \left.\frac{\partial G\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\xi}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}$$

$$L'_\xi = L'_\xi\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial L\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\xi}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}.$$

Also, G'_σ and L'_σ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\sigma}$ at $\hat{\sigma}$ and expressed as

$$G'_\sigma = G'_\sigma\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial G\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\sigma}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}$$

$$L'_\sigma = L'_\sigma\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial L\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\sigma}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}.$$

Therefore

$$\begin{aligned}
\varepsilon(X) = {} & \tilde{A}^T\left(L'_\xi \tilde{\xi} + L'_\sigma \tilde{\sigma} + h\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right) \\
& + \hat{A}^T\left(L'_\xi \tilde{\xi} + L'_\sigma \tilde{\sigma} + h\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right) \\
& \cdot \tilde{B}^T\left(G'_\xi \tilde{\xi} + G'_\sigma \tilde{\sigma} + o\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right) \\
& - \hat{B}^T\left(G'_\xi \tilde{\xi} + G'_\sigma \tilde{\sigma} + o\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right) \\
& + \widetilde{\tilde{A}^T \hat{L} + \tilde{B}^T \hat{G}} + \varepsilon_f(X) \\
= {} & \tilde{A}^T L'_\xi\left(\overset{*}{\xi} - \hat{\xi}\right) + \tilde{A}^T L'_\sigma\left(\overset{*}{\sigma} - \hat{\sigma}\right) \\
& + \tilde{A}^T h\left(X, \tilde{\xi}, \tilde{\sigma}\right) + \hat{A}^T L'_\xi \tilde{\xi} + \hat{A}^T L'_\sigma \tilde{\sigma} \\
& + \hat{A}^T h\left(X, \tilde{\xi}, \tilde{\sigma}\right) + \tilde{B}^T G'_\xi\left(\overset{*}{\xi} - \hat{\xi}\right) \\
& + \tilde{B}^T G'_\sigma\left(\overset{*}{\sigma} - \hat{\sigma}\right) + \tilde{B}^T o\left(X, \tilde{\xi}, \tilde{\sigma}\right) \\
& + \hat{B}^T G'_\xi \tilde{\xi} + \hat{B}^T G'_\sigma \tilde{\sigma} + \hat{B}^T o\left(X, \tilde{\xi}, \tilde{\sigma}\right) \\
& + \widetilde{\tilde{A}^T \hat{G} + \tilde{B}^T \hat{G}} + \varepsilon_f(X) \\
= {} & \tilde{A}^T\left(\hat{L} - L'_\xi \hat{\xi} - L'_\sigma \hat{\sigma}\right) + \hat{A}^T\left(L'_\xi \tilde{\xi} + L'_\sigma \tilde{\sigma}\right) \\
& + \tilde{B}^T\left(\hat{G} - G'_\xi \hat{\xi} - G'_\sigma \hat{\sigma}\right) \\
& + \hat{B}^T\left(G'_\xi \tilde{\xi} + G'_\sigma \tilde{\sigma}\right) + d_f
\end{aligned}$$

where

$$d_f = \tilde{A}^T\left(L'_\xi \overset{*}{\xi} + L'_\sigma \overset{*}{\sigma}\right) + \overset{*}{A}^T h\left(X, \tilde{\xi}, \tilde{\sigma}\right) + \tilde{B}^T\left(G'_\xi \overset{*}{\xi} + G'_\sigma \overset{*}{\sigma}\right) + \overset{*}{B}^T o\left(X, \tilde{\xi}, \tilde{\sigma}\right) + \varepsilon_f(X).$$

It can easily be proved that higher order terms are bounded by

$$\begin{aligned}
\left\|o\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right\| &= \left\|\tilde{G} - G'_\xi \tilde{\xi} - G'_\sigma \tilde{\sigma}\right\| \\
&\le \left\|\tilde{G}\right\| - \left\|G'_\xi\right\| \left\|\tilde{\xi}\right\| - \left\|G'_\sigma\right\| \left\|\tilde{\sigma}\right\| \\
&\le c_1 + c_2\left\|\tilde{\xi}\right\| + c_3\left\|\tilde{\sigma}\right\| \\
\left\|h\left(X, \tilde{\xi}, \tilde{\sigma}\right)\right\| &= \left\|\tilde{L} - L'_\xi \tilde{\xi} - L'_\sigma \tilde{\sigma}\right\| \\
&\le \left\|\tilde{L}\right\| - \left\|L'_\xi\right\| \left\|\tilde{\xi}\right\| - \left\|L'_\sigma\right\| \left\|\tilde{\sigma}\right\| \\
&\le c_4 + c_5\left\|\tilde{\xi}\right\| + c_6\left\|\tilde{\sigma}\right\|
\end{aligned}$$

where c_1, c_2, c_3, c_4, c_5, and c_6 are some bounded constants due to the fact that the FBF and its derivative are always bounded by constants, which is demonstrated in the Appendix.

Let $\bar{\xi}, \bar{\sigma}, \bar{A}$, and $\bar{B}$ be constants satisfying $\|\overset{*}{\xi}\| \le \bar{\xi}$, $\|\overset{*}{\sigma}\| \le \bar{\sigma}$, $\|\overset{*}{A}\| \le \bar{A}$, and $\|\overset{*}{B}\| \le \bar{B}$. Based on the following facts

$$\begin{aligned}
\left\|\tilde{\xi}\right\| &\le \left\|\overset{*}{\xi}\right\| + \left\|\hat{\xi}\right\| \le \bar{\xi} + \left\|\hat{\xi}\right\| \\
\left\|\tilde{\sigma}\right\| &\le \left\|\overset{*}{\sigma}\right\| + \left\|\hat{\sigma}\right\| \le \bar{\sigma} + \left\|\hat{\sigma}\right\| \\
\left\|\tilde{B}\right\| &\le \left\|\overset{*}{B}\right\| + \left\|\hat{B}\right\| \le \bar{B} + \left\|\hat{B}\right\| \\
\left\|\tilde{A}\right\| &\le \left\|\overset{*}{A}\right\| + \left\|\hat{A}\right\| \le \bar{A} + \left\|\hat{A}\right\|
\end{aligned}$$

the term d_f can be bounded as

$$\begin{aligned}
|d_f| = {} & \Big\|\tilde{A} L'_\xi \overset{*}{\xi} + \tilde{A}^T L'_\sigma \overset{*}{\sigma} + \overset{*}{A}^T h\left(x, \tilde{\xi}, \tilde{\sigma}\right) + \tilde{B}^T G'_\xi \overset{*}{\xi} \\
& + \tilde{B}^T G'_\sigma \overset{*}{\sigma} + \overset{*}{B}^T o\left(X, \tilde{\xi}, \tilde{\sigma}\right) + \varepsilon_f(X)\Big\| \\
|d_f| \le {} & \left\|\tilde{A}\right\| \left\|L'_\xi\right\| \left\|\overset{*}{\xi}\right\| + \left\|\tilde{A}\right\| \left\|L'_\sigma\right\| \left\|\overset{*}{\sigma}\right\| \\
& + \left\|\overset{*}{A}\right\|\left(c_4 + c_5\left\|\tilde{\xi}\right\| + c_6\left\|\tilde{\sigma}\right\|\right) + \left\|\tilde{B}\right\| \left\|G'_\xi\right\| \left\|\overset{*}{\xi}\right\| \\
& + \left\|\tilde{B}^T\right\| \left\|G'_\sigma\right\| \left\|\overset{*}{\sigma}\right\| + \left\|\overset{*}{B}^T\right\| \\
& \cdot \left(c_1 + c_2\left\|\tilde{\xi}\right\| + c_3\left\|\tilde{\sigma}\right\|\right) + \overset{*}{\varepsilon} \\
|d_f| \le {} & \left(\bar{A} + \left\|\hat{A}\right\|\right) c_5 \bar{\xi} + \left(\bar{A} + \left\|\hat{A}\right\|\right) c_6 \bar{\sigma} \\
& + \bar{A} c_4 + \bar{A} c_5\left(\bar{\xi} + \left\|\hat{\xi}\right\|\right) + \bar{A} c_6\left(\bar{\sigma} + \left\|\hat{\sigma}\right\|\right) \\
& + \left(\bar{B} + \left\|\tilde{B}\right\|\right) c_2 \bar{\xi} + \left(\bar{B} + \left\|\hat{B}\right\|\right) c_3 \bar{\sigma} + \bar{B} c_1 \\
& + \bar{B} c_2\left(\bar{\xi} + \left\|\hat{\xi}\right\|\right) + \bar{B} c_3\left(\bar{\sigma} + \left\|\hat{\sigma}\right\|\right) + \overset{*}{\varepsilon} \\
|d_f| \le {} & 2c_5 \bar{A} \bar{\xi} + 2c_6 \bar{A} \bar{\sigma} + c_4 \bar{A} + \left(c_5 \bar{\xi} + c_6 \bar{\sigma}\right) \left\|\hat{A}\right\|
\end{aligned}$$

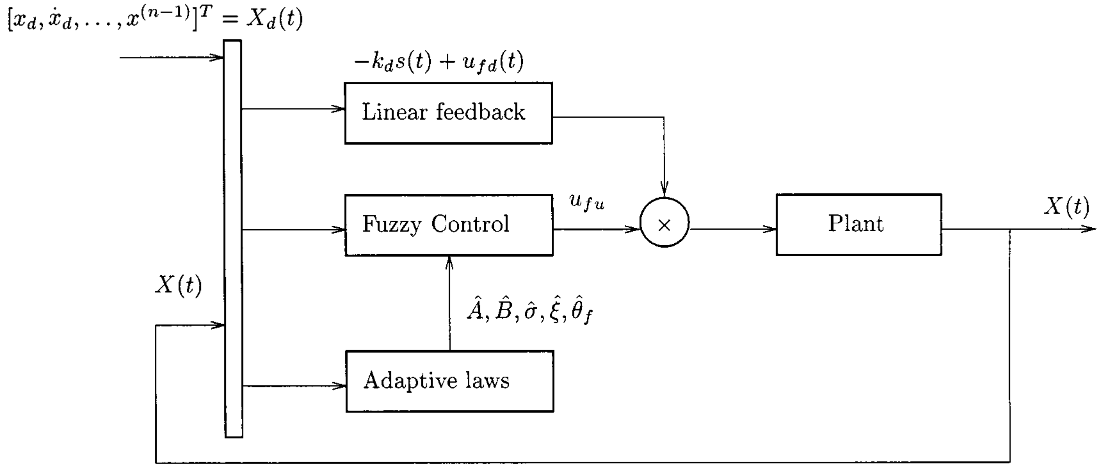

Fig. 1. Closed-loop control structure.

$$
\begin{aligned}
&+ c_5\bar{A}\left\|\hat{\xi}\right\| + c_6\bar{A}\left\|\hat{\sigma}\right\| + 2c_2\bar{B}\bar{\xi} + 2c_3\bar{B}\bar{\sigma} + c_1\bar{B} \\
&+ \left(c_2\bar{\xi} + c_3\bar{\sigma}\right)\left\|\hat{B}\right\| + c_2\bar{B}\left\|\hat{\xi}\right\| + c_3\bar{B}\left\|\hat{\sigma}\right\| + \overset{*}{\varepsilon} \\
&= \overset{*}{\theta}{}_f^T \cdot Y_f \\
&= \left[\overset{*}{\theta}_{f1}, \overset{*}{\theta}_{f2}, \overset{*}{\theta}_{f3}, \overset{*}{\theta}_{f4}, \overset{*}{\theta}_{f5}\right] \\
&\quad \cdot \left[1, \left\|\hat{A}\right\|, \left\|\hat{B}\right\|, \left\|\hat{\xi}\right\|, \left\|\hat{\sigma}\right\|\right]^T \\
\overset{*}{\theta}_{f1} &= 2c_5\bar{A}\bar{\xi} + 2c_6\bar{A}\bar{\sigma} + c_4\bar{A} + 2c_2\bar{B}\bar{\xi} + 2c_3\bar{B}\bar{\sigma} \\
&\quad + c_1\bar{B} + \overset{*}{\varepsilon} \\
\overset{*}{\theta}_{f2} &= \left(c_5\bar{\xi} + c_6\bar{\sigma}\right) \\
\overset{*}{\theta}_{f3} &= \left(c_2\bar{\xi} + c_3\bar{\sigma}\right) \\
\overset{*}{\theta}_{f4} &= c_5\bar{A} + c_2\bar{B} \\
\overset{*}{\theta}_{f5} &= c_6\bar{A} + c_3\bar{B}.
\end{aligned}
$$

Remark: The approximation error is expressed in a linearly parameterized form with respect to $\tilde{A}$, $\tilde{B}$, $\tilde{\xi}$, and $\tilde{\sigma}$, which makes the updates of $\hat{A}$, $\hat{B}$, $\hat{\xi}$, and $\hat{\sigma}$ possible. Also, note that the term d_f is not a constant and the assumption on the constant bound will not be imposed in the developed control method. This will make the developed controller more general and more applicable.

IV. Controller Design

A. Unity Control Gain

In this section, we present results pertaining to systems with the Sugeno approximator. To improve the readability we solve first the case when the control gain is a unit, i.e., $b = 1$. Such a solution contains the essential ingredients used in the more general constructions. We then modify the controller to handle the case of nonunity control gain. This gives a complete solution to the control objective.

Before the introduction of the control law, we define a filtered tracking error as

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{X}(t) \qquad \text{with } \lambda > 0 \tag{6}$$

where $s(t)$ is an error metric, which can be rewritten as $s(t) = \Lambda^T \tilde{X}(t)$ with $\Lambda^T = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, 1]$, and $\tilde{X} = X - X_d$. The tracking error vector exponentially approaches zero when $s(t) = 0$, therefore the objective is to design a controller which is able to drive $s(t)$ to zero. It can be easily proven that

$$\dot{s}(t) = -X_d^{(n)}(t) + \Lambda_v^T \tilde{X}(t) + bu - f(X) \tag{7}$$

where $\Lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, (n-1)\lambda]$. Using

$$\hat{f}(X) = \hat{A}^T L\left(X, \hat{\xi}, \hat{\sigma}\right) + \hat{B}^T G\left(X, \hat{\xi}, \hat{\sigma}\right).$$

Equation (7) can be rewritten as

$$\dot{s}(t) = -X_d^{(n)}(t) + \Lambda_v^T \tilde{X}(t) + bu - \hat{f}(X) - \varepsilon(X)$$

where $\varepsilon(X) = f(X) - \hat{f}(X)$, which is the fuzzy reconstruction error. The adaptive control law with $b = 1$ is defined as

$$u(t) = -k_d s(t) + u_{fd}(t) + u_{fu}(t) \tag{8}$$

$$u_{fd}(t) = X_d^{(n)}(t) - \Lambda_v^T \tilde{X}(t) \tag{9}$$

$$u_{fu}(t) = \hat{A}^T L\left(X, \hat{\xi}, \hat{\sigma}\right) + \hat{B}^T G\left(X, \hat{\xi}, \hat{\sigma}\right) - \hat{\theta}_f Y_f \operatorname{sgn}(s) \tag{10}$$

$$\dot{\hat{A}} = -s(t)\Gamma_1\left(\hat{L} - L'_\xi\hat{\xi} - L'_\sigma\hat{\sigma}\right) \tag{11}$$

$$\dot{\hat{\xi}} = -s(t)\Gamma_2\left(\hat{A}^T L'_\xi + \hat{B}^T G'_\xi\right)^T \tag{12}$$

$$\dot{\hat{\sigma}} = -s(t)\Gamma_3\left(\hat{A}^T L'_\sigma + \hat{B}^T G'_\sigma\right)^T \tag{13}$$

$$\dot{\hat{B}} = -s(t)\Gamma_4\left(\hat{G} - G'_\xi\hat{\xi} - G'_\sigma\hat{\sigma}\right) \tag{14}$$

$$\dot{\hat{\theta}}_f = |s(t)|\Gamma_5 Y_f \tag{15}$$

where $\Gamma_1, \ldots, \Gamma_5$ are symmetric positive definite matrices which determine the rates of adaptation. A block diagram of this controller structure is shown in Fig. 1 for reference.

Remarks:

1) Compared with [9] the controller given in this paper has an additional vector for each input that needs to be tuned. This will require more effort to tune the parameters, but it will enhance

the tracking performance due to the better approximation capabilities of the fuzzy approximator with first- order Sugeno consequents than those with constant consequents. As a matter of fact, the results of [9] can be thought of as a special case of the proposed approach and, therefore, the applicability of the method in [9] has been broadened.

2) Compared with [22], which also uses a Sugeno approximator, the controller design approach is quite different. In [22], an optimal controller is first designed and the fuzzy approximator was used to approximate the designed optimal controller, while our approach just approximates the unknown plant by fuzzy logic and uses this plant approximator for the controller design. In this case, no assumptions are required and control performance is superior (see simulation example).

B. Stability Analysis

The stability of the closed-loop system with the developed adaptive control law is shown by the following theorem.

Theorem 3: If the control law in (8)–(15) is applied to a plant with a unity control gain, then all states in the adaptive system will remain bounded and $X(t) \to 0$ as $t \to \infty$.

Proof: Consider the Lyapunov function candidate

$$V(t) = \frac{1}{2}\left(s^2(t) + \tilde{A}^T\Gamma_1^{-1}\tilde{A} + \tilde{\xi}^T\Gamma_2^{-1}\tilde{\xi} + \tilde{\sigma}^T\Gamma_3^{-1}\tilde{\sigma} + \tilde{B}^T\Gamma_4^{-1}\tilde{B} + \tilde{\theta}_f^T\Gamma_5^{-1}\tilde{\theta}_f\right). \tag{16}$$

Taking the derivative of both sides

$$\dot{V}(t) = s(t)\dot{s}(t) - \tilde{A}^T\Gamma_1^{-1}\dot{\hat{A}} - \dot{\hat{\xi}}^T\Gamma_2^{-1}\tilde{\xi} - \dot{\hat{\sigma}}^T\Gamma_3^{-1}\tilde{\sigma} - \tilde{B}^T\Gamma_4^{-1}\dot{\hat{B}} - \tilde{\theta}_f^T\Gamma_5^{-1}\dot{\hat{\theta}}_f. \tag{17}$$

Equation (10) can be rewritten as

$$\begin{aligned} u_{fu}(t) &= \hat{f}(X) - \hat{\theta}_f^T Y_f\,\mathrm{sgn}(s) \\ &= f(X) - \varepsilon(X) - \hat{\theta}_f^T Y_f\,\mathrm{sgn}(s). \end{aligned} \tag{18}$$

Recall that

$$\varepsilon(X) = \tilde{A}^T\left(\hat{L} - L'_\xi\hat{\xi} - L'_\sigma\hat{\sigma}\right) + \hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) + \tilde{B}^T\left(\hat{G} - G'_\xi\hat{\xi} - G'_\sigma\hat{\sigma}\right) + \hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) + d_f. \tag{19}$$

Then

$$\begin{aligned} u_{fu}(t) = f(X) &+ \tilde{A}^T\left(L'_\xi\hat{\xi} + L'_\sigma\hat{\sigma} - \hat{L}\right) - \hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) \\ &+ \tilde{B}^T\left(G'_\xi\hat{\xi} + G'_\sigma\hat{\sigma} - \hat{G}\right) - \hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) \\ &- d_f - \hat{\theta}_f^T Y_f\,\mathrm{sgn}(s). \end{aligned} \tag{20}$$

and

$$\begin{aligned} u(t) = -k_d s(t) &+ X_d^{(n)}(t) - \Lambda_v^T\tilde{X}(t) + f(x) \\ &+ \tilde{A}^T\left(L'_\xi\hat{\xi} + L'_\sigma\hat{\sigma} - \hat{L}\right) - \hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) \\ &+ \tilde{B}^T\left(G'_\xi\hat{\xi} + G'_\sigma\hat{\sigma} - \hat{G}\right) - \hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) \\ &- d_f - \hat{\theta}f^T Y_f\,\mathrm{sgn}(s). \end{aligned} \tag{21}$$

From (7) and (21), one has

$$\begin{aligned} \dot{s}(t) = -k_d s(t) &+ \tilde{A}^T\left(L'_\xi\hat{\xi} + L'_\sigma\hat{\sigma} - \hat{L}\right) - \hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) \\ &+ \tilde{B}^T\left(G'_\xi\hat{\xi} + G'_\sigma\hat{\sigma} - \hat{G}\right) - \hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) \\ &- d_f - \hat{\theta}_f^T Y_f\,\mathrm{sgn}(s). \end{aligned} \tag{22}$$

Then

$$\begin{aligned} \dot{V}(t) = &-k_d s^2(t) + s(t)\tilde{A}^T\left(L'_\xi\hat{\xi} + L'_\sigma\hat{\sigma} - \hat{L}\right) \\ &- s(t)\hat{A}^T\left(L'_\xi\tilde{\xi} + L'_\sigma\tilde{\sigma}\right) + s(t)\tilde{B}^T\left(G'_\xi\hat{\xi} + G'_\sigma\hat{\sigma} - \hat{G}\right) \\ &- s(t)\hat{B}^T\left(G'_\xi\tilde{\xi} + G'_\sigma\tilde{\sigma}\right) - s(t)\left[d_f + \hat{\theta}_f^T Y_f\,\mathrm{sgn}(s)\right] \\ &+ \tilde{A}^T\Gamma_1^{-1}\left[s(t)\Gamma_1\left(\hat{L} - L'_\xi\hat{\xi} - L'_\sigma\hat{\sigma}\right)\right] \\ &+ s(t)\left(\hat{A}^T L'_\xi + \hat{B}^T G'_\xi\right)^T \Gamma_2\Gamma_2^{-1}\tilde{\xi} \\ &+ s(t)\left(\hat{A}^T L'_\sigma + \hat{B}^T G'_\sigma\right)^T \Gamma_3\Gamma_3^{-1}\tilde{\sigma} \\ &+ \tilde{B}^T\Gamma_4^{-1}\left[s(t)\Gamma_4\left(\hat{G} - G'_\xi\hat{\xi} - G'_\sigma\hat{\sigma}\right)\right] \\ &- |s(t)|\tilde{\theta}_f^T\Gamma_5^{-1}\Gamma_5 Y_f \\ = &-k_d s^2(t) - s(t)d_f - s(t)\hat{\theta}_f^T Y_f\,\mathrm{sgn}(s) - |s(t)|\tilde{\theta}_f^T Y_f \\ = &-k_d s^2(t) - s(t)d_f - |s(t)|\hat{\theta}_f^T Y_f \\ &- \left[|s(t)|\overset{*}{\theta}{}_f^T Y_f - |s(t)|\hat{\theta}_f^T Y_f\right] \\ = &-k_d s^2(t) - s(t)d_f - |s(t)|\overset{*}{\theta}{}_f^T Y_f \\ \le &-k_d s^2(t) \end{aligned}$$

where the facts $|d_f| < \overset{*}{\theta}{}_f^T Y_f$ and $s(t)\,\mathrm{sgn}(s) = |s(t)|$ have been used. Therefore, all signals in the system are bounded. It is important to note that $s(t) \to 0$ as $t \to \infty$ has been established in [8], which completes the proof and establishes asymptotic convergence of the tracking error.

C. Nonunity Control Gain

We extend the result to plants with nonunity control gain. The following assumptions should be stated first.

1) The control gain is finite and non zero.
2) The functions $h(X) = (f(X)/b(X))$ and $g(x) = 1/(b(X))$ are bounded by known positive functions $M_0(X)$ and $M_1(X)$.
3) There exist a known positive function $M_2(X)$ such that

$$\left|\frac{d}{dt}\,g(X)\right| \le M_2(X)\|X\|.$$

Let

$$\hat{h}(X) = \hat{A}_h^T L\left(X, \hat{\xi}_h, \hat{\sigma}_h\right) + \hat{B}_h^T G\left(X, \hat{\xi}_h, \hat{\sigma}_h\right)$$

and

$$\hat{g}(X) = \hat{A}_g^T L\left(X, \hat{\xi}_g, \hat{\sigma}_g\right) + \hat{B}_g^T G\left(X, \hat{\xi}_g, \hat{\sigma}_g\right)$$

be the estimates of the optimal fuzzy approximators $\overset{*}{h}(X) = \overset{*}{A}{}_h^T L(X, \overset{*}{\xi}_h, \overset{*}{\sigma}_h) + \overset{*}{B}{}_h^T G(X, \overset{*}{\xi}_h, \overset{*}{\sigma}_h)$ and $\overset{*}{g}(X) = \overset{*}{A}{}_g^T L(X, \overset{*}{\xi}_g,$

$\overset{*}{\sigma}_g) + \overset{*}{B}_g^T G(X, \overset{*}{\xi}_g, \overset{*}{\sigma}_g)$, respectively. We can still get the following approximation error properties

$$\begin{aligned}\tilde{h} &= h - \hat{h} \\ &= \tilde{A}_h^T \left(\hat{L}_h - L'_{h\xi}\hat{\xi}_h - L'_{h\sigma}\hat{\sigma}_h\right) + \hat{A}_h^T\left(L'_{h\xi}\tilde{\xi}_h + L'_{h\sigma}\tilde{\sigma}_h\right) \\ &\quad + \tilde{B}_h^T\left(\hat{G}_h - G'_{h\xi}\hat{\xi}_h - G'_{h\sigma}\hat{\sigma}_h\right) \\ &\quad + \hat{B}_h^T\left(G'_{h\xi}\tilde{\xi}_h + G'_{h\sigma}\tilde{\sigma}_h\right) + d_h\end{aligned}$$

also

$$\begin{aligned}\tilde{g} &= g - \hat{g} \\ &= \tilde{A}_g^T \left(\hat{L}_g - L'_{g\xi}\hat{\xi}_g - L'_{g\sigma}\hat{\sigma}_g\right) + \hat{A}_g^T\left(L'_{g\xi}\tilde{\xi}_g + L'_{g\sigma}\tilde{\sigma}_g\right) \\ &\quad + \tilde{B}_g^T\left(\hat{G}_g - G'_{g\xi}\hat{\xi}_g - G'_{g\sigma}\hat{\sigma}_g\right) \\ &\quad + \hat{B}_g^T\left(G'_{g\xi}\tilde{\xi}_g + G'_{g\sigma}\tilde{\sigma}_g\right) + d_g.\end{aligned}$$

Furthermore, $|d_h| < \overset{*}{\theta}_h^T Y_h$ and $|d_g| < \overset{*}{\theta}_g^T Y_g$. The robust adaptive control law for the case of the nonunity control gain is:

$$u(t) = -k_d s(t) - \tfrac{1}{2}M_2(X)\|X\|s(t) + u_{fu}(t) \tag{23}$$

$$\begin{aligned}u_{fu}(t) &= \hat{A}_h^T L\left(X, \hat{\xi}_h, \hat{\sigma}_h\right) + \hat{B}_h^T G\left(X, \hat{\xi}_h, \hat{\sigma}_h\right) \\ &\quad + \hat{A}_g^T L\left(X, \hat{\xi}_g, \hat{\sigma}_g\right) a_r + \hat{B}_g^T G\left(X, \hat{\xi}_g, \hat{\sigma}_g\right) a_r \\ &\quad - \left(\hat{\theta}_h^T Y_h + \hat{\theta}_g^T Y_g\right)\operatorname{sgn}(s(t))\end{aligned} \tag{24}$$

$$\dot{\hat{A}}_h = -s(t)\Gamma_{h1}\left(\hat{L}_h - L'_{h\xi}\hat{\xi}_h - L'_{h\sigma}\hat{\sigma}_h\right) \tag{25}$$

$$\dot{\hat{\xi}}_h = -s(t)\Gamma_{h2}\left(\hat{A}_h^T L'_{h\xi} + \hat{B}_h^T G'_{h\xi}\right)^T \tag{26}$$

$$\dot{\hat{\sigma}}_h = -s(t)\Gamma_{h3}\left(\hat{A}_h^T L'_{h\sigma} + \hat{B}_h^T G'_{h\sigma}\right)^T \tag{27}$$

$$\dot{\hat{B}}_h = -s(t)\Gamma_{h4}\left(\hat{G}_h - G'_{h\xi}\hat{\xi}_h - G'_{h\sigma}\hat{\sigma}_h\right) \tag{28}$$

$$\dot{\hat{\theta}}_h = |s(t)|\Gamma_{h5}Y_h \tag{29}$$

$$\dot{\hat{A}}_g = -s(t)\Gamma_{g1}\left(\hat{L}_g - L'_{g\xi}\hat{\xi}_g - L'_{g\sigma}\hat{\sigma}_g\right)a_r \tag{30}$$

$$\dot{\hat{\xi}}_g = -s(t)\Gamma_{g2}\left(\hat{A}_g^T L'_{g\xi} + \hat{B}_g^T G'_{g\xi}\right)^T a_r \tag{31}$$

$$\dot{\hat{\sigma}}_g = -s(t)\Gamma_{g3}\left(\hat{A}_g^T L'_{g\sigma} + \hat{B}_g^T G'_{g\sigma}\right)^T a_r \tag{32}$$

$$\dot{\hat{B}}_g = -s(t)\Gamma_{g4}\left(\hat{G}_g - G'_{g\xi}\hat{\xi}_g - G'_{g\sigma}\hat{\sigma}_g\right)a_r \tag{33}$$

$$\dot{\hat{\theta}}_g = |s(t)|\Gamma_{g5}Y_g|a_r| \tag{34}$$

where $\hat{A}_h, \hat{\xi}_h, \hat{\sigma}_h, \hat{B}_h, \hat{A}_g, \hat{\xi}_g, \hat{\sigma}_g, \hat{B}_g$ are the estimates of $\overset{*}{A}_h, \overset{*}{\xi}_h, \overset{*}{\sigma}_h, \overset{*}{B}_h, \overset{*}{A}_g, \overset{*}{\xi}_g, \overset{*}{\sigma}_g, \overset{*}{B}_g$, $a_r = X_d^{(n)}(t) - \Lambda_v^T \tilde{X}(t)$ with $\Lambda_v^T = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \ldots, (n-1)\lambda]$, $\Gamma_{h1}, \ldots, \Gamma_{h5}$ and $\Gamma_{g1}, \ldots, \Gamma_{g5}$ are symmetric positive definite matrices which determine the rates of adaptation. The stability of the closed-loop system with nonunity control gain is established in the following theorem.

Theorem 4: If the control law in (23)–(34) is applied to a plant with a nonunity control gain, then all states in the adaptive system will remain bounded and $X(t) \to 0$ as $t \to \infty$.

Proof: Consider the following Luapunov function candidate:

$$\begin{aligned}V(t) = \tfrac{1}{2}\Big[&g(X)s^2(t) + \tilde{A}_h^T\Gamma_{h1}^{-1}\tilde{A}_h + \tilde{\xi}_h^T\Gamma_{h2}^{-1}\tilde{\xi}_h \\ &+ \tilde{\sigma}_h^T\Gamma_{h3}^{-1}\tilde{\sigma}_h + \tilde{B}_h^T\Gamma_{h4}^{-1}\tilde{B}_h + \tilde{\theta}_h^T\Gamma_{h5}^{-1}\tilde{\theta}_h \\ &+ \tilde{A}_g^T\Gamma_{g1}^{-1}\tilde{A}_g + \tilde{\xi}_g^T\Gamma_{g2}^{-1}\tilde{\xi}_g + \tilde{\sigma}_g^T\Gamma_{g3}^{-1}\tilde{\sigma}_g \\ &+ \tilde{B}_g^T\Gamma_{g4}^{-1}\tilde{B}_g + \tilde{\theta}_g^T\Gamma_{g5}^{-1}\tilde{\theta}_g\Big].\end{aligned} \tag{35}$$

Taking the derivative of both sides:

$$\begin{aligned}\dot{V} = &\tfrac{1}{2}\dot{g}(X)s^2(t) + s(t)g(X)\dot{s}(t) - \tilde{A}_h^T\Gamma_{h1}^{-1}\dot{\hat{A}}_h \\ &- \dot{\hat{\xi}}_h^T\Gamma_{h2}^{-1}\tilde{\xi}_h - \dot{\hat{\sigma}}_h^T\Gamma_{h3}^{-1}\tilde{\sigma}_h - \tilde{B}_h^T\Gamma_{h4}^{-1}\dot{\hat{B}}_h \\ &- \tilde{\theta}_h^T\Gamma_{h5}^{-1}\dot{\hat{\theta}}_h - \tilde{A}_g^T\Gamma_{g1}^{-1}\dot{\hat{A}}_g - \dot{\hat{\xi}}_g^T\Gamma_{g2}^{-1}\tilde{\xi}_g \\ &- \dot{\hat{\sigma}}_g^T\Gamma_{g3}^{-1}\tilde{\sigma}_g - \tilde{B}_g^T\Gamma_{g4}^{-1}\dot{\hat{B}}_g - \tilde{\theta}_g^T\Gamma_{g5}^{-1}\dot{\hat{\theta}}_g.\end{aligned} \tag{36}$$

Equation (7) can be rewritten as

$$g(X)\dot{s}(t) = -h(X) + u(t) - g(X)a_r. \tag{37}$$

From (23) and (37)

$$\begin{aligned}g(X)\dot{s}(t) = &-k_d s(t) - \tfrac{1}{2}M_2(X)\|X\|s(t) + u_{fu}(t) \\ &- h(X) - g(X)a_r\end{aligned} \tag{38}$$

$$\begin{aligned}u_{fu}(t) = &\hat{h}(X) - \hat{\theta}_h^T Y_h \operatorname{sgn}(s(t)) + \hat{g}(X)a_r \\ &- \hat{\theta}_g^T Y_g|a_r|\operatorname{sgn}(s(t))\end{aligned} \tag{39}$$

$$\begin{aligned}= &h(X) - \tilde{h}(X) - \hat{\theta}_h^T Y_h \operatorname{sgn}(s(t)) \\ &+ \left(g(X) - \tilde{g}(X)\right)a_r - \hat{\theta}_g^T Y_g|a_r|\operatorname{sgn}(s(t))\end{aligned} \tag{40}$$

$$\begin{aligned}= &h(X) - \tilde{A}_h^T\left(\hat{L}_h - L'_{h\xi}\hat{\xi}_h - L'_{h\sigma}\hat{\sigma}_h\right) \\ &- \hat{A}_h^T\left(L'_{h\xi}\tilde{\xi}_h + L'_{h\sigma}\tilde{\sigma}_h\right) \\ &- \tilde{B}_h^T\left(\hat{G}_h - G'_{h\xi}\hat{\xi}_h - G'_{h\sigma}\hat{\sigma}_h\right) \\ &- \hat{B}_h^T\left(G'_{h\xi}\tilde{\xi}_h + G'_{h\sigma}\tilde{\sigma}_h\right) \\ &- d_h - \hat{\theta}_h^T Y_h \operatorname{sgn}(s(t)) \\ &+ \Big[g(X) - \tilde{A}_g^T\left(\hat{L}_g - L'_{g\xi}\hat{\xi}_g - L'_{g\sigma}\hat{\sigma}_g\right) \\ &\quad - \hat{A}_g^T\left(L'_{g\xi}\tilde{\xi}_g + L'_{g\sigma}\tilde{\sigma}_g\right) \\ &\quad - \tilde{B}_g^T\left(\hat{G}_g - G'_{g\xi}\hat{\xi}_g - G'_{g\sigma}\hat{\sigma}_g\right) \\ &\quad - \hat{B}_g^T\left(G'_{g\xi}\tilde{\xi}_g + G'_{g\sigma}\tilde{\sigma}_g\right) - d_g\Big]a_r \\ &- \hat{\theta}_h^T Y_h \operatorname{sgn}(s(t)).\end{aligned} \tag{41}$$

From (35), (38), and (41)

$$\begin{aligned}\dot{V}(t) = &-k_d s^2(t) + \tfrac{1}{2}(\dot{g}(X) - M_2(X)\|X\|)s^2(t) \\ &- s(t)d_h - |s(t)|\overset{*}{\theta}_h^T Y_h - s(t)d_g a_r \\ &- |s(t)|\overset{*}{\theta}_g^T Y_g|a_r| < -k_d s^2(t)\end{aligned}$$

where the facts $|d_h| < \overset{*}{\theta}_h^T Y_h$ and $|d_g| < \overset{*}{\theta}_g^T Y_g$, $s(t)\operatorname{sgn}(s) = |s(t)|$ have been used.

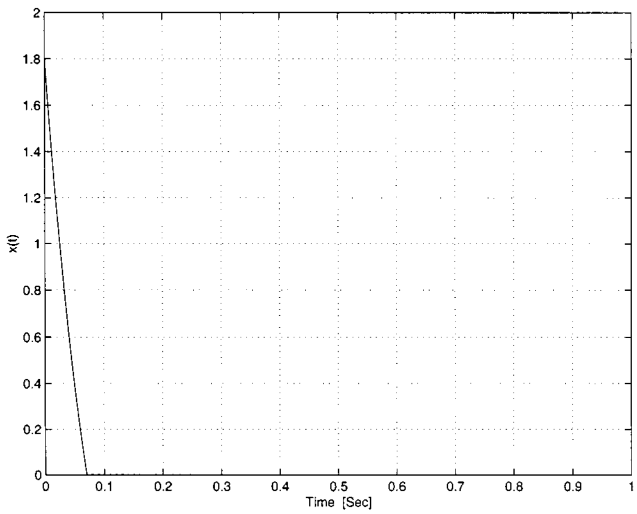

Fig. 2. Closed-loop $x(t)$ using the developed controller with seven rules.

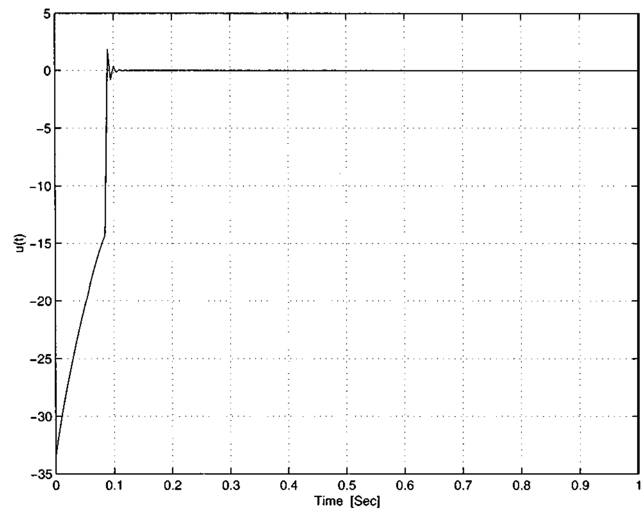

Fig. 3. Control signal$u(t)$ using the developed controller with seven rules.

V. Simulation

The effectiveness of the proposed approach is shown by applying the developed adaptive fuzzy controller to control the unstable system used in [9], [10], and [21]. The system is

$$\dot{x}(t) = \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} + u(t).$$

The adaptive fuzzy controller is used to drive the system state $x(t)$ to the origin. First, we define seven membership functions over the state space which is chosen to be $[-3, 3]$. The simulation is carried out with Sugeno first-order fuzzy rules. The values of a_k and b_k can be obtained by evaluating f at points $x = -3, -2, -1, 0, 1, 2, 3$. But they are not required here since the exact w_j^* is not required in the control law. However, with the knowledge of a_k and b_k it will be helpful for the choice of initial $\hat{A}$, $\hat{B}$, $\hat{\xi}$, $\hat{\sigma}$, and $\hat{\theta}_f$ to speed up the adaptation process. In this example, the initial values $\hat{A}$, $\hat{B}$, $\hat{\xi}$, $\hat{\sigma}$ and $\hat{\theta}_f$ are selected to be

$$\begin{aligned}
\hat{A}(0) &= [-2.04 \quad -4.08 \quad -1.02 \quad 0 \quad 0.36 \quad 1.53 \quad 2.04]^T \\
\hat{B}(0) &= [-0.8 \quad -0.6 \quad -0.4 \quad 0 \quad 0.4 \quad 0.6 \quad 0.8]^T \\
\hat{\xi}(0) &= [-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3]^T \\
\hat{\sigma}(0) &= [0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5]^T \\
\hat{\theta}_f(0) &= [4 \quad 1 \quad 1 \quad 1 \quad 1]^T.
\end{aligned}$$

We chose the initial state $x(0) = 2$. In (8), $k_d = 10$. Figs. 2 and 3 show $x(t)$ and $u(t)$. We can observe an improvement in the tracking performance compared with the results in [9], for the same number of rules using the same initial conditions. Also, we have a superior transient performance compared with [22]. We also simulated for other initial conditions, and the results were very similar; however, these results are not shown for brevity.

Figs. 4 and 5 show $x(t)$ and $u(t)$ using five rules instead of seven. The initial values are

$$\begin{aligned}
\hat{A} &= [-4.32 \quad -1.08 \quad 0 \quad 0.37 \quad 1.62]^T \\
\hat{B} &= [-0.6 \quad -0.4 \quad 0 \quad 0.4 \quad 0.6]^T \\
\hat{\xi} &= [-2 \quad -1 \quad 0 \quad 0.5 \quad 1]^T \\
\hat{\sigma} &= [0.5 \quad 0.5 \quad 0.5 \quad 0.5 \quad 0.5]^T \\
\hat{\theta}_f &= [4 \quad 1 \quad 1 \quad 1 \quad 1]^T.
\end{aligned}$$

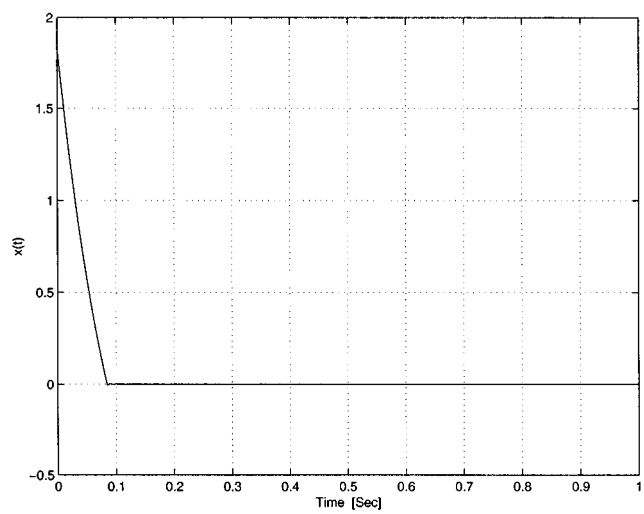

Fig. 4. Closed-loop $x(t)$ using the developed controller with five rules.

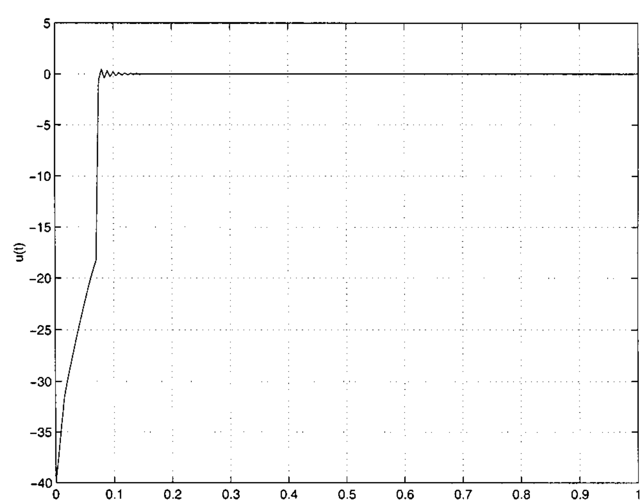

Fig. 5. Control signal $u(t)$ using the developed controller with five rules.

Also, we can observe a slight improvement on the system performance comparing it with that in [9] for different initial conditions with fewer

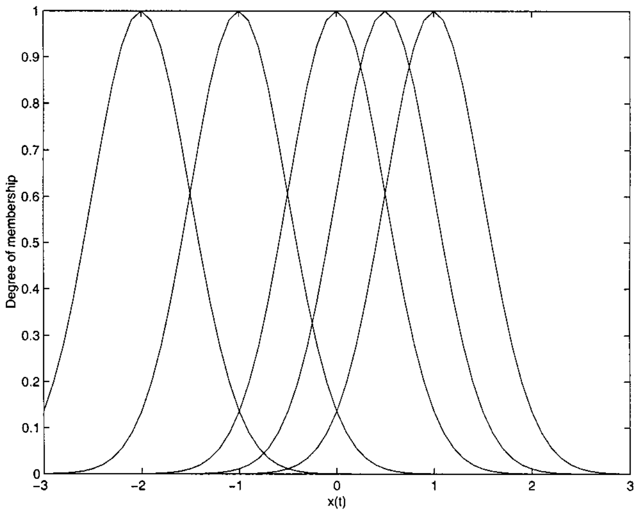

Fig. 6. Initial membership functions.

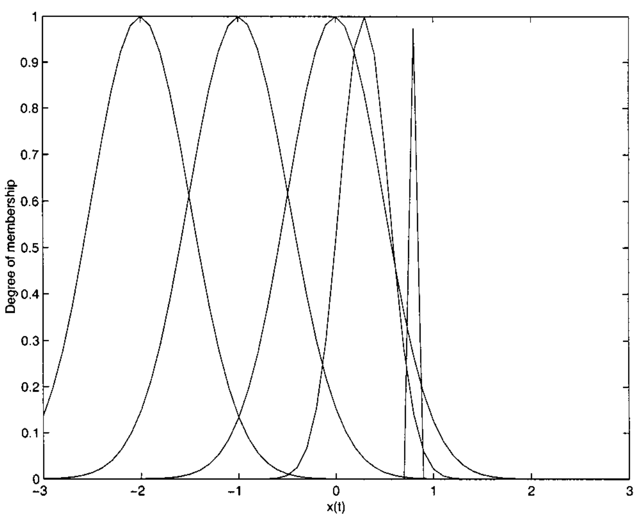

Fig. 7. Final membership functions.

rules. This improvement was expected since a Sugeno first-order consequent model has a better approximation capability than the Sugeno constant consequent model that is used in [9]. Figs. 5 and 6 show the initial and the final membership functions for the system of five rules.

VI. Conclusion

In this paper we have presented a fuzzy adaptive control law using a first- order Sugeno fuzzy approximator. Due to the better approximation capability of first-order Sugeno fuzzy approximators than fuzzy approximators with constant consequents, a better control performance has been achieved with fewer rules in the fuzzy approximator. Global boundedness of the adaptive system is established. The simulations demonstrate the effectiveness of the proposed controller.

Appendix

G'_ξ, and L'_ξ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\xi}$ at $\hat{\xi}$, expressed as

$$G'_\xi = G'_\xi\left(x, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial G\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\xi}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}$$

$$L'_\xi = L'_\xi\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial L\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\xi}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}.$$

Also, G'_σ, and L'_σ are derivatives of $G(X, \overset{*}{\xi}, \overset{*}{\sigma})$ and $L(X, \overset{*}{\xi}, \overset{*}{\sigma})$ with respect to $\overset{*}{\sigma}$ at $\hat{\sigma}$, expressed as

$$G'_\sigma = G'_\sigma\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial G\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\sigma}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}$$

$$L'_\sigma = L'_\sigma\left(X, \hat{\xi}, \hat{\sigma}\right) = \left.\frac{\partial L\left(X, \overset{*}{\xi}, \overset{*}{\sigma}\right)}{\partial \overset{*}{\sigma}}\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}$$

$$\begin{aligned}
G'_\xi &= G'_\xi\left(x, \hat{\xi}, \hat{\sigma}\right)\\
&= \left[g'_{\zeta 1}\left(\hat{\sigma}_1\left\|X - \hat{\xi}_1\right\|\right),\ g'_{\zeta 2}\left(\hat{\sigma}_2\left\|X - \hat{\xi}_2\right\|\right),\right.\\
&\qquad \left.\ldots,\ g'_{\zeta N}\left(\hat{\sigma}_N\left\|X - \hat{\xi}_N\right\|\right)\right]^T\\
g'_{\zeta j}&\left(\hat{\sigma}_j\left\|X - \hat{\xi}_j\right\|\right)\\
&= \left.\frac{\partial}{\partial \overset{*}{\xi}_j}\, g_j\left(\overset{*}{\sigma}_j\left\|X - \overset{*}{\xi}_j\right\|\right)\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}\\
&= \left.\frac{\partial}{\partial \overset{*}{\xi}_j}\left(\prod_{i=1}^{n} \exp\left[-\left(\overset{*}{\sigma}{}_j^i\left(x_i - \overset{*}{\xi}{}_j^i\right)\right)^2\right]\right)\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}\\
&= 2 \cdot g_j\left(\hat{\sigma}_j\left\|X - \hat{\xi}_j\right\|\right) \cdot \left(\hat{\sigma}_j^i\left(x_i - \hat{\xi}_j^i\right)\right) \equiv g_\zeta\\
g_\zeta &= 2 \cdot \left(\prod_{i=1}^{n} \exp\left[-\left(\hat{\sigma}_j^i\left(x_i - \hat{\xi}_j^i\right)\right)^2\right]\right)\\
&\quad \cdot \left(\hat{\sigma}_j^i\left(x_i - \hat{\xi}_j^i\right)\right)\\
&= 2 \cdot \prod_{i=1}^{n} \frac{a}{\exp^{a^2}}
\end{aligned}$$

where $a = \hat{\sigma}_j^i(x_i - \hat{\xi}_j^i)$. It is easy to show that g_ζ is bounded with the parameter, i.e., g_ζ is bounded with respect to $\hat{\xi}_j^i$

$$\begin{aligned}
G'_\sigma &= G'_\sigma\left(x, \hat{\xi}, \hat{\sigma}\right)\\
&= \left[g'_{\sigma 1}\left(\hat{\sigma}_1\left\|X - \hat{\xi}_1\right\|\right),\ g'_{\sigma 2}\left(\hat{\sigma}_2\left\|X - \hat{\xi}_2\right\|\right),\right.\\
&\qquad \left.\ldots,\ g'_{\sigma N}\left(\hat{\sigma}_N\left\|X - \hat{\xi}_N\right\|\right)\right]^T\\
g'_{\sigma j}&\left(\hat{\sigma}_j\left\|X - \hat{\xi}_j\right\|\right)\\
&= \left.\frac{\partial}{\partial \overset{*}{\sigma}_j}\, g_j\left(\overset{*}{\sigma}_j\left\|X - \overset{*}{\xi}_j\right\|\right)\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}\\
&= \left.\frac{\partial}{\partial \overset{*}{\sigma}_j}\left(\prod_{i=1}^{n} \exp\left[-\left(\overset{*}{\sigma}{}_j^i\left(x_i - \overset{*}{\xi}{}_j^i\right)\right)^2\right]\right)\right|_{\overset{*}{\xi} = \hat{\xi},\ \overset{*}{\sigma} = \hat{\sigma}}\\
&= -2 \cdot g_j\left(\hat{\sigma}_j\left\|X - \hat{\xi}_j\right\|\right) \cdot \left(\hat{\sigma}_j^i\left(x_i - \hat{\xi}_j^i\right)^2\right) \equiv g_\sigma
\end{aligned}$$

$$g_\sigma = -2 \cdot \left(\prod_{i=1}^{n} \exp\left[-\left(\hat{\sigma}_j^i \left(x_i - \hat{\xi}_j^i \right) \right)^2 \right] \right) \cdot \left(\hat{\sigma}_j^i \left(x_i - \hat{\xi}_j^i \right)^2 \right) = -2 \cdot \prod_{i=1}^{n} \frac{\left(x_i - \hat{\xi}_j^i \right)^2 b}{\exp^{(x_i - \hat{\xi}_j^i)^2 b^2}}.$$

Also we can see that g_σ is bounded with the parameter, i.e., g_σ is bounded with respect to $\hat{\xi}_j^i$.

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Estimating the Motion Direction From Brightness Gradient on Lines

J. J. Guerrero and C. Sagüés

***Abstract*—In previous works we combined feature-based techniques and optical flow methods to obtain depth or motion. The expressions relating the brightness constraint to the three-dimensional (3-D) localization and motion of a line and its projection were established. In this paper, those expressions have been used to obtain the motion direction of a camera when the rotation velocity is bounded without assumptions about the depth of lines. Our approach exploits the visibility constraint and it allows us to make use of *a priori* information about the scene or the motion. With the proposed technique, the topology (easily extracted in the image) that relates adjacent edge elements into line segments is exploited to better compute camera motion. Besides the motion direction, our method allows also to compute the rotation velocity when lines in prominent 3-D directions are available.**

***Index Terms*—Motion and structure from vision, optical flow, straight edges, visibility constraint.**

I. INTRODUCTION

Shape and motion information from vision has been usually extracted using corresponding features or optical flow measures [1]. Geometric features provide an efficient way to select, concentrate, and manipulate vision information. In particular, features like straight

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